Verification of Diffusion Mathematical Model for Long-term Materials Drying

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Abstract— The paper deals with a design of a diffusion mathematical model describing the drying processes for long-lasting desiccation of dried substances prototypically shaped as plane, cylinder, and sphere. A decrease in the substance humidity shrinkage is derived from the result in deterministic model of desiccation. The long-term desiccation of the dried substances has been simulated on the basis of COMSOL MULTIPHYSICS programme and MAPLE, where initial and marginal conditions were defined. The functional program application in MATLAB has been created in order to draw a contrast among particular results obtained from the COMSOL programme.The application enables delineating the moisture extent in substances, 3D graph of moisture extent, and a decrease in the substance humidity degree. By means of the mentioned application, it is achievable to evaluate the results even in terms of changes in some of the set parameters. The shrinkage in substance moisture can be predicted in relation to suggested mathematical models. This fact has been confirmed bythe results of long-lasting desiccation of green coffee-beans in the warehouse.

Keywords— Mathematic modeling, diffusion model, drying process, moisture, COMSOL MULTIPHYSICS software, MAPLE software, MATLAB software, drying characteristic

I. INTRODUCTION

Drying process which removes unwanted moisture from the material belongs to the most frequently occurring technological operations. Drying is one of the time-and energy-consuming operations, falling into the category of diffusion processes. The value of the diffusion coefficient determines how quickly the moisture from the material out.

The optimal drying process course can be carried out by experimental determination of drying curves of the material and by design suitable diffusion mathematical model describing the drying process.

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II. DRYING PROCESS

Drying is a process in which the heat energy removes moisture from the material by evaporation from the surface of the material and by transport of moisture in the inner texture of the material towards the surface layers. This is a fairly complicated process whereas simultaneously heat and mass transfer occurs.

The driving force of the drying process is the flow of moisture, which creates the difference of the partial pressures of water vapor p_s in the boundary layer and in the environment p_o

Generally there are three possibilities:

- if $p_s > p_o$, there is flow of moisture from the boundary layer to the surroundings - drying,
- if $p_s < p_o$, occurs on the contrary flow of moisture from the environment into the material - wet,
- if $p_s = p_o$, has reached equilibrium moisture content with the surroundings [1].

If the moisture is similar to the moisture in the boundary layer, ie the difference of the partial water vapor pressure p_s and the surroundings are not great, there is the drying process longer.

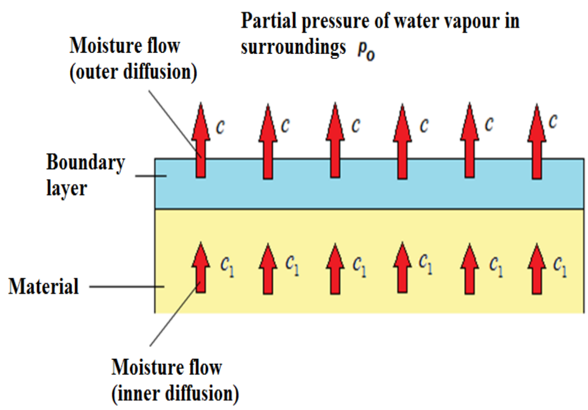


Fig. 1 Drying process – moisture transport [1]

III. MATHEMATICAL MODEL OF PLANE PLATE BODY DRYING

Mathematical description is based on the diffusion model, which assumes that the moisture inside the solid phase can be described by the diffusion equation whose solution is the moisture field inside the solid phase. We can describe for plane the second Fick's law has a form [4]

$$\frac{\partial c}{\partial t}(x,t) = \frac{\partial^2 c}{\partial x^2}(x,t) \tag{1}$$

For symmetry of the moisture field holds:

$$\frac{\partial c}{\partial x}(0,t) = 0 \tag{2}$$

Assumption of perfect air flow is described by equation:

$$c(b,t) = c_{0p}(t) \tag{3}$$

For initial condition:

$$c(x,0) = c_p \tag{4}$$

Analytical solution of the mathematical model for plane plate we obtain with using of Laplace transformation.

$$\frac{c - c_{0p}}{c_p - c_{0p}} = \frac{4}{\pi} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} e^{\frac{-D(2n+1)^2 \pi^2 t}{4l^2}} \cos(\frac{(2n+1)\pi x}{2l}) \tag{5}$$

For dimensionless values

$$C = \frac{c - c_{op}}{c_p - c_{op}}, \quad F_{0D} = \frac{Dt}{l^2}, \quad X = \frac{x}{l}, \tag{6 a,b,c}$$

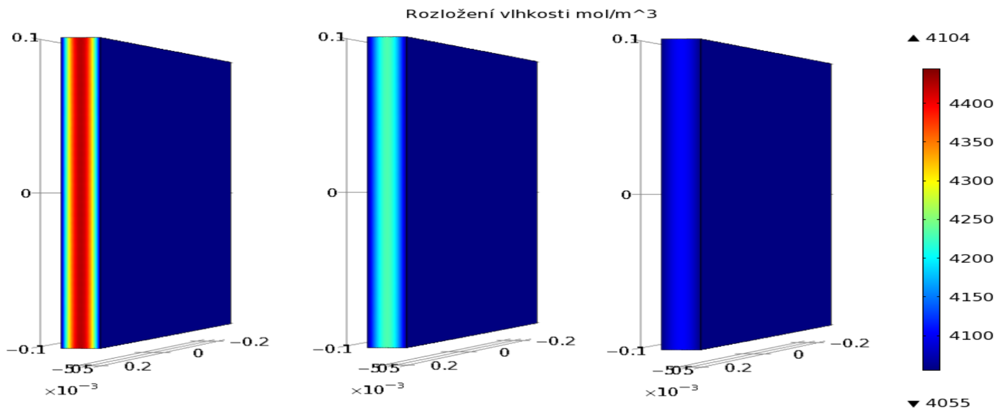


Fig. 2 Distribution of moisture in the solid a plane plate

IV. MATHEMATICAL MODEL OF CYLINDRICAL BODY DRYING

In cylindrical coordinates, the second Fick's law has a form [4]:

$$\frac{\partial c}{\partial t}(r_v,t) = \frac{1}{r_v}\frac{\partial}{\partial r_v}\left(r_v D \frac{\partial c}{\partial r_v}\right), \quad \left(0 \le r_v \le R_v, t > 0\right). \tag{7}$$

For symmetry of the moisture field holds:

$$\frac{\partial c}{\partial r_v}(0,t) = 0. \tag{8}$$

Assumption of perfect air flow is described by equation:

$$c(R_v,t) = c_{0p}(t). \tag{9}$$

Constant distribution of moisture in the solid phase is given by equation:

$$c(r_v,0) = c_p. \tag{10}$$

For dimensionless values

$$C = \frac{c - c_{op}}{c_p - c_{op}}, \quad F_{0v} = \frac{Dt}{a^2}, \quad R_v = \frac{r_v}{a_v}, \tag{11 a,b,c}$$

Analytical solution of the mathematical model (7-10), which describes moisture distribution inside dried cylindrical body, can be described by equation (12):

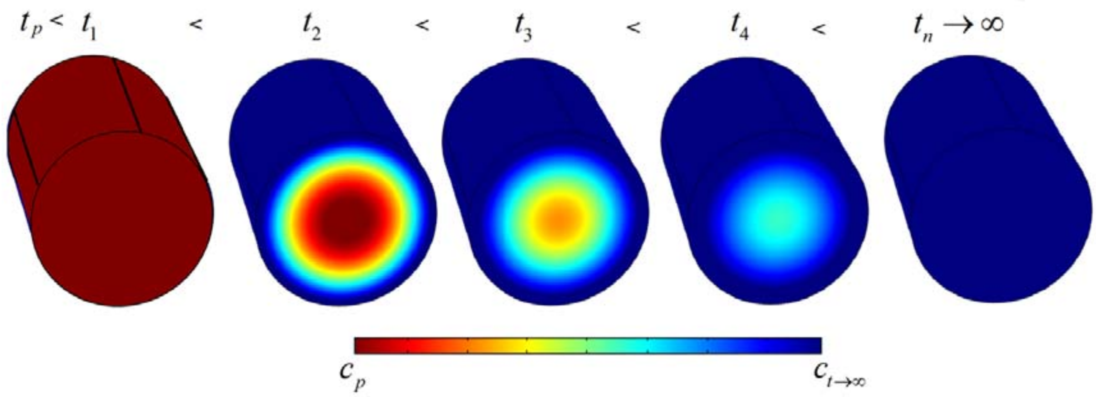


Fig. 3 Distribution of moisture in the solid cylindrical body

$$C = 1 - \frac{2}{R_v}\sum_{n=1}^{\infty}\frac{e^{-D\alpha_n^2 t} J_0\left(r_v\alpha_n\right)}{\alpha_n J_1\left(R_v\alpha_n\right)}, \qquad (12)$$

where α_n are roots of equation:

$$J_0\left(R_v\alpha_n\right) = 0 . \qquad (13)$$

With respect to a porosity of the material and fixing power of moisture, the diffusion coefficient can be computed by equation (14):

$$D_m = \frac{D}{\varepsilon\left(1+K\right)}. \qquad (14)$$

Then for the analytical solution holds:

$$C\left(X_v, Fo_v\right) = 1 - \frac{2}{R_v}\sum_{n=1}^{\infty}\frac{e^{-Fo_v R_v^2\alpha_n^2} J_0\left(X_v R_v\alpha_n\right)}{\alpha_n J_1\left(R_v\alpha_n\right)}. \qquad (15)$$

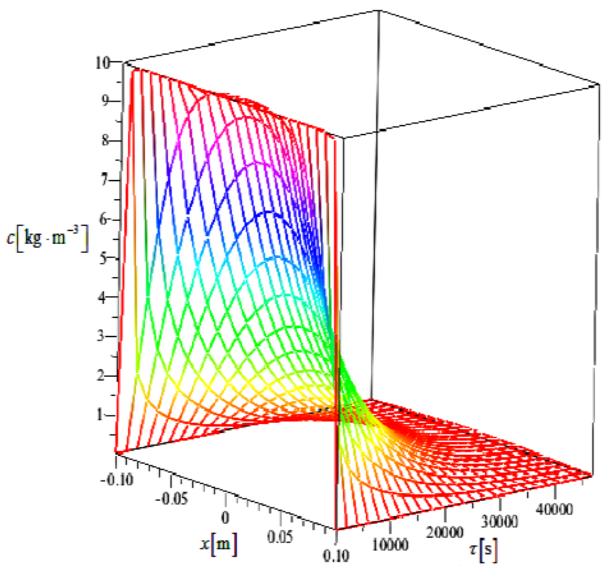


Fig. 4 3D Concentration field of moisture by drying of cylindrical body material in MAPLE application

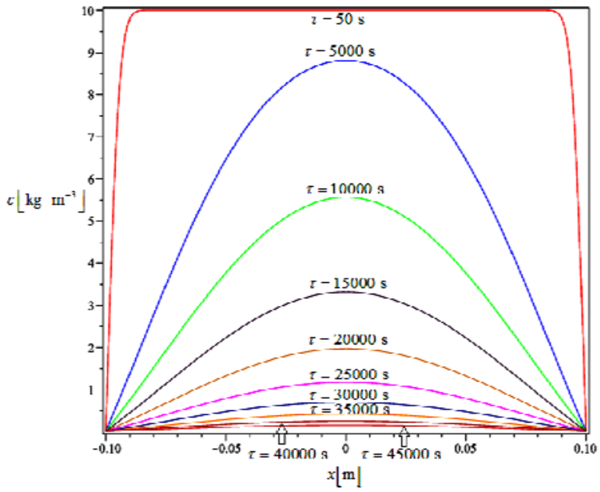


Fig. 5 Concentration field of moisture by drying of cylindrical body material

Parameters: R_v = 0,1m, D = 1.10^{-6} m^2.s^{-1}, c_p = 10 kg.m^{-3}, c_{0p} = 0,02 kg.m^{-3} , ε = 0,5, K = 10

Fig. 4 and 5 shows moisture distribution by drying of cylindrical body computed by computer simulation of analytical solution (15).

Fourier number Fo_v is dimensionless time of drying process described by equation (16):

$$Fo_v = \frac{D_m t}{R_v^2}. \qquad X_v = \frac{r_v}{R_v}. \qquad (16), (17)$$

X_v is dimensionless space coordinate:

For comparing we work out the software application for computing moisture fields in plane plate, cylindrical and spheric bodies.

MATLAB software

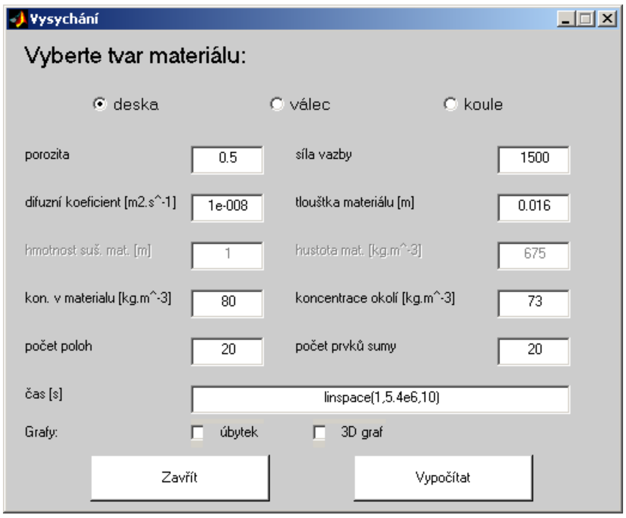


Fig. 6 User interface of software MATLAB application for modeling of drying process – inputing of values.

Part of programme application in MATLAB is foloving:

For a plane plate:

```
for o=1:grafKP
for r=1:ppoloh
poloha=linspace(-1,1,ppoloh);
for j=1:i
s1(j)=(((-1)^(j-1))/(2*(j-1)+1))*((exp(((-t(o))*((2*(j-1)+1)^2)*pi^2)/4)))*cos(((2*(j-1)+1)*pi*poloha(r))/(2));
suma1=suma1+s1(j);
end
C(r,o)=((4/pi)*suma1);
suma1=0;
end
end
```

For a cylinder:

```
for r=1:ppoloh
poloha=linspace(-1,1,ppoloh);
for j=1:i
s1(j)=((exp((-t(o))*q(j)^2))*besselj(0,(poloha(r)*q(j))))/(q(j)*besselj(1,(q(j))));
suma1=suma1+s1(j);
end
C(r,o)=real(((2)*suma1));
suma1=0;
end
for i=1:Q
x=in:0.0001:i*pi;
a=besselj(0,x);
A=[a;x];
B=find(abs(A==min(abs(a(abs(a)>=0)))));
q(i)=(A(B+1))/1;
in=q(i)+0.1;
end
```

For a sphere:

```
for o=1:grafKP
for r=1:ppoloh
poloha=linspace(-1,1,ppoloh);
for j=1:i
s1(j)=(((-1)^j)/j)*sin((j*pi*poloha(r)))*exp(-t(o)*(j^2)*(pi^2));
suma1=suma1+s1(j);
end
C(r,o)=-(((2/(pi*poloha(r)))*suma1));
suma1=0;
end
end
```

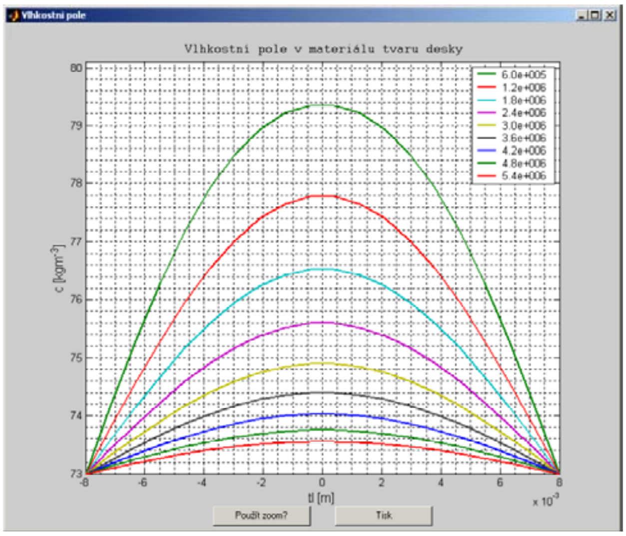


Fig. 7 Comparing of results moisture fields in MATLAB software

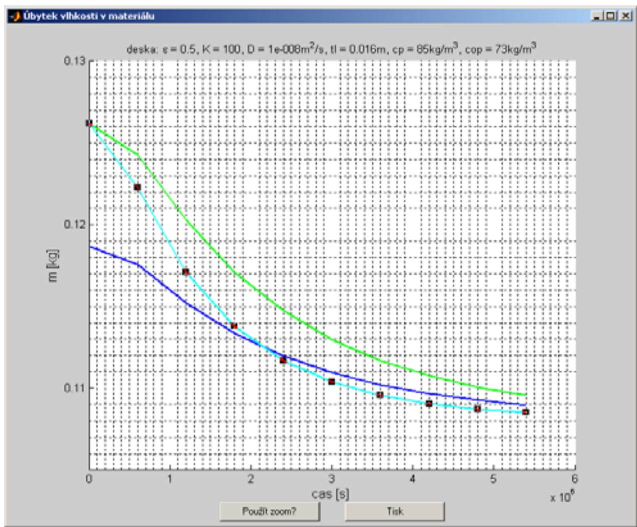


Fig. 8 The moisture loss during long-term drying calculated on the model base

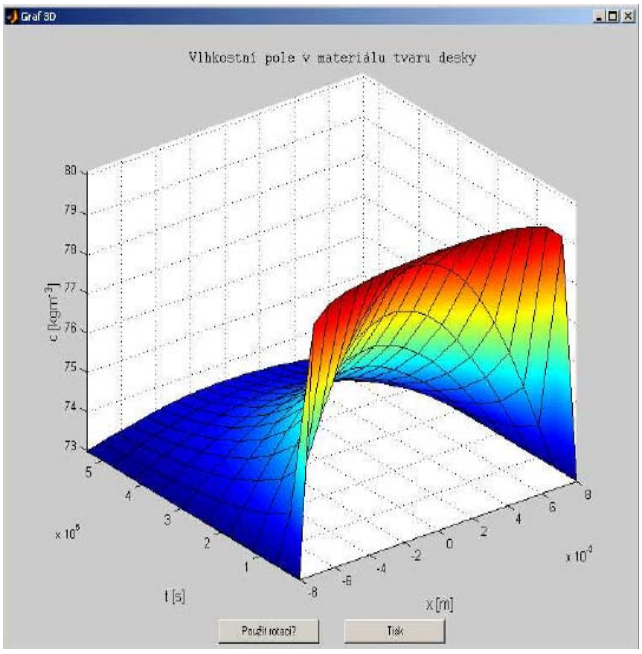


Fig. 9 Three Dimension moisture field in MATLAB software

V. Mathematical Model of Spheric Body Drying

For spheric body, the second Fick's law has a form [4]:

$$\frac{\partial c}{\partial t}(x,t) = D\left(\frac{\partial^2 c}{\partial r_k^2} + \frac{2}{r_k}\frac{\partial c}{\partial r_k}\right) \tag{18}$$

For symmetry of the moisture field holds:

$$\frac{\partial c}{\partial r_k}(0,t) = 0 \tag{19}$$

For initial condition:

$$c(r_k, 0) = c_p \tag{20}$$

Assumption of perfect air flow is described by equation:

$$c(a_k, t) = c_{0p}(t) \tag{21}$$

For dimensionless values

$$C = \frac{c - c_{op}}{c_p - c_{op}}, \quad F_{0K} = \frac{Dt}{a_k^2}, \quad R_k = \frac{r_k}{a_k} \tag{22 a,b,c}$$

Analytical solution of the mathematical model (18-21), which describes moisture distribution inside dryed spheric body, can be described by equation (23):

$$\frac{c - c_{0p}}{c_p - c_{0p}} = -\frac{2a_k}{\pi r_k} \cdot \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin\frac{n\pi r_k}{a_k} e^{\frac{-Dn^2\pi^2 t}{a_k^2}} \tag{23}$$

VI. Computer Modeling of a Long-term Green Coffee Beans Drying

The values obtained by use of computer simulation of the drying process were compared with data obtained by real measurements [3] of the moisture loss in storage of green coffee beans in bales in the warehouse. By computer modeling, the bales were approximated by cylindrical body. Weight loss was tested in samples of the initial moisture content of 12 % ot weight and a weight of approximately 1 kg.

The measured data of the moisture loss during long-term drying of green coffee beans in stock we approximated by the curves obtained using the proposed mathematical model.

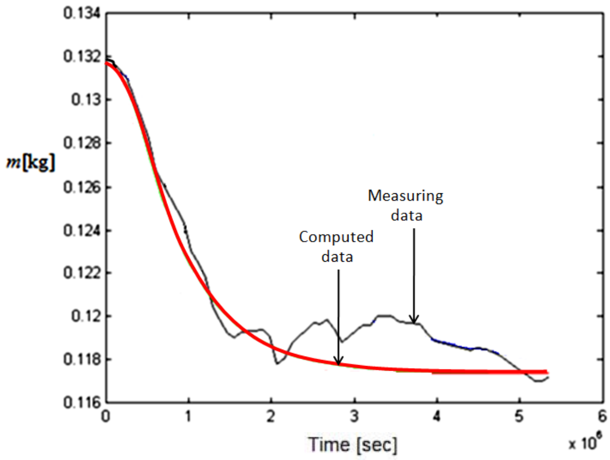
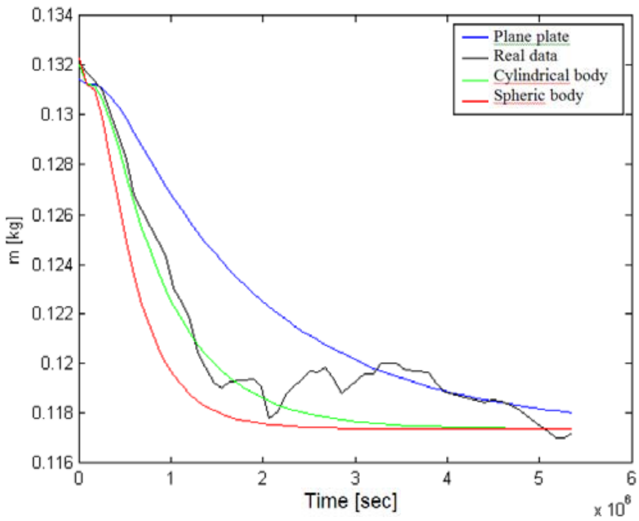
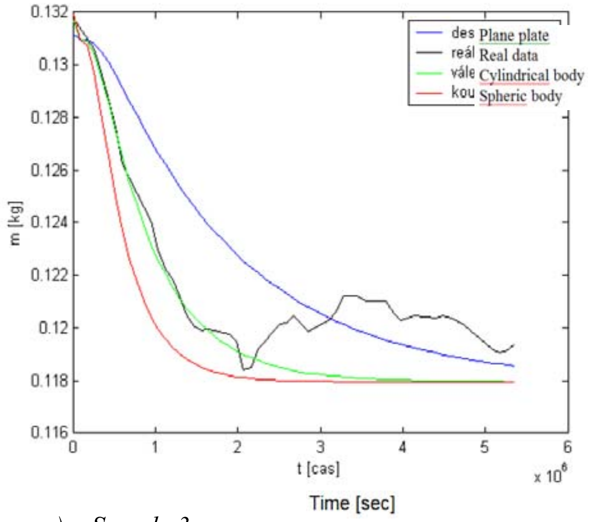


Fig. 10 Curves of the moisture loss by green coffee beans drying. The experimentally determined data were in a good accordance with computer simulation of drying process.

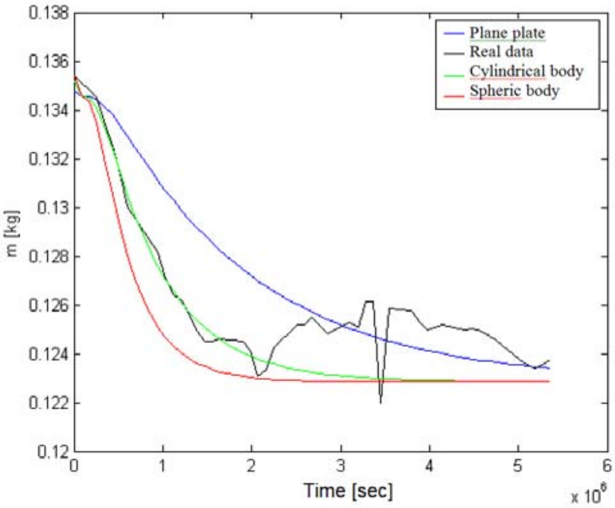
a) Sample 1:



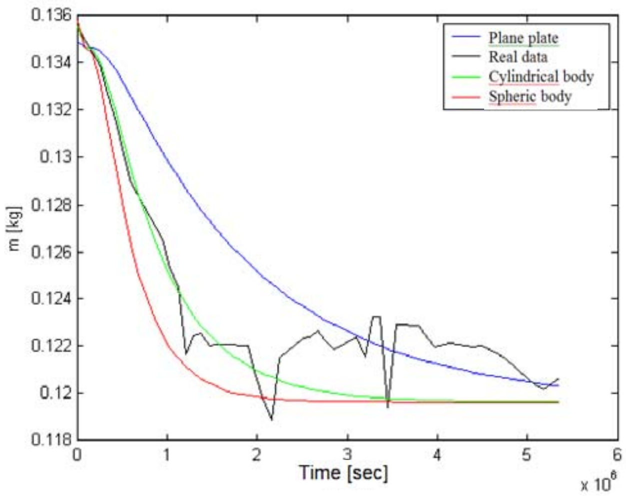
b) *Sample 2:*



c) *Sample 3:*



d) *Sample 4:*



e) *Sample 5:*

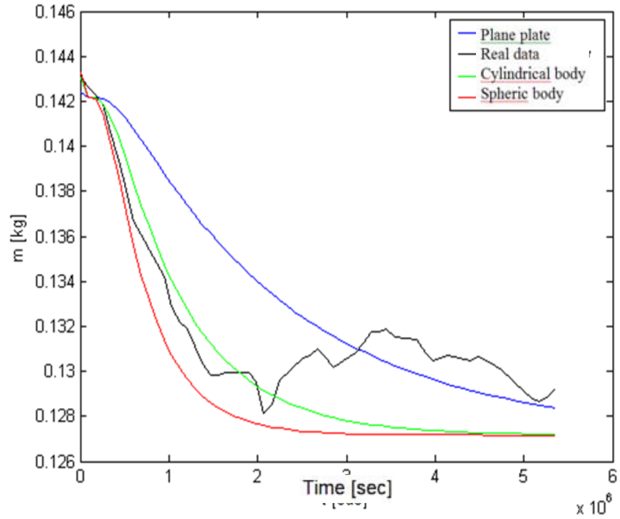
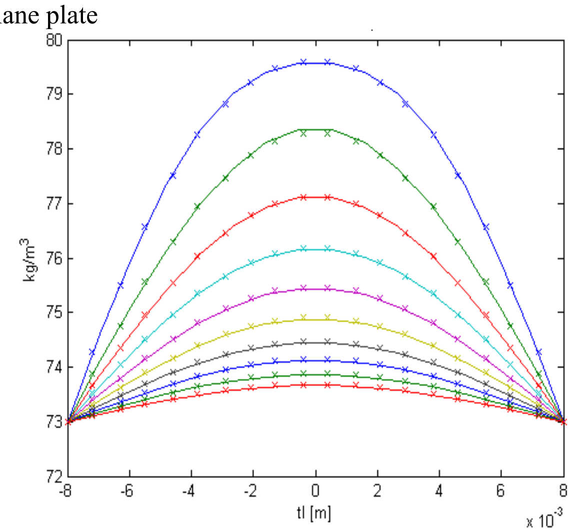


Fig. 11 *a,b,c,d,e)* Comparing of the measured and calculated course of the moisture loss during long-term drying of the coffee beans sacks - 5 samples

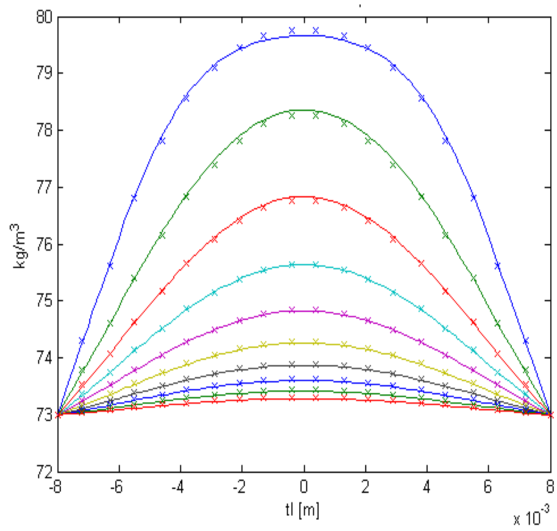
We considered the following parameters:
the effective diffusion coefficient $D = 2.5 \cdot 10^{-9}\ \mathrm{m^2 \cdot s^{-1}}$,
the initial concentration of moisture in the material $c_p = 80.5\ \mathrm{kg \cdot m^{-3}}$,
the initial concentration of the ambient moisture $c_{0p} = 0{,}01\ \mathrm{kg \cdot m^{-3}}$,
porosity of material $\varepsilon = 0.5$,
moisture fixing power in the material $K = 1$,
diameter of the bales $d = 0.1\ \mathrm{m}$.

VII. Comparison of COMSOL Multiphysics and MATLAB data

Plane plate



Cylinder:



Sphere:

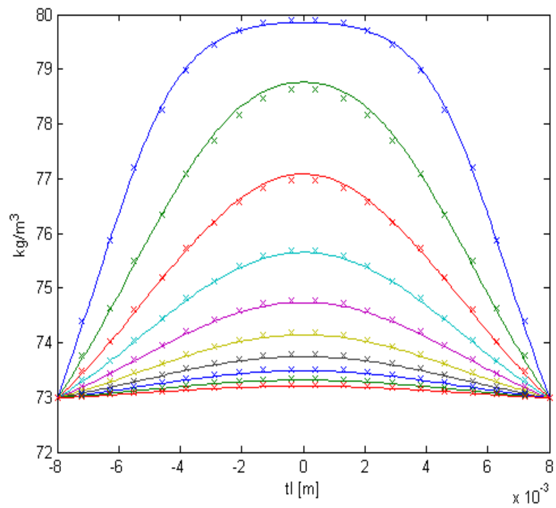


Fig. 12 *a,b,c* Comparing of moisture fields calculed in MATLAB (**symbol x**) and COMSOL(**symbol -**) software

For parameters: ε=0.5, K=1500*, l*=0.016m, D=1.10^{-9}m^{2}s^{-1}, c_p=80kgm^{-3}, c_{op}=0,01 kg.m^{-3}

The obtained theoretical results by COMSOL and MATLAB software were in a good accordance.

VIII. Conclusion

In the paper was described mathematical model suggested for description of the moisture loss in a cylindrical body during drying process. It was found, that by using experimentally determined diffusion coefficient of moisture can the suggested model predict loss of moisture in materials stored in an environment under the constant conditions.

The values obtained by computer simulation of the drying process were compared with data obtained by real measurements of the moisture loss in storage of green coffee beans in bales in the storehouse. The obtained results were in a good accordance.

List of Symbols

α_n - roots of transcendental equation [1]
c - concentration of moisture in the solid phase, [kg.m^{-3}]
c_0 - concentration of moisture in the surroundings, [kg.m^{-3}]
c_{p0} - initial concentration of moisture in the surroundings, [kg.m$_{-3}$]
c_p - initial concentration of moisture in the solid phase, [kg.m^{-3}]
C - dimensionless concentration of moisture in the solid phase, [1]
d - diameter, [m]
D - effective diffusion coefficient, [m2.s-1]
D_m - modified effective diffusion coefficient, [m^2.s^{-1}]
Fo_D - Fourier number of the plane plate body, [1]
Fo_v - Fourier number of the cylindrical body, [1]
Fo_k - Fourier number of the spheric body, [1]
J_0 - Bessel function of the first kind, of order zero, [1]
J_1 - Bessel function of the first kind, of order one, [1]
K - sorption equilibrium constant (fixing power of sorbed comp. into the solid phase), [1]
n - sequence number ,[1]
p_A - partial pressure of water vapor in the solid phase, [Pa]
p_0 - partial pressure of water vapor in the surrounding air, [Pa]
p_s - partial pressure of water vapour in the boundary layer, [Pa]
r_v - space coordinate of the cylindrical body, [m]
R_v - radius of the cylindrical body, [m]
r_k - space coordinate of the spheric body, [m]
R_k - radius of the spheric body, [m]
t time, [s]
x - coordinate, [m]
X_D - dimensionless space coordinate of the plane plate body, [1]
X_v - dimensionless space coordinate of the cylindrical body, [1]
X_k - dimensionless space coordinate of the spheric body, [1]
ε - porosity, [1]

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