Energetic Optimization with Arbitrary Terminal Moment of Electric Drives Systems

Boteanu Niculae, Popescu Marius-Constantin, Ravigan Florin

***Abstract*:** - In the paper we consider an electric drive having static load torque with a constant component and a speed proportional component, in the hypothesis of constant inertia moment and of proportionality between the electromagnetic torque and the load current. Using variational calculus, optimally condition and expression of optimal control and extremal trajectory are determined, which ensures the minimum of energy losses caused by the load current through a Joule effect in the acceleration processes. Using numerical computer we can obtain graphical representation of these variables as time functions.

***Keywords*: -** *analytic and numerical model, electric drives, variational calculus, optimal control*

I. Introduction

In the case of drives that work in continuous type service (S1), appears the necessity of achieving starting and braking processes, and in the case of those that work in uninterrupted type service with periodical change of speed (S8), appears the necessity of achieving speed variations [11], [15], [16]. To asses these processes of acceleration and deceleration, the minimization of energy loss may be considered as a quality index, and the solving of this optimization problem can be obtained by using the classical variational calculus or the Euler – Lagrange algorithm and numerical computer.

II. Mathematical model

Considering an electrical drive load with static torque, having a constant component, a component that is proportional with speed and a component that is proportional with the square of root [1], [2], [3]

Boteanu N. Niculae is currently an Associate Professor in Faculty of Engineering in Electromechanical Environment and Industrial Informatics, Electromechanical Engineering Department,, University of Craiova, ROMANIA, e-mail address nboteanu@em.ucv.ro

Marius-Constantin O.S. Popescu is currently an Associate Professor in Faculty of Engineering in Electromechanical Environment and Industrial Informatics, Electromechanical Engineering Department, University of Craiova, ROMANIA, e-mail address popescu_ctin2006@yahoo.com,

Ravigan Florin is currently a Lecturer in Faculty of Engineering in Electromechanical Environment and Industrial Informatics, Electromechanical Engineering Department, University of Craiova, ROMANIA, e-mail address fravigan@em.ucv.ro

$$M_S = M_0 + k_1\omega + k_2\omega^2 \;, \tag{1}$$

that, in the hypothesis of neglecting the electromagnetic inertia in rapport with the mechanical inertia and a constant moment of inertia this action will be rewritten by the general equation of movement.

$$M = M_S + J\,d\omega/dt \;, \tag{2}$$

and of dependence between the speed and acceleration

$$\omega = \int \dot{\omega} dt . \tag{3}$$

To expand the interpretations and the conclusions, with and for the restraint of the value intervals, will be introduced relative coordinates [16]. In this sense, considering as a reference for time the mechanical constant of time

$$T = \frac{J\omega_N}{M_N} , \tag{4}$$

and for electricity, couple and speed, their nominal values will be obtained the relative values

$$\begin{gathered} \tau = \frac{t}{T} , \quad i = \frac{i}{I_N} , \quad \mu = \frac{M}{M_N} , \quad v = \frac{\omega}{\omega_N} , \\ \mu_s = \frac{M_S}{M_N} , \mu_0 = \frac{M_0}{M_N} , k_1 = \frac{k_1\omega_N}{M_N} , k_2 = \frac{k_2\omega_N^2}{M_N} \end{gathered} \tag{5}$$

and for relative acceleration there will be the relation

$$\dot{v} = \frac{\dot{\omega}}{\omega_N / T} . \tag{6}$$

In the hypothesis of proportionality between the electromagnetic couple and the burden power, the equations (1), (2) and (3) in the relative coordinates it becomes

$$\begin{gathered} \mu_S = \mu_0 + k_1 v + k_2 v^2 , \qquad \mu_0 + k_1 + k_2 = 0, \\ i = \mu = \mu_S + \dot{v} = \mu_0 + k_1 v + k_2 v^2 + \dot{v}, \quad v = \int \dot{v} d\tau \end{gathered} \tag{7}$$

with the initial and fixed conditions

$$\tau = \tau_1 , \quad v(\tau_1) = v_1 , \quad \tau = \tau_2 , \quad v(\tau_2) = v_2 . \tag{8}$$

The multitude of the conclusions admitted and the multitude of the trajectories that will be admitted will be considered marginal and open multitudes

III. OPTIMIZATION CRITERION

Adopting as a criteria of optimization the energy loss that is caused by the burden energy through Joule effect on the duration of the acceleration process

$$\Delta w = \int_{\tau_1}^{\tau_2} \Delta p \, d\tau = \int_{\tau_1}^{\tau_2} \rho \, i^2 d\tau \quad , \tag{9}$$

and taking into account the movement equation (7), the optimization criteria that will have the expression

$$J\left[v(\tau) \right] = \int_{\tau_1}^{\tau_2} i^2 d\tau = \int_{\tau_1}^{\tau_2} \left(\mu_S + \dot{v} \right)^2 d\tau \, . \tag{10}$$

IV. FORMULATION OF OPTIMIZATION PROBLEM

The optimization problem consists in determination the admissible optimal control function $i^*(\tau)$ or $\mu^*(\tau)$, which is able to transfer the system from the initial condition $\left(\tau_1, v\left(\tau_1 \right) \right)$ to the terminal conditions $\left(\tau_2, v\left(\tau_2 \right) \right)$, on an the admissible extremale trajectory $v^*\left(\tau_1 \right)$, ensuring the minimum of the optimality criterion (10)

$$J\left[v(\tau) \right] = \int_{\tau_1}^{\tau_2} \left(\mu_s + \dot{v} \right)^2 d\tau = min \, , \tag{11}$$

for a fixed value of speed variation expressed, by the integral

$$\Delta v = v_2 - v_1 = \int_{\tau_1}^{\tau_2} \dot{v}\left(\tau \right) d\tau, \tag{12}$$

effectuated is in a given interval of time

$$\tau_2 - \tau_1 = \int_{\tau_1}^{\tau_2} 1 \, d\tau \, , \tag{13}$$

and satisfying the restrictions

$$\begin{aligned} &\left| i\left(\tau \right) \right| < i_{max} \, , \left| \mu\left(\tau \right) \right| < \mu_{max} \, , \\ &\left| v\left(\tau \right) \right| < v_{max} \, , \left| \dot{v}\left(\tau \right) \right| < \dot{v}_{max} \end{aligned} \, . \tag{14}$$

In conformity with the principle of reciprocity, the given formulation is equivalent with the formulation through which every isoperimetric condition (12) and (13) can become optimization criteria ($v_2 - v_1 = \max$, $\tau_2 - \tau_1 = \min$) or a linear combination of them [6].

So, it results a linear – quadratic optimization problem of isoperimetric extremum. To solve this issued problem, the primal problem of conditional extremum will be reduced to a dual problem of unconditional extremum by a Lagrange adjoint function based on Lagrange multiplier λ_0 [15], [16].

$$L = \left(\mu_S + \dot{v} \right)^2 + \lambda_0 \, \dot{v} = \left(\mu_0 + k_1 v + \dot{v} \right)^2 + \lambda_0 \, \dot{v} \tag{15}$$

and by determining the unconditional extremum with the functional

$$J\left[v\left(\tau \right) \right] = \int_{\tau_1}^{\tau_2} L\left(\tau \right) d\tau = min \, , \tag{16}$$

on the same extremals as those of the primal problem [15]

V. NECESSARY CONDITION OF EXTREME

The necessary condition of extremum is expressed by the Euler – Lagrange equation [15], [16],

$$\frac{\partial L}{\partial v} - \frac{d}{d\tau} \frac{\partial L}{\partial \dot{v}} = 0 \, , \tag{17}$$

that leads to the linear differential equation of the second order

$$\ddot{v} - 2k_2^2 v^3 - 3 k_1 k_2 v^2 - \left(k_1^2 + 2k_2 \mu_0 \right) v = k_1 \mu_0 \tag{18}$$

Because the condition of extremum expressed by the differential equation (17) does not contain the Lagrange multiplier λ0, it corresponds to that which might result in the case that Euler–Lagrange equation would be applied directly for the functional of the criterion (11).

VI. OPTIMAL SOLUTION WITH ARBITRARY TERMINAL MOMENT FOR STATIQUE TORQUE WITH CONSTANT COMPONENT

Case of the constant static torque, considering the Particularizations

$$\mu_0 > 0, \qquad k_1 = 0, \qquad \mu_S = \mu_0 \quad , \tag{19}$$

the condition of extremum, expressed by the differential equation (18), becomes

$$\ddot{v} = 0 \, , \tag{20}$$

and successively integrating, we obtain the trajectories family

$$\dot{v} = C_1 \, , \qquad v = C_1 \tau + C_2 \quad . \tag{21}$$

From fixed initial condition, we determine one of the integration constants

$$\tau = \tau_1, v\left(\tau_1 \right) = C_1 \tau_1 + C_2 = v_1, \Rightarrow C_2 = v_1 - C_1 \tau_1 \, , \tag{22}$$

resulting the trajectories fascicle

$$\dot{v}\left(\tau, C_1 \right) = C_1, \quad v\left(\tau, C_1 \right) = C_1 \left(\tau - \tau_1 \right) + v_1 \, . \tag{23}$$

The terminal moment being arbitrary, the terminal extremity of the trajectory $\left(\tau_2, v_2 \right)$ will be mobile on the transversal, that means

$$\nu_2 = \varphi(\tau_2) = \text{const} \quad \text{and} \quad \dot{\varphi}(\tau_2) = 0\,, \tag{24}$$

and the necessary condition of the extremum existence implies satisfying the condition of transversality [16]

$$\left[L + (\dot{\varphi} - \dot{\nu}) \frac{\partial L}{\partial \dot{\nu}} \right]_{\tau=\tau_2} = 0\,, \tag{25}$$

that leads to equation

$$\mu_0^2 - \dot{\nu}^2 = 0 \quad \text{or} \quad \mu_0^2 - C_1^2 = 0\,, \tag{26}$$

from which the second arbitrary constant is determined

$$C_1 = \pm\, \mu_0 \tag{27}$$

Taking into consideration the arbitrary constant values (27) and (24), the contact condition between the extremal trajectory and transversal implies the equation

$$\tau = \tau_2\,, \quad \nu(\tau_2) = \pm \mu_0 (\tau_2 - \tau_1) + \nu_1 = \nu_2 \;\;, \tag{28}$$

where permits determining the optimum terminal moment (fig,1).

$$\tau_2^* = \pm \frac{\nu_2 - \nu_1}{\mu_0} + \tau_1\;. \tag{29}$$

Terminal moment is inversely proportional with the value of the static torque, resulting a terminal moment which tends to infinite, when the static torque tends to zero, that means non functioning of the drive without load.

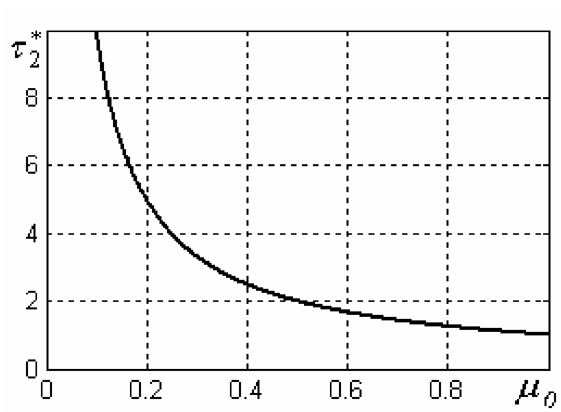

Fig. 1 Dependence of optimum terminal moment from static torque (τ_1=0 , ν_1=0 și ν_2=1)

Substituting the value of integration constant (29) in the solution (26), we can determine the extremal trajectory [3].

$$\nu^*(\tau) = \pm\, \mu_0 (\tau - \tau_1) + \nu_1\,, \dot{\nu}^*(\tau) = \mu_d^*(\tau) = \pm\, \mu_0\,, \forall \tau \in \left[\tau_1, \tau_2^*\right] \tag{30}$$

resulting a linear speed function of time and a constant acceleration (fig. 2 and fig.3).

Taking into consideration the motion equation (7) and the acceleration (30), the optimal control results

$$i^*(\tau) = \mu^*(\tau) = \mu_0 + \mu_d = = \begin{cases} 2\mu_0, & \text{acceleration} \\ 0, & \text{deceleration} \end{cases}, \forall \tau \in \left[\tau_1, \tau_2^*\right] \tag{31}$$

equal to the double of the static torque value for acceleration (fig. 2) and null for deceleration (fig. 3).

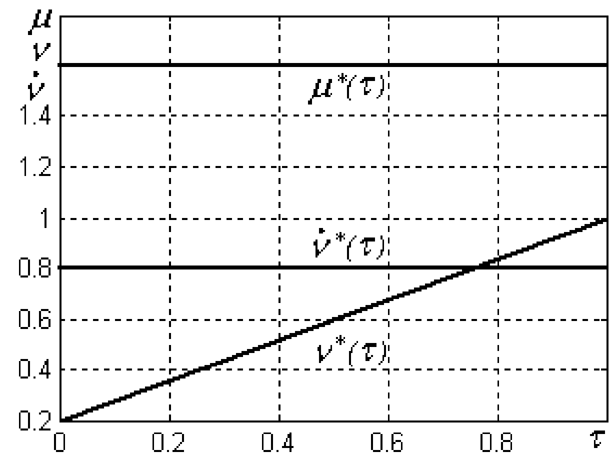

Fig.2 Optimal control and extremal trajectory during acceleration witch constant static torque
(μ_0=0.8, τ_1=0, ν_1=0.2 and *ν_2=1).*

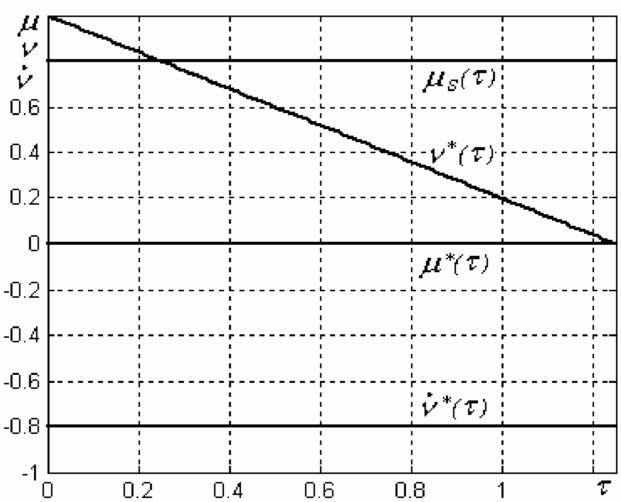

Fig. 3 Optimal control and extremal trajectory during deceleration witch constant static torque
(μ_0=0.8, τ_1=0, ν_1=1 and ν_2=0)

So, the absolute minimum of optimization criterion is ensured

$$J_{min}^*\left[\nu(\tau)\right] = \int_{\tau_1}^{\tau_2} \left(\mu_0 + \dot{\nu}^*\right)^2 d\tau = \begin{cases} 4\mu_0^2\left(\tau_2^* - \tau_1\right) \\ 0 \end{cases}$$

(32)

or, substituting the optimal terminal moment (29), we obtain

$$J^*_{min} = \begin{cases} 4\mu_0 \left(v_2 - v_1\right), & \text{acceleration} \\ 0, & \text{deceleration.} \end{cases} \quad (33)$$

VII. Optimal Solution with Arbitrary Terminal Moment for Statique Torque with Constant Component Speed Proportional Component

In the case of such a static torque, having the particularization

$$\mu_0 > 0, \qquad k_1 > 0 \ \text{and} \ \mu_0 + k_1 = 1 \quad (34)$$

and structural block diagram of electric drive (fig.4),

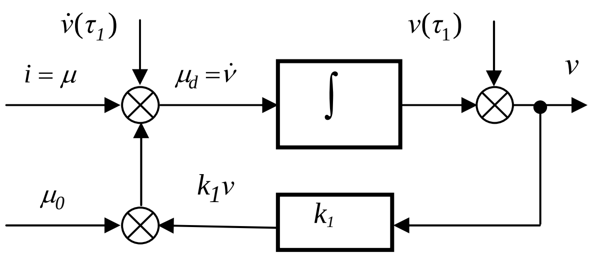

Fig. 4 Structural block diagram of electic drive

the static torque takes the form

$$\mu_S = \mu_0 + k_1 v, \quad (35)$$

and extremum condition expressed by the differential equation (20) becomes

$$\ddot{v} - k_1^2\, v = k_1\, \mu_0 \,. \quad (36)$$

Based on solving the characteristic equation attached to the homogeneous differential equation (36)

$$r^2 - k_1^2 = 0, \qquad \Rightarrow \qquad r_{1,2} = \pm\, k_1 \quad (37)$$

we obtain the general solution of homogeneous differential equation

$$v_g = C_1\, e^{k_1 \tau} + C_2\, e^{-k_1 \tau} \quad (38)$$

which, together with the particular solution of nonhomogeneous differential equation

$$v_p = -\frac{\mu_0}{k_1}, \quad (39)$$

lead to the general solution of no homogeneous differential equation, representing the family of trajectories

$$v = v_p + v_g = -\frac{\mu_0}{k_1} + C_1\, e^{k_1 \tau} + C_2 e^{-k_1 \tau} \quad (40)$$

Being fixed the initial extremity of the trajectory $\left(\tau_1, v\left(\tau_1\right)\right)$, we can determine one of the integration (arbitrary) constants

$$\begin{aligned} &\tau = \tau_1, \quad v\left(\tau_1\right) = -\frac{\mu_0}{k_1} + C_1\, e^{k_1 \tau_1} + C_2 e^{-k_1 \tau_1} = \\ &= v_1, \Rightarrow C_1 = \left(\frac{\mu_0}{k_1} + v_1\right) e^{-k_1 \tau_1} - C_2 e^{-2k_1 \tau_1} \end{aligned} \quad (41)$$

which, being substituted in the solution (40), we obtain the trajectories fascicle

$$\begin{aligned} &v\left(\tau, C_2\right) = -\frac{\mu_0}{k_1} + \left[\left(\frac{\mu_0}{k_1} + v_1\right) e^{-k_1 \tau_1} - C_2 e^{-2k_1 \tau_1}\right] \\ &e^{k_1 \tau} + C_2 e^{-k_1 \tau} \end{aligned} \quad (42)$$

and

$$\begin{aligned} &\dot{v}\left(\tau, C_2\right) = \left[\left(\mu_0 + k_1 v_1\right) e^{-k_1 \tau_1} - k_1 C_2 e^{-2k_1 \tau_1}\right] \\ &e^{k_1 \tau} - k_1 C_2 e^{-k_1 \tau} \end{aligned} \quad (43)$$

The terminal moment being arbitrary, the terminal extremity of the trajectory $\left(\tau_2, v\left(\tau_2\right)\right)$ is mobile on the transversal, that means

$$v_2 = \varphi\left(\tau_2\right) = const. \quad (44)$$

$$\text{and } \dot{\varphi}\left(\tau_2\right) = 0, \quad (45)$$

and the necessary condition of the extremum existence implies satisfying the condition of transversality [16]

$$\left[L + \left(\dot{\varphi} - \dot{v}\right) \frac{\partial L}{\partial \dot{v}}\right]_{\tau = \tau_2} = 0\,, \quad (46)$$

Having

$$L = \left(\mu_0 + k_1 v + \dot{v}\right)^2 + \lambda_0 \dot{v}\,, \quad (47)$$

$$\frac{\partial L}{\partial \dot{v}} = 2\left(\mu_0 + k_1 v + \dot{v}\right) + \lambda_0 \quad (48)$$

the equation is obtained

$$\left|\left(\mu_0 + k_1 v\right)^2 - \dot{v}^2\right|_{\tau = \tau_2} = 0\,, \quad (49)$$

or, taking into consideration speed expression (44) and the expression of the corresponding acceleration (47)

$$\left[\left(\left(\mu_0+k_1\nu_1\right)e^{-k_1\tau_1}-k_1C_2e^{-2k_1\tau_1}\right)e^{k_1\tau_2}+k_1C_2e^{-k_1\tau_2}\right]^2- \\ -\left[\left(\left(\mu_0+k_1\nu_1\right)e^{-k_1\tau_1}-k_1C_2e^{-2k_1\tau_1}\right)e^{k_1\tau_2}-k_1C_2e^{-k_1\tau_2}\right]^2=0 \qquad (50)$$

finally resulting the from

$$4k_1C_2\left[\left(\mu_0+k_1\nu_1\right)e^{-k_1\tau_1}-k_1C_2\,e^{-2k_1\tau_1}\right]=0 \qquad (51)$$

from which the second integration constant is determined

$$C_2=\begin{cases}0, & \text{acceleration}\\ \left(\dfrac{\mu_0}{k_1}+\nu_1\right)e^{k_1\tau_1}, & \text{deceleration}\end{cases} \qquad (52)$$

substituting (52) into (43) we get

$$C_1=\begin{cases}\left(\dfrac{\mu_0}{k_1}+\nu_1\right)e^{-k_1\tau_1}, & \text{acceleration}\\ 0, & \text{deceleration.}\end{cases} \qquad (53)$$

Taking into consideration the values of the arbitrary constants (52) and (53), the contact condition behveen the extremal trajectory and transversal, implies

$$\tau=\tau_2,\quad \nu\left(\tau_2\right)=-\frac{\mu_0}{k_1}+\left(\frac{\mu_0}{k_1}+\nu_1\right)e^{\pm k_1(\tau_2-\tau_1)}=\nu_2\,,(54)$$

or

$$e^{\pm k_1(\tau_2-\tau_1)}=\frac{\mu_0+k_1\nu_2}{\mu_0+k_1\nu_1}\,, \qquad (55)$$

from which, we determine, through logarithmation, the optimum terminal moment (fig.5)

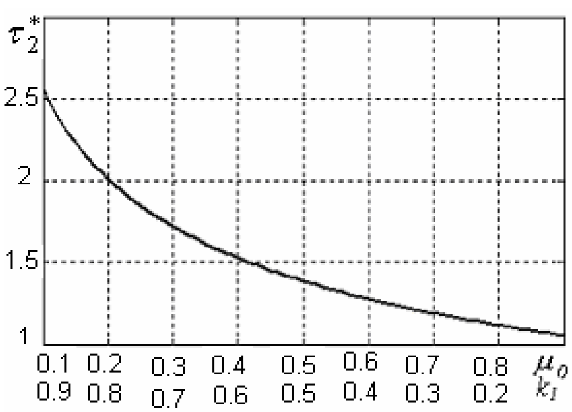

Fig. 5 Dependence of optimal terminal moment (ν_1=0 şi ν_2=1)

$$\tau_2^*=\pm\frac{1}{k_1}ln\,\frac{\mu_0+k_1\nu_2}{\mu_0+k_1\nu_1}+\tau_1\ . \qquad (56)$$

Substituting the integration constants (52) and (53) in solution (42) of the differential equation, we obtain the evolution in time (fig.6) and (fig.7) of the extremal trajectory, for speed

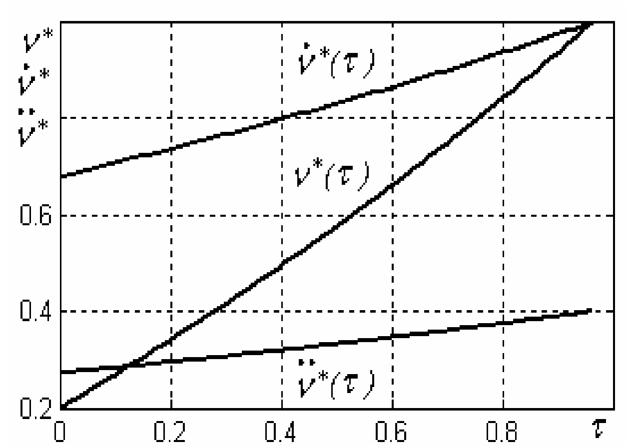

Fig. 6 Optimal control and extremal trajectory during acceleration and static torque with constant component and speed proportional component
(μ_0=0.6, k_1=0.4, ν_1=0.2 and ν_2=1)

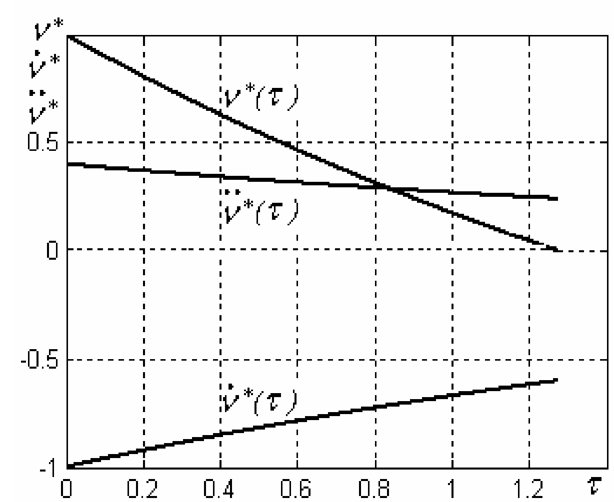

Fig.7 Optimal control and extremal trajectory during deceleration and static torque with constant component and speed proportional component
(μ_0=0.6, k_1=0.4, ν_1=1, and ν_2=0)

$$\nu^*\left(\tau\right)=-\frac{\mu_0}{k_1}+\left(\frac{\mu_0}{k_1}+\nu_1\right)e^{\pm k_1(\tau-\tau_1)}\ \ \forall\tau\in\left[\tau_1,\tau_2^*\right], \quad (57)$$

torque respectively

$$\dot{\nu}^*\left(\tau\right)=\mu_d^*\left(\tau\right)=\pm\left(\mu_0+k_1\nu_1\right)e^{\pm k_1(\tau-\tau_1)} \\ \forall\tau\in\left[\tau_1,\tau_2^*\right], \quad (58)$$

and for shock

$$\ddot{\nu}^*\left(\tau\right)=k_1\left(\mu_0+k_1\nu_1\right)e^{\pm k_1(\tau-\tau_1)}\ \ \forall\tau\in\left[\tau_1,\tau_2^*\right]. \quad (59)$$

Taking into account the speed extremal (44), load static torque will have the expression

$$\mu_s = \mu_0 + k_1 v = \left(\mu_0 + k_1 v_1\right) e^{\pm k_1(\tau-\tau_1)}, \forall \tau \in \left[\tau_1, \tau_2^*\right] \quad (60)$$

and, substituting the static torque expression (47) and dynamic torque expression (45) into motion equation (7), we can determine the optimal control variable [5]

$$\begin{aligned} i^*(\tau) &= \mu^*(\tau) = \mu_s + \mu_d = \left(\mu_0 + k_1 v_1\right) e^{\pm k_1(\tau-\tau_1)} \\ &\pm \left(\mu_0 + k_1 v_1\right) e^{\pm k_1(\tau-\tau_1)} = \\ &= \begin{cases} 2\left(\mu_0 + k_1 v_1\right) e^{k_1(\tau-\tau_1)}, & \text{acceleration} \\ 0, & \text{deceleration} \end{cases}, \forall \tau \in \left[\tau_1, \tau_2^*\right], \end{aligned} \quad (61)$$

resulting an electromagnetic torque equal to the double of the static torque value for acceleration (fig. 5) and null for deceleration (fig. 6). Absolute minimum of optimization criterion (12) is

$$\begin{aligned} J^*_{min} &= \int_{\tau_1}^{\tau_2} i^2(\tau)\, d\tau = \int_{\tau_1}^{\tau_2} \left[2\left(\mu_0 + k_1 v_1\right) e^{k_1(\tau-\tau_1)}\right]^2 d\tau = \\ &= \frac{2}{k_1}\left(\mu_0 + k_1 v_1\right)^2 \left(e^{2k_1\left(\tau_2^* - \tau_1\right)} - 1\right), \end{aligned} \quad (62)$$

and taking into consideration the optimum terminal moment (55), it becom

$$\begin{aligned} J^*_{min} &= \frac{2}{k_1}\left(\mu_0 + k_1 v_1\right)^2 \left[\left(\frac{\mu_0 + k_1 v_2}{\mu_0 + k_1 v_1}\right)^2 - 1\right] = \\ &= 4\mu_0\left(v_2 - v_1\right) + 2k_1\left(v_2^2 - v_1^2\right) \end{aligned} \quad (63)$$

Obtained results, determined for a variation of speed to the value v_1 to the value v_1, are valid - through particularization - both for starting, considering $v_1=0$, and also for braking until stopping considering $v_2= 0$.

VIII. Nature of the Extremum

In order to analyses the nature of the extremum we use Jacobi's differential equation

$$\left(\frac{\partial^2 L}{\partial v^2} - \frac{d}{d\tau}\frac{\partial^2 L}{\partial v\, \partial \dot{v}}\right)\eta - \frac{d}{d\tau}\left(\frac{\partial^2 L}{\partial \dot{v}^2}\dot{\eta}\right) = 0 \quad (64)$$

Where, having the adjoint function Lagrange (15) and calculating

$$\begin{aligned} &\frac{\partial^2 L}{\partial v^2} = 2k_1^2, \quad \frac{d}{d\tau}\frac{\partial^2 L}{\partial v\, \partial \dot{v}} = 0, \\ &\frac{d}{d\tau}\left(\frac{\partial^2 L}{\partial \dot{v}^2}\dot{\eta}\right) = 2\ddot{\eta} \end{aligned} \quad (65)$$

Jacobi's differential equation becomes

$$\ddot{\eta} - k_1^2\, \eta = 0 \quad (66)$$

Solving characteristic equation, attached to the differential equation

$$r^2 - k_1^2 = 0 \quad \Rightarrow \quad r_{1,2} = \pm k_1 \quad (67)$$

general solution of the differential equation has the form

$$\eta = C_1 e^{k_1\tau} + C_2 e^{-k_1\tau} \text{ and } \dot{\eta} = k_1\left(C_1 e^{k_1\tau} + C_2 e^{-k_1\tau}\right) \quad (68)$$

And assuming the following initial condition, we obtain the equation system

$$\begin{aligned} \tau = 0, \quad &\eta(0) = 0 \quad \Rightarrow \quad C_1 + C_2 = 0, \\ &\dot{\eta}(0) = 1 \quad \Rightarrow \quad k_1\left(C_1 - C_2\right) = 1, \end{aligned} \quad (69)$$

Which permits us to determine the integration constants

$$C_1 = \frac{1}{2k_1}, \quad C_2 = -\frac{1}{2k_1}, \quad (70)$$

Resulting finally the solution

$$\eta = \frac{1}{2k_1}\left(e^{k_1\tau} - e^{-k_1\tau}\right) = \frac{1}{k_1} sh\left(k_1\tau\right) \quad (71)$$

Because the solution of Jacobi's differential equation is not null in every point for $\tau>0$, results that the obtained extremal doesn't contain conjugated point and so Jacobi's condition is satisfied. Having the Legendre's condition, too, expressed by the inequality

$$\frac{\partial^2 L}{\partial \dot{v}^2} = 2 > 0 \quad (72)$$

results that extremal trajectory $v^*(\tau)$, determined by the optimal control $i^*(\tau) = \mu^*(\tau)$, ensures the absolute minimum for the functional – criterion (12).

IX. The Optimal Solution for the Static Couple with a Constant Component, Component Proportional with Speed and Component Proportional with the Square of Speed – Solution through Simulation

In the When there is a constant component, component proportional with speed and component proportional with the square of speed

$$\mu_0 \neq 0, \quad k_1 \neq 0, \quad k_2 \neq 0, \quad \mu_0 + k_1 + k_2 = 1 \quad 73)$$

the static couple has the form

$$\mu_s = \mu_0 + k_1 v + k_2 v^2, \quad (74)$$

and the necessary condition of extreme is expressed by the differential equation

$$\ddot{v} - 2k_2^2 v^3 - 3k_1 k_2 v^2 - \left(k_1^2 + 2k_1\mu_0\right)v = k_1\mu_0 \quad (75)$$

which, not being linear implies some difficulties for an analytical solution. For this reason, we use the solving through numerical simulation with the aid of the computer on the basis of the structural scheme (fig.8) as a problem of optimization, either bi local or with final free time.

Solving through simulation, as a problem of optimization bi local, resides in the introduction into the scheme of the triplet of values that are characteristic to the static couple (μ_0, k_1 and k_2), fixing the initial condition of acceleration $\dot{v}(\tau_1)$ and the variation interval of speed that is given by the initial value $v_1 = v(\tau_1)$) and the final value $v_2 = v(\tau_2)$, that after simulation, will be determined the evolution in time of speed $\dot{v}(\tau)$, acceleration $v(\tau)$, and the static couple $\mu_s(\tau)$ and of the couple developed by the engine $\mu(\tau)$. In this case, it will result a certain interval of time τ_2-τ_1 and a certain minimal value of the optimization criteria, corresponding to the initial acceleration that was adopted.

Solving through simulation, the problem of optimization with final free time resides in the determination of that interval of time (dependent of the initial acceleration) necessary for the evolution of the system that should ensure the smallest minimum criteria of optimization.

In this sense for simulation will be realized the following steps:

- first of all will be introduced into the scheme (fig.8) the triplet of values that are characteristic to the static couple ($\mu 0$, $k1$ and $k2$) and will be fixed the interval of variation for speed, that is the initial value v_1=v(τ_1) and the final value v_2= v (τ_2);
- as a second step, introducing different values of initial acceleration $\dot{v}(\tau_1)$ in a certain interval of values, through integration up to the final value of velocity and speed v2, will be represented the dependence of the minimal value of the optimization criteria and the final period, in conformity with the initial acceleration that was introduced, from which – for the shortest minimal of optimization criteria – will be determined the initial optimal acceleration $\dot{v}^*(\tau_1)$ (fig.9) and the optimal final time τ_2^* (fig.10) for acceleration and deceleration (fig.11);

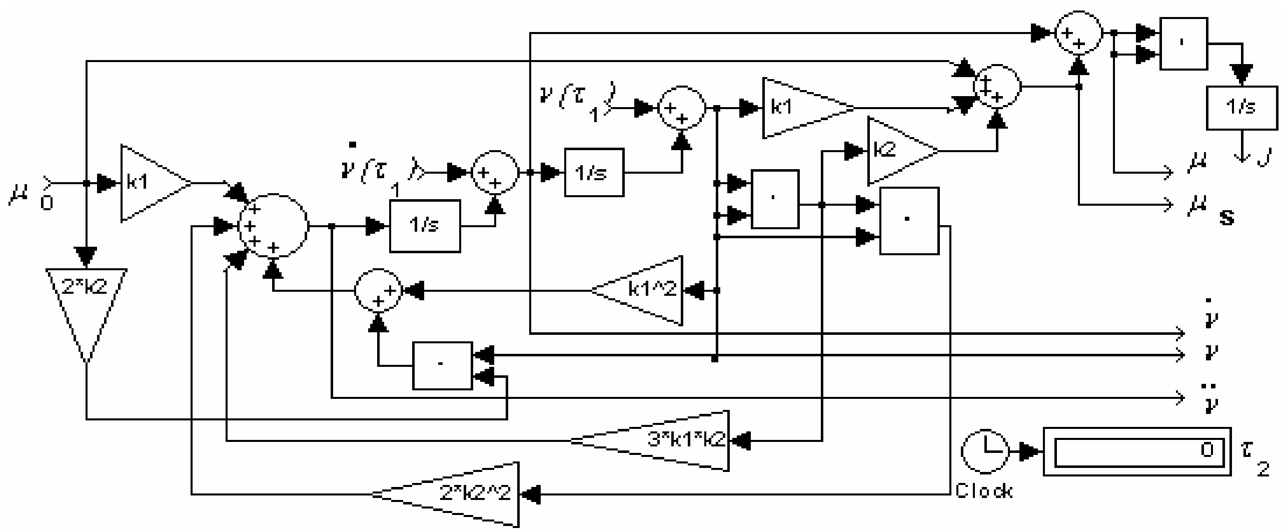

Fig.8 Structural bloc scheme for simulation in Matlab/ Simulink.

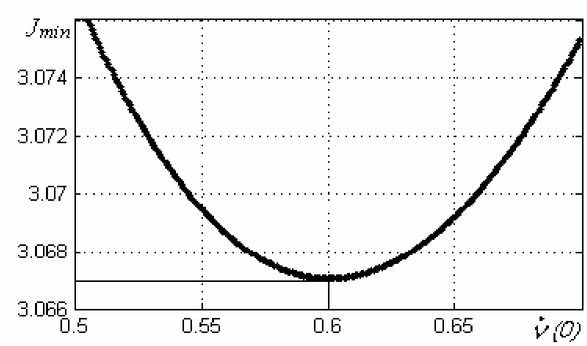

Fig.9 Dependency of the minimal value of the criteria of optimization of initial acceleration during start (μ_0=0.6, k_1=0.2, k_2=0.2, v_1=0 and v_2=1)

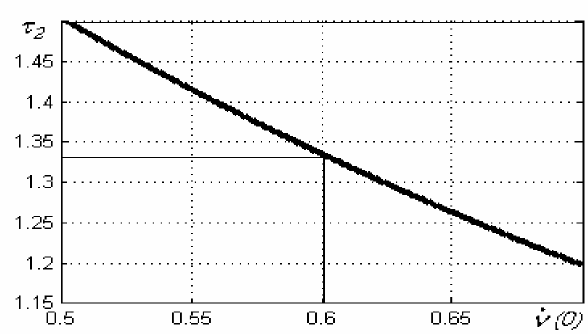

Fig.10 Dependency of the final time of initial acceleration during start (μ_0=0.6, k_1=0.2, k_2=0.2, v_1=0 and v_2=1)

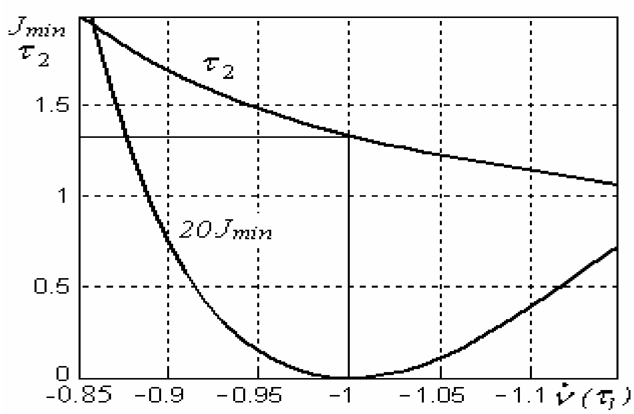

Fig.11 Dependency of the minimal value of the criteria of optimization of initial acceleration during brake operation
(μ_0=0.6, k_1=0.2, k_2=0.2, ν_1=1 şi ν_2=0)

- as a third step, introducing the optimal value of initial acceleration $\dot{\nu}^*(\tau_1)$, through integration, will be determined the evolution of the velocity extremes $\nu^*(\tau)$, the acceleration $\dot{\nu}^*(\tau)$ and shock $\ddot{\nu}^*(\tau)$ (fig.12 and fig.13);

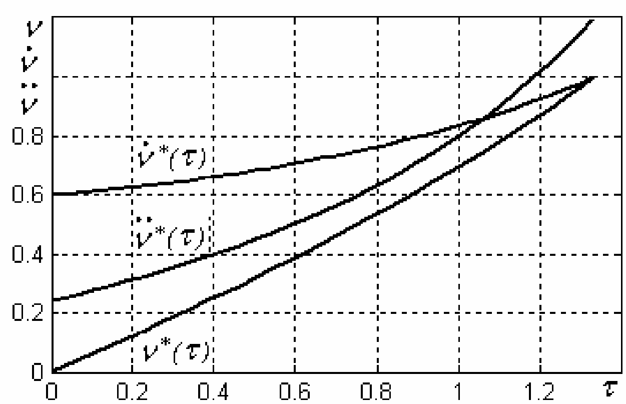

Fig. 12 Extreme trajectory of speed, acceleration and the receiving shock *(μ_0=0.6, k_1=0.2, k_2=0.2 and $\dot{\nu}$ (0)=0.6)*;

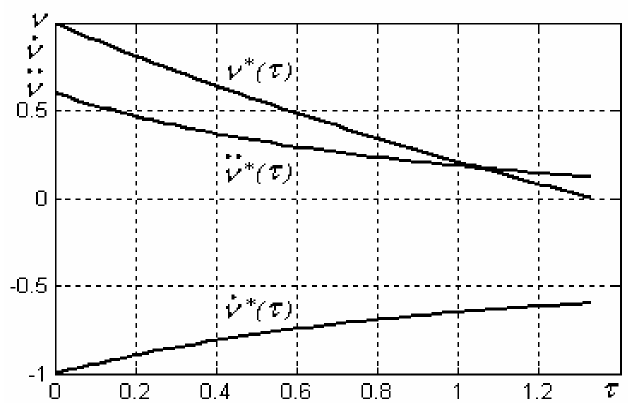

Fig. 13 Extreme trajectory of speed, acceleration and shock during brake operation
(μ_0=0.8, k_1=0.2, k_2=0.2 and $\dot{\nu}$ (0)=-1)

- as a fourth step, introducing the optimal value of initial acceleration $\dot{\nu}^*(\tau_1)$, through integration, will be determined the static couple as time function $\mu_s(\tau)$ and the evolution of the optimal command $\mu^*(\tau)$ and $i^*(\tau)$ (fig. 14 and 15).

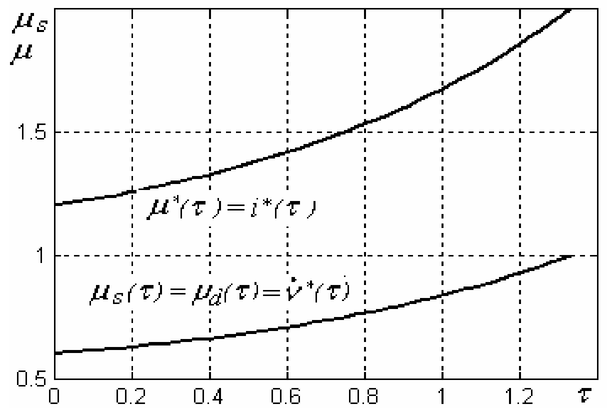

Fig. 14 Extreme command and static couple white start – up *(μ_0=0.6, k_1=0.2, k_2=0.2 şi $\dot{\nu}$ (0)=0.6)*

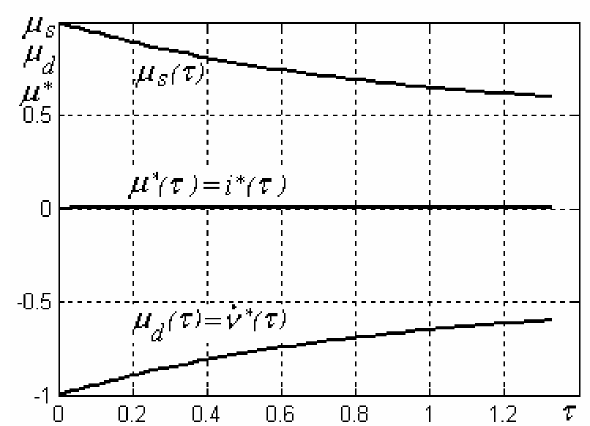

Fig. 15 Extreme command, static couple and dynamic white breaking operation
(μ_0=0.6, k_1=0.2, k_2=0.2 şi $\dot{\nu}$ (0)=-1)

From the results of the simulation, we can observe that the optimal acceleration is given by the temporary value of the static couple:

$$\dot{\nu}^*(\tau) = \pm\left(\mu_0 + k_1\nu + k_2\nu^2\right) , \qquad (76)$$

with limit values

$$\dot{\nu}^*(\tau_1) = \pm\left(\mu_0 + k_1\nu_1 + k_2\nu_1^2\right) ,\dot{\nu}^*(\tau_2) = = \pm\left(\mu_0 + k_1\nu_2 + k_2\nu_2^2\right) , \qquad (77)$$

and the extreme command is equal with the double of the static couple

$$i^*(\tau) = \mu^*(\tau) = \pm\left(\mu_0 + k_1\nu_2 + k_2\nu_2^2\right) \qquad (78)$$

in the acceleration period and is null in the deceleration period

$$i^{*}\left(\tau\right)=\mu^{*}\left(\tau\right)=0. \qquad (79)$$

The bloc scheme for simulation (fig.7) is general and – through particularizations – is available for different static couples.

The results expressed through extreme trajectory and the extreme command obtained can be used for the projection as well as for the optimal conduct of the systems with electric action with the static couple depending on speed that functions in the continuous service (S1) or uninterrupted service with the periodic modification of speed (S8), cases frequently met in the mining industry. These results, through the energy economy that is realized in the starting and breaking processes and the periodic modification of speed, lead to the increase of quality and efficiency of the systems of electric action of the installations of continuous transport on band.

X. Conclusion

For constant static torque, the speed is a linear function of time, and the others variables of the problem are constant. For static torque with a component proportional to speed, ale variables of the problem are exponential functions of time [14, 15, 16]. Electromagnetic torque of the driving motor, necessary for the optimal control, is equal to the double of the static torque in acceleration processes and is null in the deceleration processes (considering a brake with recuperation of energy). Obtained results, determined for a variation of speed, are valid-through particularization-both for starting, considering $v_1=0$, and also for braking until stopping considering $v_2=0$.

The problem in which static torque is proportional to speed ($\mu_0=0,\ k_1>0$), optimization with arbitrary terminal moment doesn't have an optimal solution in the starting or braking until stopping cases. Obtained results can be used both in designing and also in optimal control of the electric driving systems with constant static torque or depending on speed which works in service-type continuous S_1 or in uninterrupted service with periodical modification of speed S_8. Through energy savings obtained in the starting, braking and periodical modification of speed processes obtain a quality and efficiency increasing of electric driving system. Optimal control for system transportation from initial to terminal condition with minimal energy losses, is obtained in open-loop or with program which is applied to the system input. This solution presents some disadvantages concerning both implementing the program and also its exact effectuation. For this reason, we preferred a solution in closed-loop that can be obtained directly through Ricati equation.

Optimal solution with arbitrary terminal moment for statique torque with constant component, speed proportional component and a proportional component with the square of speed, it can be obtained using analytical computation or numerical simulation with the aid of the computer.

The bloc scheme for simulation (fig.8) is general and – through particularizations – is available for different static couples. The results expressed through extreme trajectory and the extreme command obtained can be used for the projection as well as for the optimal conduct of the systems with electric action with the static couple depending on speed that functions in the continuous service or uninterrupted service with the periodic modification of speed, cases frequently met in the mining industry. These results, through the energy economy that is realized in the starting and breaking processes and the periodic modification of speed, lead to the increase of quality and efficiency of the systems of electric.

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