Limit Temperatures, Spinodal Decomposition and Isospin in Heavy Ion Collisions

Armando Barrañón, Jorge Alberto López Gallardo

***Abstract*—**Spinodal decomposition signatures have been used to obtain limit temperatures for several Heavy Ion collisions. Meanwhile isospin changes, these transient temperatures remain approximately constant with a fluctuation of about 1 MeV and in the range of 10MeV. Also, a primitive breakup with equal sized fragments of a privileged fragment size equal to 6 and an excitation of 4.75MeV were found using higher order charge correlations. These transient temperatures are in the range of theoretical and experimental studies reported elsewhere and confirm the role of spinodal decomposition in the critical behavior of nuclear matter.

***Keywords*—**Molecular Dynamics, Spinodal Decomposition, Heavy Ion Collisions, Nuclear Temperature, Higher order charge correlations.

I. Introduction

HEAVY Ion Collisions are expected to experience a liquid gas phase transition due to the specifics of the internucleonic interaction, which is attractive in both the long and intermediate ranges and repulsive in the short range. It is possible that a wide zone of phase space is explored when two nuclei collide, including a region where liquid and gas phases coexist, namely the spinodal region. At this spinodal region, incompressibility is negative and uniform nuclear matter is unstable, leading to multifragmentation due to the increase of density fluctuations [1] [2]. Dynamical simulations based on Boltzmann equation, such as Landau-Vlasov (LV), Boltzmann-Uehling-Uhlenbeck (BUU) or Boltzmann-Nordheim-Vlasov (BNV), describe the time evolution of the density of a one-body system, ignoring those correlations whose order is larger than the order of binary correlations, neglecting fluctuations around the mean trajectory of the system, which altogether comes out to be quite inconvenient to study the spinodal instability zone [1]. As shown by Guarnera *et al.* [3], a spinodal decomposition produces a ”primitive breakup” where equal sized fragments have a privileged fragment size, which is related to the wave lengths of the most unstable modes of nuclear matter. Tabacaru *et al.* have obtained multifragmentation. They also computed higher order charge correlations, defined as the ratio between the number of correlated fragments and the number of uncorrelated fragments, obtaining evidence of a privileged production of equal sized fragments [4]. Chomaz *et al.* have introduced a scenario inspired on experimental data, where a gently compressed system expands and reaches thermal equilibrium approximately at the time when the system enters into the spinodal region. At this moment, density fluctuations break up the system into several hot fragments and particles. When fragments are released from the nuclear force, configuration freezes and fragments only interact with each other via coulomb force. At this moment, system has explored so much phase space that it can be described by statistical models and there is no contradiction between statistical or dynamical approximations. Statistical approximations describe the time evolution of the system and the phase diagram. Meanwhile, dynamical approximations start up in phase diagram and are more related to the thermodynamics of non extensive systems. Barrañón *et al.* have obtained computational evidence about the inverse relation between entropy and the residual size, using LATINO dynamical model to study the spinodal decomposition region of central Ni+Ni HIC at intermediate energies [5] . Tabacaru *et al.* obtained evidence about a primitive spinodal decomposition computing higher order charge correlations, defined as the ratio between the number of correlated fragments and the number of uncorrelated fragments, obtaining evidence of a privileged production of equal sized fragments [4].

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of central Ni+Ni HIC at intermediate energies [5] . In this very study, evidence is obtained about a primitive spinodal decomposition for Ni+Ni central HIC at intermediate energies.

II. Problem formulation

Heavy Ion Collisions were simulated using LATINO semiclassical model where binary interaction [6] is reproduced with a Pandharipande potential built up of Yukawa potentials linear combinations, whose coefficients are designed to both reproduce nuclear matter ground state properties and to fulfill Pauli exclusion principle [7]:

$$V_{nn} = V_{pp} = V_0\left(\frac{e^{-\mu_0 r}}{r} - \frac{e^{-\mu_0 r_C}}{r_C}\right) \quad (1)$$

and:

$$V_{np} = V_r\left(\frac{e^{-\mu_r r}}{r} - \frac{e^{-\mu_r r_C}}{r_C}\right) - V_a\left(\frac{e^{-\mu_a r}}{r} - \frac{e^{-\mu_a r_a}}{r_a}\right) \quad (2)$$

Ground states have been obtained starting from a randomly generated configuration of nucleons, confined in a parabolic potential that evolves by a frictional method until the ensemble attains its theoretical binding energy. All correlations are included so that a sufficient yield of fragments is obtained and necks are observed. Clusters are identified with an Early Cluster Recognition Algorithm that optimizes configurations in energy space. A most bound partition is obtained minimizing the sum of energies of the clusters belonging to each partition:

$$\{C_i\} = \arg\min\left[E_{\{C_i\}} = \sum_i E_{\text{int}}^{C_i}\right] \quad (3)$$

where the energy of each cluster is given by :

$$E_{\text{int}}^{C_i} = \sum_i \left[\sum_{ij\in C_i} K_j^{CM} + \sum_{j,k\in C_{iu}, j\le k} V_{jk}\right] \quad (4)$$

where the first sum includes the partition clusters, K_j^{CM} is the kinetic energy of the particle j measured in the center of mass of the cluster containing particle j, and V_{ij} is the internucleonic potential. Projectile is boosted on target with a given kinetic energy for distinct impact parameters. System evolution was simulated using a Verlet algorithm, where two Taylor expansions are subtracted, one of them forwards and the other backwards on time:

$$\vec{r}(t+\Delta t) = 2\vec{r}(t) - \vec{r}(t-\Delta t) + \vec{a}(t)h^2 \quad (5)$$

$$\vec{v}(t+\Delta t) = \vec{v}(t) + 0.5*\left[\vec{a}(t+\Delta t) + \vec{a}(t)\right]h \quad (6)$$

$$\vec{a}(t+\Delta t) = -(1/m)\nabla V\left(\vec{r}(t+\Delta t)\right) \quad (7)$$

The algorithm uses "simulated annealing" to find the most bound partition and optimizes the partition in energy space. Projectile energy is in the range of 600 to 2000 MeV and system evolves until its microscopic composition is rather frozen though some monomers are ejected. This time can be identified using the Persistence Microscopic Coefficient, defined as the probability that two particles belonging to the partition X remain bound in partition Y:

$$P[X,Y] = \frac{1}{\sum_{cluster} n_i} \sum_{cluster} n_i a_i / b_i \quad (8)$$

where b_i is equal to the number of pairs of particles belonging to the cluster C_i of partition $X \equiv \{C_i\}$ while a_i is equal to the number of particle pairs belonging to cluster C_i of partition $X \equiv \{C_i\}$ that also belong to a given cluster C_i' of partition $Y \equiv \{C_i'\}$. n_i is the number of particles in cluster C_i.

Higher order charge correlations were introduced by Moretto *et al.* [8] and are given by:

$$\left.\frac{Y(\Delta Z, < Z >)}{Y'(\Delta Z, < Z >)}\right|_M \quad (9)$$

where $Y(\Delta Z, < Z >)$ is equal to the number of fragments produced for given values of ΔZ and $< Z >$.

Adiabatic sound speed is given by:

$$V_c^2 = (1/m)\left[\left(\partial P / \partial \rho\right)_S\right] \quad (10)$$

$$= \left[(10/9) < E_K > + a(\rho/\rho_0) + b(\rho/\rho_0)^\sigma\right] \quad (11)$$

where $< E_K >$ is the mean kinetic energy, a=-358.1 MeV, b=304.8 MeV and σ =7/6 are the parameters for a soft equation of state. When $V_c^2 < 0$ nuclear matter is unstable with respect to density fluctuations, leading to dynamic instabilities [9].

Temperature is estimated using Kinetic Theory for the n nucleons in the compound:

$$\frac{3}{2} nT = K^{CM} \quad (12)$$

Excitation is computed by the temperature attained by the projectile-target compound, when the maximal compression is reached:

$$E_{x,c} / A = T^2 / 13 \quad (13)$$

This relation assumes a Fermi degenerate gas behaviour and is in better agreement with experiment results than the empirical thermometers based on isotope ratios, as shown by Moretto *et al.* in a study about evaporation in compound nucleus decay [10].

Gross *et al.* has proved that it is possible to define the change of state in finite systems by means of a Statistical Mechanics based on Boltzmann entropy definition [11]. As a matter of fact, Gibbs considered the microcanonical ensemble as the fundamental one and the canonical as an approximation [12]. According to Lebowitz it is not essential to attain the thermodynamic limit [13] neither are necessary extensivity, concavity or additivity [14] to define entropy. In this study, entropy is estimated by the classical gas:

$$S = \log\left[\left(\frac{1}{n}\right)\left(\frac{3T}{2}\right)^{3/2}\right] + S_0 \quad (14)$$

III. Simulation Results

Higher order charge correlations were obtained for central HIC Ni+Ni (Fig. 1). These higher order correlations show a peak for a fragment size equal to 6, with four equal sized fragments produced with this privileged size and an excitation equal to 4.75 MeV. This can be considered a primitive breakup signature in equal sized fragments of a privileged size. Hence, this dynamical study supports other experimental and
theoretical evidences of spinodal decomposition in HIC, such as the experimental study reported by Borderie *et al.* [15] where a fossil spinodal decomposition signature for HIC collision ^{129}Xe+^{nat}Sn at 32 AMeV was obtained. At that very experiment, liquid-gas coexistence was observed and evidence was obtained of a first order phase transition for a non extensive particle system. This HIC phase transition has been identified in previous studies with several signatures, namely Campi scattered plots [16] as well as fragment size distribution power laws [17].

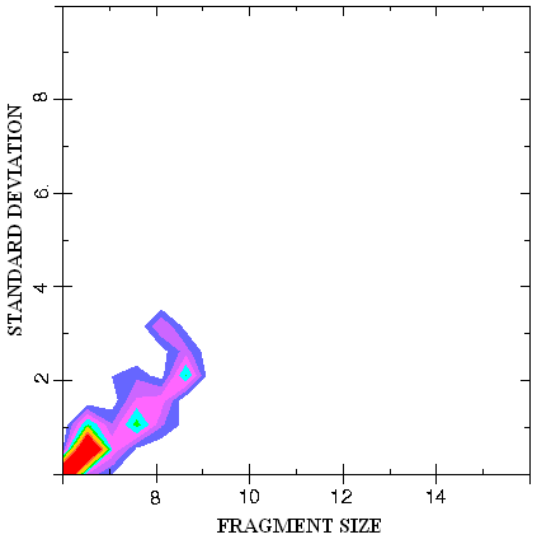

Fig. 1.- Higher order charge correlations for central HIC Ni+Ni show a peak for a fragment size equal to 6, with four equal sized fragments produced with this privileged size and an excitation equal to 4.75 MeV.

Limit spinodal temperatures are obtained for collisions with distinct isospin values but entering the spinodal region with the same entropy value. While ^{40}Ca+^{40}Ca collision attains a limit spinodal temperature equal to10.5 MeV, ^{42}Ca+^{42}Ca collision attains a limit spinodal temperature equal to 10.4MeV, and ^{44}Ca+^{44}Ca a limit spinodal temperature equal to 10.27 MeV (cf. Fig. 2). Both estimations of the nucleonic critical temperature are in the range reported by [18] restricting T_c to values greater than 10MeV. Besides these limit temperature values are also in agreement with [19] about a minimal difference between the nuclear caloric curve limit temperatures of systems poor and rich in neutrons [20].

The minimal influence of isospin in these transient temperatures can be explained in terms of Weiszäcker nuclear energy expression which contains a term related to charge balance, namely the difference between the number of neutrons and protons. Weiszäcker's expresion for nuclear energy is given by [21]:

$$E = -a_V A + a_S A^{2/3} + a_a \frac{(N-Z)^2}{A} + a_C \frac{Z(Z-1)}{A^{1/3}} \quad (15)$$

where the first term represents bulk symmetric nuclear matter energy excluding Coulombian interactions. The rest of the terms compensate nuclear energy disagreement with the asymmetric nuclear matter bulk limit. These terms are related to surface energy, to the asymmetry between neutrons N and protons Z and to coulombian repulsion, respectively. As can be seen the isospin term is rather small when compared to the first two terms. As a matter of fact, a recent study showed that this formula holds for a wide range of temperatures of the Lattice Gas Model [22].

Other studies have obtained limit temperatures in the range of 6MeV with a thermometer based on the ratio N/Z of the projectile and the isobaric ratio Y(3H)/Y(3He) which is closer to the results obtained when isotopic temperatures are used. In a recent study, Barrañón *et al.* used LATINO Model to simulate violent Heavy Ion Collisions obtaining similar values for the same isotopic temperatures, even for the early stage of these collisions and for collisions with distinct isospin values[23]. The isotopic thermometer used at that was given by:

$$T_{HHe} = \frac{14.3}{\ln[1.59(9/8)^{1/2}\frac{Y_t/Y_d}{Y_{^4He}/Y_{^3He}}]} \quad (15)$$

where Y_t, Y_d, Y_{4He} and Y_{3He} are the yields of such species. Therefore LATINO Model holds for both kinds of temperature definitions.

IV. Conclusion

Once LATINO dynamical model was applied, higher order charge correlations provided computational evidence about the spinodal decomposition in the early stage of fragmentation for central HIC Ni+Ni at intermediate energies. This confirms previous evidence reported by others of a fossil spinodal decomposition, indicating liquid-gas coexistence and a first order phase transition for non extensive central HIC Ni+Ni. Also, limit temperatures computed for heavy ion collisions with different isospins are in agreement with recent theoretical computations.

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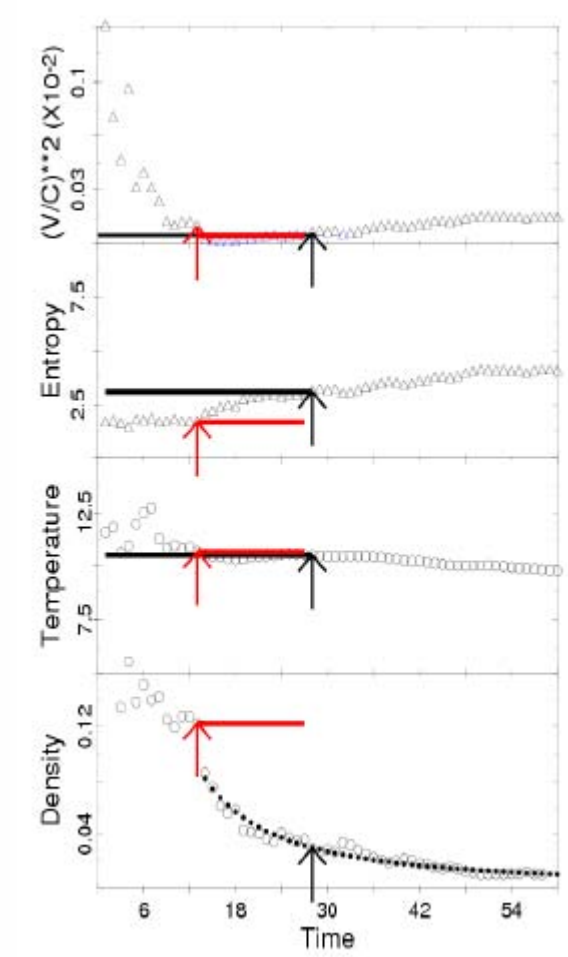

Fig. 2.- Limit spinodal temperature for $^{40}Ca+^{40}Ca$ collision. Squared sound speed is lower than zero when system enters into the spinodal region (left arrow). At this time temperature is constant and density decays following a power law. Right arrow signs the time when squared speed sound is great than zero. Transient temperature is in the range of 10MeV and density attains a value close to 0.18, which is its ground state value, when the system enters into the spinodal region.

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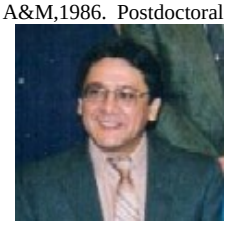

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