Some Topological Properties of Semi-Dynamical Systems

M. H. Anvari and M. Atapour

***Abstract*—Recently there has been an extensive study on Relative Semi-Dynamical Systems (RSD-Systems). In this paper, we explore some topological properties of RSD-systems. Here, in particular, minimal RSD-systems are characterized and transitive homeomorphisms are investigated. Moreover, α-level relative topological entropy is extended to RSD-systems. Finally, as a computational example, we develop an RSD-system over the polynomial function space $\mathbf{R}[x]$ based on the derivative operator; we also calculate α-level relative topological entropy for this system.**

***Index Terms*—Relative semi-dynamical system, Transitivity, Relative topological entropy**

I. Introduction

ANY scientific approach towards studying dynamics on natural systems relies on modeling (analytical, numerical, or observational). A successful mathematical modeling requires an awareness of the following alternative approaches: Synthesis: finding similarities between models that appear different Analysis:
identifying differences between models which appear similar A model is accepted or validated by evaluating its accuracy, i.e., how well the formal system describes the natural system? This can be done by matching experimental observations and/or measurements with the theory. The process of mathematical modeling has the following steps:

1 Beginning with observations, we start with a question or hypothesis, which is investigated within a conceptual framework (the model).
2 We test and validate the model with experimental data.

However all data are not crisp, also getting the facts through observational process depends on the idea of observer. So we should add the evaluation of the "thought of observer" to the above two main points and extend the fuzzy version of such a mathematical model. In order to develop a mathematical model underlying uncertainty and fuzziness in a dynamical system, any variation and/or approximation on a system should be identified by an observer. Moreover, we need a method to compare different perspectives of the observers, also to measure the complexity and/or the uncertainty of the system through viewpoint of the observers. So first, we should mathematically identify the observer. In our approach, there is a one to one correspondence between $[0,1]^X$, the set of all functions $f : X \to [0,1]$, and the observers where X denotes the base space of the system. We indicate any structure or dynamics on X in terms of μ-qualify or μ-relative which means from the viewpoint of μ. For example, μ-relative topology is the description of the topological notion on X by eyes of the observer μ. Applying this idea for describing any other geometrical structures on X such as manifolds [9], vector fields [7] and metrics [9], [8] has led to the μ-Relativism Perspective on X.

An extension of the fuzzy dynamical system [2], [4], [12], which is called relative semi-dynamical system, has been introduced in [6] to explain the dynamics on the system related to the observer's perspective. In this paper first, in section II, some necessary definitions and terminology are reviewed from [3], [6], [10], [11]. Then minimal relative semi-dynamical systems are introduced and characterized in section III. The concept of transitivity from the viewpoint of the observer is also discussed in section IV. In section V, some results regarding topological entropy, as an invariant object under the conjugate relation [6], are presented in order to classify relative semi-dynamical systems. Finally we discuss some computational examples in section VI.

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M. H. Anvari would like to thank the University of Saskatchewan for providing the financial support for the preparation of this work.

II. Definitions and Terminology

In this section some necessary definitions and terminology such as fuzzy set, fuzzy topology and relative semi-dynamical system are reviewed from [3], [6], [10], [11].

Given any nonempty set X, any function $\mu \in [0,1]^X$ is said to be a *fuzzy subset* of X. Given the fuzzy subset μ, τ_μ is said to be a *μ-relative topology* on X if τ_μ is a collection of members of $[0,1]^X$ satisfying the following conditions [11]:

i) μ, $\chi_\phi \in \tau_\mu$, where χ is the characteristic function,
ii) If $\lambda \in \tau_\mu$ then $\lambda \subseteq \mu$, i.e. $\lambda(x) \le \mu(x)$ on X,
iii) If $\lambda_1, \lambda_2 \in \tau_\mu$ then $\lambda_1 \cap \lambda_2 \in \tau_\mu$, where $(\lambda_1 \cap \lambda_2)(x) = \inf\{\lambda_1(x), \lambda_2(x)\}$,
iv) If $\{\lambda_i : i \in \Gamma\} \subseteq \tau_\mu$ then $\cup_{i\in\Gamma}\lambda_i \in \tau_\mu$, where $\cup_{i\in\Gamma}\lambda_i = \sup\{\lambda_i(x)\}_{i\in\Gamma}$.

Note that τ_μ is, in some sense, a fuzzy model of the topology on X from the viewpoint of the observer μ. In fact, for a given $\alpha \in (0,1]$, if we let

$$\lambda_\alpha = \{x \in X \ : \ \lambda(x) > \alpha\}$$

and

$$(\tau_\mu)_\alpha = \{\lambda_\alpha \ : \ \lambda \in \tau_\mu\}$$

then $(\mu_\alpha, (\tau_\mu)_\alpha)$ can be considered as a crisp topological space.

With the above notations, let (X, τ_μ) denote a μ-relative topological space; a mapping $f : X \to X$ is called (μ, μ)-*continuous* if

$$f^{-1}(\eta) \cap \mu \in \tau_\mu$$

for all $\eta \in \tau_\mu$, where $f^{-1}(\eta)(x) = \eta(f(x))$. Moreover the triple (f, X, τ_μ) is called a *relative semi-dynamical system* or briefly an *RSD-system*.

III. Minimality on RSD-Systems

Given an $\alpha \in [0, 1]$, let (f, X, τ_μ) be an RSD-system. for any $x \in X$, the set

$$O^f_\mu(x) := \{f^n(x) \ : \ n = 0, 1, 2, \cdots\}$$

is called an *orbit* of x. In this section the so called μ_α-minimal RSD-system (f, X, τ_μ) is defined so that any real value function g on μ_α is determined by its values on the μ_α-minimal.

Definition 1: An RSD-system (f, X, τ_μ) is called a *minimal relative semi-dynamical system* on μ_α or briefly μ_α*-minimal* if:

i) $f(\mu_\alpha) \subset \mu_\alpha$,
ii) For all $x \in \mu_\alpha$, the set $\{f^n(x) : n = 0, 1, 2, ...\}$, $O^f_\mu(x)$, is a dense subset of μ_α, where the topology of μ_α is $(\tau_\mu)_\alpha$.

The following theorem shows how to identify any (f, X, τ_μ) as a μ_α-minimal.

Theorem 2: Given an $\alpha \in [0, 1]$, let (f, X, τ_μ) be an RSD-system. The following statements are equivalent.

i) f is μ_α-minimal.
ii) Let $f(\mu_\alpha) \subset \mu_\alpha$. If C is a closed subset of the topological space $(\mu_\alpha, (\tau_\mu)_\alpha)$ such that $f(C) \subset C$, then $C = \mu_\alpha$ or $C = \emptyset$.
iii) Let $f(\mu_\alpha) \subset \mu_\alpha$. If $O \in (\tau_\mu)_\alpha$ is a nonempty open set, then $\mu_\alpha = \cup^0_{n=-\infty} f^n(O)$, where $f^0(O) = O$.

Proof:
i)$\Longrightarrow$ii)
Let $f(\mu_\alpha) \subset \mu_\alpha$. Suppose that C is a nonempty closed subset of μ_α and $f(C) \subset C$. There exists $x \in C$ such that

$$\mu_\alpha = \overline{\{f^n(x) \ : \ n = 1, 2, \cdots\}} \subset C.$$

Therefore $C = \mu_\alpha$.
ii)$\Longrightarrow$iii)
If $f(\mu_\alpha) \subset \mu_\alpha$ and $O \in (\tau_\mu)_\alpha$, then there exists $\lambda \in \tau_\mu$ such that $O = \lambda_\alpha$. A straightforward calculation shows that $f^{-1}(O) = (f^{-1}(\lambda))_\alpha$. Therefore $C = \mu_\alpha - \cup^0_{n=-\infty} f^n(O)$ is a closed subset of μ_α. Moreover $f(C) \subset C$ and C is a nonempty set.
iii)$\Longrightarrow$i)
Suppose that $x \in \mu_\alpha$ is given and O is a nonempty open subset of μ_α; then $x \in f^{-n}(O)$ for some $n \in \{0\} \cup N$. Therefore $f^n(x) \in O$. Thus we can see that

$$\overline{\{f^n(x) \ : \ n = 1, 2, \cdots\}} = \mu_\alpha.$$

□

We recall from [6] that a subset D of X is called an invariant for the RSD-system (f, X, τ_μ) if $f(D) \subset D$. An invariant subset D of μ_α is called μ_α-minimal if $f : D \longrightarrow D$ is D-minimal. Given an RSD-system (f, X, τ_μ), the necessary conditions for the the existence of a μ_α-minimal set are presented in the following theorem.

Theorem 3: Let (f, X, τ_μ) be an RSD-system. Also let $\alpha \in [0, 1]$ be given such that $f(\mu_\alpha) \subset \mu_\alpha$ and $\mu_\alpha \neq \emptyset$. Then f has a μ_α-minimal set.

Proof: Let M denote the set of all nonempty closed invariant subsets of f; then Cantor's intersection property and Zorn's lemma imply that M as an ordered set under inclusion has a minimal set which is the required $\mu_\alpha-$minimal set. □

Theorem 4: let $f : X \longrightarrow X$ be a $\mu_\alpha-$minimal and $g : \mu_\alpha \longrightarrow \mathbf{R}$ be a continuous function such that $gof = g$, where the topology of μ_α is $(\tau_\mu)_\alpha$. Then g must be constant.

Proof: The condition $gof = g$ implies that $gof^n = g$ for all $n \in \mathbf{N}$. Thus for a given $x \in \mu_\alpha$, we have

$$g(O^f_\mu(x)) = \{g(x)\},$$

where $O^f_\mu(x)$ is the orbit of x. Moreover, the continuity of g implies that

$$g(\mu_\alpha) = g(\overline{O^f_\mu(x)}) = \overline{g(O^f_\mu(x))} = \{g(x)\}.$$

□

IV. Transitivity on RSD-Systems

The goal of this section is to investigate those RSD-systems (f, X, τ_μ) which are invariant only on the dense subsets of μ_α. More precisely,

Definition 5: Any RSD-system f is called μ_α-transitive if there exists an orbit in X, $O^f_\mu(x)$, such that $\mu_\alpha \cap O^f_\mu(x)$ is a dense subset of μ_α. Such an orbit is called μ_α-transitive.

Theorem 6: Let f be a (μ, μ)-homeomorphism. Then the following statements are equivalent:

i) f is μ_α-transitive.
ii) If U is a nonempty open subset of μ_α, where $f(U) = U$, then U is dense in μ_α.
iii) Let V and W be two nonempty open sets in μ_α. Then there exists an $n \in \mathbf{Z}$ such that $f^n(V) \cap W \neq \emptyset$.
iv) If $O_\alpha(x) = O^f_\mu(x) \cap \mu_\alpha$, then the set

$$\{x \in X \ : \ O_\alpha(x) \text{ is dense in } \mu_\alpha\} \cap \mu_\alpha$$

can be written as an intersection of countable collection of the open dense subsets of μ_α.

Proof:
i)$\Longrightarrow$ii)
f is μ_α-transitive so there exists $x \in X$ such that

$$O_\alpha(x) = \{x, f(x), f^2(x), \cdots, f^n(x), \cdots\} \cap \mu_\alpha$$

is dense in μ_α. Thus $f^n(x) \in U$ for some $n \in \mathbf{Z}$. Let $f^k(x) \in O_\alpha(x)$, then $f^k(x) \in f^k(f^{-n}(U)) \subset U$. So $O_\alpha(x) \subset U$. Therefore, U is dense in μ_α.
ii)$\Longrightarrow$iii)
The set $U = \bigcup_{n\in\mathbf{Z}} f^n(V)$ is an open subset of μ_α and $f(U) = U$. So U is dense in μ_α. Thus $U \cap W \neq \emptyset$. Hence there exists an $n \in \mathbf{Z}$ such that $f^n(V) \cap W \neq \emptyset$.
iii)$\Longrightarrow$iv)

Let $\{D_n\}_{n\in\mathbf{N}}$ be a family of dense subsets in μ_α. We can see that:

$$O_\alpha(x)\bigcap\mu_\alpha=\bigcap_{n=0}^{\infty}\bigcap_{k=1}^{\infty}\Big(\bigcup_{l=-\infty}^{\infty}f^l\big(B(D_n,1/k)\big)\Big),$$

where $B(D_n,1/k)=\big\{y\in X\cap\mu_\alpha : d(y,D_n)<\frac{1}{k}\big\}$.
iv)$\Longrightarrow$i)
$\{x\in X : O_\alpha(x)$ is dense in $\mu_\alpha\}\cap\mu_\alpha$ is a dense subset of μ_α so this set is nonempty. Therefore, there exists $O_\alpha(x)$ such that $O_\alpha(x)$ is dense in μ_α. □

V. Relative Topological Entropy

In this section we want to compare the levels of complexity of any given RSD-system (f,X,τ_μ) from the viewpoint of different observers. For this purpose, here the α-level relative topological entropy is introduced and a method is presented for calculating the entropy index.

Suppose that τ_μ is a μ-relative topology on X. Let $\alpha\in(0,1)$ be given such that (X,μ) is a compact (α,μ)-Hausdorff space [6]. Moreover, let

$$\Theta=\big\{\lambda_\alpha^i\ :\ \lambda^i\in\tau_\mu,\ i=1,\cdots,n\big\}$$

be an open cover for μ_α. Regarding the above notations, the open cover Σ is called a *subcover* of Θ if $\Sigma\subset\Theta$.

Definition 7: The relative topological entropy of the open cover Θ with the level α is defined to be $H_\alpha(\Theta):=\log N(\Theta)$, where $N(\Theta)$ is the smallest number of open sets which can be used in any subcover of Θ.
Let $\big\{\Theta^r=\{(\lambda_r^1)_\alpha,\cdots,(\lambda_r^{Nr})_\alpha\} : r=1,\cdots,k\big\}$ be a family of open covers for μ_α. An *α-refinement* of this family is the open cover $\bigvee_{r=1}^k\Theta^r$, which is defined by:

$$\Big\{(\lambda_1^{i_1})_\alpha\cap(\lambda_2^{i_2})_\alpha\cap\cdots\cap(\lambda_k^{i_n})_\alpha : (\lambda_j^{i_j})_\alpha\in\Theta^j, j\le k\Big\}$$

Lemma 8: Let X be a compact (α,μ)-Hausdorff space and $f : X\longrightarrow X$ be an RSD-system. Moreover, let $\Theta=\{\lambda_\alpha^1,\cdots,\lambda_\alpha^n\}$ be an open cover for μ_α. Then

$$f^{-1}\Theta=\big\{(\mu\cap f^{-1}\lambda^1)_\alpha,\cdots,(\mu\cap f^{-1}\lambda^n)_\alpha\big\}$$

would be an open cover for $(\mu\cap f^{-1}\mu)_\alpha$.

Proof: Since Θ is an open cover for μ_α, we have $\mu_\alpha\subset\bigcup_{i=1}^n\lambda_\alpha^i$. Now if $x\in((f^{-1}\mu)\cap\mu)_\alpha$, then $\min\{\mu(f(x)),\mu(x)\}>\alpha$. So $f(x)\in\mu_\alpha$. Hence there exists $1\le m\le n$ such that $f(x)\in\lambda_\alpha^m$. Thus $\lambda^m(f(x))>\alpha$. So $(f^{-1}\lambda^m)(x)>\alpha$. Moreover we have $\mu(x)>\alpha$. Therefore $x\in(\mu\cap f^{-1}\lambda^m)_\alpha$. Hence $f^{-1}(\Theta)$ is a cover for $(\mu\cap f^{-1}\mu)_\alpha$. Since f is (μ,μ)-continuous, $\mu\cap f^{-1}\lambda^i, i=1,\cdots,n$ are open sets in τ_μ. So $f^{-1}(\Theta)$ is an open cover for $(\mu\cap f^{-1}(\mu)_\alpha$. □

Theorem 9: Let Θ and Σ be two finite open covers for μ_α. Then the following inequality holds.

$$H_\alpha(\Theta\vee\Sigma)\le H_\alpha(\Theta)+H_\alpha(\Sigma).$$

Proof: Let

$$\Theta\supseteq\Theta'=\{\lambda_\alpha^1,\cdots,\lambda_\alpha^n\}$$

and

$$\Sigma\supseteq\Sigma'=\{\gamma_\alpha^1,\cdots,\gamma_\alpha^m\}$$

be respectively the subcovers of Θ and Σ such that $H_\alpha(\Theta)=\log n$ and $H_\alpha(\Sigma)=\log m$. Now $\Theta'\vee\Sigma'$ is a subcover of $\Theta\vee\Sigma$. So

$$H_\alpha(\Theta\vee\Sigma)\le\log(nm)=\log n+\log m=H_\alpha(\Theta)+H_\alpha(\Sigma).$$

□

Theorem 10: Let $f : X\longrightarrow X$ be an RSD-system and Θ be a finite open cover for μ_α. The following inequality holds:

$$H_\alpha(\Theta)\ge H_\alpha(f^{-1}\Theta).$$

Proof: Let $\Theta'=\{\lambda_\alpha^1,\cdots,\lambda_\alpha^n\}\subseteq\Theta$ be an open over such that $H_\alpha(\Theta)=\log n$. Then

$$f^{-1}\Theta'=\big\{(\mu\cap f^{-1}\lambda^1)_\alpha,\cdots,(\mu\cap f^{-1}\lambda^n)_\alpha\big\}$$

is an open cover for $(\mu\cap f^{-1}\mu)_\alpha$. Thus

$$N(f^{-1}\Theta)\le N(f^{-1}\Theta')\le n=N(\Theta).$$

Therefore, $H_\alpha(\Theta)\ge H_\alpha(f^{-1}\Theta)$. □

Theorem 11: Given a compact (α,μ)-Hausdorff space X, let $f : X\longrightarrow X$ be an RSD-system. Moreover, let Θ be an open cover for μ_α, where $\alpha\in(0,1)$. The following limit exists.

$$\lim_{n\to\infty}\frac{1}{n}H_\alpha\big(\bigvee_{i=0}^{n-1}f^{-i}(\Theta)\big)$$

Proof: Let $x_n:=H_\alpha\big(\bigvee_{i=0}^{n-1}f^{-i}(\Theta)\big)$. Then for all $n,m\in\mathbf{N}$ we have:

$$\begin{aligned}x_{n+m}&=H_\alpha\Big(\bigvee_{i=0}^{n+m-1}f^{-i}(\Theta)\Big)\\&\le H_\alpha\Big(\bigvee_{i=0}^{n-1}f^{-i}(\Theta)\Big)+H_\alpha\Big(f^{-n}\big(\bigvee_{j=0}^{m-1}f^{-j}(\Theta)\big)\Big)\\&\le H_\alpha\Big(\bigvee_{i=0}^{n-1}f^{-i}(\Theta)\Big)+H_\alpha\Big(\bigvee_{j=0}^{m-1}f^{-j}(\Theta)\Big)\\&=x_n+x_m.\end{aligned}$$

Thus $\lim_{n\to\infty}\frac{x_n}{n}$ exists, since $\{x_n\}_{n\in N}$ is a sub-additive sequence. □

Regarding the above theorem, the α-level relative topological entropy for the RSD-system $f : X\longrightarrow X$ associated to the open cover Θ is defined by:

$$h_\alpha(f,\Theta)=\lim_{n\to\infty}\frac{1}{n}H_\alpha(\bigvee_{i=0}^{n-1}f^{-i}\Theta)\cdot$$

where X is a compact (α,μ)-Hausdorff space.

Definition 12: The α-level relative topological entropy for f is defined by:

$$h_\alpha(f)=\sup\{h_\alpha(f,\Theta) : \Theta \text{ is a finite cover of } \mu_\alpha\}$$

Referring to [6], two RSD-systems (f,X,τ_μ) and (g,X,τ_μ) are called μ-conjugate if there exists a μ-homeomorphism $\varphi : X\longrightarrow X$ such that $\varphi of=go\varphi$. The next theorem shows that the relative topological entropy is invariant under μ-conjugate relation.

Theorem 13: If $f : X\longrightarrow X$ and $g : X\longrightarrow X$ are μ-conjugate then $h_\alpha(f)=h_\alpha(g)$ for all $\alpha\in(0,1)$.

Proof: Because of the μ-conjugate relation, there exists a μ-homeomorphism $\varphi : X \longrightarrow X$ such that $\varphi \circ f = g \circ \varphi$. Now, let $\alpha \in (0,1)$ and Θ be a finite open cover for μ_α, then:

$$\begin{aligned} h_\alpha(g,\Theta) &= \lim_{n\to\infty} \frac{1}{n} H_\alpha\Big(\bigvee_{i=0}^{n-1} g^{-i}(\Theta)\Big) \\ &= \lim_{n\to\infty} \frac{1}{n} H_\alpha\Big(\bigvee_{i=0}^{n-1} g^{-i}\big(\varphi^{-1}(\Theta)\big)\Big) \\ &= h_\alpha(f,\varphi^{-1}\Theta). \end{aligned}$$

So $h_\alpha(g) = h_\alpha(f)$ Since φ is a μ-homeomorphism. □

VI. COMPUTATIONAL EXAMPLE

Recently progress has been made in the development of algorithms for optimizing polynomials on an arbitrary semi-algebraic set, which is the set defined by Boolean combination of polynomial equations and inequalities.. The main idea being stressed is that reducing problem to an easier problem involving semi-definite programming. Lesserre, in [5], describes an extension of the method for minimizing a polynomial on an arbitrary semi-algebraic set. However, the study of semi-algebraic sets is based mainly on the slicing technique, which makes it possible to decompose them into the finite number of subsets semi-algebraically homomorphic to an open hypercube. This composition allows us to investigate semi-algebraically connected components for every semi-algebraic set with finite cover. That is just one of the reasons to care the notion of connectedness and compactness for polynomial function spaces.

Our approach to the above problem is to develop the μ-relative semi-dynamical system over one variable polynomial function space $\mathbf{R}[x]$ based on the derivative operator. we are going to use orbits as the μ-open sets to decompose $\mathbf{R}[x]$. This topic may be interesting for further independent research subject on semi-definite programming. But here, we have just looked over it as an example.

Let $X = \mathbf{R}[x]$ and define $\mu : X \longrightarrow [0,1]$ by:

$$\mu(f) = \begin{cases} \frac{1}{\deg(f)} & \text{if } \deg(f) \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Also let $\lambda^i : X \longrightarrow [0,1]$ be defined by:

$$\lambda^i(f) = \begin{cases} \frac{1}{i} & \text{if } \deg(f) = i \\ 0 & \text{otherwise} \end{cases}$$

Since $\lambda^i \cap \lambda^j = \chi_\emptyset$ for $i \neq j$ and $\bigcup_{i\in\mathbf{N}} \lambda^i = \mu$, we can consider τ_μ as the μ-topology generated by $\{\lambda^i : i \in \mathbf{N}\}$. We also have the following results:
$\alpha = 1 \Rightarrow \mu_1 = \chi_\emptyset$
$\alpha = 0 \Rightarrow \mu_0 = \{f \in \mathbf{R}[x] : \deg(f) \geq 1\}$
If $\alpha \in (0,1)$ then

$$\mu_\alpha = \begin{cases} \{f \in \mathbf{R}[x] : \deg(f) \leq [\frac{1}{\alpha}]\} & \text{if } \frac{1}{\alpha} \notin \mathbf{N} \\ \{f \in \mathbf{R}[x] : \deg(f) \leq [\frac{1}{\alpha}] - 1\} & \text{if } \frac{1}{\alpha} \in \mathbf{N} \end{cases}$$

Suppose that $F : X \longrightarrow X$ is the derivation map, i.e. $F(f) = f'$. Then F is (μ,μ)-continuous since:

$$\begin{aligned} F^{-1}(\lambda^i)(f) &= \lambda^i(F(f)) \\ &= \lambda^i(f') = \begin{cases} \frac{1}{i} & \text{if } \deg(f') = i \\ 0 & \text{otherwise} \end{cases} \\ (\mu \cap F^{-1}(\lambda^i))(f) &= \min\{\mu(f), F^{-1}(\lambda^i)(f)\} \\ &= \begin{cases} \frac{1}{i+1} & \text{if } \deg(f) = i+1 \\ 0 & \text{otherwise} \end{cases} \\ &= \lambda^{i+1}(f). \end{aligned}$$

So $\mu \cap F^{-1}(\lambda^i) = \lambda^{i+1} \in \tau_\mu$. Thus (F, X, τ_μ) is a relative semi-dynamical system.
Now for any $k \in \mathbf{Z}$ we consider the orbit of the element $f(x) = x^k$ in X as follows:

$$\begin{aligned} O(f) &= \{F^n(f) : n \in \{0,1,2,\ldots\}\} \\ &= \{x^k, kx^{k-1}, k(k-1)x^{k-2}, \ldots, k!, 0\} \end{aligned}$$

The structure of μ-topology on X implies that $O(f)$ is dense in μ_α. Moreover, μ_α is compact and (F, X, τ_μ) is μ_α-minimal for all $\alpha \in (0,1)$. In fact, the derivation map F is μ_α-transitive when $\alpha \neq 0$.
It is easy to see that $h_\alpha(F) = 0$ for all $\alpha \neq 0$, but the presented computational method seems complicated for calculating $h_0(F)$. So an alternative method for calculation of the relative topological entropy is needed; that would be our next research goal.

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