A General Optimization Scheme of Line-Source Pattern Synthesis and Its Applications to Monopulse Tracking Array Antennas

Keum Sook Ha and Eui Joon Park

***Abstract*—**An iterative nonlinear inversion method in the line source pattern synthesis problem is newly presented for extracting the source distribution functions corresponded to the arbitrarily prescribed sum and difference pattern sidelobe levels. In this method, the generalized Fourier transform pair is first formulated which enables the desired patterns and the distributions to be simultaneously extracted by the optimum perturbation of pattern null positions. A significant result is that the difference patterns with the individually specified sidelobe levels are optimally synthesized by appropriately updating the Taylor line source sum pattern formula. Furthermore the method can be extended to the synthesis of the discrete linear array antennas. The scheme for extracting the excitation current weights of discrete array is also based on the perturbations of pattern nulls which are represented by the complex root locations on Schelkunoff's unit circle. Numerical examples show the flexibility and effectiveness of the proposed procedures.

***Keywords*—**Array antennas, line source, null positions, sum and difference patterns

I. Introduction

IN the designing monopulse tracking array antennas, the sum pattern and the difference pattern have to be synthesized. Sum pattern must have high gain and low sidelobe levels (SLLs) for long range target detection and interference mitigation. The difference pattern is also required for fine angular tracking, thus should have a large slope at boresight for accurate angle tracking. A variety of methods have been developed for the synthesis of the patterns [1]–[4]. A review of them reveals that the formulas of Taylor sum pattern [5] and Bayliss difference pattern [6] have been basically used for these antenna performance characteristics, evoking the synthesis problem on the continuously distributed line source antennas with arbitrary patterns. Since the patterns are expressed by an integral equation in which the integrand stands for the source distribution, the efficient inverse transform is crucial to recovering the distributed source potential corresponding to a desired pattern. Generally, the analytical solution of the potential is not easy to find, especially in the case of the patterns with arbitrary SLLs. So the previous numerical inverse transform methods have required an iterative sampling method, and/or the overall optimization routines for fitting to the objective pattern window [7].

In order to overcome the above specific and complex methods, this paper presents a generalized design scheme for both sum and difference patterns with arbitrary SLLs. In this scheme, the relationship between the patterns and the source distribution functions is analytically established by formulating an generalized Fourier transform pair, and then the source distribution is numerically adapted to the specified patterns by using an appropriate perturbation of null positions inherent to the pair. Especially, the difference patterns with the desired individual SLLs, which have more rapid sidelobe decay rates than Bayliss difference pattern, are optimally synthesized by appropriately modifying the Taylor line source sum pattern formula. Furthermore the concept of null perturbation can be applied to the discrete array antennas with the relative excitation current weights of antenna elements. The procedure is based on the optimum perturbation of null points which are inherent to the Schelkunoff's polynomial [8] represented for the pattern array factor. Accordingly, opposite to the conventional method [8], [9] in which the excitation weights are directly optimized, this method is advantageous in that the patterns with the desired individual SLLs and the corresponding excitation weights are easily synthesized by the control of null positions. It is showed that two types of difference patterns can be synthesized as imposing the different initial values of null positions in the optimization process. Some numerical results show the validity and usefulness of the

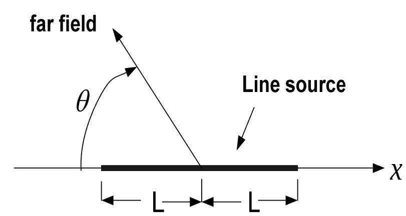

Fig. 1 Geometry of line source

Manuscript received December 17, 2007: Revised version received March 16, 2008. This paper was supported by Research Fund, Kumoh National Institute of Technology.

Keum Sook Ha is with the Division of Computer Information and Electronics, Kumi College, Gyungbook, KOREA (corresponding author to provide e-mail: ksha@ kumi.ac.kr).

Eui Joon Park is with the School of Electronic Engineering, Kumoh National Institute of Technology, Gyungbook, KOREA (e-mail: ejpark@kumoh.ac.kr).

proposed procedures.

II. ALGORITHMS FOR PATTERN SYNTHESIS

Radiation from a continuous line source is assumed and the appropriate geometry is showed in Fig. 1 where $2L$ is the line source length and θ is the angle measured from the endfire direction to a far field point. Let λ be the wavelength. Defining $p = \pi x / L$ and $z = (2L/\lambda)\cos\theta$, the continuous line source have a distribution function $g(p)$. Then the related space factor pattern $F(z)$ is given by the following Fourier transform relationship [5], [10].

$$F(z) = \int_{-\pi}^{\pi} g(p)\exp(-jpz)\,dp \qquad (1)$$

Expanding $g(p)$ as $g(p) = \sum_{n=0}^{N}(a_n \cos(np) + b_n \sin(np))$, (1) can be rewritten by the restricted set of sampling functions as follows.

$$F(z) = \sum_{n=0}^{N} \pi \cdot a_n \left(sa(\pi(z-n)) + sa(\pi(z+n))\right) - j\sum_{n=1}^{N} \pi \cdot b_n \left(sa(\pi(z-n)) - sa(\pi(z+n))\right) \qquad (2)$$

where $F(n) = \pi(a_n - jb_n)$ and $F(0) = 2\pi a_0$. The real and imaginary parts are even and odd in the z domain, respectively. The problem we now are considering is how to take full advantage of $F(z)$ for patterns with arbitrary lobe heights for cases not only sum pattern but also difference pattern.

For generality, we first introduce the Taylor line source sum pattern [4] compatible with the real part of (2), and then modify it in order to activate the case of $b_n \neq 0$, based on the Orchard's ripple making theory [9], [11]. We choose to keep all null positions at the integer location for $|z| \geq \bar{n}$, and to move those for $|z| \leq \bar{n} - 1 = N$ near the locations z_n that would produce the nearly constant sidelobes near the main beam. The result is as follows:

$$F(z) = A \cdot Sa(\pi z) \cdot \prod_{n=1, n \supseteq m}^{N} \left(\frac{n^2}{n^2 - z^2}\right)\left(1 - \frac{z}{z_n - jv_m}\right) \cdot \left(1 + \frac{z}{z_n + jv_m}\right) \qquad (3)$$

whose real and imaginary parts are even and odd in the entire z domain. That is, $F(z) = F_e(z) + F_o(z)$ where $F_o(0) = 0$. So, the framework of (3) is consistent with that of (2). Here, z_n is the null or dip position to be perturbed in the z domain. v_m causes a dip in position. z_n and v_m are optimally perturbed for

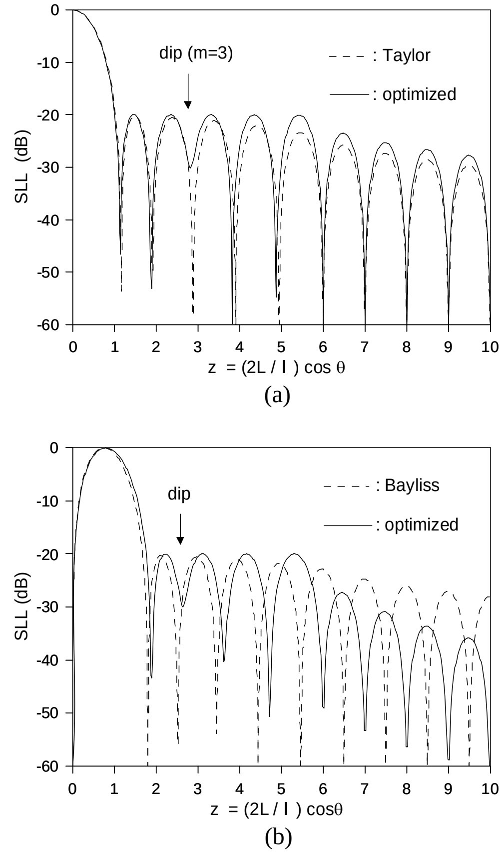

Fig. 2 Conventional sum and difference patterns compared by optimized patterns. (a) sum pattern, (b) difference pattern

magnitude of $F(z)$, which has the individually prescribed N lobe heights with the furthest lobes exponentially decaying in level according to the coefficient A.

For example, the dotted line in Fig. 2 (a) is Taylor sum pattern with 20 dB sidelobe peaks in the case that $\bar{n} = N + 1 = 6$.

For a difference pattern, it is required that $F(0) = 0$ and $F(z)$ have no deviation against the regular pattern at $z \neq 0$. Thus, $F_e(z)$ must become odd for $F_e(0) = 0$. Once the alteration is done, $F_o(z)$ must become even and then the position of two deviations must be interchanged to maintain the framework, which is consistent with the need to have $g(p)$ be real. Letting the altered $F(z)$ be $F^A(z) = F_o^A(z) + jF_e^A(z)$ and defining $F'(z) = jF^A(z)$, $F'(z) = -F_e^A(z) + jF_o^A(z)$ yields. After some algebra on the even $\leftrightarrow$ odd alteration in (2), the following relationships are derived:

$$F_e^A(z) = F_o(z) + D_1 \qquad (4)$$

$$F_o^A(z) = F_e(z) - D_0 - D_2 \qquad (5)$$

where

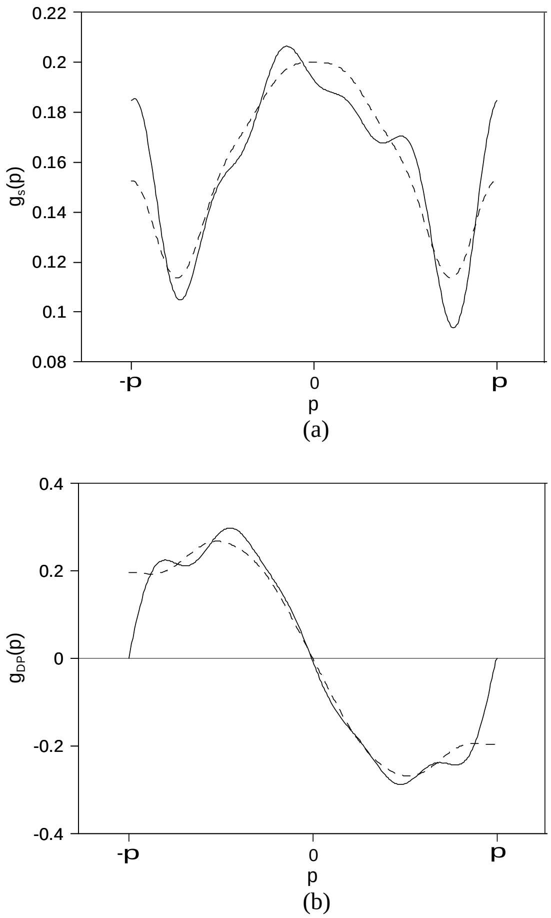

Fig. 3 Distribution functions corresponding to Fig.2. (a) for sum pattern, (b) for difference pattern

$$D_0 = 2\pi a_0 \cdot Sa(\pi z) \tag{6}$$

$$D_1 = 2\pi \sum_{n=1}^{N} b_n \cdot Sa(\pi(z-n)) \tag{7}$$

$$D_2 = 2\pi \sum_{n=1}^{N} a_n \cdot Sa(\pi(z+n)) \tag{8}$$

In (5), the subtraction by D_0 removes the numerical problem found in the constant A in the case of difference pattern. Accordingly, when b_n^A and a_n^A are the updated b_n and a_n, the following relationships are obtained for sum and difference patterns, respectively.

$$F_s(z) \leftrightarrow g_s(p) = \sum_{n=0}^{N} (a_n \cos(np) + b_n \sin(np)), \tag{9}$$

$$F_{DP}(z) \leftrightarrow g_{DP}(p) = \sum_{n=1}^{N} (b_n^A \cos(np) + a_n^A \sin(np)) \tag{10}$$

For example, the dotted line in Fig. 2 (b) is Bayliss difference pattern with 20 dB sidelobe peaks in the case that $\bar{n} = N + 1 = 6$. As expected, the nulls for $|z| \geq \bar{n}$ are located at half points between the integer locations.

Table I Extracted coefficients for distribution functions of the optimized patterns (Fig. 2)

n	Sum pattern		Difference pattern	
	a_n $(\times 10^{-3})$	b_n $(\times 10^{-3})$	a_n^A $(\times 10^{-3})$	b_n^A $(\times 10^{-3})$
0	159.210			
1	32.497	-0.927	-293.617	3.576
2	13.417	-1.360	17.996	-0.753
3	-16.943	-9.602	-27.983	10.526
4	16.136	1.592	28.308	1.301
5	-11.434	-0.666	-21.981	-0.583

If the antenna elements are uniformly and linearly spaced, the far field patterns are represented by the following array factor instead of space factor.

$$AF(w) = \sum_{m=0}^{M} I_m w^m = \prod_{m=1}^{M} (w - w_m) \tag{11}$$

in which I_m is the relative excitation current weights of the m'*th* element and $w = \exp(j\varphi)$, with $\varphi = (2\pi d / \lambda)\cos\theta$, where d is the interelement spacing. If the roots w_m are effectively derived from (11), current weights are automatically calculated by comparing coefficients. The roots are expressed as $w_m = \exp(j2\pi z_m / M)$ in which z_m are the null positions to be optimally perturbed. In this paper, opposite to the direct optimization of weights [8], [12], the nulls z_m are optimized.

III. OPTIMIZATION AND VALIDATIONS

Letting the general notation for $F(z)$ and $AF(z)$ be F_G, the error function $E(\mathbf{X})$ for null positions adapted to the prescribed *m'th* sidelobe peak values is defined by the least square method as follows and the iteration for the minimization with stopping condition $E \leq \varepsilon$ goes along with the conjugate gradient method.

$$E(\mathbf{X}) = \sum_{\substack{p,d=1 \\ p \supseteq d}}^{N} ((\ln(F_G^p(\mathbf{X}) / S^p))^2 + (\ln(F_G^d(\mathbf{X}) / S^d))^2) \tag{12}$$

where $\mathbf{X} = (\mathbf{Z}, \mathbf{V})$ in which $\mathbf{Z} = [z_1, \cdot\cdot, z_N]$ and $\mathbf{V} = [v_1, \cdot\cdot, v_N]$. Minimization of E is achieved by updating $\mathbf{X}$ to reduce the logarithmic difference between the performances F_G during the updating process and the specifications S which represent the prescribed objective *p'th* peak value and *d'th* dip value.

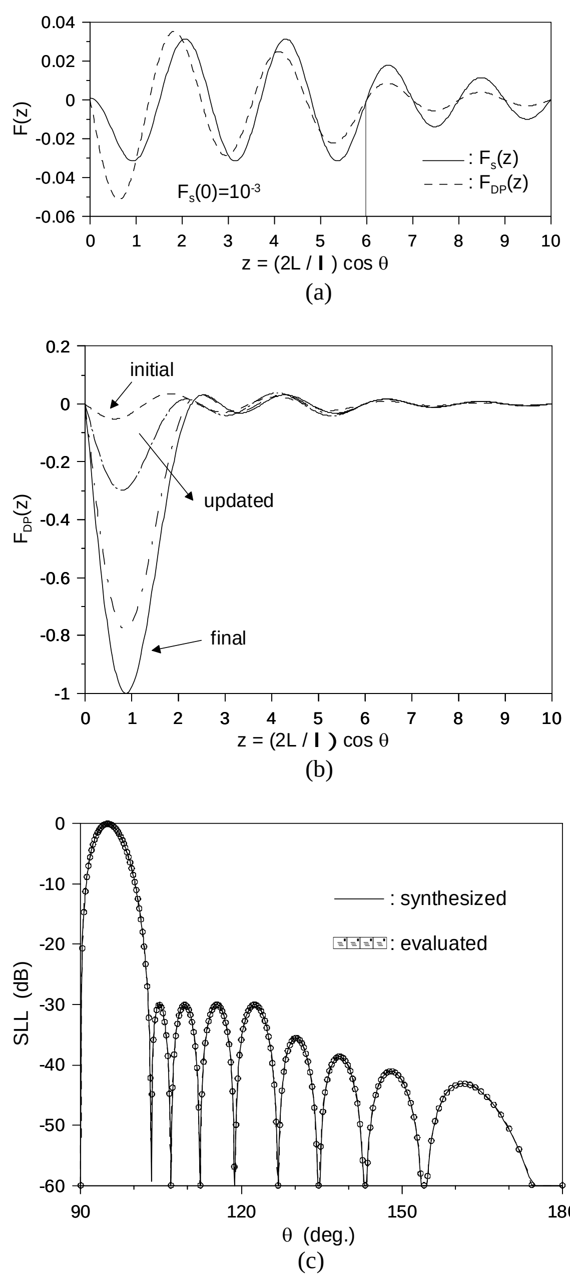

Fig. 4 Synthesis procedures for difference patterns. (a) initial pattern, (b) update process, (c) evaluated pattern.

The summary of the algorithm mentioned above is as follows.

1) Determine the desired sidelobe number N to be controlled, individual sidelobe levels and dip values.
2) Synthesize the magnitude of pattern satisfying the prescribed goal by optimally perturbing the z_n and v_m from (3).
3) Calculate a_n and b_n from the relationships that $F(n) = \pi(a_n - jb_n)$, $F(0) = 2\pi a_0$ and then calculate $g_s(p)$ from (9) .

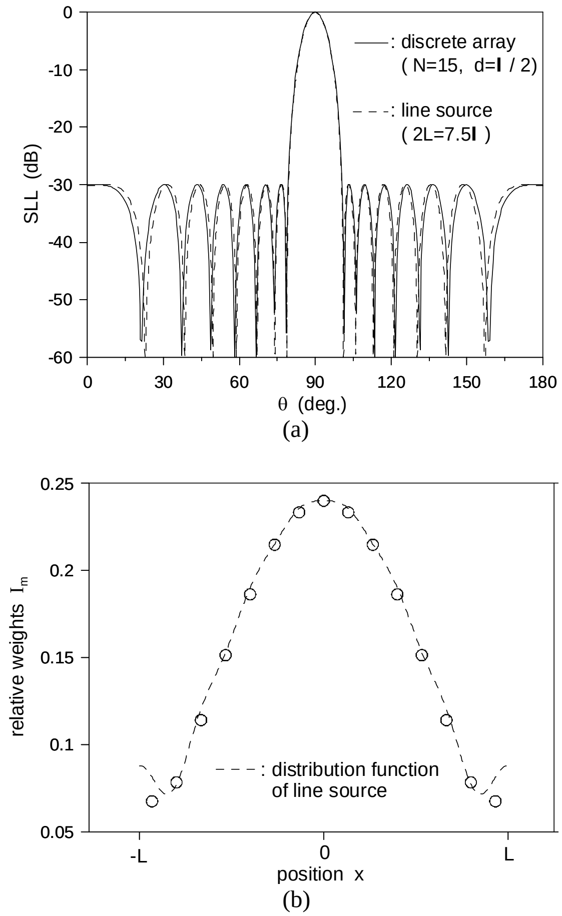

Fig. 5 Examples of synthesized sum patterns. (a) optimized patterns. (b) the corresponding distribution and weights

4) For difference patterns, perform the step 2)-3) by setting $A \to \delta$ (for instance, $\delta = 10^{-3}$) for the convergence consideration. In this process, $F_e(z)$ and $F_o(z)$ are replaced by $F_o^A(z)$ and $F_e^A(z)$, respectively, in the manner of (4)-(8) at each iteration. When the goal set by step 1) is satisfied, the final updated a_n^A and b_n^A are derived and then $g_{DP}(p)$ is calculated from (10).

The solid line of Fig. 2 (a) is the pattern with 30 dB dip ($m = 3$), which is the result performed by the optimization process mentioned above. The solid line of Fig. 2 (b) is the pattern with 30 dB dip ($m = 3$), which is the result performed by (9) and (10) through the optimization process. This figure shows that the proposed difference pattern has nulls at the integer locations for $|z| \geq \bar{n}$, and the sidelobe decay rate is faster than the conventional difference pattern. Fig. 3 shows the corresponding source distribution functions calculated by (9) and (10). From these figures, the null positions of patterns synthesized from (10) are different from those of optimized Bayliss pattern as mentioned above, showing more rapid decay of SLLs in the range $n \geq 6$. And $g_{DP}(p) = 0$ at the edge of

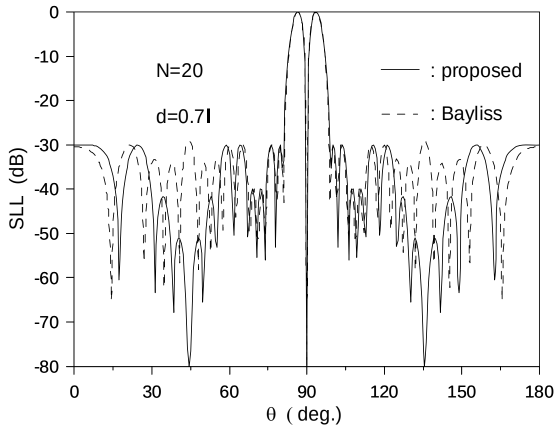

Fig. 6 Optimized difference patterns in the case of discrete array

Table II Optimized nulls and relative weights of Fig. 6

		Bayliss	Proposed
Optimized Null Positions	z_1	-9.3836	-10.0046
	z_2	-8.4927	-9.0018
	z_3	-7.5444	-8.0408
	z_4	-6.4339	-6.6520
	z_5	-5.2352	-5.3745
	z_6	-4.5321	-4.6455
	z_7	-3.8540	-3.9314
	z_8	-2.8285	-2.8925
	z_9	-2.2147	-2.2573
Extracted Current Weights	I_1	0.2658	0.1830
	I_2	0.3724	0.3904
	I_3	0.6085	0.5423
	I_4	0.8400	0.8194
	I_5	0.9700	0.9640
	I_6	0.9936	0.9857
	I_7	1.0000	1.0000
	I_8	0.8271	0.8431
	I_9	0.5581	0.5518
	I_{10}	0.1842	0.2024

line source, satisfying the physical realization. Table I shows the extracted Fourier coefficients for distribution functions of the optimized sum and difference pattern of Fig. 2.

The solid line of Fig. 4 (a) is the pattern of $F_s(z)$ with -30 dB SLLs in the case that $b_n = 0$, $A \to 10^{-3}$, and $\bar{n} - 1 = 5$. From the optimization process, the pattern was calculated for the synthesis of $F_{DP}(z)$. The dotted line shows the initial $F_{DP}(z)$ to be updated by (4)-(8). The update process from this initial pattern is showed in Fig. 4 (b). In Fig. 4 (c), the synthesized pattern is compared with the evaluated pattern which is calculated by the extracted a_n^A of (10), showing the good consistency in the visible region in the case of $2L = 10\lambda$.

Fig. 5 (a) shows the optimized sum patterns of continuous line source with $\bar{n} = 8$, and discrete array antenna with $d = \lambda / 2$ and $M = 15$. The corresponding distribution function and weights are shown in Fig. 4 (b). In the case of discrete array, the roots w_m are effectively optimized from (12) and then the corresponding current weights are automatically calculated by comparing coefficients from (11). The roots are expressed as $w_m = \exp(j2\pi z_m / M)$ in which the null position z_m are optimally perturbed. Each initial values of nulls are simply set as $\mathbf{Z}^0 = [1,2,\cdots,7]$. Since $M = 15$ and $d = \lambda / 2$, the number of nulls are M-1=14 in the visible region.

Fig. 6 shows the optimized difference patterns with $N = 20$ and $d = 0.7\lambda$ in the case of the discrete arrays which have prescribed SLLs of -30, -40, -40, -30, and -30 dB near the main beam. It is showed that two types of difference patterns can be synthesized as imposing the different initial values of null positions in the optimization process. The initial values of nulls are given by nulls extracted by the optimizations for continuous line source with same prescribed SLLs. In Table II, The corresponding weights are shown with the optimized nulls. When the values are substituted in (11), the resulting patterns are consistent with Fig. 6. Since $d = 0.7\lambda$, the w-excursion is 1.4 revolutions around the Schelkunoff unit circle. Fig. 6 also shows that the SLLs of the proposed difference pattern have more rapid decay rate than the optimized Bayliss pattern.

IV. CONCLUSION

A general iterative scheme was newly presented to synthesize simultaneously both the sum and difference patterns with arbitrary sidelobe levels. The scheme is based on the generalized Fourier transform pair which enables the pattern nulls to be easily optimized for the prescribed patterns. The line source difference patterns are synthesized from well-known sum pattern formula by the developed Fourier transform pair. The concept of these synthesis procedures can be applied to extracting excitation weights for the discrete array antenna pattern with arbitrary SLLs. It was also showed that two types of difference patterns can be synthesized as imposing the different initial values of null positions in the optimization process.

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Keum Sook Ha (M’08) received the B.S., M.S., and Ph.D. degrees from Kyungpook National University, Korea, in 1983, 1990, and 2003, respectively, all in electrical and computer engineering.

From 1983 to 1985, she was a Research Engineer with the Hankook Computer Co., Seoul, Korea. From 1986 to 1992, she was a Research Assistant with the Department of Electronic Engineering, Kyungpook National University, Korea. Since February 1992, she has been with Kumi College, Gyungbook, Korea, where she is currently an Associate Professor with the Division of Computer Information and Electronics. Her research interests include optimization algorithms, parallel processing, multimedia, RFID, and simulation of electronic devices.

Eui Joon Park (M’08) received the B.S. degree in electronic engineering from Dong-A University, Busan, Korea, in 1983, and the M.S. and Ph.D. degrees in electronic engineering from Korea University, Seoul, Korea, in 1986 and 1990, respectively.

Since April 1990, he has been with Kumoh National Institute of Technology, Gyungbook, Korea, where he is currently a Professor with the School of Electronic Engineering. His research interests include computational electromagnetics, RF/microwave circuits, electromagnetic scattering and wave propagation, antenna design, and inverse-scattering problems.