The Dynamics of Persuasion

John P. Curtis and Frank T. Smith

***Abstract*—**The act of persuading other people to adopt an acceptable opinion, and perhaps as a result, to do something we want them to, forms a huge part of our lives. Indeed this skill and the equally important one of being willing to listen to reason and be persuaded could be said to be at the core of what we term society. To what extent can we represent this social process mathematically? The concerned reader will doubtless be greatly relieved to hear that the authors believe that this goal is still very far distant. Nonetheless, some simple models discussed here may show some promising avenues to explore. Following some assumptions about propensity to be persuaded, discrete and continuous models are investigated and solved analytically for two people, and then generalised to an arbitrary number of people. The three person continuous model is then examined in some detail. Possible applications are discussed.

***Keywords*—** Persuasion, Opinion, Discrete Model, Continuous Model.

I. Introduction

We are all constantly subjected to efforts to persuade us to adopt a particular opinion, usually with a view to encouraging us to act in some desired manner. At the start of her novel *Persuasion* [1], Jane Austen's heroine Anne Elliot had been discouraged by a well-intentioned friend from marrying her fiancé Captain Wentworth and was living to regret being swayed. One can immediately think of advertising and politics, as familiar prime examples of efforts to persuade people.

Of course, people hold opinions on a wide variety of issues, from whether they prefer tea or coffee to how they will vote in general elections. Here we suppose that their opinions may be represented by a number between zero and unity, with these values representing extremes of opinion and intervening values representing shades of opinion. This approach to the representation of opinion follows Carletti *et al.* [2], who investigated how to make an efficient propaganda, using a statistical approach. However, they supposed that all members of the population were of equal susceptibility to the propaganda. This was represented by considering random meetings of pairs of the population, supposing that the change in opinion of a person depended linearly on the difference in opinion between the other member of the pair and the person's own opinion just before the meeting. The constant of proportion was taken to be fixed. Some very interesting structures in the data resulted, with clumps of opinion values establishing themselves, very reminiscent of political viewpoints, left-wing, centre and right-wing. The opinion as a continuous variable has been investigated by a number of previous authors [3-8]. In particular, the formation of public opinion has been the focus of recent research [9, 10]. These models have generally taken a stochastic approach.

In this paper we depart from the statistical approach and instead suppose that all members of our population (just two at first, then an arbitrary number) are influenced simultaneously by all others to a greater or lesser extent depending on the value of their interpersonal constant with each other person – what we may term their 'propensity to persuasion'. We consider both discrete and continuous formulations. We demonstrate in both formulations that, in the case of two people that have non-zero propensities for persuasion, both opinions converge on a common value. We see that this common value favours the opinion of the less accommodating member of the duo, and illustrate this with some examples. We then state the general problems for an arbitrary number of mutually interacting people. We thereafter focus on the continuous problem with three people, which offers some interesting features. Having accomplished these investigations, we discuss the many possible ways in which this work could be extended and applied.

II. Discrete Two Person Model

Let us define person N's opinion on some issue, whether e.g. a personal preference, political persuasion, or even level of education, as being a number O_N in the closed interval $0 \le O_N \le 1$. In reality everyone has a huge number of opinions on the same huge number of topics. However, we are mathematicians, and it is our opinion that it will be best (certainly for us) to focus on building a model for a single issue first. We suppose that we can define this person N's susceptibility or propensity to be persuaded by person M, σ_{NM} as a rate constant of proportionality μ_{NM} times the interval Δt of time over which the process of persuasion occurs. This could easily be modelled as variable, but let us keep matters simpler for now and suppose Δt remains constant. We have

Manuscript received Jan. 15, 2008; Revised version received March 19, 2008. J. P. Curtis is a Royal Society Industry Fellow and Honorary Senior Research Fellow in the Mathematics Department, Gower Street, University College London, United Kingdom. (tel: 44-207-679-2831, e-mail: curtis@ math.ucl.ac.uk). F.T. Smith F.R.S. is Goldsmid Professor of Applied Mathematics in the Mathematics Department, Gower Street, University College London, United Kingdom. (tel: 44-207-679-2837; e-mail: frank@ math.ucl.ac.uk).

$$\sigma_{NM} = \mu_{NM}\,\Delta t\,. \tag{1}$$

Let us now further define the opinion of person N at the end of time interval i as $X_N^{(i)}$, where $0 \le X_N^{(i)} \le 1$. Let us suppose that this opinion changes during the time interval $i+1$ as a result of interaction with person M (and for simplicity at this stage, just person M) according to the simple rule

$$X_N^{(i+1)} = X_N^{(i)} + \sigma_{NM}\left(X_M^{(i)} - X_N^{(i)}\right). \tag{2}$$

This rule states that the change in person N's opinion is linearly related to the difference in opinion between him or her and the influencing person M. This rule was applied by Carletti et al. in their analysis and would seem a natural first choice to explore. We immediately see that it is sensible to constrain σ_{NM} to lie in the interval $0 \le \sigma_{NM} \le 1$. The persuader is most unlikely to persuade someone to adopt an opinion beyond his or hers, i.e. to start below in value and end above, or vice-versa.

Let us now consider the case of just two people, numbered 1 and 2, mutually influencing each other. We have two equations of the form (2) to consider

$$X_1^{(i+1)} = X_1^{(i)} + \sigma_{12}\left(X_2^{(i)} - X_1^{(i)}\right), \tag{3}$$

$$X_2^{(i+1)} = X_2^{(i)} + \sigma_{21}\left(X_1^{(i)} - X_2^{(i)}\right). \tag{4}$$

Let us assign initial opinions $X_1^{(0)}, X_2^{(0)}$ to persons 1 and 2. Define

$$d_i = X_2^{(i)} - X_1^{(i)}\,. \tag{5}$$

Then, from equations (3) and (4),

$$\begin{aligned} d_{i+1} &= X_2^{(i+1)} - X_1^{(i+1)} \\ &= \left(X_2^{(i)} - X_1^{(i)}\right)\left(1-\sigma_{12}-\sigma_{21}\right) \\ &= \left(1-\sigma_{12}-\sigma_{21}\right)d_i\,. \end{aligned} \tag{6}$$

It follows that the solution for the difference in opinion is

$$d_i = \left(1-\sigma_{12}-\sigma_{21}\right)^i d_0\,. \tag{7}$$

Equations (3) and (4) thus become the recurrence relations

$$X_1^{(i+1)} = X_1^{(i)} + \sigma_{12}\left(1-\sigma_{12}-\sigma_{21}\right)^i, \tag{8}$$

$$X_2^{(i+1)} = X_2^{(i)} + \sigma_{21}\left(1-\sigma_{12}-\sigma_{21}\right)^i. \tag{9}$$

The solutions are

$$\begin{aligned} X_1^{(i)} = X_1^{(0)} + \sigma_{12}\, d_0 \{&\left(1-\sigma_{12}-\sigma_{21}\right)^i \\ &+ \left(1-\sigma_{12}-\sigma_{21}\right)^{i-1} \\ &+ \ldots\ldots + 1\} \end{aligned} \tag{10}$$

and

$$\begin{aligned} X_2^{(i)} = X_2^{(0)} + \sigma_{21}\, d_0 \{&\left(1-\sigma_{12}-\sigma_{21}\right)^i \\ &+ \left(1-\sigma_{12}-\sigma_{21}\right)^{i-1} \\ &+ \ldots\ldots + 1\}. \end{aligned} \tag{11}$$

Summing the geometric progressions yields the following solutions

$$\begin{aligned} X_1^{(i)} = X_1^{(0)} + \frac{\sigma_{12}}{\left(\sigma_{12}+\sigma_{21}\right)}&\left(X_2^{(0)} - X_1^{(0)}\right)\times \\ &\left\{1-\left(1-\sigma_{12}-\sigma_{21}\right)^i\right\}, \end{aligned} \tag{12}$$

$$\begin{aligned} X_2^{(i)} = X_2^{(0)} + \frac{\sigma_{21}}{\left(\sigma_{12}+\sigma_{21}\right)}&\left(X_1^{(0)} - X_2^{(0)}\right)\times \\ &\left\{1-\left(1-\sigma_{12}-\sigma_{21}\right)^i\right\}. \end{aligned} \tag{13}$$

It is interesting to explore the behaviour of both solutions as $i \to \infty$. Assuming that $\left|1-\sigma_{12}-\sigma_{21}\right| < 1$, in the limit

$$X_1^{(i)}, X_2^{(i)} \to \frac{\left(\sigma_{12}X_2^{(0)} + \sigma_{21}X_1^{(0)}\right)}{\left(\sigma_{12}+\sigma_{21}\right)} \tag{14}$$

as $i \to \infty$. Thus both opinions converge together. For later comparison with the continuous case, note that, from equation (1), the limit (14) reduces to

$$\frac{\left(\mu_{12}X_2^{(0)} + \mu_{21}X_1^{(0)}\right)}{\left(\mu_{12}+\mu_{21}\right)}\,. \tag{15}$$

III. Continuous Two Person Model

Suppose that the time at the end of the time interval i is t, so that the time at the end of the $i+1$th is $t+\Delta t$. Let us set

$$x_j(t) = X_j^{(i)}, \quad x_j(t+\Delta t) = X_j^{(i+1)}, \quad j = 1, 2\,. \tag{16}$$

Equations (3) and (4) may be written as

$$x_1(t+\Delta t) = x_1(t) + \mu_{12}\Delta t\left(x_2(t) - x_1(t)\right), \tag{17}$$

$$x_2(t+\Delta t) = x_2(t) + \mu_{21}\Delta t\left(x_1(t) - x_2(t)\right), \tag{18}$$

where we have availed ourselves of equation (1). Dividing by Δt and taking the limit as $\Delta t \to 0$, we may thus express equations (17) and (18) as coupled first-order differential equations

$$\dot{x}_1 = \mu_{12}(x_2 - x_1), \tag{19}$$

$$\dot{x}_2 = \mu_{21}(x_1 - x_2). \tag{20}$$

Let us now introduce the difference in opinions

$$u = x_2 - x_1. \tag{21}$$

Subtraction of equation (19) from equation (20) and use of equation (21) yields

$$\dot{u} = -(\mu_{12} + \mu_{21})u, \tag{22}$$

which has the solution

$$u = Ae^{-(\mu_{12}+\mu_{21})t}. \tag{23}$$

Defining

$$x_1(0) = x_{10}, \quad x_2(0) = x_{20}, \tag{24}$$

equations (21) and (23) and conditions (24) yield

$$A = x_{20} - x_{10}. \tag{25}$$

Equation (19) then gives

$$\dot{x}_1 = \mu_{12}(x_{20} - x_{10})e^{-(\mu_{12}+\mu_{21})t} \tag{26}$$

Equation (26) is readily integrated to yield

$$x_1 = -\frac{\mu_{12}(x_{20} - x_{10})}{(\mu_{12} + \mu_{21})}e^{-(\mu_{12}+\mu_{21})t} + C_1, \tag{27}$$

where C_1 is a constant. The first initial condition (24) supplies this constant, so that the solution for x_1 is given by

$$x_1 = x_{10} + \frac{\mu_{12}(x_{20} - x_{10})}{(\mu_{12} + \mu_{21})}\left(1 - e^{-(\mu_{12}+\mu_{21})t}\right). \tag{28}$$

Similarly, the solution for x_2 is given by

$$x_2 = x_{20} + \frac{\mu_{21}(x_{10} - x_{20})}{(\mu_{12} + \mu_{21})}\left(1 - e^{-(\mu_{12}+\mu_{21})t}\right). \tag{29}$$

Both solutions converge on the limit

$$x_1, x_2 \to \frac{(\mu_{12}x_{20} + \mu_{21}x_{10})}{(\mu_{12} + \mu_{21})} \tag{30}$$

and this is identical to the limit in the discrete case (15).

IV. Analysis Of Final Opinion Change

Let the limit (30) (or (15)) be denoted by l. Then the final change in opinion of person 1 is given by

$$\begin{aligned} l - x_{10} &= \frac{\mu_{21}x_{10} + \mu_{12}x_{20} - \mu_{12}x_{10} - \mu_{21}x_{10}}{\mu_{12} + \mu_{21}} \\ &= \frac{\mu_{12}}{(\mu_{12} + \mu_{21})}(x_{20} - x_{10}). \end{aligned} \tag{31}$$

Similarly the final change in the opinion of person 2 is given by

$$l - x_{20} = \frac{\mu_{21}}{(\mu_{12} + \mu_{21})}(x_{10} - x_{20}). \tag{32}$$

Let us take the moduli of these changes as follows:

$$|l - x_{10}| = \frac{\mu_{12}}{(\mu_{12} + \mu_{21})}|x_{20} - x_{10}|, \tag{33}$$

$$|l - x_{20}| = \frac{\mu_{21}}{(\mu_{12} + \mu_{21})}|x_{20} - x_{10}|. \tag{34}$$

Thus the size of the change in person 1's opinion exceeds or is less than the size of the change in person 2's opinion depending on whether $\mu_{12} > \mu_{21}$ or $\mu_{12} < \mu_{21}$. As we would expect, the mathematics confirms that the person with the greater propensity to be persuaded changes his or her opinion more.

In the extreme case where one of the people, say person 1, has zero inclination to be persuaded from his or her present opinion, i.e. where $\mu_{12} = 0$, it is apparent from equation (30) that person 2's opinion will in this model converge on person 1's opinion, i.e. $x_2 \to x_{10}$.

V. Some Examples

In Fig. 1 we show a simple example where we have taken well separated initial opinions of both people and allowed them slightly different levels of propensity to be persuaded μ_{12} and μ_{21}. It may be seen how the two opinions converge as predicted to the value 0.54 given by equation (15). Note that this value is displaced towards the person with the lesser propensity to persuasion, namely person 1.

What happens when we assign person 1 zero propensity to be persuaded, i.e. change μ_{12} from 0.1 to 0.0? The result of this change is shown in Fig. 2. Person 1 remains intransigent, keeping the same opinion at value 0.7. Person 2's opinion asymptotes to person 1's. In this formulation it seems that person 1 is rewarded by his opinion being adopted gradually by person 2.

In our final numerical example, shown in Fig. 3, we repeat the discrete run of figure 1, this time showing the comparison with the continuous case.

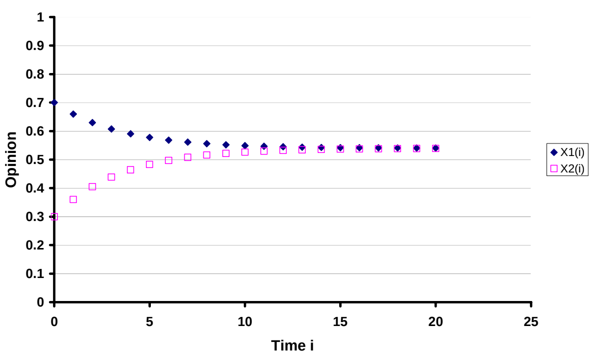

Fig. 1. Convergence of opinions in discrete two person case. Person 1's opinion is shown with solid diamonds, person 2's with open squares. The parameters take the following values

$$\mu_{12} = 0.1,\ \mu_{21} = 0.15,\ \Delta t = 1.0,$$
$$X_1^{(0)} = 0.7,\ X_2^{(0)} = 0.3$$

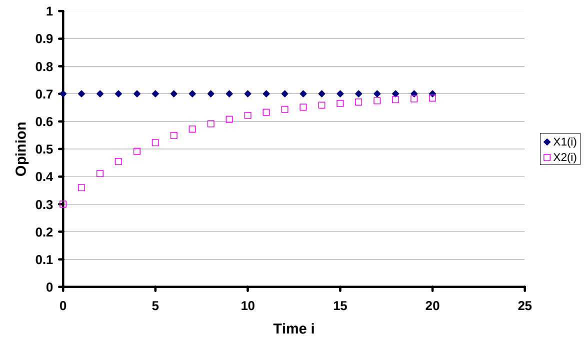

Fig. 2. Convergence of opinions in discrete two person case, with key and parameters as for figure 1, except that here person 1's intransigence, with $\mu_{12} = 0$, is rewarded with compliance to his or her opinion by person 2.

For the unit time interval $\Delta t = 1$ in the discrete case it is apparent that there is little difference between the two cases. We would expect this difference to become greater as Δt is increased, basically reflecting the increasingly poor representation of the continuous case by the discrete one. However, note that the discrete case with a large and indeed varying time interval may be better suited to the representation of real interactions.

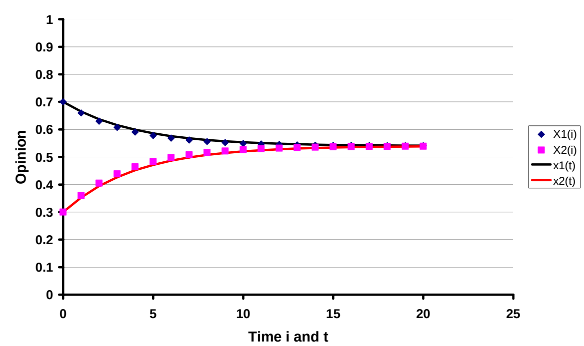

Fig. 3. Convergence of opinions in discrete and continuous two person cases, with parameters as for figure 1. It may be seen that the two approaches give almost identical results.

VI. Generalisation To More People

Models for both the discrete and continuous cases for a general number N of people are straightforward to state, if more complex to investigate numerically.

In the discrete case, equations (3) and (4) are replaced by

$$X_j^{(i+1)} = X_j^{(i)} + \sum_{k \neq j}^{N} \sigma_{jk} \left(X_k^{(i)} - X_j^{(i)} \right), \tag{35}$$

where $j = 1, 2, ..., N.$

In the continuous case, equations (19) and (20) become

$$\dot{x}_j = \sum_{k \neq j}^{N} \mu_{jk} \left(x_k - x_j \right), \tag{36}$$

where again $j = 1, 2, ..., N.$

Note that these equations must be applied with greater care because there is the possibility that the sums can become large enough to render the discrete change in opinion large enough to make the new opinion exceed unity, and in the continuous case, to make the derivative of the opinion so great that the opinion exceeds unity at the next time step.

Equations (35) and (36) may readily be applied to any network of people, such as a communications network, or a management structure. For active links between people the values of μ_{jk} will be non-zero; where people are not influenced by particular people the corresponding values of μ_{jk} will be zero.

The three person case is addressed in detail in the next section.

VII. Continuous Three Person Model

For three people, equations (36) may be written as

$$\dot{x}_1 = a\left(x_2 - x_1\right) + b\left(x_3 - x_1\right), \tag{37}$$

$$\dot{x}_2 = c\left(x_1 - x_2\right) + d\left(x_3 - x_2\right), \tag{38}$$

$$\dot{x}_3 = e\left(x_1 - x_3\right) + f\left(x_2 - x_3\right). \tag{39}$$

where we have introduced

$$\begin{aligned} a &= \mu_{12}, \quad b = \mu_{13}, \quad c = \mu_{21}, \\ d &= \mu_{23}, \quad e = \mu_{31}, \quad f = \mu_{32}, \end{aligned} \tag{40}$$

to render the manipulations which follow more readable. If we now proceed by analogy with Section III and subtract equation (37) from (38), and equation (38) from equation (39), we obtain after some simple manipulations the equations

$$\begin{aligned} \dot{x}_2 - \dot{x}_1 = &-(a+b+c)(x_2 - x_1) \\ &+ (d-b)(x_3 - x_2), \end{aligned} \tag{41}$$

$$\begin{aligned} \dot{x}_3 - \dot{x}_2 = &-(d+e+f)(x_3 - x_2) \\ &+ (c-e)(x_2 - x_1). \end{aligned} \tag{42}$$

It is convenient to introduce the differences

$$X = x_2 - x_1, \tag{43}$$

$$Y = x_3 - x_2, \tag{44}$$

and new constants

$$\begin{aligned} A &= -(a+b+c), \quad B = d - b, \\ C &= -(d+e+f), \quad D = c - e. \end{aligned} \tag{45}$$

Then equations (41) and (42) become

$$\dot{X} = AX + BY \tag{46}$$

and

$$\dot{Y} = CY + DX. \tag{47}$$

Elimination of X yields the second-order linear differential equation

$$\ddot{Y} - (A+C)\dot{Y} - (BD - AC)Y = 0. \tag{48}$$

With a few exceptions as discussed below, this has solution

$$Y = Y_1 e^{-\lambda_1 t} + Y_2 e^{-\lambda_2 t}, \tag{49}$$

where λ_1 and λ_2 are the roots of the quadratic

$$\lambda^2 + (A+C)\lambda - (BD - AC) = 0, \tag{50}$$

given by

$$\lambda = \frac{-(A+C) \pm \sqrt{(A-C)^2 + 4BD}}{2}, \tag{51}$$

and

$$Y_1 = -\frac{\left\{D\left(x_{20} - x_{10}\right) + \left(\lambda_2 + C\right)\left(x_{30} - x_{20}\right)\right\}}{\left(\lambda_1 - \lambda_2\right)}, \tag{52}$$

$$Y_2 = x_{30} - x_{20} - Y_1. \tag{53}$$

The solution for X is

$$X = X_1 e^{-\lambda_1 t} + X_2 e^{-\lambda_2 t}, \tag{54}$$

where

$$X_1 = -\frac{Y_1(\lambda_1 + C)}{D} \tag{55}$$

and

$$X_2 = -\frac{Y_2(\lambda_2 + C)}{D}. \tag{56}$$

Having found X and Y it is convenient to write equations (37)-(39) in the forms

$$\dot{x}_1 = (a+b)X + bY, \tag{57}$$

$$\dot{x}_2 = -cX + dY, \tag{58}$$

$$\dot{x}_3 = -eX - (e+f)Y. \tag{59}$$

The right-hand sides being known, we may integrate equations (57)-(59) and apply the initial conditions

$$x_1(0) = x_{10}, \quad x_2(0) = x_{20}, \quad x_3(0) = x_{30}, \tag{60}$$

to obtain the solutions

$$\begin{aligned} x_1 = x_{10} &+ \frac{1}{\lambda_1}\{(a+b)X_1 + bY_1\}\left(1 - e^{-\lambda_1 t}\right) \\ &+ \frac{1}{\lambda_2}\{(a+b)X_2 + bY_2\}\left(1 - e^{-\lambda_2 t}\right), \end{aligned} \tag{61}$$

$$\begin{aligned} x_2 = x_{20} &+ \frac{1}{\lambda_1}(-cX_1 + dY_1)\left(1 - e^{-\lambda_1 t}\right) \\ &+ \frac{1}{\lambda_2}(-cX_2 + dY_2)\left(1 - e^{-\lambda_2 t}\right), \end{aligned} \tag{62}$$

$$\begin{aligned} x_3 = x_{30} &- \frac{1}{\lambda_1}\{eX_1 + (e+f)Y_1\}\left(1 - e^{-\lambda_1 t}\right) \\ &- \frac{1}{\lambda_2}\{eX_2 + (e+f)Y_2\}\left(1 - e^{-\lambda_2 t}\right). \end{aligned} \tag{63}$$

Provided λ_1 and λ_2 are positive the opinions will asymptote to the same limiting value, as is shown by equations (49) and (54). Note that for positive propensities to be persuaded, we have $-(A+C) > 0$, and the condition that both roots are positive is, from equation (51),

$$(A-C)^2 + 4BD < (A+C)^2, \tag{64}$$

which reduces to

$$BD - AC < 0. \tag{65}$$

This is equivalent to

$$\begin{aligned} bc + de + ad + bd + ae \\ + ce + af + bf + cf > 0, \end{aligned} \tag{66}$$

which is always satisfied for positive propensities to be persuaded.

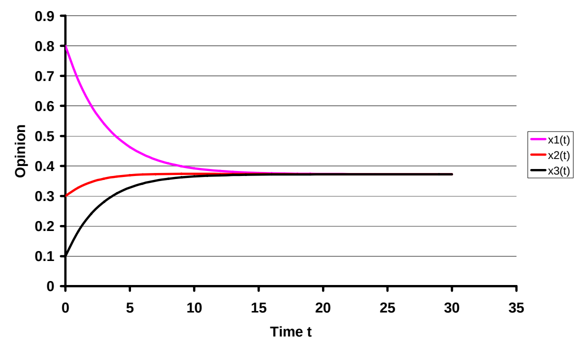

Fig. 4. Convergence of opinions in the continuous three person case, with parameters as follows:

$$a = 0.1,\ b = 0.12,\ c = 0.13,$$
$$d = 0.15,\ e = 0.1,\ f = 0.05,$$
$$x_{10} = 0.8,\ x_{20} = 0.3,\ x_{30} = 0.1.$$

Fig. 4 shows the convergence of the three opinions in an example of the general case with solution given by equations (61)-(63).

An example of an interesting exceptional case arises when either $B = 0$ or $D = 0$. Under these circumstances

$$\lambda_1 = -C, \quad \lambda_2 = -A. \tag{67}$$

Let us investigate the case where $D = 0$. From equation (47)

$$Y = Y_3 e^{Ct} \tag{68}$$

where

$$Y_3 = x_{30} - x_{20}. \tag{69}$$

It is straightforward to show that

$$X = X_3 e^{At} + K e^{Ct}, \tag{70}$$

where

$$K = \frac{BY_3}{(C-A)}, \quad X_3 = x_{20} - x_{10} - K. \tag{71}$$

The solution for x_1 is then determined in the same way as before, and is

$$\begin{aligned} x_1 = x_{10} &- \frac{(a+b)X_3}{A}\left(1-e^{At}\right) \\ &- \frac{1}{C}\left\{\frac{(a+b)BY_3}{(C-A)} + bY_3\right\}\left(1-e^{Ct}\right). \end{aligned} \tag{72}$$

The other opinions are then readily evaluated as

$$x_2 = x_1 + X, \tag{73}$$

$$x_3 = x_2 + Y. \tag{74}$$

An even more special case arises when, in addition to $D = 0$, we have $A = C$. Equation (68) still holds and on eliminating C equation (46) is now

$$\dot{X} = AX + BY_3 e^{At}. \tag{75}$$

The solution is

$$X = (BY_3 t + q)e^{At}, \tag{76}$$

where

$$q = x_{20} - x_{10}. \tag{77}$$

Equation (76) appears to offer the possibility under particular conditions of an initial divergence of opinion before an eventual asymptotic convergence. The corresponding solutions for x_1, x_2, x_3 are

$$\begin{aligned} x_1 = x_{10} &+ \frac{(a+b)BY_3}{A^2}\left(Ate^{At} + 1 - e^{At}\right) \\ &- \frac{\{(a+b)q + bY_3\}}{A}\left(1-e^{At}\right), \end{aligned} \tag{78}$$

$$x_2 = x_1 + X, \tag{79}$$

and

$$x_3 = x_2 + Y. \tag{80}$$

Results very similar to those derived in equations (67) – (80) inclusive hold when $B = 0$.

VIII. Discussion

The first comment to make on the above analysis is the obvious one that relationships between people are far more complex than can possibly be captured using models like equations (3) and (4). There is difficulty too in deciding what numerical value to ascribe to a particular opinion, and some opinions are likely simply not to be amenable to this kind of representation. Therefore we stress that the modeling here is naïve and the mathematics trivial, if only for ease of presentation; but based on the present approach the modeling can readily be improved and made more realistic, perhaps involving nonlinear effects; that will soon make the mathematics become very nontrivial indeed.

We have shown that the three-person case has resulted in a quadratic for the exponential time constants. We foresee, but have not as yet shown, that a polynomial equation of degree one less than the number of people will be needed for greater numbers of people. There may be interesting interpretations in this context of known properties of polynomial equations of cubic, quartic, quintic or higher orders. While it is likely that one would turn to numerical methods for large numbers of people, where analytical solutions can be determined they will provide very useful verification for such schemes.

As a further example of a generalization leading to more sophisticated mathematics, one can easily envisage vectors of opinions describing a given person’s characteristics. One might allow more general dependency of an opinion on one issue on the same person’s own opinions on other issues, and others’ opinions on relevant issues. This would give greater flexibility at cost of much greater complexity.

As another such example, we remark that as yet no account of a person being prepared to make a change in opinion to some extent, but not beyond a particular value, has been taken. This is likely to be easily treated using the discrete approach, but may cause troublesome discontinuities in derivatives in the continuous formulation.

This leads us to the general issue of which type of model to apply under which circumstances. There may, for example, be situations where the rate of response of the other people to the first person, as well as the plain differences in opinion, affects his or her opinion. Such feedback situations may be very amenable to the continuous modelling approach. By contrast, where persuasion is occurring on a sporadic basis, the continuous approach is likely to be less useful, and event-based simulation may be more appropriate.

It is believed that both methods are most likely to bear fruit where there is a clear-cut metric of some opinion. They appear to represent, but we think not explain, the process by which people who are intransigent are successful at getting more persuadable people to change their opinions to conform. This is going on all the time in society and it is perhaps an ethical issue to be debated, whether exploring this mathematically is in the interests of society, or even dangerous.

IX. Conclusion

We have first established some very simple mathematical models for the mutual persuasion of two people by each other. The models rely on the most basic metric of opinion, a scale from 0 to 1. They show that, if open to persuasion, both parties converge on an intermediate opinion value that lies towards the person having the lesser inclination or propensity to be persuaded. We think this bears some resemblance to everyday experience.

The models developed are of two kinds, a discrete model and a continuous one. This latter is considered to be something of an idealization, but the close agreement between discrete and continuous models for reasonably small and regular time intervals has been demonstrated. Under these circumstances the continuous approach may be beneficial, e.g. in treating feedback situations. Other situations, where persuasion occurs sporadically, are likely to render the continuous approach unsuitable.

We have then examined the continuous model for three people in some detail and again shown convergence on a common intermediate opinion. For three people there are some interesting special cases where it appears that differences of opinion can initially grow before converging asymptotically to the common limiting value.

We are aware we are entering an arena with this work where ethical considerations are to be respected, and indeed are somewhat fearful that the work, by its very nature, could be abused. But intransigent folk already know only too well what they are up to, and the authors hope that this paper will in some way help the reasonable man on the fabled Clapham Omnibus to achieve the reasonable and civilized society we all aspire to.

Let us end on a lighter and more hopeful note, by asking what became of our heroine Anne Elliot? It is a happy ending and we commend you to find out why yourselves, if you have not yet read *Persuasion.*

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