Portfolio Selection Problem with Maximum Downside Deviation Measure: A Stochastic Programming Approach

K. Ibrahim, A.A. Kamil, and A. Mustafa

Abstract—In this paper, single stage stochastic programming two stage stochastic programming models are proposed for portfolio selection problems. The uncertainty of the future returns is the most important character within this optimization problem. The concept of risk is used in the Markowitz Mean-Variance model that identifies risk with the volatility (variance) of the random objective. Since then, mean-risk optimization paradigm received extensive development both theoretically and computationally. We utilize probabilistic methods alongside with optimization techniques and develop the stochastic programming models for portfolio selection problem minimizing maximum downside deviation from the expected return. We develop **single stage and two stage stochastic programming with recourse**. The models were developed for risk averse investors and the objective of the stochastic programming models is to minimize the **maximum downside deviation**. We use the so-called "**Here-and-Now**" approach where the decision-maker makes decision "now" before observing the actual outcome for the stochastic parameter. We compare the optimal portfolios between the single stage and two stage models with the incorporation of the deviation measure. The models are applied to the optimal selection of stocks listed in Bursa Malaysia and the return of the optimal portfolio is compared between the two stochastic models. The results show that the two stage model outperforms the single stage model in the optimal and in-sample analysis

Keywords— Portfolio optimization, Maximum negative deviation, Downside risk, Stochastic Linear Programming.

I. INTRODUCTION

PORTFOLIO optimization has been one of the important research fields in financial decision making. The most important character within this optimization problem is the uncertainty of the future returns. To handle such problems, we utilize probabilistic methods alongside with optimization techniques. Stochastic programming is our approach to deal with uncertainty. This approach can deal the management of portfolio risk and the identification of optimal portfolio simultaneously. Stochastic programming models explicitly consider uncertainty in some of the model parameters, and provide optimal decisions which are hedged against such uncertainty.

Stochastic Programming is a branch of mathematical programming where the parameters are random. The objective of stochastic programming is to find the optimum solution to problems with uncertain data.

In the deterministic framework, a typical mathematical programming problem could be stated as

Minimize $f(x)$

Subject to   (1)

s.t $g_i(x) \leq 0,$ $i = 1, \ldots m,$

where x is from R^n or Z^n.

Uncertainty, usually described by a random element $\xi(\omega)$, where ω is a random outcome from a space Ω, leads to situation where instead of just $f(x)$ and $g_i(x)$ one has to deal with $f(x, \xi(\omega))$ and $g_i(x, \xi(\omega))$. Traditionally, the probability distribution of ξ is assumed to known or can be estimated and is unaffected by the decision vector x. The problem becomes decision making under uncertainty where decision vector x has to be chosen before the outcome from the distribution of $\xi(\omega)$ can be observed.

The methodology for portfolio selection problems dates back to the work of [1]. [1,2] used the concept of risk into the problem and introduced mean-risk approach that identifies risk with the volatility (variance) of the random objective. Since 1952, mean-risk optimization paradigm received extensive development both theoretically and computationally. [3] proposed the mean absolute deviation from the mean as the risk measure to estimate the nonlinear variance-covariance of the stocks in the mean-variance model. It transforms the portfolio selection problem from a quadratic programming into a linear programming problem. At the same time, the popularity of downside risk among investors is growing and mean-return-downside risk portfolio selection models seem to oppress the familiar mean-variance approach. The reason for the success of the former models is that they

K. Ibrahim is with the Universiti Teknologi Mara Terengganu, 23000, Dungun, Terengganu, Malaysia (phone: 019-282-7104; e-mail: khalipah60@ yahoo.com).

A. A. Kamil is with the School of Distance Education, Universiti Sains Malaysia, 11800, Penang, Malaysia (e-mail: anton@usm.my http://pppjj.usm.my/).

A. Mustaffa is with the School of Mathematical Sciences, Universiti Sains Malaysia, 11800, Penang, Malaysia (e-mail: adli@cs.usm.my http://www..usm.my/).

separate return fluctuations into downside risk and upside potential. This is especially relevant for asymmetrical return distributions, for which mean-variance model punish the upside potential in the same fashion as the downside risk. This led Markowitz [1] to propose downside risk measures such as (downside) semi variance to replace variance as the risk measure. Consequently, one observes growing popularity of downside risk models for portfolio selection [4].

[5] introduced another linear programming model which maximize the minimum return or minimize the maximum loss (minimax) over time periods and applied to the stock indices from eight countries, from January 1991 until December 1995. The analysis showed that the model performs similarly with the classical mean-variance model. In addition, [5] argues that, when data is log-normally distributed or skewed, the minimax formulation might be more appropriate method, compared to the classical mean-variance formulation, which is optimal for normally distributed data. [6] also considered the minimax model but it was analyzed with the maximum semi deviation.

[7] and independently [8] suggested an approach to stochastic programming and termed as stochastic programming with recourse. Recourse is the ability to take corrective action after a random event has taken place. The main innovation is to amend the problem to allow the decision maker the opportunity to make corrective actions after a random event has taken place. In the first stage a decision maker a here and now decision. In the second stage the decision maker sees a realization of the stochastic elements of the problem but he is allowed to make further decisions to avoid the constraints of the problem becoming infeasible.

Stochastic programming is becoming more popular in finance as computing power increases. There have been numerous applications of Stochastic Programming methodology to real life problems over the last two decades. The applicability of stochastic programs to financial planning problems was first recognized by [9]. [10] and [11] developed multistage stochastic programs with recourse to address portfolio management problems with fixed-income securities under uncertainty in interest rates. The models integrate stochastic programming model for the selection of portfolios with Monte Carlo simulation models of the term structure of interest rates. [12], [13], and [14] also applied stochastic programs to fixed-income portfolio management problems. [15] modelled a portfolio selection problem with transaction costs as a two-stage stochastic programming problem and evaluated the model using historical data obtained from the Taiwan Stock Exchange. Their results show that the model outperforms the market and the MV and MAD model.

In this paper we develop **single stage and two stage stochastic programming with recourse** for portfolio selection problem and the objective is to minimize the **maximum downside deviation** measure of portfolio returns from the expected return. We use the so-called "**Here-and-Now**" approach where the decision-maker makes decision "now" before observing the actual outcome for the stochastic parameter. The portfolio optimization problem considered in this paper follows the original Markowitz formulation and is based on a single period model of investment. At the beginning of a period, an investor allocates capital among various securities. Assuming that each security is represented by a variable, this is equivalent to assigning a nonnegative weight to each variable. During the investment period, a security generates a random rate of return. The change of capital invested observed at the end of the period is measured by the weighted average of the individual rates of return.

The main objective of this study is to compare the optimal portfolio selected using two different stochastic programming models. We compared the optimal portfolios between the single stage and two stage models with the incorporation of deviation measure. This method is applied to the optimal selection of stocks listed in Bursa Malaysia and the return of the optimal portfolio from the two models is compared.

The remainder of the paper is organized as follows. In the next section we discuss the maximum downside deviation measure and we include its deviation properties introduced by [16]. Then we formulate single stage stochastic programming model and the equivalent stochastic linear programming model using an auxiliary variable for portfolio selection problem. Then we extend the single stage model to two stage stochastic programming with recourse model. Section 3 devoted to the experimental analysis on real-life data from Bursa Malaysia. Finally, some concluding remarks are given in section 4.

II. METHODOLOGY

Consider a set of securities $I = \{i : i = 1,2,...,n\}$ for an investment. At the end of a certain holding period the assets generate returns, $\tilde{r} = (\tilde{r}_1, \tilde{r}_2,...,\tilde{r}_n)^T$. The returns are unknown at the beginning of the holding period, that is, at the time of the portfolio selection and are treated as random variables. Denote their mean value by, $\bar{r} = E(\tilde{r}) = (\bar{r}_1, \bar{r}_2,...,\bar{r}_n)^T$. At the beginning of the holding period the investor wishes to apportion his budget to these assets by deciding on a specific allocation $x = (x_1, x_2,...,x_n)^T$ such that $x_i \geq 0$ (i.e., short sales are not allowed) and $\sum_{i \in I} x_i = 1$ (budget constraint). We use boldface character to denote vectors, and ~ to denote random variables.

The uncertain return of the portfolio at the end of the holding period is $\tilde{R} = R(x,\tilde{r}) = x^T \tilde{r}$. This is a random variable with a distribution function, say F, that is, $F(x,\mu) = P\{R(x,\tilde{r}) \leq \mu\}$. We assume that the distribution function F does not depend on the portfolio composition x. The expected return of the portfolio is

$$\bar{R} = E[\tilde{R}] = E[R(x,\tilde{r})] = \bar{R}(x,\tilde{r}).$$

Suppose the uncertain returns of the assets, $\tilde{r}$, are represented by a finite set of discrete scenarios $\Omega = \{\omega : \omega = 1,2,...,S\}$, whereby the returns under a particular scenario $\omega \in \Omega$ take the values $r_\omega = (r_{1\omega}, r_{2\omega},...,r_{n\omega})^T$ with associated probability $p_\omega > 0$, $\sum_{\omega \in \Omega} p_\omega = 1$. The mean return of the assets is $\bar{r} = \sum_{\omega \in \Omega} p_\omega r_\omega$. The portfolio return under a

particular realization of asset return r_ω is denoted by $R_\omega = R(x,r_\omega)$. The expected portfolio return is expressed as

$$\overline{R} = \overline{R}(x,r_\omega) = E[R(x,r_\omega)] = \sum_{\omega \in \Omega} p_\omega R(x,r_\omega).$$

Let $M[R(x,r_\omega)]$ be the minimum of the portfolio return. The *maximum downside deviation* measure is defined as

$$\kappa(x) = MM[R(x,r_\omega)] \qquad (2)$$
$$= [E[R(x,r_\omega)] - \text{Min}[R(x,r_\omega)]]$$

Maximum downside deviation risk $MM[R(x,r_\omega)]$ is a very pessimistic risk measure related to the worst case analysis. It does not take into account the distribution of outcomes other than the worst one.

A. Properties of the $MM[R(x,\tilde{r})]$ measures

Since being introduced in [17], the axiomatic approach to construction of risk measures has been repeatedly employed by many authors for development of other types of risk measures, tailored to specific preferences and applications (see [16]. [18], [19] and [20]).

Proposition 1: $MM[R(x,\tilde{r})]$ measure is a deviation measure

Proof:

1) *Subadditivity:* $\kappa(X_1 + X_2) \le \kappa(X_1) + \kappa(X_2)$.

$MM[R_1(x,\tilde{r}) + R_2(x,\tilde{r})]$

$= max\{ E[R_1(x,\tilde{r}) + R_2(x,\tilde{r})]$

$\quad - [R_1(x,\tilde{r}) + R_2(x,\tilde{r})]\}$

$= max\{(E[R_1(x,\tilde{r})] - R_1(x,\tilde{r}))$

$\quad + (E[R_2(x,\tilde{r})] - R_2(x,\tilde{r})\}$

$\le max\{ E[R_1(x,\tilde{r})] - R_1(x,\tilde{r})\}$

$\quad + max\{ E[R_2(x,\tilde{r})] - R_2(x,\tilde{r})\}$

$\le MM[R_1(x,\tilde{r})] + MM[R_2(x,\tilde{r})]$

2) Positive Homogeneity

$MM[0] = max(E[0] - 0) = 0$

$MM[\lambda(R(x,\tilde{r})]$

$= max\{ E[\lambda R(x,\tilde{r})] - \lambda R(x,\tilde{r})\}$

$= \lambda max\{ E[R(x,\tilde{r})] - R(x,\tilde{r})\}$

$= \lambda MM[R(x,\tilde{r})]$, for all $\lambda > 0$

3) Translation invariance:

$\kappa(X + \alpha) = \kappa(X) - \alpha$, for all real constant α.

$MM[(R(x,\tilde{r}) + \alpha]$

$= max\{ E([R(x,\tilde{r}) + \alpha] - [R(x,\tilde{r}) + \alpha])\}$

$= max\{ E[R(x,\tilde{r})] + \alpha - R(x,\tilde{r}) - \alpha\}$

$= max\{ E[R(x,\tilde{r})] - R(x,\tilde{r})\}$

$= MM[(R(x,\tilde{r})]$

4) Convexity:

$\kappa[\lambda X_1 + (1-\lambda)X_2] \le \lambda\kappa(X_1) + (1-\lambda)\kappa(X_2)$

for all $\lambda \in [0,1]$.

$MM[\lambda R_1(x,\tilde{r}) + (1-\lambda)R_2(x,\tilde{r})]$

$= max\{ E[\lambda R_1(x,\tilde{r}) + (1-\lambda)R_2(x,\tilde{r})]$

$\quad - [\lambda R_1(x,\tilde{r}) + (1-\lambda)R_2(x,\tilde{r})]\}$

$= max\{(E[\lambda R_1(x,\tilde{r})] + E[(1-\lambda)R_2(x,\tilde{r}))]$

$\quad - \lambda R_1(x,\tilde{r}) + (1-\lambda)R_2(x,\tilde{r})\}$

$= max\{ \lambda(E[R_1(x,\tilde{r})] - R_1(x,\tilde{r}))$

$\quad + (1-\lambda)(E[R_2(x,\tilde{r})] - R_2(x,\tilde{r}))\}$

$\le \lambda max\{(E[R_1(x,\tilde{r})] - R_1(x,\tilde{r}))\}$

$\quad + (1-\lambda)max\{ E[R_2(x,\tilde{r})] - R_2(x,\tilde{r}))\}$

$\le \lambda MM[R_1(x,\tilde{r})] + (1-\lambda)MM[R_2(x,\tilde{r})]$

B. Single Stage Stochastic Programming Portfolio Optimization Model with MM deviation measure

The portfolio selection optimization model is formulated as a single stage stochastic programming model as follows:

Definition 1: S_MM

The stochastic portfolio optimization problem where the difference between the expected portfolio return and the maximum of minimum portfolio returns is minimized and constraining the expected portfolio return is:

$$\text{Minimize } max[\overline{R}(x,r_\omega) - R(x,r_\omega)] \qquad (3a)$$

Subject to:

$$R(x,r_\omega) = \sum_{i \in I} x_i r_{\omega i} \qquad \forall \omega \in \Omega \qquad (3b)$$

$$\overline{R}(x,r_\omega) = \sum_{\omega \in \Omega} p_\omega R(x,r_\omega) \qquad (3c)$$

$$\overline{R}(x,r_\omega) \ge \alpha \qquad (3d)$$

$$\sum_{i \in I} x_i = 1 \qquad (3e)$$

$$L_i \le x_i \le U_i \qquad \forall i \in I \qquad (3f)$$

Model S_MM minimizes the maximum downside deviation of portfolio returns from the expected portfolio return at the end of the investment period. Equation (3b) defines the total portfolio return under each scenario ω. Equation (3c) defines the expected return of the portfolio at the end of the horizon, while equation (3d) constraints the expected return by the target return α. Equation (3e) insures that the total weights of all investments sum to one, that is, budget constraints ensuring full investment of available budget. Finally equation (3f) insures that the weights on assets purchased are nonnegative, disallowing short sales and place upper bound on the weights.

Solving the above parametric programs for different values of the expected portfolio return α yields the MM-efficient frontier.

C. Linear programming formulation for S_MM

Model S_MM has a non linear objective function and a set of linear constraints. Thus it is a non linear stochastic programming model. However the model can be transformed to a linear stochastic programming model as discussed below.

For every scenario $\omega \in \Omega$, let an auxiliary variable,

let
$$\eta = \max_{\omega \in \Omega} [\overline{R}(x, r_\omega) - R(x, r_\omega)]$$

(4)   Subject to

$$\eta \geq \max_{\omega \in \Omega} [\overline{R}(x, r_\omega) - R(x, r_\omega)] \text{ for } \forall \omega \in \Omega$$

Then, we have

$$MM[R(x, r_\omega)] = \eta \qquad (5)$$

Subject to $\eta \geq \max_{\omega \in \Omega} [\overline{R}(x, r_\omega) - R(x, r_\omega)]$ for $\forall \omega \in \Omega$

Substituting (5) in portfolio optimization models (3) resulting in the following stochastic linear programming model:

Minimize η (6a)

Subject to :

$$R(x, r_\omega) = \sum_{i \in I} x_i r_{\omega i} \qquad (6b)$$

$$\overline{R}(x, r_\omega) = \sum_{\omega \in \Omega} p_\omega R(x, r_\omega) \qquad (6c)$$

$$\overline{R}(x, r_\omega) \geq \alpha \qquad (6d)$$

$$\overline{R}(x, r_\omega) - R(x, r_\omega) \leq \eta \qquad (6e)$$

$$\sum_{i \in I} x_i = 1 \qquad (6f)$$

$$L_i \leq x_i \leq U_i \qquad \forall i \in I \qquad (6g)$$

Theorem 1

If x^* is an optimal solution to (3), then (x^*, η^*) is an optimal solution to (6), where $\eta = \max_{\omega \in \Omega} [\overline{R}(x, r_\omega) - R(x, r_\omega)]$. On the other hand, if (x^*, η^*) where $\eta = \max_{\omega \in \Omega} [\overline{R}(x, r_\omega) - R(x, r_\omega)]$ is an optimal solution to (6), then x^* is an optimal solution to (3)

Proof:

If x^* is an optimal solution to (3), then (x^*, η^*) is a feasible solution to (6), where $\eta = \max_{\omega \in \Omega} [\overline{R}(x, r_\omega) - R(x, r_\omega)]$. If (x^*, η^*) is not an optimal solution to (6), then there exists a feasible solution (x, η), to (6) where $\eta = \max_{\omega \in \Omega} [\overline{R}(x, r_\omega) - R(x, r_\omega)]$ such that $\eta \leq \eta^*$.

Noticing that $\max_{\omega \in \Omega} [\overline{R}(x, r_\omega) - R(x, r_\omega)] \leq \eta$, then we have

$$\max_{\omega \in \Omega} [\overline{R}(x, r_\omega) - R(x, r_\omega)] \leq \eta < \eta^*$$

$$< \max_{\omega \in \Omega} [\overline{R}(x^*, r_\omega) - R(x^*, r_\omega)]$$

which contradicts that x^* is an optimal solution to (3).

On the other hand, if (x^*, η^*) is an optimal solution to (6), where $\eta = \max_{\omega \in \Omega} [\overline{R}(x, r_\omega) - R(x, r_\omega)]$ then x^* is an optimal solution to (3). Otherwise, there exists a feasible solution x to (3), such that

$$\max_{\omega \in \Omega} [\overline{R}(x, r_\omega) - R(x, r_\omega)] < \max_{\omega \in \Omega} [\overline{R}(x^*, r_\omega) - R(x^*, r_\omega)]$$

Denote by $\eta = \max_{\omega \in \Omega} [\overline{R}(x, r_\omega) - R(x, r_\omega)]$, then we have

$$\eta = \max_{\omega \in \Omega} [\overline{R}(x, r_\omega) - R(x, r_\omega)]$$

$$< \max_{\omega \in \Omega} [\overline{R}(x^*, r_\omega) - R(x^*, r_\omega)]$$

$$< \eta^*$$

which contradicts that (x^*, η^*) is an optimal solution to (6).

□

D. Two Stage Stochastic Programming Model with recourse

Here a dynamic model is introduced where not only the uncertainty of the returns is included in the model but future changes, recourse, to the initial compositions is allowed. The portfolio optimization model is formulated by assuming the investor can make corrective action after the realization of random values by changing the composition of the optimal portfolio. This can be done by formulating the single period stochastic linear programming models with the maximum downside deviation measure as a two-stage stochastic programming problem with recourse. The two-stage stochastic programming problem allows a recourse decision to be made after uncertainty of the returns is realized.

Consider the case when the investor is interested in a first stage decision x that hedges against the *risk* of the second-stage action. At the beginning of the investment period, the investor selects the initial composition of the portfolio, x. The first stage decision x is made when there is a known distribution of future returns. At the end of the planning horizon, once a particular scenario of return is realized, the investor rebalances the composition by either purchasing or selling the selected stocks. In addition to the initial, or first stage decision variables, x, let a set of second stage variables, $y_{i,\omega}$ to represent the composition of stock i after rebalancing is done, i.e., $y_{i,\omega} = x_i + P_{i,\omega}$ or $y_{i,\omega} = x_i - Q_{i,\omega}$ where $P_{i,\omega}$ and $Q_{i,\omega}$ are the quantity purchased and sold respectively. $y_{i,\omega}$ is selected after the uncertainty of returns is realized.

E. Linear representation of MM

Before the two stage stochastic programming models for portfolio optimization problem to minimize the second stage risk measure is formulated, lets formulate maximum downside deviation of portfolio returns from the expected return in terms of the second stage variables y.

Let $\kappa(R(y_\omega, r_\omega)) = MM[R(y_\omega, r_\omega)]$

$$= \max_{\omega \in \Xi} [\overline{R}(y_\omega, r_\omega) - R(y_\omega, r_\omega)] \qquad (7)$$

For every scenario $\omega \in \Omega$, let the auxiliary variable

$$\eta = \max_{\omega \in \Omega} [\overline{R}(\, y_\omega, r_{,\omega}) - R(\, y_\omega, r_\omega)]$$

Subject to

$$\eta \geq \max_{\omega \in \Omega} [\overline{R}(\, y_\omega, r_\omega) - R(\, y_\omega, r_\omega)] \text{ for } \forall \omega \in \Omega \qquad (8)$$

Then , we have

$$\text{MM}[R(\, x, r_\omega)] = \eta \qquad (9)$$

Subject to

$$\eta \geq \max_{\omega \in \Omega} [\overline{R}(\, y_\omega, r_\omega) - R(\, y_\omega, r_\omega)] \text{ for } \forall \omega \in \Omega$$

F. Two Stage Stochastic Linear Programming formulation of 2S_MM

The two stage stochastic linear programming model for portfolio optimization problem that hedge against second stage MM is formulated as follows:

Definition 2: 2S_MM

The stochastic portfolio optimization problem where the maximum downside deviation of portfolio returns from the expected return is minimized and constraining the expected portfolio return is:

Minimize $\quad \eta \qquad\qquad\qquad\qquad\qquad$ (10a)

Subject to

$$\sum_{i \in I} x_i = 1 \qquad\qquad\qquad\qquad (10b)$$

$$\sum_{i \in I} y_{\omega i} = 1 \qquad\qquad \forall \omega \in \Omega \qquad (10c)$$

$$\overline{R}(\, x, r_\omega) + R(\, y_\omega, r_\omega) \geq \alpha \quad \forall \omega \in \Omega \qquad (10d)$$

$$L_i \leq x_i \leq U_i \qquad\quad \forall i \in I \qquad\qquad (10e)$$

$$L_{\omega i} \leq y_{\omega i} \leq U_{\omega i} \quad \forall i \in I, \, \forall \omega \in \Omega \qquad (10f)$$

$$R(\, y_\omega, r_\omega) \geq \eta \quad \forall \omega \in \Omega \qquad\qquad (10g)$$

Model (10) minimizes the maximum downside deviation of portfolio return from the expected portfolio return of the second stage variables, y, at the end of the investment period. Equation (10b) insures that the total weights of all investments in the first stage sum to one, and equation (10c) insures that the total weights of all investments in the second stage under each scenario ω sum to one that is, budget constraints ensuring full investment of available budget. Equation (10d) constraints the expected return by the target return, α, while equations (10e) and (10f) insures that the weights on assets purchased are nonnegative, disallowing short sales and place upper bound on the weights in the first stage and second stage respectively. Finally equations (10g) and (10h) define the maximum downside deviation of portfolio returns from the expected portfolio return in the second stage and the auxiliary variables for the linear representation of the deviation measure.

III. NUMERICAL ANALYSIS

The models were tested on ten common stocks listed on the main board of Bursa Malaysia. These stocks were selected at random from a set of stocks that were already listed on December 1989 and still in the list on May 2004. The closing prices were obtained from Investors Digest. At the beginning, sixty companies were selected at random. Then, ten stocks were selected and the criterion used to select the ten stocks in the analysis is described as follows:

i) Those companies which do not have complete closing monthly price during the analysis period are excluded.

ii) Since the portfolios are examined on the basis of historical data, those with negative average returns over the analysis period are excluded.

The empirical distributions computed from past returns are use as equiprobable scenarios. Observations of returns over N_S overlapping periods of length Δt are considered as the N_S possible outcomes (or scenarios) of the future returns and a probability of $\frac{1}{N_s}$ is assigned to each of them. Assume that we have T historical prices of the stocks under consideration. For each point of time, the realized return vector over the previous period of 1 month is computed, which will be further be considered as one of the N_S scenarios for the future returns on the assets. Thus, for example, a scenario r_{is} for the return on asset i is obtained as

$$r_{is} = \frac{P_i(t+1) - P_i(t)}{P_i(t)} \qquad\qquad (11)$$

For each stock, 100 scenarios of the overlapping periods of length 1 month are obtained, i.e. N_S.

To evaluate the performance of the two models, the portfolio returns resulting from applying the two stochastic optimization models are examined. The comparison between S_MM and 2S_MM models is made by analyzing the optimal portfolio returns in-sample portfolio returns and out-of-sample portfolio returns over 60-month period from to 06/1998 to 05/2004. At each month, the historical data from the previous 100 monthly observations is used as scenarios to solve the resulting optimization models and the return of the optimal portfolio is recorded. Then the in-sample realized portfolio return is calculated. The clock is advanced one month and the out-of-sample realized return of the portfolio is determined from the actual return of the assets. The same procedure is then repeated for the next period and the average returns were computed for in-sample and out-of-sample realized portfolio return. A minimum monthly required return α equals to one is used in the analysis for both the S_MM and 2S_MM models.

A. Comparison of Optimal Portfolio returns between S_MM and 2S_MM

Fig. 1 presents the graphs of optimal portfolio returns resulting from solving the two models; S_MM and 2S_MM.
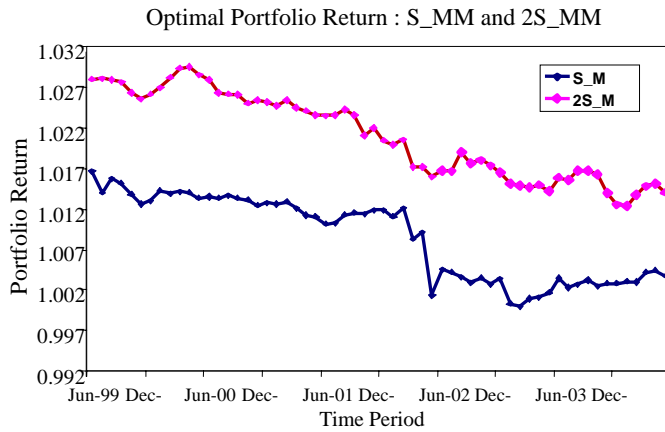
Fig. 1 comparison of optimal portfolio returns between single stage S_MM and two stage 2S_MM models

The optimal portfolio returns of the two models exhibit the same pattern. There is a decreasing trend in the optimal returns in both models. However, in Fig. 1, it can be   seen that the optimal portfolio returns from the two stage stochastic programming with recourse model, 2S_MM are higher than the optimal portfolio returns from the single stage stochastic programming model, S_MM in all testing periods.   This shows that an investor can make a better decision regarding the selection of stocks in a portfolio when he takes into consideration both making decision facing the uncertainty and the ability of making correction actions when the uncertain returns are realized compared to considers only making decision facing the uncertainty alone.

B.   Comparison of Average In-Sample Portfolio returns between S_MM and 2S_MM

The average realized returns are used to compare the in-sample portfolio returns between S_MM model and 2S_MM model and the results are presented in Fig. 2.
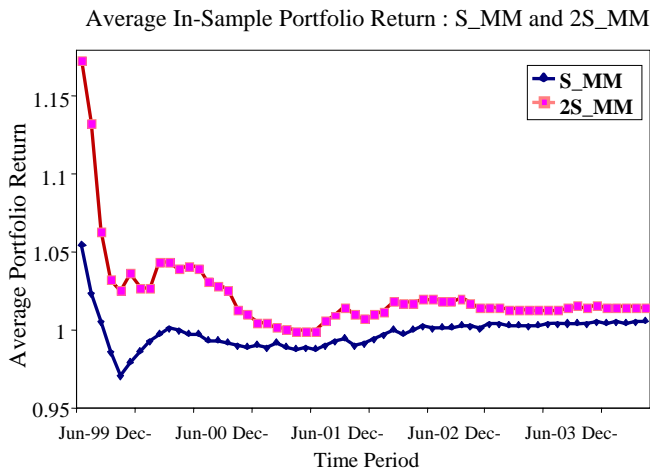

Fig. 2 comparison of the average in-sample portfolio return between single stage S_MM and two stage 2S_MM models

There is an increasing trend in the months from December 1999 until April 2000, then decreasing trend until June 2001. Starting from June 2001 until May 2004, both averages show an increasing trend. The average in-sample portfolio returns of 2S_MM are higher than the average in-sample portfolio returns in all testing periods.

C.   Comparison of Out-Of-Sample Portfolio returns between S_MM and 2S_MM models

In real-life environment, model comparisons are usually done by means of out-of-sample analysis. Several approaches can be used in order to compare models. One of the most commonly applied methods is based on the representation of the out-of-sample returns of the selected portfolios over a given period and on their comparison against a required level of return.

The comparison of out-of-sample portfolio returns between the single stage stochastic programming model, S_MM and the two stage stochastic programming with recourse model, 2S_MM are also done using the average return. The results of Out-Of-Sample analysis are presented in Fig. 3.
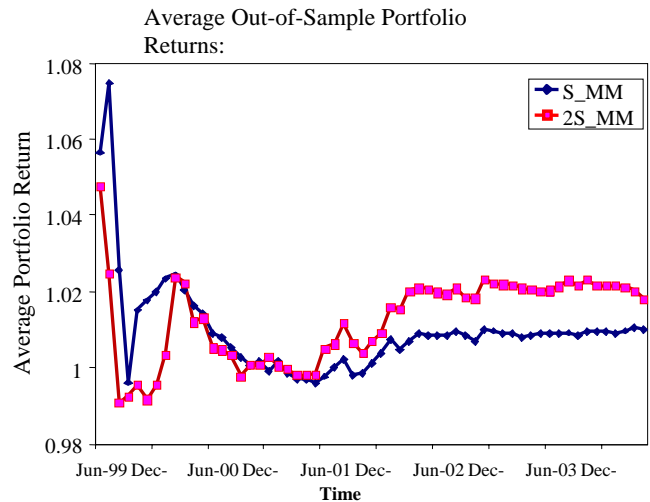

Fig. 3 comparison of out-of-sample portfolio returns between single stage S_MM and two stage 2S_MM models

Throughout the testing periods, the average returns from the two models show similar patterns. There is an increasing trend in the months from December 1999 until December 2000, then decreasing trend until June 2001. Starting from June 2001, both averages show an increasing trend. The average out-of-sample of the two-stage model, 2S_MM is higher than those of single stage model, S_MM. Certainly, the models have been applied directly to the original historical data treated as future returns scenarios thus loosing the trend information.   Possible application of some forecasting procedures prior to the portfolio optimization models, we consider, seems to be an interesting direction for future research. For references on scenarios generation see [21].

IV. CONCLUSION

In this paper, a portfolio selection of stocks with maximum downside semi deviation measure is modeled as a single stage

and a two stage stochastic programming models. Single stage model incorporates uncertainty in the model and in the two stage model the uncertainty is incorporated in the models and at the same time considers rebalancing the portfolio composition at the end of investment period. The comparison of the optimal portfolio returns, the in-sample portfolio returns and the out-of-sample portfolio returns shows that the performance of the two stage model is better than that of the single stage model. Here, we use historical data as scenarios of future returns. In our future research we will generate scenarios of future asset returns using appropriate scenario generation method before applying to our developed models.

REFERENCES

[1] H.M. Markowitz, "Portfolio Selection," *Journal of Finance.* vol. 8**,** pp. 77-91, 1952.

[2] H.M. Markowitz, *Portfolio Selection:Efficient Diversification of Investmen*t. New York: Wiley& Sons, 1959.

[3] H. Konno, and H. Yamazaki, "Mean-Absolute Deviation Portfolio Optimization Model and Its Application to Tokyo Stock Market," *Management Science*. vol. 7, pp. 519-531, 1991.

[4] F.A. Sortino and H.J. Forsey, "On the Use and Misuse of Downside Risk," *Journal of Portfolio Management*. Winter : 35-42, 1996.

[5] M.R. Young, "A minimax portfolio selection rule with linear programming solution," *Management Science*. vol. 44, pp. 673-683,1998**.**

[6] W. Ogryszak, "Multiple Criteria Linear Programming Model for Portfolio Selection," *Annals of Operations Research*, vol. 97, pp. 143-162, 2000.

[7] G.B. Dantzig, "Linear programming under uncertainty,". *Management Science.* vol. 1, pp. 197-206, A16,1955.

[8] E.M.L.Beale, "On Minimizing a Convex Function Subject to Linear Inequalities," *Journal of the Royal Statistical Society Bell Syst. Tech. J.*, Series B 17, pp. 173-184, 1955.

[9] D.B. Crane, " A stochastic programming model for commercial bank bond portfolio management," *Journal of Financial and Quantitative Analysis*. vol. 6, pp. 955–976, 1971.

[10] K.J. Worzel,K.J., C. Vassiadou-Zeniou S.A. Zenios, A. (1994). Integrated simulation and optimization models for tracking fixed-income securities. *Operations Research*. vol. 42, no. 2, pp. 223–233, 1994.

[11] S.A. Zenios, M.R. Holmer, R. McKendall and C. Vassiadou-Zeniou, "Dynamic models for fixed income portfolio management under uncertainty," *Journal of Economic Dynamics and Control*. vol. 22, pp. 1517–1541, 1998.

[12] R.S. Hiller and J. Eckstein, "Stochastic dedication: Designing fixed income portfolios using massively parallel Benders decomposition," *Management Science*. vol. 39, no. 11, pp. 1422–1438, 1994.

[13] S.A. Zenios, " Asset/liability management under uncertainty for fixed income securities," *Annals of Operations Research*. vol. 59, pp. 77-98, 1995.

[14] A. Consiglio and S.A. Zenios, " Integrated simulation and optimization models for tracking international fixed income indices," *Mathematical Programming*. vol. B89 , no. 2, pp. 311–339, 2001.

[15] K.W. Chang, H.J. Chen and C.Y. Liu, "A Stochastic Programming Model for Portfolio Selection.,"*Journal of Chinese Institute of Industrial Engineers*, vol. 19, no. 3, pp. 31-41,2002.

[16] R.T. Rockafellar, S. Uryasev and M. Zabarankin, " Deviation Measures in Risk Analysis and Optimization," ISE Dept., University of Florida, Tech. Rep. 7, *2002,*

[17] P. Artzner, F. Delbaen, J.M. Eber. and D. Heath, " Coherent measures of risk," *Mathematical Finance*. vol. 9, no. 3, pp. 203-228, 1999.

[18] R.T. Rockafellar, S. Uryasev and M. Zabarankin, "Generalized Deviation in Risk Analysis," ISE Dept., University of Florida. Tech. Rep. 4, 2004.

[19] C. Acerbi "Spectral measures of risk: A coherent representation of subjective risk aversion," *Journal of Banking Finance*. vol. 26, no. 4, pp. 1487-1503, 2002.

[20] A. Ruszczynski and A. Shapiro, " Optimization of convex risk functions," Available: http:// www.optimization~online.org. //04

[21] D.R. Carino, D.H. Myers and W.T. Ziemba, " Concepts, Technical Issues and Use of the Russel-Yasuda Kasai Financial Planning Model" *Operations Research*. vol.46, pp. 450-463,1998.