Simulation methods for the heat distribution systems

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Abstract-This study deals with the issues of heat consumption in the urban agglomeration and problems linked with modeling and simulation of systems providing heat supply. In this article we will call such system SHDC (System of Heat Distribution and Consumption). The key problem in the controlling mechanism of such systems consists in transport delay of transferring heat media. Therefore the control mechanism must operate in prediction kind of mode. The two control parameters, or control variables, are temperature of heat carrier and its flow rate. Their time behavior must be predicted for efficient control of the whole heat energy supply. Main focus will be placed on description of the proposed and implemented computer model of the heat distribution system in the selected agglomeration. This model is proposed as a discrete simulation model. The simulation is one of the methods, which can be effectively used for the analysis of large and complex dynamic systems properties such as the municipal heating networks - distribution and heat consumption system. The model is implemented in the form of computer applications and tested on real operational data.

Keywords- Consumption, distribution, heat, modeling, simulation.

I. INTRODUCTION

THEquestions of distribution and consumption of heat energy in the urban agglomeration are very topical, especially in the context of finite worldwide classic energy resources and the constant increase in energy prices. There is also important ecological aspect, because obtaining and using of energy generally has mostly negative environmental impact. Especially the coal-fired power is considerable burden on nature. On the other side, nuclear power plants represent unacceptable risk for many countries. Therefore, it is necessary to seek all paths leading to energy production, distribution and consumption efficiency [2].

The effective management of distribution and subsequent management of production of the heat energy is one of the factors that can lead to savings of energy. Heat energy must be transported to the place of consumption in time when it is required and in the expected quantity and quality. Quality of supplied heat energy is expressed in the temperature of heat transferring media. The correct delivery time, quantity and quality of heat energy must go hand in hand with minimal distribution costs [5]. It is obvious that the heat distribution is inextricably linked to its consumption and therefore we can talk about the management of the heat distribution and consumption.

This paper deals with the issues of heat consumption in the urban agglomeration with centralized heat energy source – heating plant - and a large distribution network. Such system is in this paper called SHDC (System of Heat Distribution and Consumption) and its principal scheme is shown in Fig. 1.

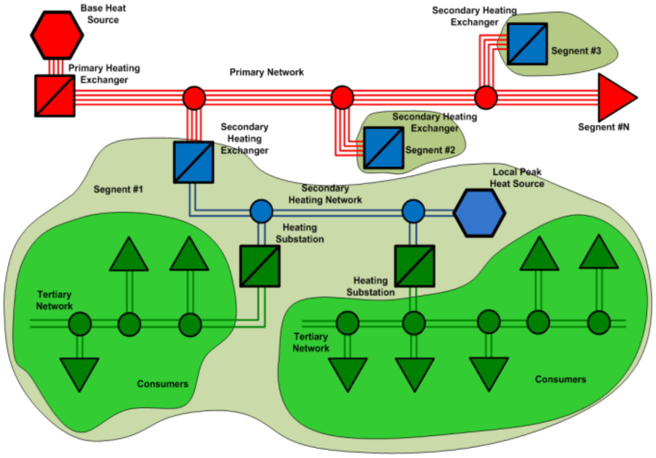

Fig. 1Schematic diagram of SHDC [11]

All three areas - energy production, distribution and consumption - are closely related and is optimal when they work in mutual energy balance. However, it must be said that it is the energy balance in the time interval, not the energy balance at each time point. This means that the amount of thermal energy that is consumed for a certain time period Δt starting in time t_i, must be produced in time advance and transported from place of production to place of consumption, see Fig. 2. The distribution system has a specific, and usually not small, ability to accumulate heat energy and it is able to compensate immediate differences between the amount of heat produced and consumed energy. The size of accumulation is determined by the amount of heat energy bound in heat transfer media - usually water - in the distribution system.

For the simplification, assume the same length time interval Δt for produced and consumed heat. Denote the$\delta(t)$ as the time interval between heat production and consumption (transport delay), $P_p(t-\delta(t))$ as production heat power in time$(t-\delta)$, $P_c(t)$ consumption heat power in time t and considering the time interval Δt, the following equation for heat energy balance can be expressed:

$$\int_t^{t+\Delta t}(P_P(t-\delta(t))dt = \int_t^{t+\Delta t} P_C(t)dt + \int_t^{t+\Delta t} A(t)dt \tag{1}$$

which can be simply written

$$Q_P = Q_C + A \tag{1a}$$

There:

Q_p is the heat produced during the time interval Δt, however started in time t $-\delta$, prior to the consumption time.

Q_c is the heat consumed during the time interval Δt and

A is the change of heat energy accumulation in the distribution network in the time interval Δt.

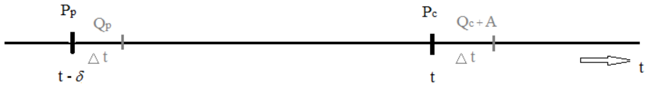

Fig. 2 Discretized timeline

For the effective procedure of management such system, i.e. procedure which will lead to savings in heat energy, it must respect the dependencies expressed in equation (1). The goal is to reach a situation where changes of accumulation, which results in changing the temperature of the heat transfer medium at the outlet of the distribution network, were minimal. Thus, the temperature of the returned water should fluctuate as little as possible.

It is obvious that the analytical solution satisfying equation (1) is very difficult, especially for the following reasons:

1) The time course of consumed heat Q_c is not possible to accurately predict because it is a stochastic variable depending on many factors, Highlights from the outdoor temperature at the point of consumption, time of day and day type (weekday, weekend day or holiday). The prediction of the external temperature is itself also a stochastic variable with relatively high variance.

2) Determination of transport timing delay, which is also variable and stochastic variable, it is also very complicated. Its size generally depends on the sizes of Q_p and Q_c, which are active in times. Moreover the Q_c, generally consists of partial consumptions Q_{ci} in different parts of the distribution network and the activity time is different for each Q_{ci}.

This is why our research was focused on implementation of the computer model of the distribution system of heat consumption SHDC. This model is designed as a discrete simulation model with a number of freely usable parameters

There are many different approaches for simulation models and operational optimization of heating networks [6]; [2] and heat-load modeling [7]. Our approach is to use data mining combined with simple model of heating network [7]; [9]. The adaptive parts of the model utilize real data measured in distribution systems to set up its internal structures for subsequent use in prediction and regulation.

For our model, the distribution network of chosen city was simplified and model was trained on the real measured data [5]. The main aim of all experiments faces the question: When and how much heat energy should be produced and what temperature is to set-up for hot water supply. Several experiments, presented in this article lately, have confirmed importance of finding days whose outside temperature is close to one we are just need to control. Thus, for “tomorrow” we need to know weather forecast to choose right intervention in heating water temperature. Based on the weather forecast the database can be scan to select day with likely the same values of outside temperature. The found day becomes the base for “tomorrow” heat supply proposal. The expectations are that the data from that day are telling us consumption needs and also provide information about trace in time.

Simulation and control can be described in the following steps:

1) *obtain weather forecast for day to propose*,
More frequent updates and increased accuracy for a particular location is an advantage

2) *seek and choose best matching day from the past*
Looking for days of ancient history has no sense, e.g. previous years, heating season, because the system is constantly changing. Days from the surrounding area should be preferred. It is also advisable to monitor the previous days, because if it is such a day following a significant change in the weather, the behavior of consumers is considerably in an unstable state)

3) *train the model*
Behavior of consumers is such nonlinear that find a general function describing its behavior is practically impossible. Better is to identify a particular period and model optimization for a given situation.)

4) *predict behavior for proposing day*
System trained on a similar day can learn from any mistakes and optimize the management of individual variables to the optimal operation)

II. SIMULATION MODEL

The simulation is one of the few methods, which can be effectively used for the analysis of large and complex systems. For the simulation is characteristic to create a model of the system, usually an abstract model and today almost exclusively a computer model. With this model are performed experiments and their results are then applied to the original system [7].

The simulation model is usually created in several steps:

1) *Analysis of the original system and the creation of a conceptual model.* Here you decide what type of model used (continuous or discrete, deterministic or stochastic, etc.), open or sealed model, what parameters that can influence and modify the properties of the model will be included in the model.

2) *Collection of data for model validation.* If the modeled and simulated system exists, it is mostly a historical operational data, where we know the values of the input data, and system behavior (output data)

3) *Programming a computer simulation model.* The model can be programmed in either a universal language, currently probably in object-oriented language, or you can use any available simulation language or a general simulation system.

4) *Verification and validation of created simulation model* using test data collected.

So created verified and validated simulation model is prepared to subsequent using.

The use of a simulation model in this case is fundamentally twofold:

1) *offline model using*
   It is possible to analyze the properties and behavior of simulated system as a wide dynamic system
2) *online model using*
   Simulation model can be integrated into the control system simulated system as an instrument to prediction of its behavior at a certain (limited) time in the future.

Proposed model of SHDC is, in contrast to the commonly used continuous models, discretized in time - time is running in simulation discontinuous steps, with a sampling period. The size of the time period depends on the speed of change of monitored values. Therefore, if this model will be used for analysis by going to the SHDC run in normal mode and not going with great speed changes, such as surges in the piping, etc., it is possible to work with the time period of several minutes. For this kind of analysis is used concept of the simulation model justified.

This relatively long sampling period is also important for the second of described ways of using the simulation model, because in the time period of several minutes we can perform numerous calculations, which are usually needed for predicting the behavior of SHDC in the near future, i.e. online model using.

A. Model description

The distribution network can be presented as a set of sources of heat energy (supply heating stations) and heat consumers, which are cross connected through piping. The pipes and heat consumers are concentrated for model in sections, which are connected in nodes. Section starts and ends in the node and can be divided in several elements of the distribution net (pipe lines and heat consumers). Each this element has its own constant characteristics from the point of view flow and heat transfer. One simple example of one part of distribution network is presented in the Fig. 3,

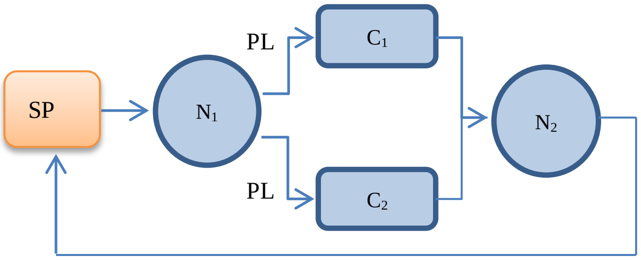

Fig. 3 Schematic of the distribution network elements

where:

- C consumer,
- PL pipe line
- N node,
- S section,
- SP supply (source)

Simulation time is running in discrete time intervals constant length signed as Δt. Time interval Δt is identical to the sampling time interval and Δt_j determine the simulation step *j*. As basic "moving" element ("transaction" in simulation terminology) consider "discrete flow quantum" DFQ of fluid (water). The DFQ flows through the network and gradually loses its energy, depending on the current position. The volume of the quanta is determined by the quantity of water entering into the distribution network for the time interval Δt in given step of simulation. Amount of heat energy in DFQ is based on its quantity and its temperature [5].

Proposed model is used for modeling of two closely connected processes.

B. Flow modeling

To monitor the flow quantum passing through the distribution network, it is of course necessary to respect the fundamental physical laws applicable to the fluid flow and heat energy transfer - conservation of mass and energy and the law of continuity. Based on these laws was defined the rules for describing the mass flow in network nodes and in interconnecting pipes. Used rules are as follows:

1) While passing through the section, the DFQ don't alter its volume, its length *L* in a pipe varies depending on the diameter *D* of the current pipe line.
2) Each section is divided into pipe lines, as noted above. While DFQ_i is passing two consecutive pipe lines *p* and *q* of the section, DFQ_i is split into the two new DFQ- DFQ_{ip} and DFQ_{iq}. DFQ_{ip} is the part of DFQ_i, which remains in the pipe line *p* - does not reach the border between pipe lines *p* and *q*, DFQ_{iq} is contrary of DFQ_{ip} - part of DFQ_i which passed transition of *p* and *q* in current simulation step and switched into *q* pipe line.
3) The above mentioned DFQ_i's splitting rules are used also for each DFQ_i, which is entering node *k*. Each DFQ_i is split into two parts – DFQ_{ip} and DFQ_{iq}. DFQ_{iq} is the part of DFQ_i, which reached the node in particular simulation step and DFQ_{ip} is the part which does not reach it – remains in the pipe.
4) In node *k*, is for each output section *j*, i.e.section which is outgoing from node *k*, a new DFQ_j is created. From the law of continuity we can use equation:

$$\sum Vol(DFQ_{iq}) = \sum Vol(DFQ_j) \tag{4}$$

where:

- *Vol(DFQ)* describe function, which presents particular part of flow quantum

- $\sum$ on the left is processed for all section, which allows flow to come into the node
- $\sum$ on the right is processed for all sections, which allows flow to come out from the node

According to these rules is then modeled the mass flow in the distribution network. In each simulation step the flow quantum, denoted ${}^{j}DFQ_i$ in the network is monitored. For illustration the ${}^{j}DFQ_i$ in pipe is shown in the Fig.4,

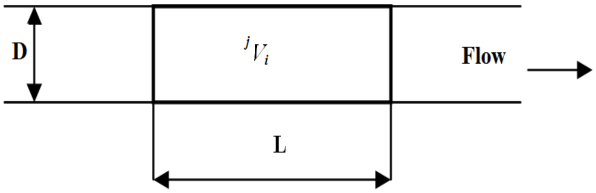

Fig.4 Discreet flow quantum ${}^{j}DFQ_i$

where:

- index i describe particular quantum,
- index j describe time period for ${}^{j}DFQ_i$ analysis, i.e. the simulation step,
- D is pipe diameter in current ${}^{j}DFQ_i$location,
- L is current ${}^{j}DFQ_i$ length,
- ${}^{j}V_i$ is volume of ${}^{j}DFQ_i$

We can apply simple terms:

$$ {}^{j}V_i = \pi * D^2 / 4 * L \qquad (2) $$

and

$$ {}^{j}V_0 = M_j * \Delta t_j \qquad (3) $$

where:

- ${}^{j}V_0$ is volume of DFQ on input into distribution network
- M_j mass flow in simulation step j

C. Heat transfer modeling

For each flow quantum, which is at a given time in the distribution network, its heat balance is calculated in every simulation step. The heat balance is based on law of the preservation of the thermal energy. As mentioned above, the time interval Δt may have length of the few minutes, so it was considered appropriate to model the temperature T of DFQ according to the formulas for cooled object. For it is valid differential equation.

$$ \frac{dT}{dt} = K * (T - T_{ext}) \qquad (4) $$

where

- K is the constant describing thermal characteristics for the particular element of the distribution network and heating medium (e.g. K for the pipe line depends on pipe wall material, its isolation, its diameter, velocity and specific capacity of the heating medium - hot water)
- T is the current temperature DFQ in the particular simulation step j,
- T_{ext} is the current outside temperature

Solving the equation (4), the (5) is obtained

$$ T_1 = exp\,(-K*t) * (T_0 - T_{ext}) + T_{ext} \qquad (5) $$

where:

- T_0 and T_1 are the water temperature at the beginning and end of the time interval Δt

The amount of the ΔQ, transferred in given time interval is then function of the temperature difference T_0 and T_1, i.e.:

$$ \Delta Q = c_p * V * (T_0 - T_{ext}) * (1 - exp\,(-K * \Delta t)) \qquad (6) $$

III. Model Use and Its Applicability

It is expected that the proposed simulation model will be used in the control system SHDC for the following purposes:

A. Identification of model parameters for the selected time period

Significant for the modeling approach to SHDC is to determine the function $s_r(...)$, which describes heat consumption for the consumer r. As the analysis of the real data show, the amount of heat consumed is mostly affected by the external temperature and time of the day. With using of (7), which describes the dependences on the T_{ext}, is possible for simplicity write

$$ s_r(...) = \lambda_r * ({}^{i}T_j - T_{ext}) * (1 - exp(-K * \Delta t)) * k_h \qquad (7) $$

where:

- ${}^{i}T_j$ is the current temperature DFQ_i for the particular simulation step j
- k_h is coefficient which corrects heat consumption oscillations during a day
- λ_r is the coefficient including other factors (e.g. another meteorological influences – sun intensity, wind etc.) which affect the consumption in r element of the distribution network

The coefficient k_h is considered as a function of time, for which discrete set of values (usually in one hour intervals) is specified. Assuming the daily frequency of calculation of consumption of heat, this is probably a reasonable assumption, so it is necessary to determine 24 coefficients k_h. When we use function $s_r(...)$ in form of equation (7) then for each k_h it is possible in the time between hours to use the value from the previous hour or use interpolated values. It is possible to use linear or other more complex interpolation. For our purpose of use k_h and needful accuracy, the linear interpolation is fully suitable.

To determine values of function k_h we can use the following procedure, which uses the simulation model in combination with genetic algorithm:

1) To select the appropriate length of time period, longer than one day. For the simplicity at the beginning of each simulation experiment we assume that the distribution network is empty. Therefore, for the beginning of the simulation, we must simulate the preceding interval, to let distribution network fills with hot water and let the state of the network become stabilized. This period is given at least as time delay for transportation, which means that all water from the source must have enough time to pass the system and come back to source again. Then we must follow at least one day period to obtain discrete values of k_h for each hour of the day.
2) For the chosen time interval to select and verify necessary historical data. For this task the following data are required (measured in time interval Δt):
    - Temperature of heating water T_v, measured on source output. This is also considered as an input for distribution network. For our purpose only one source is expected, but model is generally capable to use various combinations of sources and consumers.
    - Temperature of returned water T_{vv} – output of distribution network ergo reentry into source. It should be noted that this value is implicitly influenced by the size of the transport delay for distribution network but this is already covered in the structure of simulation model.
    - Total mass flow M in distributive network, again measured on the network input (output of the source)
    - Air temperature measured for particular points of distributive network. Quantities of these values will depend on the density of the network, its measuring points for the outside temperature at that location. If measuring points are located at different places than the landmarks of the distribution network, it is also possible to use interpolated values, in this case, interpolation in the area, i.e. two-dimensional. Also, if there are meteorological data measured in a different time period than Δt, the value interpolated in the timeline has to be used.

    Following data, if available, can help refine the whole calculation, e.g.:

    - Temperature T_v a T_{vv} in different parts of distributing network.
    - Pressure situation in the major individual network points. This is particularly important in cases where the topology of distribution networks is complicated. According to the pressure values the value of the mass flow in different sections of the distribution network may be determined - without the knowledge of pressure values can be mass flow in each section only estimated.
3) To determine searched values k_h for 24 points of timeline is possible to use several methods based on principles allowing us to find a function(s) which should have the best course approximating analyzed variables. One option is for example to use genetic algorithms. In the presented solution was the method PSO (Particle Swarm Optimization) used - see [8]. This method has been lately compared with other methods, such as SOMA, neural networks [10], and Levenberg-Marquard algorithms for nonlinear methods of least squares. It was found that the results, achieved in terms of accuracy and speed of convergence, are similar. PSO is therefore comparable for the determination of the correction factors and we use it.
4) The main idea of calculation is that in every simulation step the value of temperature of returned water T_{vv} is calculated. Calculated values are considered as points of approximating function for variable T_{vv}.
5) For calculating the approximating function the simulation model will be used. For each simulation experiment, we set sought coefficient k_h for selected points in the timeline to values generated by the PSO algorithm. With every set of parameters one simulation experiment is processed for the selected time slot according to point 1. Rate of the quality of approximation is calculated according to equation (9), i.e.

$$F = \sum ({}^{j}T_{vvmeas} - {}^{j}T_{vvcalc})^2 \tag{9}$$

where:

- F is value of approximation accuracy (fitness),
- ${}^{j}T_{vvmeas}$ is measured value of returned water temperature for the time interval Δt_j and
- ${}^{j}T_{vvcalc}$ is calculated value of returned water temperature for the simulation step j

The sum is performed for the entire simulated period in each simulation step. It is thus the sum of squares of deviations between the measured values T_{vv} (the values measured on the real system) and the calculated values T_{vv} (the values calculated using the simulation model) in terms of the search parameters - the values of k_h in 24 points of the timeline.

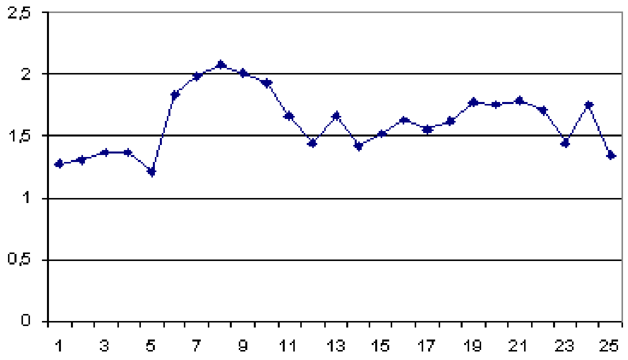

Fig. 5 Calculated k_h coefficient

The calculated F value is transmitted into PSO algorithm, which will use it to generate the next set of values of the coefficients of k_h, with which the process repeats. After a certain number of iterations (from practical results, there is a few thousand iterations necessary), the calculated values T_{vv} are close to measured values T_{vv} with sufficient accuracy. So designated coefficients k_h can be then used in the simulation model as the parameters for its further use - see section II in chapter IV.

An example result of the calculation of the coefficient k_h was shown in Fig. 5.

An example for comparison of measured and predicted values of T_{vv} in identified model, based on calculated coefficients, is shown in Fig. 6.

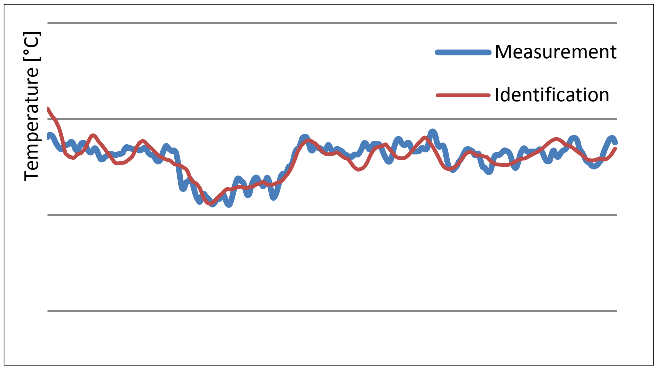

Fig. 6 Identification of SHDC - calculated and measured temperature of T_{vv}.

It is obvious, that the described procedure can be reused if the typical pattern of consumption of heat will vary in different types of days - a working day, weekend day, holiday, etc. In this case, it is necessary to repeat the procedure for each typical day, with the use of appropriate choice of time period in accordance with point 1. Then, for each type of day we obtain own set of values k_h that the model "switch" according to the type of the day which is being modeled.

These procedures are thus used to identify the model, i.e. to find such values of model parameters that provide the most accurate approximation of the function characterizing the system behavior - in the case of SHDC it is the timing of T_{vv}.

It is also apparent that this procedure can be similarly used to seek the values of correction coefficients for other variables that affect the consumption of heat, like any other meteorological variable - sunshine, wind direction and magnitude, relative humidity, etc. [4]. Also, the procedure may be easily modified, if we assume a different shape and form of the function $s_r(\ldots)$.

B. *Prediction of appropriate timing of the supplied amount of heat*

It should be noted that the idea of building a predictor in the project gradually evolved and changed. At the beginning of the project, the opinion prevailed that it will be enough to identify a model for a longer period and once identified, the model could be used for this period without any modifications. But it turned out that this approach does not lead to sufficiently accurate results, the characteristics of the system are not sufficiently "stable". It is probably due to both the details of the processed model and major stochastic nature of the whole system. Therefore, the idea was abandoned, and the work moved in the direction of the procedure described below. Given the fact, that the work on the project is not yet finished, described process reflects the present state of the solution and may not be definitive.

The proposed procedure is based on several fundamental ideas:

1) SHDC will behave similarly under similar conditions.
2) Predicted period is suitable to choose short and necessary calculations (including simulation) to perform for the shorter period repeatedly. This is depending, as already mentioned, on speed changes inside the SHDC and size of sampling period Δt. If this period is few minutes, it is easy to perform calculations repeatedly.
3) To control the production and distribution of heat, there are two control variables - temperature T_v and mass flow M. It will be necessary to find and use an appropriate cost function, which allows the required amount of supplied heat to optimally divide to the parts obtained mass flow M and the temperature T_v. Search for this objective function is the task for other parts of the project and is not in this article further discussed.

IV. MODEL IMPLEMENTATION

Introduced model was implemented in the form of a software application. The program modules are written in Java, data – historical operational data, configuration and description data for distribution network, simulation results – are stored in database. Connection to database is realized through JDBC interface and all queries to database are defined by SQL statements. These chosen software tools and solution allow easy portability to different SW environments.

The basic class is class SIMULATOR, which realize separately one complete simulation experiment, i.e. one simulation run for whole simulated time period. This gives the possibility to write application as multithreaded, so that there are running in parallel several instances of the class SIMULATOR, everyone in their own thread. It gives higher performance for calculation, especially in case, when many simulation experiments must be provided.

V. Practical Experiments

The experiments for presented research took place in two areas. Both are medium size towns (consider central Europe scale), one with about 50 thousand people and second with about 70 thousand. However, the main difference is in the distribution system parts, which we monitor. For the smaller town we prepare model for secondary distribution. It covers systems for urban location, typically housing estate or groups of family houses. Such systems deal with distribution between heat exchanger station and individual door stations. In this case the model has resolution – "single houses".

The second model (bigger town) is broadly conceived. It covers primary distribution where the resolution is – "housing estates".

The models can bind together and cover most of the significant parts of the distribution systems.

A. *Primary distribution network simulation*

Based on similar day identification, the prediction was prepared. Because the system has a time delay of several hours, and already there have been several experiments, the prediction has been prepared for two days with a gradual refining. Following pictures show predicted and obtained values.

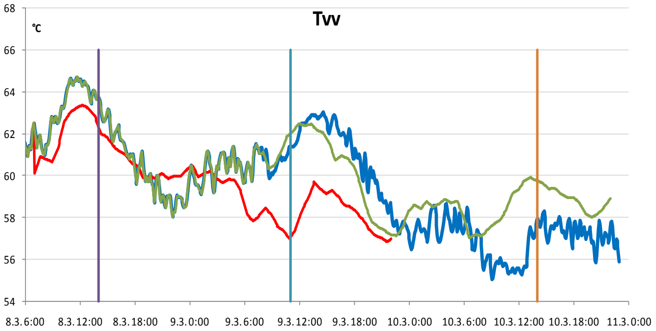

Fig. 7 Predicted and measured values T_{vv}(returned water)

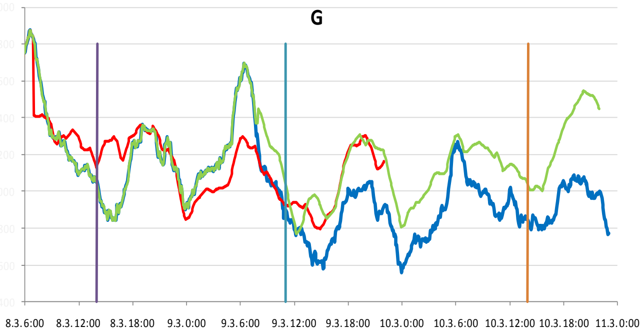

Fig. 8 Predicted and measured values G (mass flow)

where:

green curves show measured course,
red curves are predicted values and
blue curves refine the previous prediction and extend it to the next day.

B. *Secondary distribution network simulation*

With regard to proposed model requirements, the similar period were identified. Compliance between measured and output data after model identification is shown in Fig. 9.

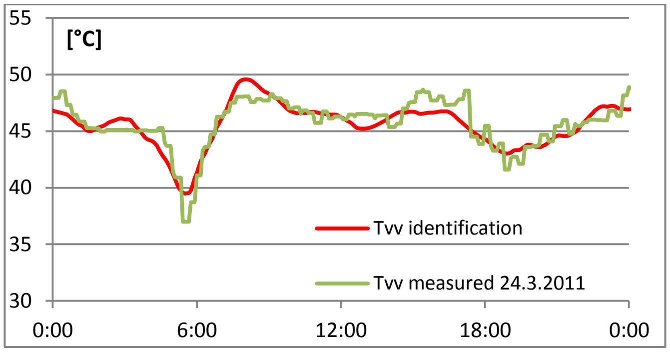

Fig. 9 Identification results

Successfully identified model were after that used for system behavior prediction. The result from the model was compared with the real system responses. Obtained results are shown in following pictures, where red curves are predicted and green measured courses.

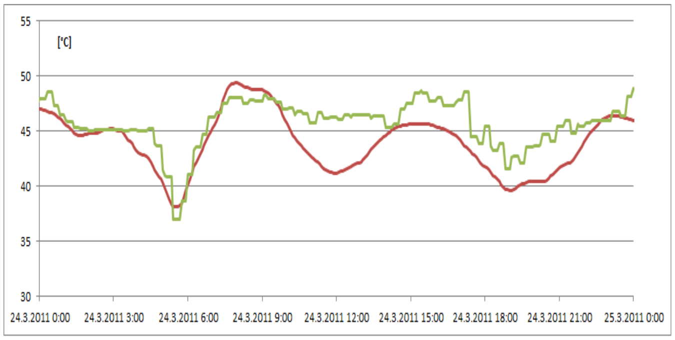

Fig. 10 Predicted and measured values T_{vv}(returned water)

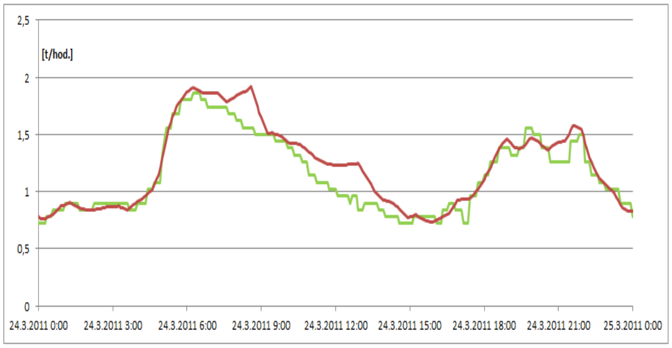

Fig. 11 Predicted and measured values G (mass flow)

VI. CONCLUSION

The results obtained during two experiments show, that the proposed simulation model is a well suited tool for analyzing the properties and behavior of SHDC. It also appears that the prediction accuracy of the individual variables is strongly dependent on the accuracy of weather forecasts especially the outdoor air temperature is very essential.

Introduced simulation model can be used also in different manner. It can be incorporated into the control system to predict the behavior of SHDC at a certain (limited) time in the future, to streamline the management of SHDC.

Application of the presented simulation model will be especially interesting for large systems, where is the large distance between production and consumption of heat, which means that a long time is needed to transfer thermal energy from source to consumer.

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