Mixing of two different temperature water flows as feedback controlled system mathematically modeled and simulated in MS Excel spreadsheet

S. Hubalovsky

***Abstract*—**One of the most important methods in current scientific and technological research is process of modeling and simulation of real experiment as well as modeling and simulation of of feedback regulated systems. System approach, modeling and simulation are discipline with its own theory and research methodology.

The paper focuses to the theory of the process of modeling and simulation, visualization of feedback controlled system. Multidisciplinary approach is point out too.

Step by step there will be shown the process of creation of static, dynamic and feedback controlled mathematical model. Mathematical model is supplemented by the simulation model realized in MS Excel spreadsheet. Visualization of the simulation model is realized in MS Excel *XY* chart.

***Keywords*—**Modeling, computer simulation, mathematic model, simulation model, static characteristic, dynamic characteristic, feedback controlled system.

I. Introduction

THE terms *system*, *model*, *simulation*, *multidisciplinary approach* are important in current approach to scientific, technological and professional practice. Many universities are realizing that modeling and simulation is becoming an important tool in solving and understanding numerous and diverse problems.

Modeling and simulation, is becoming one of the academic programs of choice for students in all disciplines – see e.g. in [1], [2], [3], [4]. Modeling and simulation is a discipline with its own body of knowledge, theory, and research methodology.

To engage modeling and simulation the mathematical model of the real system has to be first created. Models are approximations for the *real system*. The model is then followed by *simulation*, which allows for the repeated observation of the model. After the simulations model is verified, a third step takes place and that is *visualization* of the model and the real system.

The ability to define system, to build up mathematical model and to create simulation model develops logical thinking skills and imagination and is an inseparable part of a student's study skills for those studying the specializations "Applied Informatics".

In this paper we first briefly introduce to theory of modeling and simulation as a method of multidisciplinary investigation of feedback controlled system. Similar introduction can be found in [5], [6], [7].

Secondly we introduce a case study illustrating step by step process modeling and simulations of a feedback regulated system – *system of mixing of cold and hot water* in flow-throw water tank. First the mathematical model of the static, dynamic and feedback regulated system will be introduced. Finally the computer simulation model via MS Excel spreadsheet will be shown.

II. Modeling and simulation as multidisciplinary educational tool

A. Modeling

Modeling is a method that is often used in professional and scientific practice in many fields of human activity.

The main goal of modeling is describe the content, structure and behavior of the real system representing a part of the reality.

The models are always only approaching of the reality, because the real systems are usually more complex than the models are. The system homomorphism is applied in the process of modeling, which means that each element and interaction between the elements of the model corresponds to one element and interaction of the modeled real system, but the reverse is not true. The model is always to be understood as simplification of the original. If the relation of isomorphism is between the model and real system the original model we could not distinguish between the model and the original, which is discussed e.g. in [8].

The first step in the process of computer simulation is creation of mathematical model of the studied real system. The model can be obtained either theoretically based on basic physical properties of the system, or numerically by means of the measured values. Determination of parameters of theoretical model developed from empirical data is called system identification.

The mathematical model must adequately describe the dependency system outputs on its inputs. Models of physical systems are usually established as a system of mathematical equations as will be shown in the following paragraphs of this paper.

B. Simulation

The process of modeling is closely related to the simulation. Simulation can be understood as process of executing the model. Simulation enables representation of the modeled real system and its behavior in real time by means of computer. The simulation enables also visualization and editing of the model.

A typical simulation model can be written both through specialized programming languages that were designed specifically for the requirements of simulations, or the simulation model can be created in standard programming languages and spreadsheets (MS Excel).

From the above considerations, it is clear that simulation is a process that runs on the computer. In some publications, therefore, can be found the term "computer simulation". It generally is valid that computer simulation is a computer-implemented method used for exploring, testing and analysis of properties of mathematical models that describe the behavior of the real systems which cannot be solved using standard analytical tools, se e.g. [9].

The simulation models represented by executable computer program have to be isomorphic with the mathematical model that is a representation. It means that the mathematical model and simulation model have to represent the real system, its elements, internal interactions and external interaction with the environment in the same way.

In our paper the real system is simulated in MS Excel spreadsheet and visualized in MS Excel *XY* dependency charts.

C. Simulation of feedback regulated system

Many real systems are constructed so that they are able control themselves by means of the feedback.

Feedback is a mechanism or process that is looped back to control a system within itself. Such a loop is called a feedback loop. In the system containing the feedback loop the input characteristics or signals are influenced by output characteristics or signals. There are two types of the feedback:

- *positive feedback*, where increase of value of outputs increases value of inputs and conversely;
- *negative feedback*, where increase of value of outputs decreases value of inputs and conversely.

Feedback of the regulated system in engineering practice takes place through the controller. The controller is a device for influencing the regulated system, automated control and for achievement and maintaining its desired state. Typically is used in negative feedback of the system. The controller's input is usually not directly monitored output value of the entire system, but only deviation from the desired value. The controller controls the system to either the complete elimination of deviation, or to keep deviation within the prescribed limits. The controller reads the states of the system, either directly or, if unobtainable, is reconstructing the states from the model. The reading of the values of system is in time based on either continuous. Controlling of the system can be either analog, digital or step.

The computer simulation is essential method not only for representation of the mathematical model but can also be used for investigation of the feedback controlled systems. Simulation allows:

- realization of situation that cannot be investigated using conventional real experimental devices;
- setting the parameter of the feedback and its optimization as before the real control system is made up;
- visualization of the behavior of the feedback controlled system.

In our paper the real feedback controlled system is simulated in MS Excel spreadsheet and visualized in MS Excel *XY* dependency charts.

D. Significant function of the simulation

Simulation has from the scientific point of view several functions – see e.g. [9].

We will focus in this paper two of them and they are:

- replacing the real experiment;
- development of educational process.

1) Replacement of the real experiment

This is an important and indispensable feature of simulations and simulation model because it allows realize a situation that cannot be investigated using conventional real experimental devices. The main advantage of simulations is that simulations model allows changing of input parameters, visualization and optimization the effects of the real experiment. The simulation is usually safety and cheaper.

2) Development educational process

The simulation is very useful from educational point of view. Using the simulation model and visualization of simulation results on the screen, students can better understand the basic processes and systems and develop their intuition. It is also essential that the teaching by means of simulation is much cheaper and faster than the teaching carried by real experiment. In some cases providing the real experiment cannot be feasible.

Despite the fact that experimental education in the laboratory cannot be completely replaced (because students acquire manual dexterity, they learn to work with real laboratory instruments, they learn to plan, implement and evaluate realistic experiment), the simulations is a part and basic methods of scientific knowledge. Students can easily learn theoretical foundations of the laboratory tasks. The simulation model of the real laboratory task can help them to check some of the operations performed in the laboratory. This can reduce the direct lessons in the laboratory only to necessary time for their own experimental measurements. Alternatively, lessons can be realized only by the simulation models. In this case, it is important to note that students are deprived of contact with the real device, so that they will not get a full picture of the implementation of the experimental measurements.

The case study of this paper is typical example of the simulation of the real feedback controlled system suitable for educational purpose. The presented simulation application is created in MS Excel spreadsheet and visualized in MS Excel *XY* dependency charts.

E. Model verification and validation

Verification and validation are important aspects of the process modeling and simulation. They are essential prerequisites to the credible and reliable use of a model and its results [10].

1) Verification

In modeling and simulation, verification is typically defined as the process of determining if executable simulation model is consistent with its specification – e.g. mathematical model. Verification is also concerned with whether the model as designed will satisfy the requirements of the intended application. Verification is concerned with transformational accuracy, i.e., it takes into account simplifying assumptions executable simulation model. Typical questions to be answered during verification are:

- Does the program code of the executable simulation model correctly implement the mathematical model?
- Does the simulation model satisfy the intended uses of the model?
- Does the executable model produce results when it is needed and in the required format?

2) Validation

In modeling and simulation, validation is the process of determining the degree to which the model is an accurate representation of the real system. Validation is concerned with representational accuracy, i.e., that of representing the real system in the mathematical model and the results produced by the executable simulation model. The process of validation assesses the accuracy of the models. The accuracy needed should be considered with respect to its intended uses, and differing degrees of required accuracy may be reflected in the methods used for validation. Typical questions to be answered during validation are:

- Is the mathematical model a correct representation of the real system?
- How close are the results produced by the simulation executable model to the behavior of the real system?
- Under what range of inputs are the model's results credible and useful?

Validation and verification are both ultimately activities that compare one thing to another. Validation compares real system and mathematical model. Verification compares mathematical model and executable simulation model. Sometimes validation and verification are done simultaneously in one process.

Validation of the mathematical model as well as verification of the simulation model of our real system – seven storey rectification column – are to be done simultaneously by the comparison of the dependencies of the quantities theoretically calculated from the simulation model with the dependencies of quantities experimentally measured on rectification column.

The whole process of transformation from a real system, the simulation model and its visualization is shown in Fig. 1.

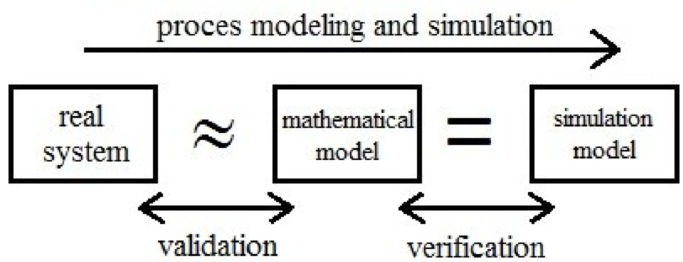


Fig. 1 Process modeling and simulation

Here again let us summarize that the mathematical model that reflects the real system has some limitations and simplifying assumptions (the real system and mathematical model are in homomorphic relation).

In contrast, the simulation model is only the computer expression of the mathematical model (the mathematical model and simulation model are in isomorphic relationship).

F. Multidisciplinary approach

Another important benefit associated with the process of modeling and simulation of real experiments is a multidisciplinary approach, without which the process of identifying the real system using mathematical and simulation model and cannot be realized. This is also emphasized in this paper.

Multidisciplinary approach generally means that specialized disciplines are applied in a study of real system. These disciplines provide partial analysis of the real system. These mono-disciplinary analyses are integrated to overall solution by integrating the solver who has basic multi-disciplines knowledge.

In our case study four disciplines are integrated, namely, experimental physics, thermodynamics, mathematics and computer science.

III. CASE STUDY – MODELING AND SIMULATION OF REAL SYSTEM

A. Problem definition

As it was already mentioned in the previous section, it is essential to emphasize modeling and simulation as one of the important educational tool, which helps students develop their logical thinking, intuition and can support understanding of the real process and experiment.

In the following parts of the paper case study used in education of subject Automation and Mechatronics in University of Hradec Kralove will be described. Within the case study the process modeling and simulation of feedback regulated system will be shown. Similar examples can be found e.g. in [1], [4], [5], [6], [11].

B. Description of the real experimental system

Real experimental system that is simulated in this paper is schematically shown on Fig. 2.

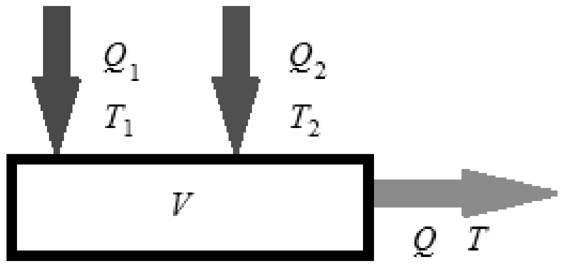


Fig. 2 Real experimental system

The system consists of the water container with volume V, two water inflows – the first of cold water with temperature T_1 and flow rate Q_1 and the second of warm water with temperature T_2 and flow rate Q_2. Overflow from the tank discharges water at the temperature T and flow rate Q. The water is perfectly mixed. The system enables changes the temperatures T_1 and T_2 as well as the flow rates Q_1 and Q_2.

This simple system enables represent step by step process of the creation of a mathematical model and simulation model for visualization of:

- static characteristics;
- transient characteristics;
- process of feedback.

The visualization of the characteristic is via the tables of the appropriate value characteristic dependencies and via the *XY* dependency charts. Detailed descriptions of the dependencies are a scope of the following parts.

IV. Mathematical model

In this section the transformation of above described real system (seven-storey rectification column) will be described. From multidisciplinary point of view the disciplines like physics and mathematics will be involved in.

The derivation of the common mathematical model is e.g. based on [12].

A. Static model

To measure the static properties of the systems it is necessary to realize the number of measurements so that the given value of the input variable u (input signal) is applied to the system input and after the system stabilization, i.e., after termination the transient phenomenon, we measure the value of the output variables y (output signal). Measurement is repeated for different values of the input signal. This provides the set of *XY* points - the dependency of the output variables on input variables – $y(u)$. By charting this dependency in *XY* graph the static characteristics of the system can be reached - see Fig. 3.

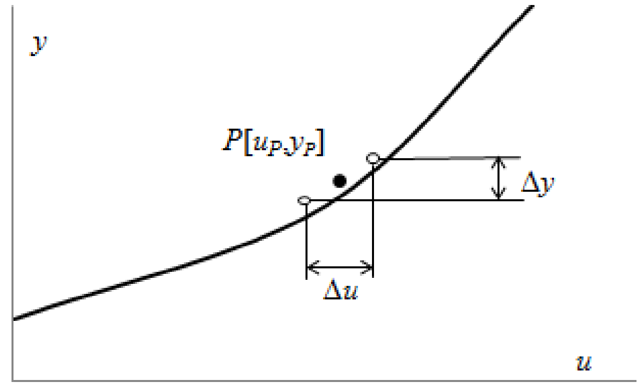


Fig. 3 *XY* graph of the static characteristic

The transition function which describes the static properties of the system can be obtained by approximation of the measured data via suitable mathematical method. Usually the transfer function is approximated by polynomial function of the given order. This function is understood as static mathematical model in the analytical form. This mathematical model allow for any value of the input signal (from measured range) calculate the corresponding output value.

Static properties of our discussed system can be described by following equations:

$$Q = Q_1 + Q_2 , \tag{1}$$

$$TQ = T_1 Q_1 + T_2 Q_2 , \tag{2}$$

where the input variables are T_1, T_2, Q_1 and Q_2, and output variables T and Q – for the description see par. III.

A very important parameter, which describes the static behavior of the regulated system, is *action gain Z* of the system. It is defined as the ratio of output value y to the value of the input variable u in steady state.

If there is a regulated system, at the point P (see Figure 2), the action gain of the system the point P can be approximately calculated as the ratio of change of the output signal - Δy, and the change of the input signal - Δu:

$$Z = \frac{\Delta y}{\Delta u} . \tag{3}$$

If the dependence $y(u)$ is described analytically, the gain of the system can be calculated as the value of the first derivative of this function at given operating point P.

$$Z = \frac{dy}{du} . \tag{4}$$

If we know the value of Z for given system and required value of output signal, the value of input signal, which has to be applied to input of the system, can be calculated to achieve the required output values.

In our discussed system the gain of output variable T can be studied depending on one of the output variables T_1, T_2, Q_1 and Q_2.

B. Dynamic model – transition characteristic

The measurements of dynamic characteristics of the system mean the investigation of the transition state of the system. The input signal as an appropriate function of time is applied to the system input and the response of the system is measured, i.e. time dependency of the output signal is measured. The time dependence of the input signal is usually selected in the form of a step function when the input signal is change from the initial value $u_0(t)$ to the value $u_S(t)$. This step value change can be expressed by following formulas:

$$u(t)=\begin{cases} u_0(t=0) \\ u_S(t\geq 0)=u_0(t=0)+\Delta u \end{cases}. \tag{5}$$

Time dependence of the output signal $y(t) = f(u(t))$ after the step change of the input signal $u(t)$ shown on the Fig. 4 is called as *transition characteristic*. The output signal is stabilizes to the value of $y_S(t)$.

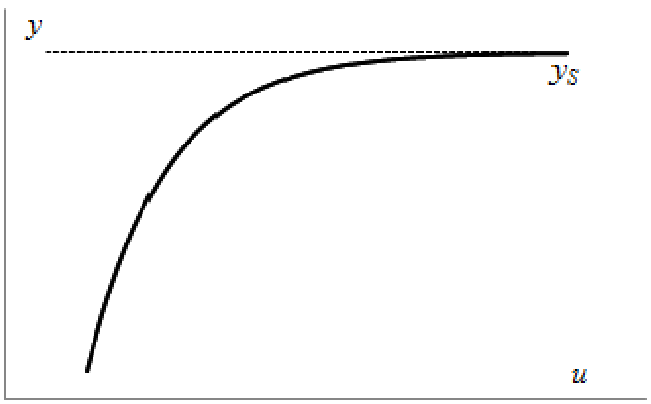


Fig. 4 *XY* graph of the transition characteristic

The dynamic characteristics of systems are mathematically described by differential equations, usually linear differential equations of n^{th} order with constant coefficients and constant initial conditions.

A dynamic model of the system can therefore be represented either graphically by the transient characteristic or analytically by the differential equation. The result of the differential equation is the time dependence of the output value to the step change of the input value.

The dynamic properties of our discussed system can be described by the following equation:

$$TQ+\frac{d(VT)}{dt}=T_1Q_1+T_2Q_2\,. \tag{6}$$

For a constant value of the tank volume V the equation can be rewritten:

$$TQ+V\,\frac{d(T)}{dt}=T_1Q_1+T_2Q_2\,. \tag{7}$$

The solution of this differential equation is:

$$T=T_\infty+\left(T_0-T_\infty\right)e^{-\frac{t}{\tau}}, \tag{8}$$

or

$$T=T_\infty\left(1-e^{-\frac{t}{\tau}}\right)+T_0\,e^{-\frac{t}{\tau}}, \tag{9}$$

where

$$\tau=\frac{V}{Q_1+Q_2}, \tag{10}$$

is time constant of the water tank.

$$T_0=\frac{T_1Q_1+T_2Q_2}{Q_1+Q_2},\quad t=0, \tag{11}$$

is steady state temperature of water at the beginning of measurement (before step change of the input values T_1, T_2, Q_1 and Q_2) and

$$T_\infty=\frac{T_1Q_1+T_2Q_2}{Q_1+Q_2},\quad t\geq 0, \tag{11}$$

is steady state temperature of water after the measurement of the transient characteristic (for the actual input values T_1, T_2, Q_1 and Q_2).

The differential equation (9) can be converted to numerical recurrent equitation:

$$T(k+1)=T_\infty\left(1-e^{-\frac{t_S}{\tau}}\right)+T(k)e^{-\frac{t_S}{\tau}}, \tag{12}$$

where $T(k)$ is steady state temperature of water in previous interval of measurement, $T(k+1)$ steady state temperature in the following time interval and t_S is sampling time interval.

The ratio of values of y_S and u_s equals to the gain of the system, which is identical with the thickness calculated from the static characteristics of the system.

The ratio

$$Z(t\geq 0)=\frac{y_S}{u_S} \tag{13}$$

equals the action gain of static characteristic given by equation (3).

C. Mathematical model of the feedback controlled system

The regulator or controller is integrated part of control circuit, which includes a converter, a summation element, a central control element that controls the activity of the control circuit and power amplifier. Hereafter, under the term controller will be understood the central element.

Controllers can be divided based on energy income to either *direct*, which take power supply directly from control circuit or *indirect*, which has the external power supply.

According to the character of the media, which carry the control signal the controllers can be divided into *mechanic*, *pneumatic*, *hydraulic* or *electric*.

According the character of the control signal we distinguish *analog controllers* and *digital controllers*.

Analog controllers can be from the transmission dynamics point of view divided to *proportional*, *integral* and *derivative*. In the control technology are widely used are proportional–integral–derivative controllers (PID controllers).

In the following part the mathematical description of the controllers' dynamics will be discussed. Let's assume ideal controllers working without any delay.

The most important characteristic used for mathematical description of the control systems are:

1. Error difference between required value (set-point) – $y_R(t)$ and current measured output value – $y(t)$:

$$e(t) = y_R(t) - y(t) \tag{14}$$

2. Controlled input action signal – $u(t)$.

It is also important to appreciate that the control action u counteracts the change in the controlled variable y, i.e. that y is function of u – $y(u)$.

1) Proportional (P) controller:

Response of the P controller is described by following equitation:

$$u(t) = r_0 e(t), \tag{15}$$

where constant r_0 is proportional gain of P controller.

Regarding the fact, that the action signal u is multiple of the error difference e, it is clear that the proportional controller is always working with the permanent control deviation.

2) Integral (I) controller:

Response of the I controller is described by following equitation:

$$u(t) = r_{-1} \int_0^t e(\tau) d\tau, \tag{16}$$

where constant r_{-1} is integral gain of I controller. I controller controls with zero control deviation.

3) Derivative (D) controller:

Response of the D controller is described by following equitation:

$$u(t) = r_1 \frac{de(t)}{dt}, \tag{17}$$

where constant r_1 is derivative gain of D controller. D controller controls with zero control deviation.

Itself D controller cannot be use for control. Usually combinations of PD, respectively PID are used. Proportional and integral terms affects the quality of regulation in steady-state, derivative term affects process of the control especially in the transition state.

4) PID analog controller:

Response of the PID controller is described by following equitation:

$$u(t) = r_0 e(t) + r_{-1} \int_0^t e(\tau) d\tau + r_1 \frac{de(t)}{dt}, \tag{18}$$

or

$$u(t) = r_0 \left[e(t) + \frac{1}{K_i} \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \right], \tag{19}$$

where K_i is integral constant and K_d is derivative constant.

PID controller is used to control complex dynamic systems.

5) PID digital controller:

Despite the fact the regulated systems are analog system, their control can be realized by ***digital controller*** usually implemented by digital computer.

Digital controller at certain time intervals – the *sampling interval*, scans the values of controlled variables y and transmits the corresponding values of control variable u.

Mathematical description of the response of digital PID controller can be derived from (19), where integration term is approximated by numeric summation and derivative term is approximated by the first numeric differences. The PID digital controller can be in k^{th} control step described by following equitation:

$$u(k) = r_0 \left(\begin{array}{l} e(k) + \\ + \dfrac{1}{K_i} K_t \displaystyle\sum_{j=0}^{k} \dfrac{e(j) + e(j+1)}{2} + \\ + K_d \dfrac{e(k) - e(k-1)}{K_t} \end{array} \right), \tag{20}$$

where K_t is sampling interval.

The equitation (20) can be rewritten to the incremental form used in computer simulation:

$$\begin{aligned} u(k) = q_0 e(k) + q_1 e(k-1) + \\ + q_2 e(k-2) + u(k-1), \end{aligned} \tag{21}$$

where u(k) value of control action signal in k^{th} step, and $e(k) = y_R(k) - y(k)$ is error difference between desired value and required value in k^{th} step. The parameters q_0, q_1, q_2 has the following form:

$$q_0 = r_0\left(1+\frac{K_t}{2K_i}+\frac{K_d}{K_t}\right), \tag{22}$$

$$q_1 = -r_0\left(1-\frac{K_t}{2K_i}+2\frac{K_d}{K_t}\right), \tag{23}$$

$$q_2 = r_0\frac{K_d}{K_t}. \tag{24}$$

Above mentioned general equitations of PID control system can be applied to our discussed real system - of mixing of cold and hot water in flow-throw water tank. In this case the control action signal u is given by one of the input values T_1, T_2, Q_1 or Q_2; required value y_R is given by required temperature T_R and current measured value y is given by current temperature T.

In the case the controlled action signal is given by Q_2 the equitation (21) has a form:

$$Q_2(k) = q_0e(k)+q_1e(k-1)+q_2e(k-2)+Q_2(k-1), \tag{25}$$

where $e(k)$ is given by:

$$e(k) = T_R(k)-T(k) \tag{26}$$

and q_0, q_1, q_2 are calculated from formulas (22), (23), (24).

Similar equations are valid if controlled action signal is given by Q_1, T_1 or T_2.

6) Properties of the closed control regulated system

For optimal realization of the regulation of the feedback controlled system it is necessary to explore not only the static and dynamic properties of the regulated system but also the behavior of a closed control circuit. The interest is taken in:

- stability of the controlled feedback system;
- response of the controlled input signal u and measured output signal y during the transition from the initial to the new operating point (during the step change of the required value y_R);
- eliminating the influence of random disturbances that affect the feedback.

There are a number of criteria for assessing the quality of the control process. In practice, however, a very simple assessment of controlled regulation process of step transition is used, see Fig. 5.

Quality of the control regulated process is characterized by:

- over-deviation of the measured output signal Δy;
- deviation period T_m;
- period of the regulated control step $T_{2\%}$.

All these three values are minimized in optimal control regulated process. In the synthesis of regulatory process it is necessary to:

- select the appropriate controller (P, D, I, PID);
- optimally the parameters of the controlled system.

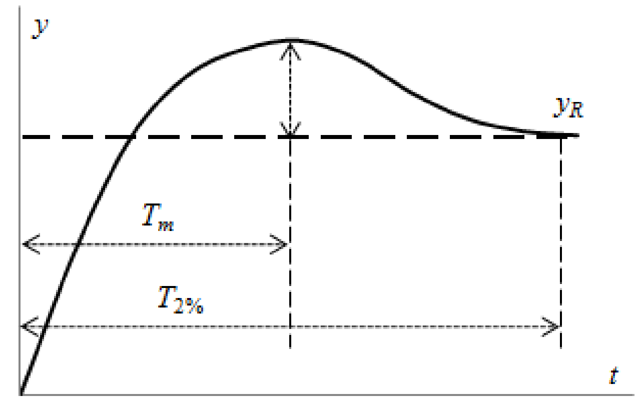


Fig. 5 Quality quantyties of the regulation process

V. Computer simulation model

In this section the above mentioned mathematic models (1. *static*; 2. *transition dynamic*; 3. *feedback regulated*) of the real system will be transformed to the computer simulation model. The computer model is to be realized in MS Excel spreadsheet. Visualization of the static and dynamic behavior of the characteristic as well as the behavior of the characteristic during the regulation control process is to be done by drawing the appropriate characteristic dependency in MS Excel *XY*-chart.

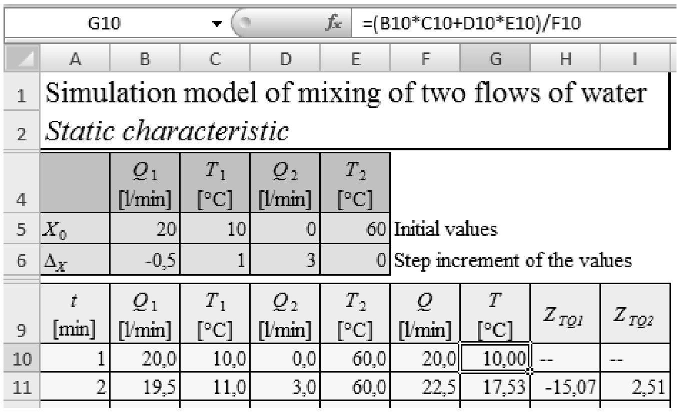
The basic time unit of the simulation models is taken as 1 minute. The output values of the simulation model are calculated from input values based on numerical calculations directly in MS Excel cells. Simulation models are constructed so that the given row of the MS table contains the input values and calculated output values corresponding to the given time from the beginning of the simulation.

A. Static simulation model

The input variables of the static simulation model are the flow rates Q_1 and Q_2 and the corresponding flow water temperatures T_1 and T_2. The input values are entered by their initial values at the beginning of the simulation and step increment of this values during one minute.

The output variables are Q, T and Z. Q is calculated according to equation (1), T is calculated according to the equation (2). As an example, the gain Z of temperature T depending on the Q_1 and Q_2 is determined.

The table with the input and output values is shown on the Fig. 6. The input values are entered in the rows 5 and 6. The calculations of output values needed for simulation model are carried out in the cells down from row 10. In our particular case, we have limited the calculation of the static simulation model in the time interval $\langle 0, 20\rangle$ min.


G10 =(B10*C10+D10*E10)/F10

Simulation model of mixing of two flows of water

Static characteristic

	Q_1 [l/min]	T_1 [°C]	Q_2 [l/min]	T_2 [°C]	
X_0	20	10	0	60	Initial values
Δ_X	-0,5	1	3	0	Step increment of the values

t [min]	Q_1 [l/min]	T_1 [°C]	Q_2 [l/min]	T_2 [°C]	Q [l/min]	T [°C]	Z_{TQ1}	Z_{TQ2}
1	20,0	10,0	0,0	60,0	20,0	10,00	--	--
2	19,5	11,0	3,0	60,0	22,5	17,53	-15,07	2,51

Fig. 6 Simulation model of static characteristics in MS Excel table

The visualization of the simulation model via the *XY* chart - the time dependence of the output values T and input values T_1, T_2 is shown on the Fig. 7, time dependence Q, Q_1 and Q_2 is shown on the Fig. 8.

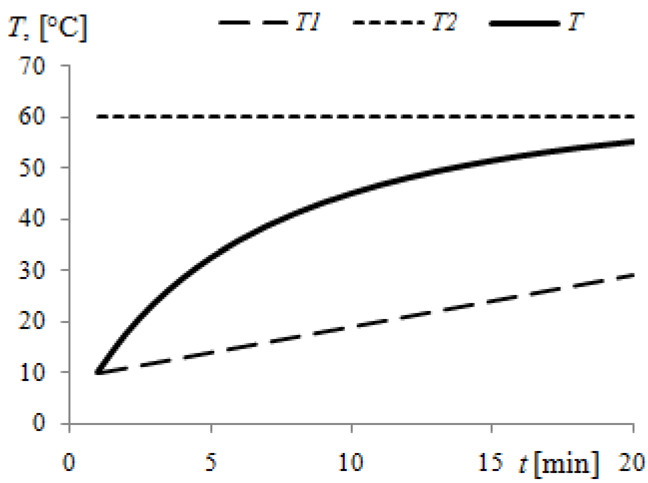


Fig. 7 Simulation model of static characteristic – time dependence of flow rates

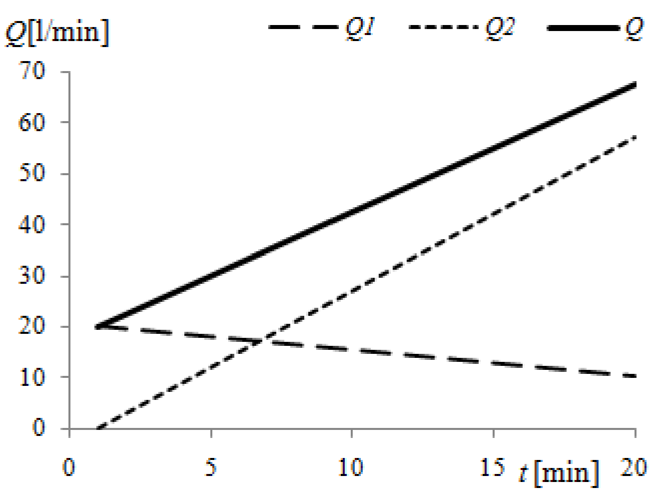


Fig. 8 Simulation model of static characteristic – time dependence of flow rates

Fig. 9 show the gain Z of temperature T depending on flow rates Q_1 and Q_2.

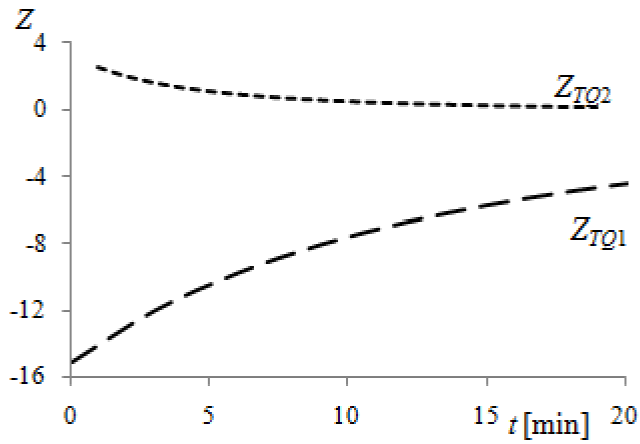


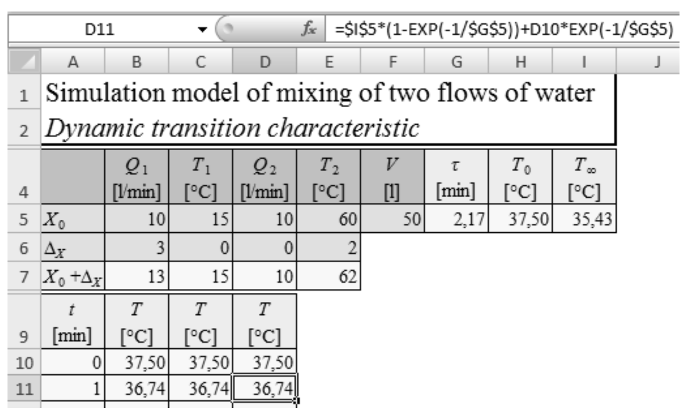
Fig. 9 Simulation model of static characteristic – time dependence of Z_{TQ1} and Z_{TQ2}

B. Dynamic simulation model – transition characteristic

The input variables of the dynamic simulation model are the flow rates Q_1 and Q_2, the corresponding flow water temperatures T_1 and T_2 and step change of this values ΔQ_1, ΔQ_2, ΔT_1 and ΔT_2. The input values are entered by their initial value at the beginning of the measurement of the transition characteristic and step change of this values. To calculate time constant of the water tank the volume V is other value needed to be entered.

The output is temperature of the water in the tank T calculated according both – analytical model given by (9) and numeric recurrent calculation model given by (12).

Fig. 10 shows the MS Excel table to which cells the input values are entered (rows 5 and 6).


D11 =$I$5*(1-EXP(-1/$G$5))+D10*EXP(-1/$G$5)

Simulation model of mixing of two flows of water

Dynamic transition characteristic

	Q_1 [l/min]	T_1 [°C]	Q_2 [l/min]	T_2 [°C]	V [l]	τ [min]	T_0 [°C]	T_∞ [°C]
X_0	10	15	10	60	50	2,17	37,50	35,43
Δ_X	3	0	0	2				
$X_0+\Delta_X$	13	15	10	62				

t [min]	T [°C]	T [°C]	T [°C]
0	37,50	37,50	37,50
1	36,74	36,74	36,74

Fig. 10 Simulation model of dynamic transition characteristics in MS Excel table

The calculation of transition characteristic of temperature T is carried down from the row 10. In our particular case, we have limited the calculation of the transition characteristic in the time interval $\langle 0, 30 \rangle$ min – see Fig. 11.

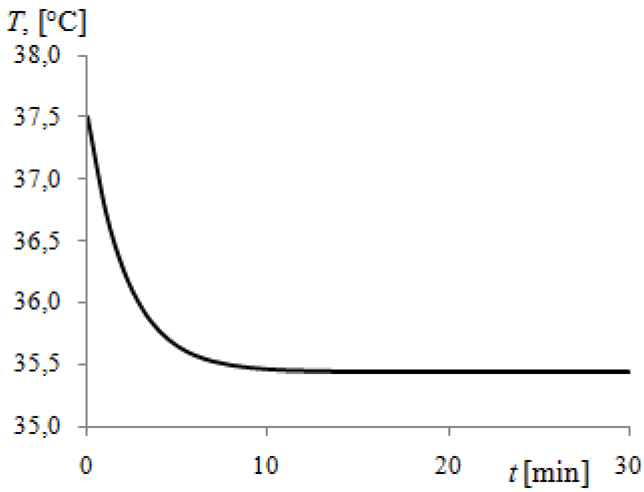


Fig. 11 Simulation model of time dependency of transition characteristic T

C. Simulation model of the feedback controlled system

Simulation model of the controlled regulated system is represented by time dependencies of input and output variables. The model is created in the MS Excel spreadsheet and dependencies are visualized by means of MS Excel *XY* chart. In our particular case, we have limited the simulation of the feedback control in the time interval ⟨0, 50⟩ min.

The main principles of the simulation model will be described in the following text.

1) Setting the action signal

The controlled input action signal can be one of the input quantities Q_1, Q_2, T_1 or T_2.

If e.g. controlled input action signal is Q_2 only its value is changing during the simulation. Other inputs Q_1, T_1, T_2 are set in the beginning and are during simulation of the control process constant.

Setting the input action signal is given by radio buttons – see Fig. 12.

D18 =IF($A$13=2;$J$6*K17+$K$6*K16+$L$6*K15+D17;$D$6)

	A	B	C	D	E	F	G	H	I	J	K	L
1	Simulation model of mixing of two flows of water											
2	*Feedback control (PID Controller)*											
5	V [l]	Q_1 [l/min]	T_1 [°C]	Q_2 [l/min]	T_2 [°C]	r_0	K_i	K_d	K_T	q_0	q_1	q_2
6	50	20	5	10	50	0,50	1,0	2,0	1,0	1,75	-2,25	1,00

	A	B	C	D	E
8		T_R [°C]		*Qty of Steps*	*Actual Step*
9		⟨9-29⟩ min	⟨30–50⟩ min		
10	Initial:	30,0	20,0	30	0
11	Final:	10,0	40,0		
12	Actual:	30,0	20,0		

Controlled Input Signal
○ Q1 ○ T1
◉ Q2 ○ T2

14	t [min]	Q_1 [l/min]	T_1 [°C]	Q_2 [l/min]	T_2 [°C]	Q [l/min]	τ [min]	T_S [°C]	T [°C]	T_R [°C]	$e = T_R - T$
15	0	20,0	5,0	10,0	50,0	30,0	1,67	20,00	20,00	20,0	0,00
16	1	20,0	5,0	10,0	50,0	30,0	1,67	20,00	20,00	20,0	0,00
17	2	20,0	5,0	10,0	50,0	30,0	1,67	20,00	20,00	20,0	0,00
18	3	20,0	5,0	10,0	50,0	30,0	1,67	20,00	20,00	20,0	0,00

Fig. 12 Simulation model of PID controller in MS Excel table

Initial value of the action signal (e.g. Q_2) and other input values (Q_1, T_1, T_2) are entered into the cells B6:E6.

Value of Q_2 (or other action signal Q_1 or T_1 or T_2) is calculated in the range B15:E65 of the sheet based on (24). The calculation formula entered e.g. in cells D18 is as follows:

=IF($A$1=2;$J$6*K17+$K$6*K16+$L$6*K15+D17;$D$6).

The condition $A$1=2 checks if Radio Button Q_2 is checked. If yes, Q_2 is action signal and its value is calculated based on (25), otherwise the value of the Q_2 is constant and equals values of the cell D6 - see Fig. 12.

2) Setting the required temperature TR

Other input quantity is required temperature T_R of the mixed water in the container. Value of T_R is changed stepwise at given time of the control feedback process. Step change of value T_R influences the change of the value of action signal (e.g. Q_2).

In our particular case at the beginning of the simulation – in time interval ⟨0÷8⟩ min we assume that $T_R = T_S$, where T_S in temperature of steady state given by (11). Rest time interval of the simulation ⟨9÷50⟩ min is split to two parts - ⟨9÷29⟩ min and ⟨30÷50⟩ min. Two step changes of the value of T_R are arisen at the beginning of those time intervals. These step changes influence value of the input action signal (e.g. Q_2) by feedback mechanism and time dependence of this signal is visualized on the *XY* chart.

Step changes of the value T_R are in our presented simulation model realized by specific way that enables animation of the simulation chart. The principle of this animation is that the value of the step change ΔT_R can step by step alter from initial to final value and each step by step change is visualized as a new time dependence of input and output variables in MS Excel *XY* simulation chart.

The principle of an animation is recording of a sequence of images which slightly differ. Principle of the animation in MS Excel chart is that the chart has to gradually change by changing of the chart source data, namely T_R.

Initial setting of the values based on the T_R is step by step changed is in our particular simulation model realized in the range of cells A8:E12 – see Fig. 12.

The gradually change of the T_R is realized by iterative recalculation of the value of appropriate cell in MS Excel. In our particular case the circular reference formula

=IF(E10>=D10;0;E10+1)

is entered into the cell E10.

This formula expresses that the value of the cell E10 after one iterative recalculation step increases by 1. Iterative recalculation starts by pressing of **F9** key. By repeated pressings, eventually by holding down the key **F9,** the value of cell E10 will step by step increase by 1 up to value entered in cell D10, which determine number of the animation steps. The value of the cell E10 is step animation parameter based on actual value of T_R – cells B12, C12 resp. is calculated by the formula

=B10+(B11-B10)/$D$10*$E$10

for the cell B12.

Actual value of T_R from the cell B10 is copied to the cells J24:J44 (time interval ⟨9÷29⟩ min) and actual value of T_R from the cell C10 is copied to the cells J45:J65 (time interval ⟨30÷50⟩ min).

3) Other input parameters

The other constant that has to be initialized in the beginning of the simulation are V, K_t, K_i, K_d and r_0. The value of these constants determines the type of the control process (P, PI, PD or PID controller), because regarding (19) and (20) they enable neglected some of the terms of this equitation. Table I shows possible values of these constant for given type of the control process.

Table I Values of constant r_0, K_i, K_d for given type of control process

	r_0	K_i	K_d
P controller	0,5	100000	0
PI controller	0,5	1	0
PD controller	0,5	100000	2
PID controller	0,5	1	2

Input values V, K_t, K_i, K_d and r_0 are entered to the cells in the row 6.

4) Output variable

The output variable is the actual temperature T of the mixture water. The value of the temperature T is calculated in the column I of the Excel simulation model based on (12).

5) Examples of simulation models

The first example of the simulation model is shown on the Fig. 13 and Fig. 14.

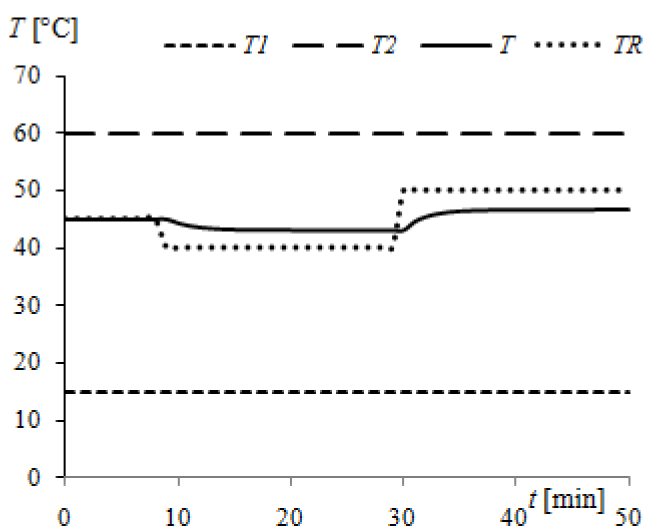


Fig. 13 Simulation model of P controller – time dependencies of $T_1(t)$, $T_2(t)$, $T_R(t)$ and $T(t)$

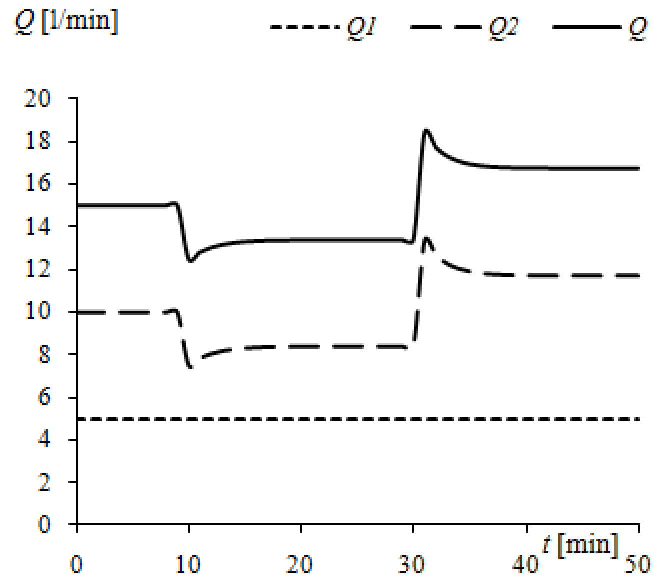


Fig. 14 Simulation model of P controller – time dependencies of action signal $Q_2(t)$, $Q_1(t)$ and $Q(t)$

It is the model of P type of controller – the values $K_i = 10000$ and value $K_d = 0$. The action input signal is $Q_2 = 10$ l/min. The other input values are $Q_1 = 5$ l/min, $T_1 = 15$ °C and $T_1 = 60$ °C. The required temperature $T_R = \{45, 40, 50\}$ °C.

Fig. 13 shows time dependencies of the temperatures $T_1(t)$, $T_2(t)$, $T_R(t)$ and $T(t)$; Fig. 14 shows the time dependencies of action signal $Q_2(t)$, steady flow rate $Q_1(t)$ and flow rate $Q(t)$.

The second example of the simulation model is shown on the Fig. 15 and Fig. 16.

It is the model of PID type of controller for the same initial values as in previous example. Only the values $K_i = 1$ and value $K_d = 2$.

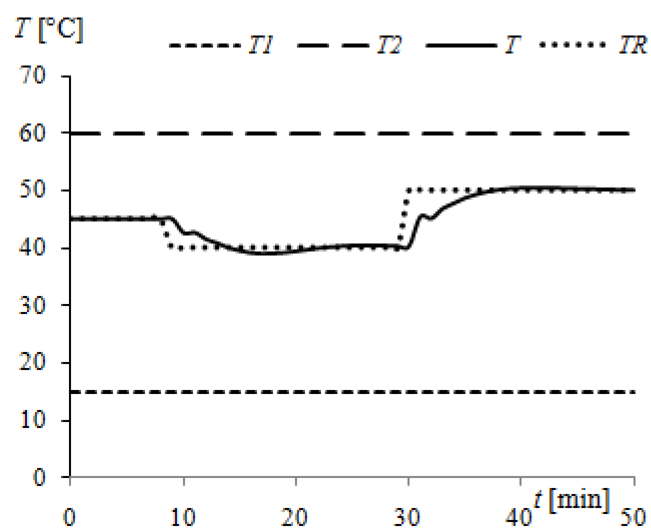


Fig. 15 Simulation model of PID controller – time dependencies of $T_1(t)$, $T_2(t)$, $T_R(t)$ and $T(t)$

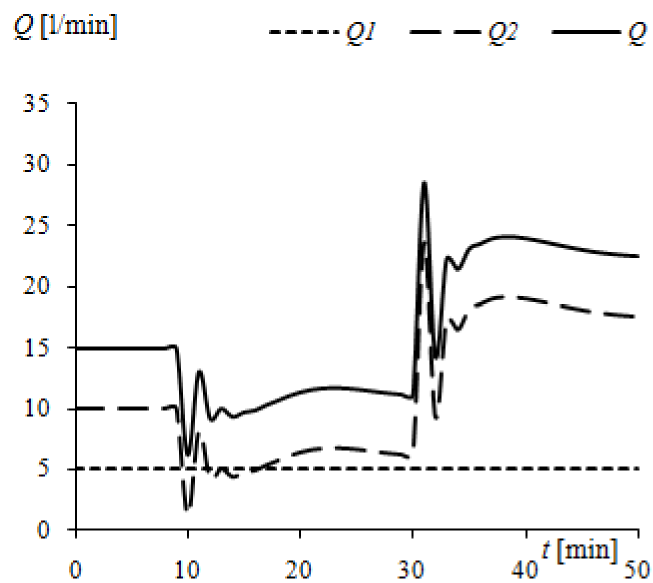


Fig. 16 Simulation model of PID controller – time dependencies of action signal $Q_2(t)$, $Q_1(t)$ and $Q(t)$

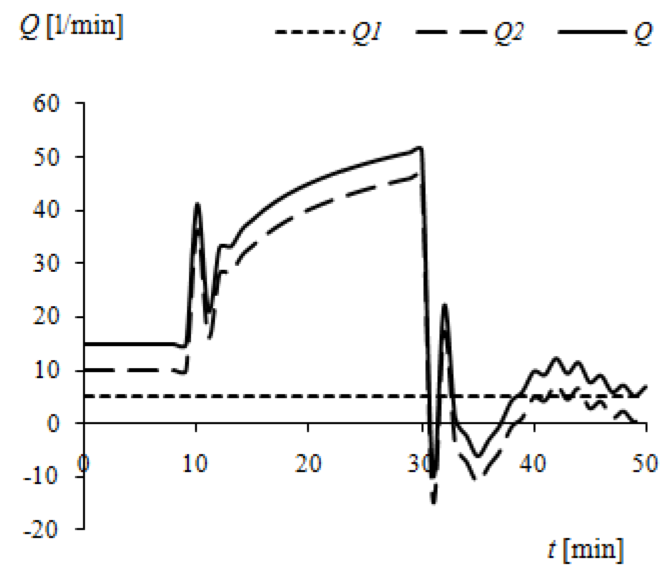


Fig. 18 Simulation model of PID controller – time dependencies of action signal $Q_2(t)$, $Q_1(t)$ and $Q(t)$ of non-controlable system

The last example is the simulation model of non-controllable system - see Fig. 17 and Fig. 18.

It is the model of PID type of controller – the values $K_i = 10000$ and value $K_d = 0$. The action input signal is $Q_2 = 10$ l/min. The other input values are $Q_1 = 5$ l/min, $T_1 = 0$ °C and $T_1 = 60$ °C. The required temperature $T_R = \{40, 55, 20\}$ °C.

In this case the optimal control regulated process is not reached, the value over-deviation of the measured output signal Δy, deviation period T_m and period of the regulated control step $T_{2\%}$ has no minimum.

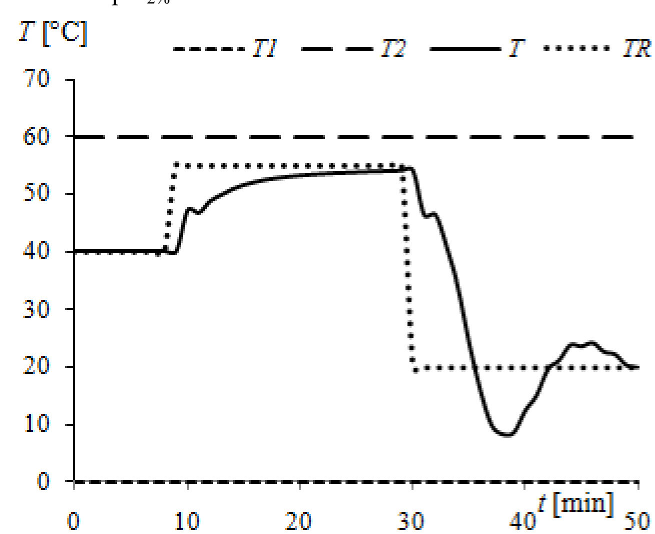


Fig. 17 Simulation model of PID controller – time dependencies of $T_1(t)$, $T_2(t)$, $T_R(t)$ and $T(t)$ of non-controllable system

VI. Conclusion

Modeling and computer simulation provides new methodology of experimental research in current science and high school education – see e.g. [13]

In the paper there is offered the case study of process modeling and simulation of feedback regulated system – mixing of cold and hot water in flow-throw water tank. Step by step there is shown the process of mathematical modeling of static, dynamic and feedback regulation of the system and process of creation of the simulation model in MS Excel spreadsheet and visualization in MS Excel *XY* chart.

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Stepan Hubalovsky was born in Trutnov, Czech Republic in 1970, he obtained master degree in education of mathematics, physics and computer science in 1995 and doctor degree in theory of education in physics in 1998 both in Faculty of Mathematics and Physics, Charles university in Prague, Czech Republic.

He worked 5 years as master of mathematics, physics and computer science on several secondary schools. He works as assistant professor on University of Hradec Kralove from 2006. He interested in algorithm development, programming and computer modelling.

RNDr. Stepan Hubalovsky, Ph.D. is member of Union of Czech Mathematicians and Physicist.