The Averaging Model of a Six-Pulse Diode Rectifier Feeding Paralleled Buck Converters

T.Sopaprim, K-N. Areerak*, K-L. Areerak

***Abstract*—**Power converter models are time-varying in nature because of their switching behaviors. This paper presents the averaging methods called DQ and generalized state-space averaging modeling methods that are used to eliminate the switching actions to achieve the time-invariant models. The DQ modeling method is used to analyze the dynamic model of a three-phase rectifier including the transmission line on AC side, while the generalized state-space averaging modeling method is applied to derive the dynamic model of a buck converter. Intensive time-domain simulations via the well-known software packages with the exact topology models are used to validate the proposed models. The simulation results show that the proposed mathematical models provide high accuracies in both transient and steady-state responses. The reported models require the very fast simulation time compared with the full topology model of commercial software packages. Therefore, the averaging model is suitable for the system design via the searching algorithms in which the repeating calculation is needed during the searching process.

***Keywords*—**Three-phase diode rectifier; Controlled buck converter; DQ modeling method; Generalized state-space averaging method; Modeling and simulation.

I. Introduction

THE power converter models are time-varying in nature due to their switching behaviors. It is very complicated to use the time-varying model for the system analysis and design. Therefore, there are several approaches commonly used for eliminating the switching actions to achieve a time-invariant model. Then, the classical linear control theory can be easily applied to the model for a system analysis and design.

The first method is the generalized state-space averaging (GSSA) modeling method. This method has been used to analyze many power converters in DC distribution systems [1]-[3], as well as uncontrolled and controlled rectifiers in single-phase AC distribution systems [4],[5] and 6- and 12-pulse diode rectifiers in three phase systems [6]. The second is an average-value (AV) modeling method, which has been used for 6- and 12- pulse diode rectifiers in many publications [7]-[9], as well as generators with line-commutated rectifiers [10]-[14]. These rectifiers can be modeled with good accuracy as a constant DC voltage source. However, this method is not easily applicable to analyze the general AC power system with multi-converter power electronic systems. Another technique widely used for AC system analysis is that of DQ-transformation theory [15]-[17], in which power converters can be treated as transformers. The DQ modeling method can also be easily applied for modeling a power system comprising vector-controlled converters where the GSSA and AV models are not easily applicable. The DQ models of three-phase AC-DC power systems have been reported in the previous works for stability studies of the power system including a constant power load (CPL) [18]-[20]. The DQ method for modeling the three-phase uncontrolled and controlled rectifier has been reported in [18] and [21], respectively.

From the literature reviews, this paper presents the combination between the DQ modeling approach and the GSSA modeling method to derive the mathematical model of a three-phase rectifier feeding both resistive load and paralleled buck converters in which it has not been reported in the previous publications. According to the advantages of DQ and GSSA methods, the DQ method is selected to analyze the three-phase diode rectifier including the transmission line components on AC side, while the GSSA method is used to analyze the buck converters with their controls. The proposed model derived from both DQ and GSSA methods is validated by the intensive time-domain simulation via the exact topology model. The results show that the proposed mathematical models provide high accuracies in both transient and steady-state responses. In the future work, the reported models will be used for stability studies of the system due to the effect of a CPL.

The paper is structured as follows. In Section II, the considered system is described. Deriving the dynamic model of the considered power system is fully explained in Section III. The model in Section III is a nonlinear model derived from both DQ and GSSA methods called DQ+GSSA model. Therefore, the linearization technique using the first order term of Taylor's series expansion including the steady-state value calculation is fully explained in Section IV. In Section V, the model validation using the small-signal simulation is illustrated. Finally, Section VI concludes and discusses the advantages of the DQ and GSSA modeling methods to derive

Manuscript received August 3, 2011: Revised version received September 11, 2011. This work was supported by Suranaree University of Technology (SUT) and by the office of the Higher Education Commission under NRU project of Thailand.

T. Sopapirm is the master student in the PeMC research group, Suranaree University of Technology, Nakhon Ratchasima, 30000, THAILAND (email: tay_livesut@hotmail.com)

*K-N. Areerak is the lecturer in the PeMC research group, Suranaree University of Technology, Nakhon Ratchasima, 30000, THAILAND (phone: +66-4422-4520; fax: +66-4422-4601; email: kongpan@sut.ac.th)

K-L. Areerak is the assistant professor in the PeMC research group, Suranaree University of Technology, Nakhon Ratchasima, 30000, THAILAND (email: kongpol@sut.ac.th)

the model of the AC-DC power system with multi-converter power electronic loads.

II. Considered Power System

The considered system is depicted in Fig. 1. It consists of a balanced three-phase voltage source, transmission line, three-phase diode rectifier, and DC-link filters feeding a resistive load (R_{dc}) and controlled buck converters. The buck converters with their controls behave as CPLs to regulate the output voltage of the resistive loads (R_1 and R_2). It is assumed that the diode rectifier and the buck converter are operated under a continuous conduction mode (CCM) and the higher harmonics of the fundamental are neglected.

III. Deriving The Dynamic Model

In this paper, the DQ modeling method is firstly selected to derive the dynamic model of a three-phase diode rectifier feeding uncontrolled buck converters (open-loop operation) in which such rectifier can be treated as a transformer [18]. As a result, the equivalent circuit of the power system as shown in Fig. 1 (without controllers of buck converters) can be represented in the DQ frame as depicted in Fig. 2. Note that the equivalent circuit in Fig. 2 is simplified by fixing the rotating frame on the phase of the diode rectifier switching function (ϕ_1=ϕ).

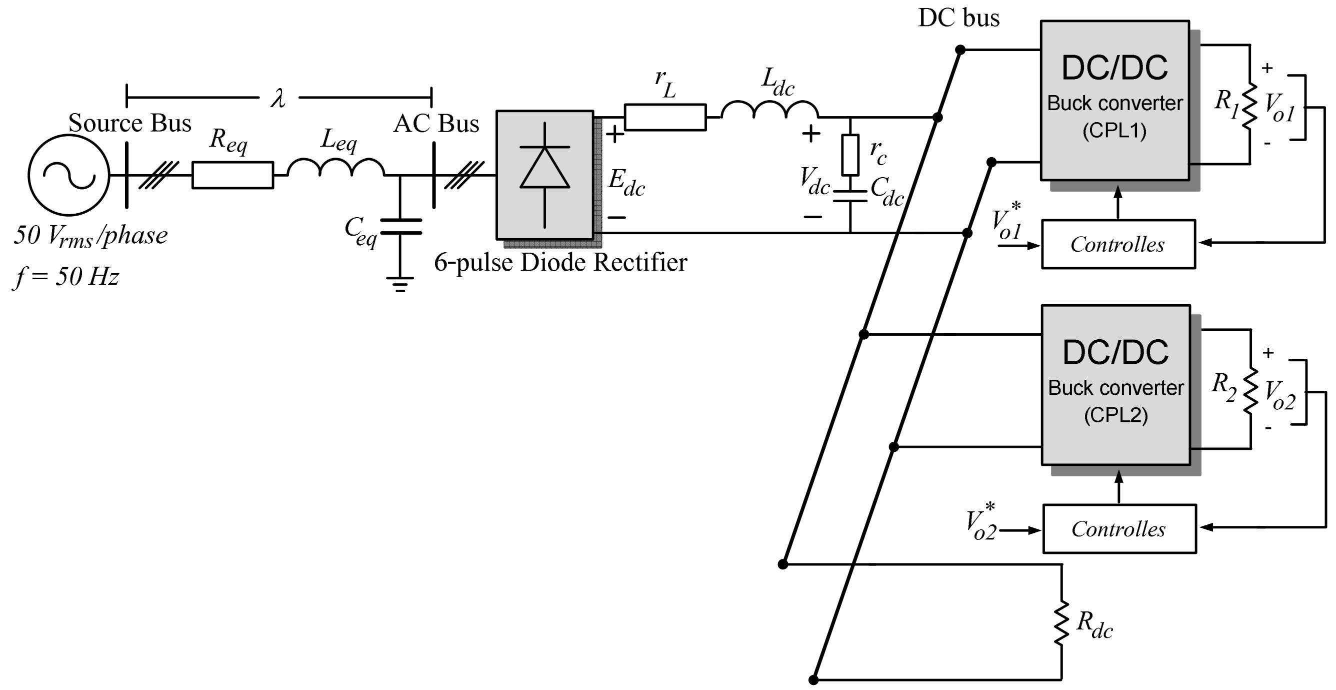

Fig. 1 Considered Power System

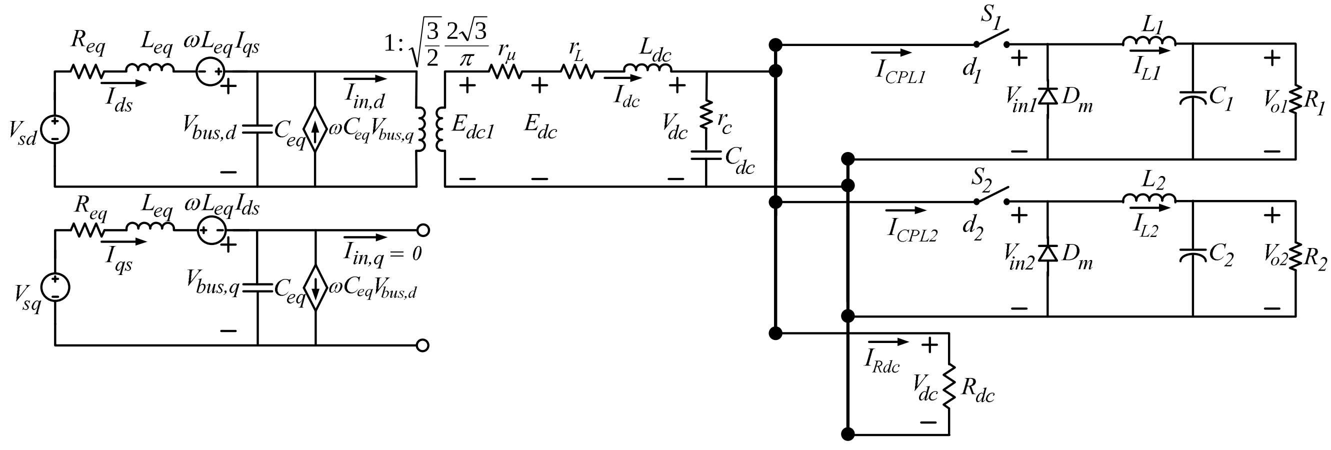

Fig. 2 The equivalent circuits of the considered power system on DQ frame under open-loop operation of buck converters

In Fig. 2, the three-phase diode rectifier including the transmission line on AC side is transformed into the DQ frame via the DQ modeling method. Notice that the diode rectifier can be modeled as the transformer in which it can provide the time-invariant model. The GSSA modeling method is then used to eliminate the switching action of the uncontrolled buck converter. The dynamic model of the system in Fig. 2 using GSSA modeling method can be expressed as:

$$
\begin{cases}
\dot{I}_{ds} = -\frac{R_{eq}}{L_{eq}} I_{ds} + \omega I_{qs} - \frac{1}{L_{eq}} V_{bus,d} + \frac{1}{L_{eq}} V_{sd} \\
\dot{I}_{qs} = -\omega I_{ds} - \frac{R_{eq}}{L_{eq}} I_{qs} - \frac{1}{L_{eq}} V_{bus,q} + \frac{1}{L_{eq}} V_{sq} \\
\dot{V}_{bus,d} = \frac{1}{C_{eq}} I_{ds} + \omega V_{bus,q} - \sqrt{\frac{3}{2}} \frac{2\sqrt{3}}{\pi C_{eq}} I_{dc} \\
\dot{V}_{bus,q} = -\omega V_{bus,d} + \frac{1}{C_{eq}} I_{qs} \\
\dot{I}_{dc} = \sqrt{\frac{3}{2}} \frac{2\sqrt{3}}{\pi L_{dc}} V_{bus,d} - \left(\frac{r_\mu + r_L + r_c}{L_{dc}} \right) I_{dc} - \left(\frac{R_{dc} - r_c}{L_{dc} R_{dc}} \right) V_{dc} + \frac{r_c d_1}{L_{dc}} I_{L1} + \frac{r_c d_1}{L_{dc}} I_{L1} \\
\dot{V}_{dc} = \frac{1}{C_{dc}} I_{dc} - \frac{1}{C_{dc} R_{dc}} V_{dc} - \frac{d_1}{C_{dc}} I_{L1} - \frac{d_2}{C_{dc}} I_{L2} \\
\dot{I}_{L1} = \frac{d_1}{L_1} V_{dc} - \frac{1}{L_1} V_{o1} \\
\dot{V}_{o1} = \frac{1}{C_1} I_{L1} - \frac{1}{R_1 C_1} V_{o1} \\
\dot{I}_{L2} = \frac{d_2}{L_2} V_{dc} - \frac{1}{L_2} V_{o2} \\
\dot{V}_{o2} = \frac{1}{C_2} I_{L2} - \frac{1}{R_2 C_2} V_{o2}
\end{cases}
\tag{1}
$$

where d_1 and d_2 are the duty cycle of the buck converters.

For deriving the dynamic model of the power system including the controllers of buck converters, the schematic of the controllers is considered. It can be seen that the PI controllers of the current loop (inner loop) and the voltage (outer loop) for each buck converter are represented by K_{pv1}, K_{iv1}, K_{pi1}, K_{ii1}, K_{pv2}, K_{iv2}, K_{pi2}, and K_{ii2} respectively. From Fig. 3, d^* can be derived and given in (2).

$$
\begin{cases}
d_1^* = -K_{pi,1} I_{L1} - K_{pv,1} K_{pi,1} V_{o1} + K_{iv,1} K_{pi,1} X_{v1} \\
\quad + K_{ii,1} X_{i1} + K_{pv,1} K_{pi,1} V_{o1}^* \\
d_2^* = -K_{pi,2} I_{L2} - K_{pv,2} K_{pi,2} V_{o2} + K_{iv,2} K_{pi,2} X_{v2} \\
\quad + K_{ii,2} X_{i2} + K_{pv,2} K_{pi,2} V_{o2}^*
\end{cases}
\tag{2}
$$

It can be seen that when PI controllers are considered, the X_v of the voltage loop control and the X_i of the current loop control for each buck converter are set as the state variables of the model. Moreover, when the buck converter is regulated, the d_1 and d_2 in (1) becomes d_1^* and d_2^* as given in (2). Therefore, applying d_1^* and d_2^* from (2) into d_1 and d_2 in (1) and adding the state variables of the PI controllers, the dynamic model of the system in Fig. 3 derived by using the DQ and GSSA methods can be expressed as:

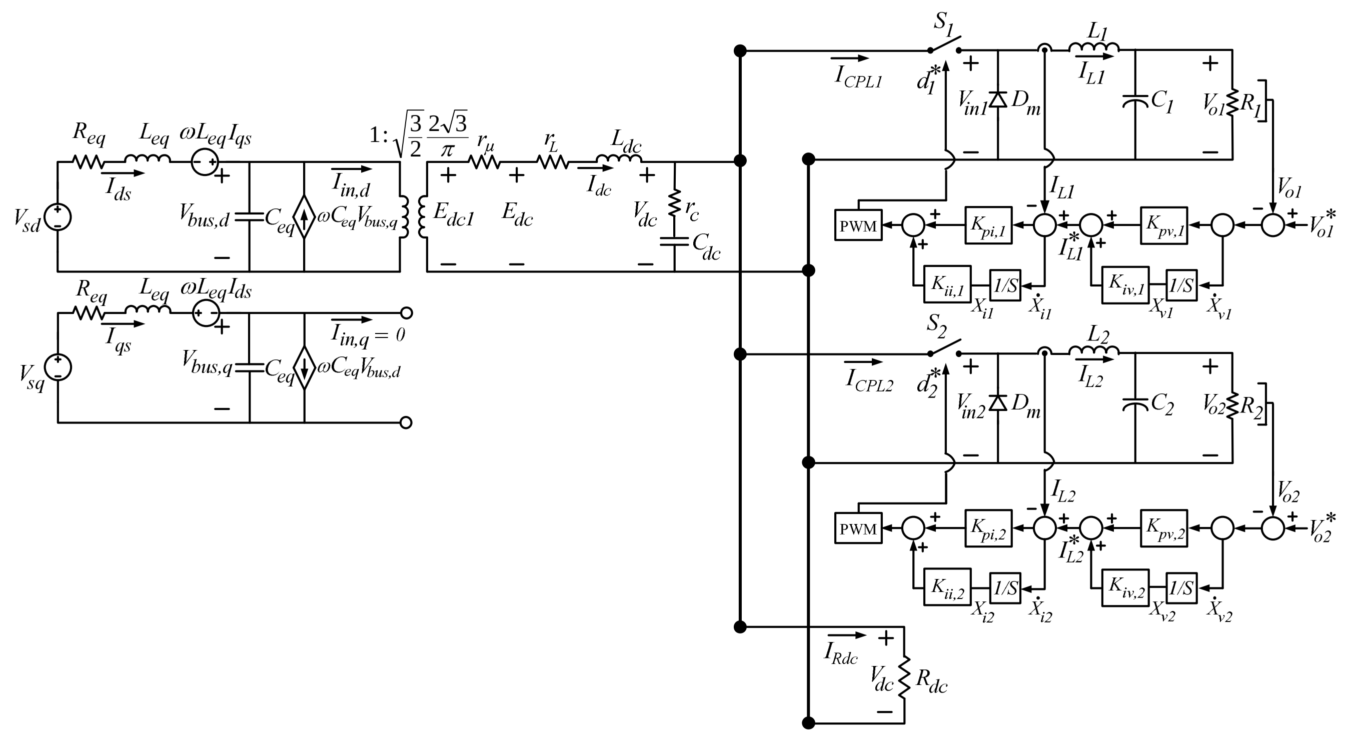

Fig. 3 The equivalent circuit of the considered power system on DQ frame including controllers of buck converters

$$
\left\{
\begin{aligned}
\dot{I}_{sd} &= -\frac{R_{eq}}{L_{eq}} I_{sd} + \omega I_{sq} - \frac{1}{L_{eq}} V_{bus,d} + \frac{1}{L_{eq}} \sqrt{\frac{3}{2}} V_m \cos\lambda \\
\dot{I}_{sq} &= -\omega I_{sd} - \frac{R_{eq}}{L_{eq}} I_{sq} - \frac{1}{L_{eq}} V_{bus,q} + \frac{1}{L_{eq}} \sqrt{\frac{3}{2}} V_m \sin\lambda \\
\dot{V}_{bus,d} &= \frac{1}{C_{eq}} I_{sd} + \omega V_{bus,q} - \sqrt{\frac{3}{2}} \cdot \frac{2\sqrt{3}}{\pi C_{eq}} I_{dc} \\
\dot{V}_{bus,q} &= -\omega V_{bus,d} + \frac{1}{C_{eq}} I_{sq} \\
\dot{I}_{dc} &= \sqrt{\frac{3}{2}} \cdot \frac{2\sqrt{3}}{\pi L_{dc}} V_{bus,d} - \left(\frac{r_\mu + r_L + r_C}{L_{dc}} \right) I_{dc} - \left(\frac{R_{dc} - r_c}{L_{dc} R_{dc}} \right) V_{dc} - \frac{r_c K_{pi,1}}{L_{dc}} I_{L1}^2 \\
&\quad - \frac{r_c K_{pv,1} K_{pi,1} I_{L1}}{L_{dc}} V_{o1} + \frac{r_c K_{iv,1} K_{pi,1} I_{L1}}{L_{dc}} X_{v1} + \frac{r_c K_{ii,1} I_{L1}}{L_{dc}} X_{i1} + \frac{r_c K_{pv,1} K_{pi,1} I_{L1}}{L_{dc}} V_{o1}^* \\
&\quad - \frac{r_c K_{pi,2}}{L_{dc}} I_{L2}^2 - \frac{r_c K_{pv,2} K_{pi,2} I_{L2}}{L_{dc}} V_{o2} + \frac{r_c K_{iv,2} K_{pi,2} I_{L2}}{L_{dc}} X_{v2} + \frac{r_c K_{ii,2} I_{L2}}{L_{dc}} X_{i2} + \frac{r_c K_{pv,2} K_{pi,2} I_{L2}}{L_{dc}} V_{o2}^* \\
\dot{V}_{dc} &= \frac{1}{C_{dc}} I_{dc} + \frac{1}{R_{dc} C_{dc}} V_{dc} + \frac{K_{pi,1}}{C_{dc}} I_{L1}^2 + \frac{K_{pv,1} K_{pi,1} I_{L1}}{C_{dc}} V_{o1} - \frac{K_{iv,1} K_{pi,1} I_{L1}}{C_{dc}} X_{v1} \\
&\quad - \frac{K_{ii,1} I_{L1}}{C_{dc}} X_{i1} - \frac{K_{pv,1} K_{pi,1} I_{L1}}{C_{dc}} V_{o1}^* + \frac{K_{pi,2}}{C_{dc}} I_{L2}^2 + \frac{K_{pv,2} K_{pi,2} I_{L1}}{C_{dc}} V_{o2} - \frac{K_{iv,2} K_{pi,2} I_{L1}}{C_{dc}} X_{v2} \\
&\quad - \frac{K_{ii,2} I_{L1}}{C_{dc}} X_{i2} - \frac{K_{pv,2} K_{pi,2} I_{L1}}{C_{dc}} V_{o2}^* \\
\dot{I}_{L1} &= -\frac{K_{pi,1} V_{dc}}{L_1} I_{L1} - \frac{\left(K_{pv,1} K_{pi,1} V_{dc} + 1 \right)}{L_1} V_{o1} + \frac{K_{iv,1} K_{pi,1} V_{dc}}{L_1} X_{v1} + \frac{K_{ii,1} V_{dc}}{L_1} X_{i1} + \frac{K_{pv,1} K_{pi,1} V_{dc}}{L_1} V_{o1}^* \\
\dot{V}_{o1} &= \frac{1}{C_1} I_{L1} - \frac{V_{O1}}{R_1 C_1} \\
\dot{X}_{v1} &= -V_{o1} + V_{o1}^* \\
\dot{X}_{i1} &= -I_{L1} - K_{pv,1} V_{o1} + K_{iv,1} X_{v1} + K_{pv,1} V_{o1}^* \\
\dot{I}_{L2} &= -\frac{K_{pi,2} V_{dc}}{L_2} I_{L2} - \frac{\left(K_{pv,2} K_{pi,2} V_{dc} + 1 \right)}{L_2} V_{o2} + \frac{K_{iv,2} K_{pi,2} V_{dc}}{L_2} X_{v2} + \frac{K_{ii,2} V_{dc}}{L_2} X_{i2} + \frac{K_{pv,2} K_{pi,2} V_{dc}}{L_2} V_{o2}^* \\
\dot{V}_{o2} &= \frac{1}{C_2} I_{L2} - \frac{V_{O2}}{R_2 C_2} \\
\dot{X}_{v2} &= -V_{o2} + V_{o2}^* \\
\dot{X}_{i2} &= -I_{L2} - K_{pv,2} V_{o2} + K_{iv,2} X_{v2} + K_{pv,2} V_{o2}^*
\end{aligned}
\right. \tag{3}
$$

It can be seen in (3) that when we consider the controlled buck converter, the dynamic model of the system described in (1) becomes to (3) having the PI controller parameters. In addition, the state variables X_{v1}, X_{i1}, X_{v2}, and X_{i2} are also included. Equation (3) is the nonlinear differential equations. Therefore, (3) can be linearized using the first order terms of the Taylor expansion so as to achieve a set of linear differential equations around an equilibrium point. The details of the DQ+GSSA linearized model of (3) are given in Section IV.

IV. DQ+GSSA Linearized Model And Steady-State Value Calculation

As mentioned in Section III, (3) can be linearized using the first order terms of the Taylor expansion so as to achieve a set of linear differential equations around an equilibrium point.

The DQ+GSSA linearized model of (3) is then of the form in (4).

$$
\left\{
\begin{aligned}
\delta \dot{\mathbf{x}} &= \mathbf{A}(\mathbf{x_o}, \mathbf{u_o}) \delta \mathbf{x} + \mathbf{B}(\mathbf{x_o}, \mathbf{u_o}) \delta \mathbf{u} \\
\delta \mathbf{y} &= \mathbf{C}(\mathbf{x_o}, \mathbf{u_o}) \delta \mathbf{x} + \mathbf{D}(\mathbf{x_o}, \mathbf{u_o}) \delta \mathbf{u}
\end{aligned}
\right. \tag{4}
$$

where

$$
\begin{aligned}
\delta \mathbf{x} = \big[& \delta I_{ds} \;\; \delta I_{qs} \;\; \delta V_{bus,d} \;\; \delta V_{bus,q} \;\; \delta I_{dc} \;\; \delta V_{dc} \;\; \delta I_{L1} \\
& \delta V_{o1} \;\; \delta X_{v1} \;\; \delta X_{i1} \;\; \delta I_{L2} \;\; \delta V_{o2} \;\; \delta X_{v2} \;\; \delta X_{i12} \big]
\end{aligned}
$$

$$
\delta \mathbf{u} = \left[\delta V_m \;\; \delta V_{o1}^* \;\; \delta V_{o2}^* \right]^T
$$

$$
\delta \mathbf{y} = \left[\delta V_{dc} \;\; \delta V_{o1} \;\; \delta V_{o2} \right]^T
$$

$$
\mathbf{A}(\mathbf{x_o},\mathbf{u_o})=\begin{bmatrix}
-\frac{R_{eq}}{L_{eq}} & \omega & -\frac{1}{L_{eq}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-\omega & -\frac{R_{eq}}{L_{eq}} & 0 & -\frac{1}{L_{eq}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\frac{1}{C_{eq}} & 0 & 0 & \omega & \sqrt{\frac{3}{2}}\cdot\frac{2\sqrt{3}}{\pi C_{eq}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \frac{1}{C_{eq}} & -\omega & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \sqrt{\frac{3}{2}}\cdot\frac{2\sqrt{3}}{\pi L_{eq}} & 0 & -\left(\frac{r_\mu+r_L+r_c}{L_{dc}}\right) & -\left(\frac{R_{dc}-r_c}{L_{dc}R_{dc}}\right) & a(5,7) & -\frac{r_cK_{pv,1}K_{pi,1}I_{L1}}{L_{dc}} & \frac{r_cK_{iv,1}K_{pi,1}I_{L1}}{L_{dc}} & \frac{r_cK_{ii,1}I_{L1}}{L_{dc}} & a(5,11) & -\frac{r_cK_{pv,2}K_{pi,2}I_{L2}}{L_{dc}} & \frac{r_cK_{iv,2}K_{pi,2}I_{L2}}{L_{dc}} & \frac{r_cK_{ii,2}I_{L2}}{L_{dc}}\\
0 & 0 & 0 & 0 & \frac{1}{C_{dc}} & 0 & a(6,7) & \frac{K_{pv,1}K_{pi,1}I_{L1}}{C_{dc}} & -\frac{K_{iv,1}K_{pi,1}I_{L1}}{C_{dc}} & -\frac{K_{ii,1}I_{L1}}{C_{dc}} & a(6,11) & \frac{K_{pv,2}K_{pi,2}I_{L2}}{C_{dc}} & -\frac{K_{iv,2}K_{pi,2}I_{L2}}{C_{dc}} & -\frac{K_{ii,2}I_{L2}}{C_{dc}}\\
0 & 0 & 0 & 0 & 0 & a(7,6) & -\frac{K_{pi,1}V_{dc,0}}{L_1} & -\frac{K_{pv,1}K_{pi,1}V_{dc,0}+1}{L_1} & \frac{K_{iv,1}K_{pi,1}V_{dc,0}}{L_1} & \frac{K_{ii,1}V_{dc,0}}{L_1} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{C_1} & -\frac{1}{R_1C_1} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -1 & -K_{pv,1} & K_{iv,1} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & a(11,6) & 0 & 0 & 0 & 0 & -\frac{K_{pi,2}V_{dc,0}}{L_2} & -\frac{K_{pv,2}K_{pi,2}V_{dc,0}+1}{L_2} & \frac{K_{iv,2}K_{pi,2}V_{dc,0}}{L_2} & \frac{K_{ii,2}V_{dc,0}}{L_2}\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{C_2} & -\frac{1}{R_2C_2} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -K_{pv,2} & K_{iv,2} & 0
\end{bmatrix}_{14\times 14}
$$

$$
a(5,7)=-\frac{2r_cK_{pi,1}I_{L1,0}}{L_{dc}}-\frac{r_cK_{pv,1}K_{pi,1}V_{o1,0}}{L_{dc}}+\frac{r_cK_{iv,1}K_{pi,1}X_{v1,0}}{L_{dc}}+\frac{r_cK_{ii,1}X_{i1,0}}{L_{dc}}+\frac{r_cK_{pv,1}K_{pi,1}V^*_{o1,0}}{L_{dc}},\ a(5,11)=-\frac{2r_cK_{pi,2}I_{L2,0}}{L_{dc}}-\frac{r_cK_{pv,2}K_{pi,2}V_{o2,0}}{L_{dc}}+\frac{r_cK_{iv,2}K_{pi,2}X_{v2,0}}{L_{dc}}+\frac{r_cK_{ii,2}X_{i2,0}}{L_{dc}}+\frac{r_cK_{pv,2}K_{pi,2}V^*_{o2,0}}{L_{dc}}
$$

$$
a(6,7)=\frac{2K_{pi,1}I_{L1,0}}{C_{dc}}+\frac{K_{pv,1}K_{pi,1}V_{o1,0}}{C_{dc}}-\frac{K_{iv,1}K_{pi,1}X_{v1,0}}{C_{dc}}-\frac{K_{ii,1}X_{i1,0}}{C_{dc}}-\frac{K_{pv,1}K_{pi,1}V^*_{o1,0}}{C_{dc}},\ a(6,11)=\frac{2K_{pi,2}I_{L2,0}}{C_{dc}}+\frac{K_{pv,2}K_{pi,2}V_{o2,0}}{C_{dc}}-\frac{K_{iv,2}K_{pi,2}X_{v2,0}}{C_{dc}}-\frac{K_{ii,2}X_{i2,0}}{C_{dc}}-\frac{K_{pv,2}K_{pi,2}V^*_{o2,0}}{C_{dc}}
$$

$$
a(7,6)=-\frac{K_{pi,1}i_{L1,0}}{L}-\frac{K_{pv}K_{pi,1}V_{o1,0}}{L}+\frac{K_{iv,1}K_{pi,1}X_{v1,0}}{L}+\frac{K_{ii,1}X_{i1,0}}{L}+\frac{K_{pv,1}K_{pi,1}V^*_{o1,0}}{L},\ a(7,11)=-\frac{K_{pi,2}i_{L2,0}}{L}-\frac{K_{pv}K_{pi,2}V_{o2,0}}{L}+\frac{K_{iv,2}K_{pi,2}X_{v2,0}}{L}+\frac{K_{ii,2}X_{i2,0}}{L}+\frac{K_{pv,2}K_{pi,2}V^*_{o2,0}}{L}
$$

$$
\mathbf{B}(\mathbf{x_o},\mathbf{u_o})=\begin{bmatrix}
\sqrt{\frac{3}{2}}\cdot\frac{\cos(\lambda_o)}{L_{eq}} & 0 & 0\\
\sqrt{\frac{3}{2}}\cdot\frac{\sin(\lambda_o)}{L_{eq}} & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & \frac{K_{pv,1}K_{pi,1}V_{dc,0}}{L_1} & 0\\
0 & 0 & 0\\
0 & 1 & 0\\
0 & K_{pv,1} & 0\\
0 & 0 & \frac{K_{pv,2}K_{pi,2}V_{dc,0}}{L_2}\\
0 & 0 & 0\\
0 & 0 & 1\\
0 & 0 & K_{pv,2}
\end{bmatrix}_{14\times 3}
$$

$$
\mathbf{C}(\mathbf{x_o},\mathbf{u_o})=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0\end{bmatrix}_{1\times 14}
$$

$$
\mathbf{D}(\mathbf{x_o},\mathbf{u_o})=\begin{bmatrix}0 & 0 & 0\end{bmatrix}_{1\times 3}
$$

According to DQ+GSSA linearized model in (4), the model needs to define $V_{dc,o}$, λ_o, $V_{o1,o}$, $V_{o2,o}$, $I_{L1,o}$, $I_{L2,o}$, $X_{v1,o}$, $X_{v2,o}$, $X_{i1,o}$ and $X_{i2,o}$. In the paper, the power equation can be first applied to determine the steady state values at the AC side, here are $V_{bus,o}$ and λ_o. Other steady-state values can be calculated by:

$$
\begin{cases}
V_{dc,o}=\dfrac{3\sqrt{3}}{\pi}\cdot(\sqrt{2}V_{bus,o})-\dfrac{3L_{eq}\omega}{\pi}I_{dc,o}-r_LI_{dc,o}\\
V_{o1,o}=V^*_{o1},\ \ V_{o2,o}=V^*_{o2}\\
I_{L1,o}=\dfrac{V_{o1,o}}{R_1},\ \ I_{L2,o}=\dfrac{V_{o2,o}}{R_2}\\
X_{V1,o}=\dfrac{I_{L1,o}}{K_{iv,1}},\ \ X_{V2,o}=\dfrac{I_{L2,o}}{K_{iv,2}}\\
X_{i1,o}=\dfrac{V_{o1}}{K_{ii,1}V_{dc,o}},\ \ X_{i2,o}=\dfrac{V_{o2}}{K_{ii,2}V_{dc,o}}
\end{cases}
\tag{5}
$$

where

$$
\begin{cases}
I_{dc,o}=\dfrac{\sqrt{3}\left|\dfrac{V_se^{j0}-V_{bus,o}e^{-j\lambda_o}}{Ze^{j\gamma}}\right|}{\sqrt{\dfrac{3}{2}}\left(\dfrac{2\sqrt{3}}{\pi}\right)}\\
Z=\sqrt{R_{eq}^2+(\omega L_{eq})^2},\quad \gamma=\tan^{-1}\left(\dfrac{\omega L_{eq}}{R_{eq}}\right)
\end{cases}
$$

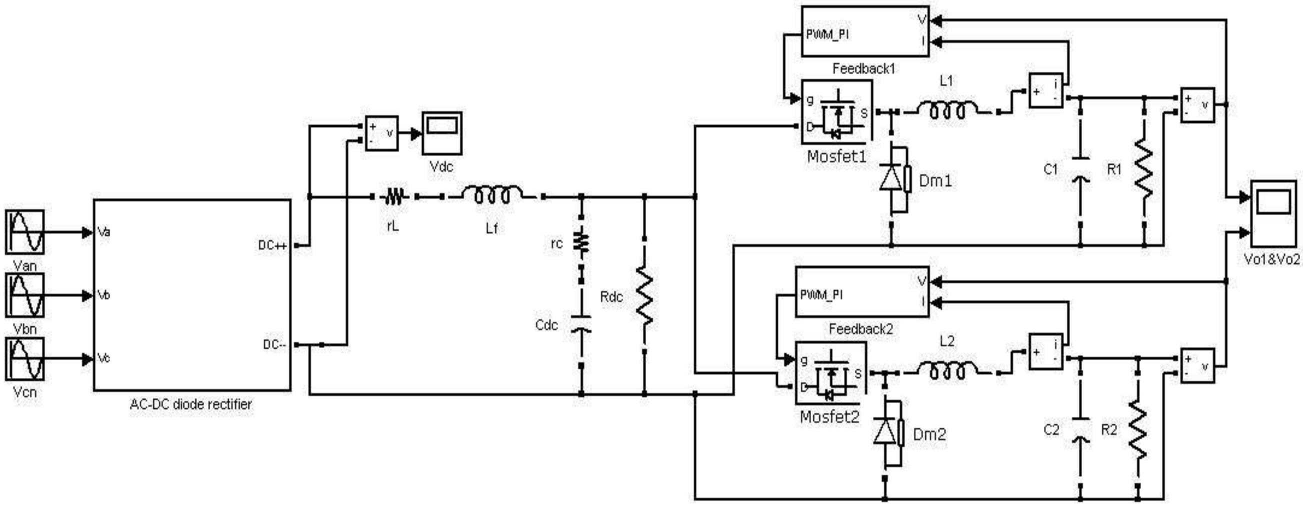

Fig. 4 The exact topology model (SPS^{TM} of SIMULINK)

V. SMALL-SIGNAL SIMULATION

The DQ+GSSA linearized model in (4) is simulated for small-signal transients against a corresponding exact topology model as shown in Fig.4. The set of system parameters is given in Table 1 with the voltage loop controllers $K_{pv1} = K_{pv2}$= 0.05 and $K_{iv1} = K_{iv2}$ = 50 ($\omega_{n,voltage}$ = 64 Hz, ζ_v = 1.0), and the current loop controllers $K_{iv1} = K_{iv2}$ = 0.7728 and $K_{ii1} = K_{ii2}$ = 11040 ($\omega_{n,current}$ = 3200 Hz, ζ_i = 0.7).

Table 1: Parameters of the Power System in Fig.1

Parameter	Value
V_s	50 $V_{rms/phase}$
ω	2πx50 rad/sec.
R_{eq}	0.1 Ω
L_{eq}	24 μH
C_{eq}	2 nF
r_L	0.01 Ω
r_c	0.4 Ω
L_{dc} ($\Delta I_{dc} \leq$ 1.5 A)	50 mH
C_{dc} ($\Delta V_{dc} \leq$ 10 V)	500 μF
R_{dc}	500 Ω
L_1=L_2 ($\Delta I_L \leq$ 0.5 A)	14.168 mH
C_1=C_2 ($\Delta V_o \leq$ 50 mV)	125 μF
R_1= R_2	20 Ω

Fig. 5 shows the V_{dc}, V_{o1} and V_{o2} responses of the system in Fig. 1 to a step change of V_{o1}^* and V_{o2}^* from 5 V to 15 V that occurs at t = 0.6 second and t = 1.5 second, respectively. Similarly, Fig. 6 shows the voltage responses of the system in Fig. 1 to a step change of V_{o1}^* and V_{o2}^* from 15 V to 25 V.

From the result in Fig. 5 and Fig. 6, an excellent agreement between both models is achieved under the small-signal simulation. It confirms that the mathematical model of the power system with paralleled controlled buck converters and a resistive load derived from both DQ and GSSA methods provide a good accuracy. The DQ+GSSA linearized model in the paper will be also used for the stability analysis due to the effect of CPL in the future work. In addition, it is well known that simulations of power electronic system using software packages (such as MATLAB, PSIM, and etc.) via the exact topology models consume a huge simulation time due to a switching behavior. It is not easily applicable for simulation of complex systems. The simulation time when the system was simulated via the proposed model coding in MATLAB requires 0.156 sencond, while the full topology model of SPS^{TM} as shown in Fig. 4 consumes 165 second. Hence, the averaging model of the power electronic based system derived by the proposed modeling method in the paper can be used to reduce the simulation time. The reported model is also suitable for the optimal controller design using the artificial intelligence (AI) techniques [22]-[25] because the very fast simulation time can be obtained.

VI. CONCLUSION

This paper presents how to derive the dynamic model of the three-phase diode rectifier feeding multi-converter power electronic loads with their controls. The DQ and GSSA modeling methods are used to eliminate the switching behaviour of the power converter in which the DQ method is

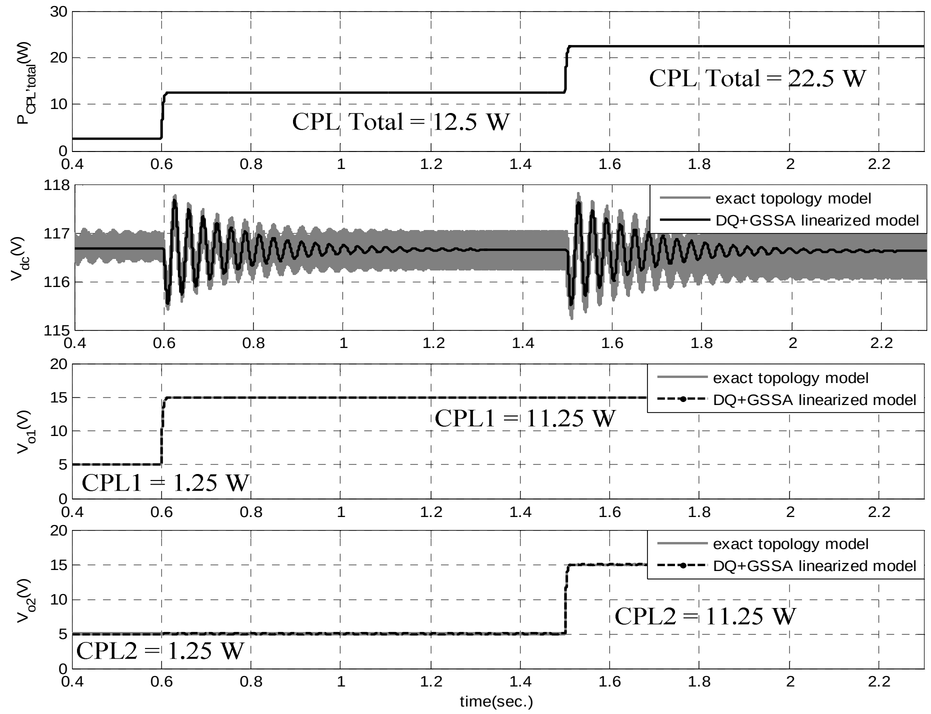

Fig.5. V_{dc}, V_{o1} and V_{o2} responses of the system in Fig. 1 to a step change of V_{o1}^* and V_{o2}^* from 5 V to 15 V

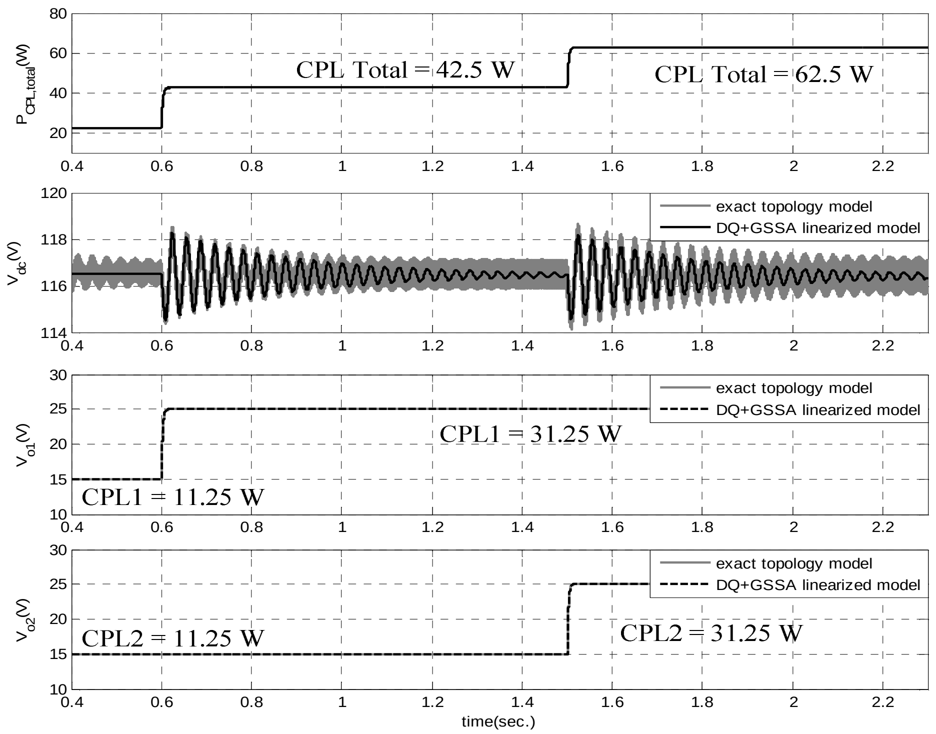

Fig.6. V_{dc}, V_{o1} and V_{o2} responses of the system in Fig. 1 to a step change of V_{o1}^* and V_{o2}^* from 15 V to 25 V

used to analyze the three-phase rectifier and the GSSA method is also applied to the buck converter. The proposed models are suitable for the system design and simulation. Moreover, it is well known that when the power converters are regulated, they behave as a CPL. This CPL can significantly degrade power system stability margins. Therefore, the dynamic model of the power system is very important. The DQ+GSSA linearized model in the paper can be also used for stability studies in the future work. Accoring to the fast simulation time of the proposed model, it can be used with the AI searching algorithms for the optimal controller design to achieve the best output system responses.

ACKNOWLEDGMENT

This work was supported by Suranaree University of Technology (SUT) and by the office of the Higher Education Commission under NRU project of Thailand.

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T. Sopapirm was born in Saraburi, Thailand, in 1988. He received the B.S. degree in electrical engineering from Suranaree University of Technology (SUT), Nakhon Ratchasima, Thailand, in 2009, where he is currently studying toward the M.Eng. degree in electrical engineering. His main research interests include stability analysis, modeling of power electronic system, digital control, FPGA, and AI application.

K-N. Areerak received the B.Eng. and M.Eng. degrees from Suranaree University of Technology (SUT), Nakhon Ratchasima, Thailand, in 2000 and 2001, respectively and the Ph.D. degree from the University of Nottingham, Nottingham, UK., in 2009, all in electrical engineering. In 2002, he was a Lecturer in the Electrical and Electronic Department, Rangsit University, Thailand. Since 2003, he has been a Lecturer in the School of Electrical Engineering, SUT. His main research interests include system identifications, artificial intelligence application, stability analysis of power systems with constant power loads, modeling and control of power electronic based systems, and control theory.

K-L. Areerak received the B.Eng, M.Eng, and Ph.D. degrees in electrical engineering from Suranaree University of Technology (SUT), Thailand, in 2000, 2003, and 2007, respectively. Since 2007, he has been a Lecturer and Head of Power Quality Research Unit (PQRU) in the School of Electrical Engineering, SUT. He received the Assistant Professor in Electrical Engineering in 2009. His main research interests include active power filter, harmonic elimination, AI application, motor drive, and intelligence control system.