

# Time-Domain Simulation of Nonuniform Multiconductor Transmission Lines in Matlab

Lubomír Brančík, Břetislav Ševčík

**Abstract**—The paper presents a technique for the time-domain simulation of nonuniform multiconductor transmission lines (MTL) based on an implicit Wendroff method. This technique falls into a class of the finite-difference time-domain (FDTD) methods useful to solve various electromagnetic systems. Its basic version is extended to enable solving both voltage and/or current distributions along the MTL's wires and their sensitivities w.r. to lumped and distributed parameters. An experimental error analysis is done on the Thomson cable, a single transmission line with known analytical solutions. The examples of simulation of both uniform and nonuniform MTLs are shown and compared with other methods, some results for nonlinear MTLs are also presented. All computations were performed in the Matlab language, and the computational efficiency is assessed by means of detailed CPU times evaluation.

**Keywords**—Multiconductor transmission line; transient analysis; sensitivity; Wendroff method; Matlab; simulation.

## I. INTRODUCTION

THE implicit Wendroff formula falls into a class of finite-difference time-domain methods (FDTD) useful to solve various electromagnetic systems in time domain [1]–[3]. As is shown e.g. in [4], [5], the method can successfully be used for solving transient phenomena on transmission lines in field of a power engineering, or its generalized version in [6] enables to solve transients and sensitivities of uniform multiconductor transmission lines (MTL). The paper follows [7], where the Wendroff formula has been adapted for solving nonuniform and/or nonlinear MTLs, and presents further results including detailed computational efficiency by using Matlab.

A simple system with an  $(n+1)$ -conductor transmission line, terminated by multiports, left (L), right (R), is shown in Fig. 1.

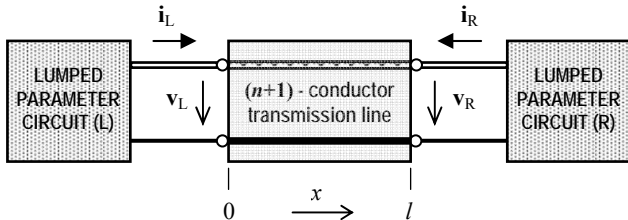


Fig. 1 Simple system with  $(n+1)$ -conductor transmission line

First, think of a linear MTL defined by its length  $l$  and per-unit-length (p.-u.-l.)  $n \times n$  matrices  $\mathbf{R}_0(x)$ ,  $\mathbf{L}_0(x)$ ,  $\mathbf{G}_0(x)$ ,  $\mathbf{C}_0(x)$ ,

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i.e. nonuniform in general. The MTL telegraphic equations are

$$\begin{aligned} -\frac{\partial \mathbf{v}(t, x)}{\partial x} &= \mathbf{R}_0(x) \mathbf{i}(t, x) + \mathbf{L}_0(x) \frac{\partial \mathbf{i}(t, x)}{\partial t}, \\ -\frac{\partial \mathbf{i}(t, x)}{\partial x} &= \mathbf{G}_0(x) \mathbf{v}(t, x) + \mathbf{C}_0(x) \frac{\partial \mathbf{v}(t, x)}{\partial t}, \end{aligned} \quad (1)$$

where  $\mathbf{v}(t, x)$  and  $\mathbf{i}(t, x)$  are  $n \times 1$  column vectors of voltages and currents of  $n$  active wires at the distance  $x$  from the MTL's left end, respectively [8]. The equations above are supplemented by boundary conditions reflecting terminating lumped-parameter circuits, e.g. by means of their generalized Thévenin or Norton equivalents, as will be shown later.

## II. IMPLICIT WENDROFF FORMULA APPLICATION

The principle of the implicit Wendroff formula lies in following operations on (1). For the  $j$ -th time step and  $k$ -th spatial coordinate, (1) is modified by substitutions

$$\begin{aligned} \mathbf{u}(t, x) \Big|_{j, k} &\doteq \frac{1}{4} (\mathbf{u}_{k+1}^j + \mathbf{u}_k^j + \mathbf{u}_{k+1}^{j-1} + \mathbf{u}_k^{j-1}), \\ \frac{\partial \mathbf{u}(t, x)}{\partial t} \Big|_{j, k} &\doteq \frac{1}{2} \left( \frac{\mathbf{u}_k^j - \mathbf{u}_k^{j-1}}{\Delta t} + \frac{\mathbf{u}_{k+1}^j - \mathbf{u}_{k+1}^{j-1}}{\Delta t} \right), \\ \frac{\partial \mathbf{u}(t, x)}{\partial x} \Big|_{j, k} &\doteq \frac{1}{2} \left( \frac{\mathbf{u}_{k+1}^j - \mathbf{u}_k^j}{\Delta x} + \frac{\mathbf{u}_{k+1}^{j-1} - \mathbf{u}_k^{j-1}}{\Delta x} \right), \end{aligned} \quad (2)$$

where  $\mathbf{u}(t, x)$  denotes either the voltage  $\mathbf{v}(t, x)$  or current  $\mathbf{i}(t, x)$  vectors. The indexes have ranges  $k = 1, 2, \dots, K$ , and  $j = 1, 2, \dots, J$ , with  $K$  and  $J$  as the numbers of intervals  $\Delta x = l/K$  and  $\Delta t = T/J$  in space and time, respectively, and where  $l$  denotes an MTL's length and  $T$  an upper limit of the time interval of interest. So we have chosen equidistant intervals to simplify our notations, although they could vary in general. Substituting (2) into (1) leads to systems of equations

$$\begin{aligned} \mathbf{v}_k^j - \mathbf{v}_{k+1}^j + \mathbf{A}_{vk} (\mathbf{i}_k^j + \mathbf{i}_{k+1}^j) &= -\mathbf{v}_k^{j-1} + \mathbf{v}_{k+1}^{j-1} + \mathbf{B}_{vk} (\mathbf{i}_k^{j-1} + \mathbf{i}_{k+1}^{j-1}), \\ \mathbf{A}_{ik} (\mathbf{v}_k^j + \mathbf{v}_{k+1}^j) + \mathbf{i}_k^j - \mathbf{i}_{k+1}^j &= \mathbf{B}_{ik} (\mathbf{v}_k^{j-1} + \mathbf{v}_{k+1}^{j-1}) - \mathbf{i}_k^{j-1} + \mathbf{i}_{k+1}^{j-1}, \end{aligned} \quad (3)$$

with coefficients matrices

$$\begin{aligned} \mathbf{A}_{vk} &= -(\mathbf{R}_{0k}/2 + \mathbf{L}_{0k}/\Delta t) \Delta x, \quad \mathbf{B}_{vk} = (\mathbf{R}_{0k}/2 - \mathbf{L}_{0k}/\Delta t) \Delta x, \\ \mathbf{A}_{ik} &= -(\mathbf{G}_{0k}/2 + \mathbf{C}_{0k}/\Delta t) \Delta x, \quad \mathbf{B}_{ik} = (\mathbf{G}_{0k}/2 - \mathbf{C}_{0k}/\Delta t) \Delta x. \end{aligned} \quad (4)$$

Here  $\mathbf{R}_{0k} = \mathbf{R}_0(\zeta_k)$ ,  $\mathbf{L}_{0k} = \mathbf{L}_0(\zeta_k)$ ,  $\mathbf{G}_{0k} = \mathbf{G}_0(\zeta_k)$ , and  $\mathbf{C}_{0k} = \mathbf{C}_0(\zeta_k)$ , with  $\zeta_k \in (x_k, x_{k+1})$ , mostly  $\zeta_k = (x_k + x_{k+1})/2$ . Defining further

$$\mathbf{v}^j = [\mathbf{v}_1^{jT}, \mathbf{v}_2^{jT}, \dots, \mathbf{v}_{K+1}^{jT}]^T, \quad \mathbf{i}^j = [\mathbf{i}_1^{jT}, \mathbf{i}_2^{jT}, \dots, \mathbf{i}_{K+1}^{jT}]^T, \quad (5)$$

as column vectors of the order  $n(K+1) \times 1$ , and finally

$$\mathbf{x}^j = [\mathbf{v}^{jT}, \mathbf{i}^{jT}]^T \quad (6)$$

as the  $2n(K+1) \times 1$  column vector, with  $^T$  as a transposition, we can write a recursive formula

$$\mathbf{x}^j = \mathbf{A}^{-1} (\mathbf{B}\mathbf{x}^{j-1} + \mathbf{D}^j). \quad (7)$$

The formula (7) expresses a solution in incoming time  $t_j$ , based on the values in preceding time  $t_{j-1}$ . The matrices  $\mathbf{A}$  and  $\mathbf{B}$  are formed by (4) and by boundary conditions, the column vector  $\mathbf{D}^j$  depends on values of external sources taken in time  $t_j$ . The constitution of these matrices can be explained by (8), when the MTL is divided only on  $K = 3$  parts.

$$\begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \\ \mathbf{v}_4 \\ \mathbf{i}_1 \\ \mathbf{i}_2 \\ \mathbf{i}_3 \\ \mathbf{i}_4 \end{pmatrix} = \begin{pmatrix} \mathbf{I} & -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{v1} & \mathbf{A}_{v1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{v2} & \mathbf{A}_{v2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{A}_{v3} & \mathbf{A}_{v3} \\ \mathbf{A}_{i1} & \mathbf{A}_{i1} & \mathbf{0} & \mathbf{0} & \mathbf{I} & -\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{i2} & \mathbf{A}_{i2} & \mathbf{0} & \mathbf{0} & \mathbf{I} & -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{i3} & \mathbf{A}_{i3} & \mathbf{0} & \mathbf{0} & \mathbf{I} & -\mathbf{I} \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{R}_{iL} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{R}_{iR} \end{pmatrix} \times \begin{pmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \\ \mathbf{v}_4 \\ \mathbf{i}_1 \\ \mathbf{i}_2 \\ \mathbf{i}_3 \\ \mathbf{i}_4 \end{pmatrix}^{j-1} + \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix}^j \quad (8)$$

The terminating circuits are supposed to be linear resistive, and they can be replaced by their generalized Thévenin equivalents with matrices of internal resistances  $\mathbf{R}_{iL}$  and  $\mathbf{R}_{iR}$ , and vectors of internal voltages  $\mathbf{v}_{iL}(t)$  and  $\mathbf{v}_{iR}(t)$ . They are located in two downmost lines of (8) corresponding to the boundary conditions incorporation. In more general case, however, when reactive elements are present, ordinary differential equations should have to be considered instead of the algebraic ones. Finally,  $\mathbf{I}$  and  $\mathbf{0}$  are the  $n$ -th order identity and zero matrices, respectively. (7) will now be used to derive a procedure for evaluation of sensitivities. Let us consider a parameter  $\gamma$  which can be some element of MTL p.-u.-l. matrices, an MTL length or a lumped parameter of terminating circuits. Then, after some arrangements, we have

$$\frac{\partial \mathbf{x}^j}{\partial \gamma} = \mathbf{A}^{-1} \left( \mathbf{B} \frac{\partial \mathbf{x}^{j-1}}{\partial \gamma} - \frac{\partial \mathbf{A}}{\partial \gamma} \mathbf{x}^j + \frac{\partial \mathbf{B}}{\partial \gamma} \mathbf{x}^{j-1} + \frac{\partial \mathbf{D}^j}{\partial \gamma} \right). \quad (9)$$

It is clear that (7) is also needed in (9) thus their simultaneous recursive processing leads to both voltage/current distributions and their sensitivities. Firstly, if we exclude sensitivities w.r. to voltages of the external sources,  $\partial \mathbf{D}^j / \partial \gamma = \mathbf{0}$ . The derivatives  $\partial \mathbf{A} / \partial \gamma$  and  $\partial \mathbf{B} / \partial \gamma$  are done submatrix-wise as is obvious from decomposed forms shown in (8). Finally,  $\partial \mathbf{R}_{iL(R)} / \partial \gamma = \mathbf{0}$  if  $\gamma$  is a distributed parameter, see Table I, or it is stated easily if  $\gamma$  is a lumped parameter.

TABLE I  
DERIVATIVES OF MATRICES (4) W.R. TO PARAMETERS  $\gamma$

| Parameter                    | $\frac{\partial \mathbf{A}_{vk}}{\partial \gamma}$                            | $\frac{\partial \mathbf{B}_{vk}}{\partial \gamma}$                            | $\frac{\partial \mathbf{A}_{ik}}{\partial \gamma}$                            | $\frac{\partial \mathbf{B}_{ik}}{\partial \gamma}$                            |
|------------------------------|---|---|---|---|
| $\gamma \in \mathbf{C}_{0k}$ | $\mathbf{0}$  | $\mathbf{0}$  | $-\frac{\partial \mathbf{C}_{0k}}{\partial \gamma} \frac{\Delta x}{\Delta t}$ | $-\frac{\partial \mathbf{C}_{0k}}{\partial \gamma} \frac{\Delta x}{\Delta t}$ |
| $\gamma \in \mathbf{L}_{0k}$ | $-\frac{\partial \mathbf{L}_{0k}}{\partial \gamma} \frac{\Delta x}{\Delta t}$ | $-\frac{\partial \mathbf{L}_{0k}}{\partial \gamma} \frac{\Delta x}{\Delta t}$ | $\mathbf{0}$  | $\mathbf{0}$  |
| $\gamma \in \mathbf{G}_{0k}$ | $\mathbf{0}$  | $\mathbf{0}$  | $-\frac{\partial \mathbf{G}_{0k}}{\partial \gamma} \frac{\Delta x}{2}$        | $\frac{\partial \mathbf{G}_{0k}}{\partial \gamma} \frac{\Delta x}{2}$         |
| $\gamma \in \mathbf{R}_{0k}$ | $-\frac{\partial \mathbf{R}_{0k}}{\partial \gamma} \frac{\Delta x}{2}$        | $\frac{\partial \mathbf{R}_{0k}}{\partial \gamma} \frac{\Delta x}{2}$         | $\mathbf{0}$  | $\mathbf{0}$  |
| $\gamma \equiv l$            | $\frac{\mathbf{A}_{vk}}{l}$   | $\frac{\mathbf{B}_{vk}}{l}$   | $\frac{\mathbf{A}_{ik}}{l}$   | $\frac{\mathbf{B}_{ik}}{l}$   |

### III. EXPERIMENTAL ACCURACY EVALUATION

An accuracy of the method will experimentally be assessed through known analytical solutions for voltage and current distributions on a Thomson cable, an uniform single TL with negligible  $L_0$  and  $G_0$  primary parameters. The cable is driven from a unit step voltage source as shown in Fig. 2.

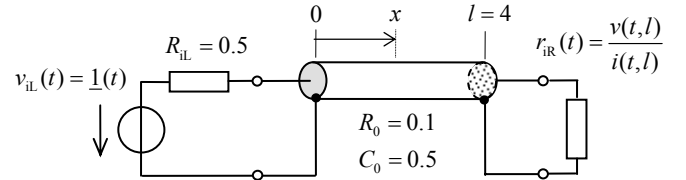


Fig. 2 Transmission line system with Thomson cable

For an infinitely long cable, when no reflected waves exist, analytical solutions have the forms [9]

$$\begin{aligned} i(t, x) &= \underline{1}(t) / R_{iL} \cdot e^{a(t)(a(t)+2b(t, x))} \cdot \operatorname{erfc}(a(t) + b(t, x)), \\ v(t, x) &= \underline{1}(t) \cdot \operatorname{erfc}(b(t, x)) - R_{iL} i(t, x), \\ a(t) &= \frac{1}{R_{iL}} \cdot \sqrt{\frac{R_0 t}{C_0}}, \quad b(t, x) = \frac{x}{2} \cdot \sqrt{\frac{R_0 C_0}{t}}, \end{aligned} \quad (10)$$

with  $\operatorname{erfc}$  as a complementary error function. To ensure a validity of the above equations at the TL of a finite length  $l$ , it

is terminated by a time-dependent resistance  $r_{iR}(t)$  matching it perfectly in the time interval of interest. The relative errors evaluated in case of  $K = J = 256$  are shown in Fig. 3.

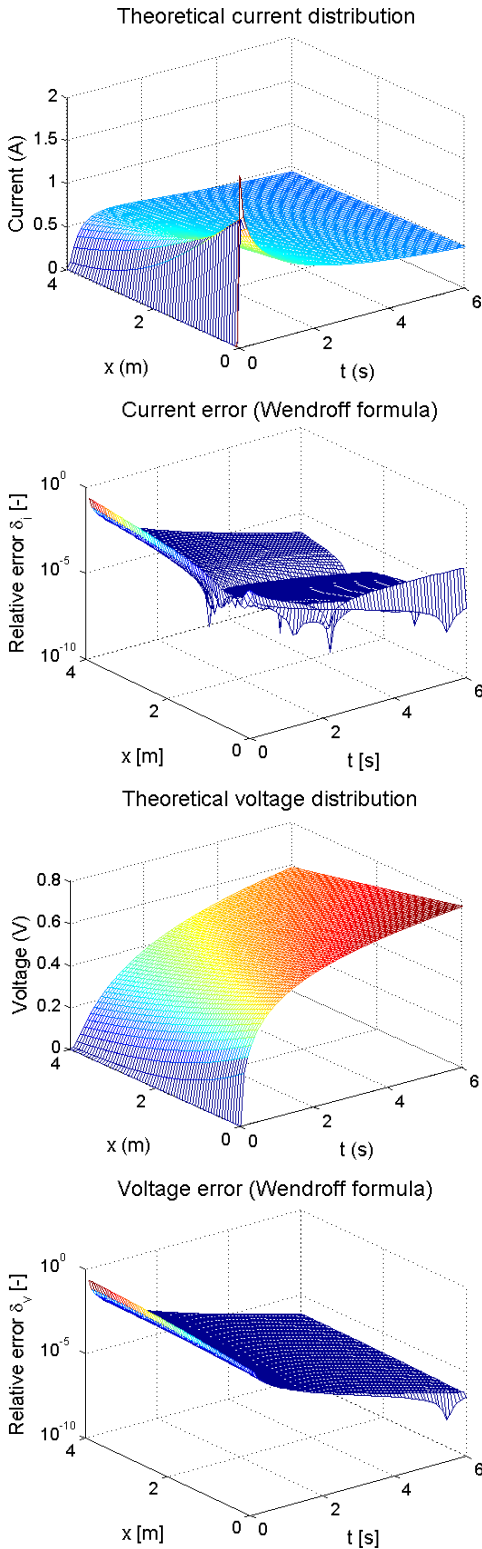


Fig. 3 Thomson cable relative errors for Wendroff method

Following (10) the semirelative sensitivities have been derived and summarized in Table II.

TABLE II  
THOMSON CABLE'S SEMIRELATIVE SENSITIVITIES

| Parameter              | $f(t,x)$ | Semirelative sensitivity $S_\gamma = \gamma \frac{\partial f(t,x)}{\partial \gamma}$           |
|------------------------|----------|--|
| $\gamma \equiv R_0$    | $i(t,x)$ | $-1(t) \frac{e^{-b^2(t,x)}}{R_{iL} \sqrt{\pi}} [a(t) + b(t,x)] + a(t) [a(t) + 2b(t,x)] i(t,x)$ |
|                        | $v(t,x)$ | $1(t) \frac{e^{-b^2(t,x)}}{\sqrt{\pi}} a(t) - R_{iL} a(t) [a(t) + 2b(t,x)] i(t,x)$             |
| $\gamma \equiv C_0$    | $i(t,x)$ | $1(t) \frac{e^{-b^2(t,x)}}{R_{iL} \sqrt{\pi}} [a(t) - b(t,x)] - a^2(t) i(t,x)$                 |
|                        | $v(t,x)$ | $-1(t) \frac{e^{-b^2(t,x)}}{\sqrt{\pi}} a(t) + R_{iL} a^2(t) i(t,x)$                           |
| $\gamma \equiv R_{iL}$ | $i(t,x)$ | $1(t) \frac{2e^{-b^2(t,x)}}{R_{iL} \sqrt{\pi}} a(t) - (2a(t) [a(t) + b(t,x)] + 1) i(t,x)$      |
|                        | $v(t,x)$ | $-1(t) \frac{2e^{-b^2(t,x)}}{\sqrt{\pi}} a(t) + 2R_{iL} a(t) [a(t) + b(t,x)] i(t,x)$           |

Examples of voltage semirelative sensitivities are presented in Fig. 4, including relative errors determined on a basis of Table II, with  $K = J = 512$  stated. The results are comparable with those in Fig. 3, the errors increase generally at beginning of a time interval where discontinuities occur. In this special case, when a terminating resistance  $r_{iR}(t)$  is time dependent, as is defined in Fig. 2, the matrix  $\mathbf{A}$  in (7) has to be evaluated repeatedly in each time step with the help of (10), for  $x = l$  substituted. It is similarly valid for  $\partial \mathbf{A} / \partial \gamma$  in (9), if sensitivities are calculated, when  $\partial r_{iR}(t) / \partial \gamma$  is stated on a basis of Table II.

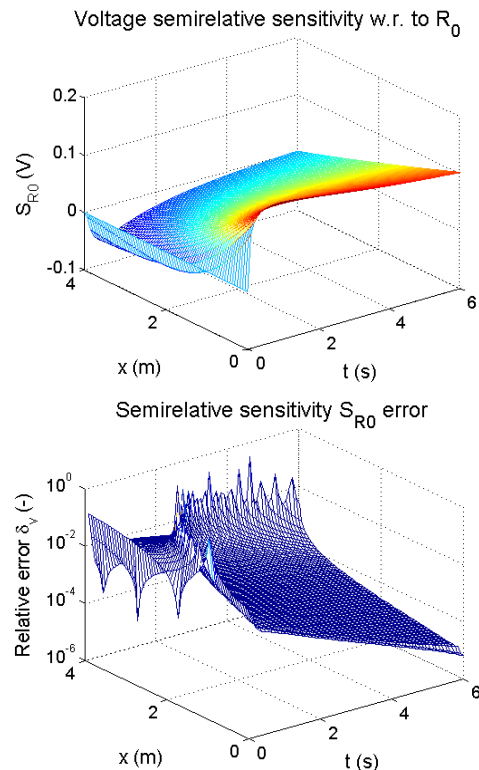


Fig. 4 Thomson cable semirelative sensitivities and relative errors

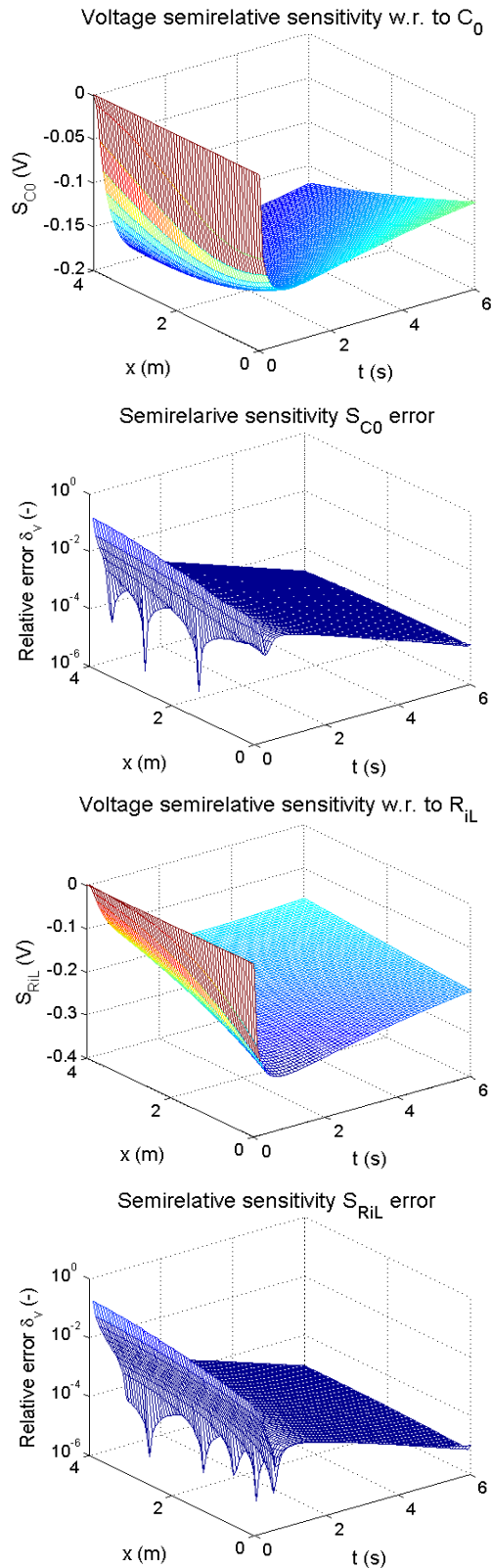


Fig. 4 Thomson cable semirelative sensitivities and relative errors (continuation)

IV. EXAMPLES OF MTL SIMULATION

In the following a simple (2+1)-conductor transmission line system will be considered according to Fig. 5.

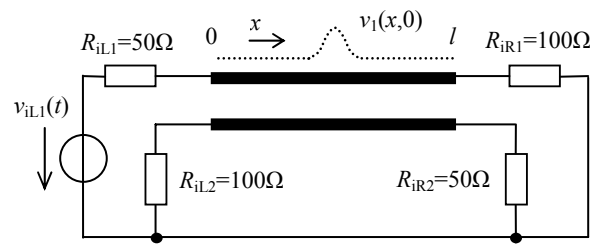


Fig. 5 Simple (2+1)-conductor transmission line system

First, the MTL is considered under zero initial conditions, i.e.  $v(x,0) = 0$  and  $i(x,0) = 0$ , while the feeding voltage is

$$v_{iL1}(t) = \sin^2(\pi t / (2 \cdot 10^{-9})), \text{ if } 0s \leq t \leq 2 \cdot 10^{-9}s, \quad (11)$$

$$v_{iL1}(t) = 0, \text{ otherwise.}$$

The generalized Thévenin internal matrices and vectors are

$$\mathbf{R}_{iL} = \begin{bmatrix} R_{iL1} & 0 \\ 0 & R_{iL2} \end{bmatrix}, \mathbf{v}_{iL} = \begin{bmatrix} v_{iL1} \\ 0 \end{bmatrix}, \mathbf{R}_{iR} = \begin{bmatrix} R_{iR1} & 0 \\ 0 & R_{iR2} \end{bmatrix}, \mathbf{v}_{iR} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (12)$$

The MTL's length is  $l = 0.4m$ , and its p.-u.-l. matrices  $\mathbf{P}_0(x) \in \{\mathbf{R}_0(x), \mathbf{L}_0(x), \mathbf{G}_0(x), \mathbf{C}_0(x)\}$  have unified forms

$$\mathbf{P}_0(x) = \mathbf{P}_0 e^{px} = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} e^{px}, \quad (13)$$

with particular primary parameters shown in Table III [10].

TABLE III  
PRIMARY PARAMETERS FOR MTL IN FIG. 5

| $\mathbf{P}_0$ | $\mathbf{R}_0$ ( $\Omega/m$ ) | $\mathbf{L}_0$ (nH/m) | $\mathbf{G}_0$ (S/m) | $\mathbf{C}_0$ (pF/m) |
|----------------|-------------------------------|-----------------------|----------------------|-----------------------|
| $P_{11}$       | $R_{11} = 0.1$                | $L_{11} = 494.6$      | $G_{11} = 0.1$       | $C_{11} = 62.8$       |
| $P_{12}$       | $R_{12} = 0.02$               | $L_{12} = 63.3$       | $G_{12} = -0.01$     | $C_{12} = -4.9$       |
| $P_{22}$       | $R_{22} = 0.1$                | $L_{22} = 494.6$      | $G_{22} = 0.1$       | $C_{22} = 62.8$       |

If  $p = 0$  in (13), for all the p.-u.-l. matrices, the MTL becomes uniform. An inhomogeneity is introduced by the value  $p = \ln(2)/l \approx 1.733$  to get two-time greater p.-u.-l. parameters at the MTL's end compared to its beginning. The examples of voltage distributions for (non)uniform MTLs are in Fig. 6.

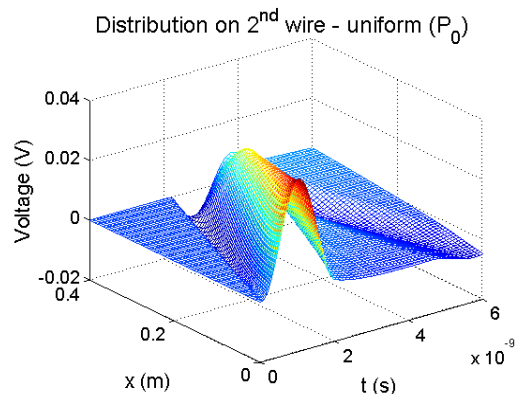


Fig. 6 Voltage distributions for (non)uniform MTLs

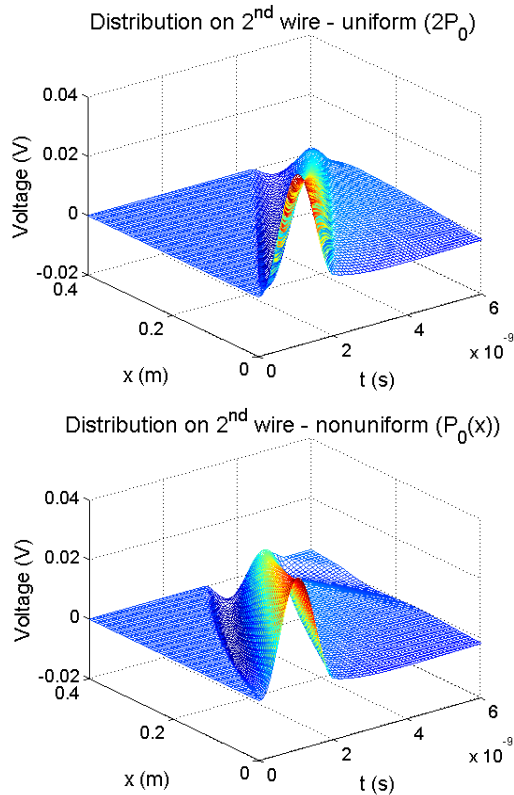


Fig. 6 Voltage distributions for (non)uniform MTLs (continuation)

The examples of voltage semirelative sensitivity distributions for (non)uniform MTLs are in Fig. 7.

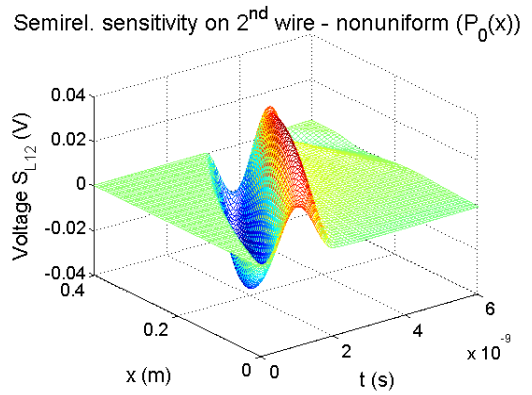


Fig. 7 Voltage semirelative sensitivity distribution w.r. to  $L_{12} \in \mathbf{L}_0(x)$  for (non)uniform MTL (continuation)

It can be seen when doubling all p.-u.-l. parameters ( $2\mathbf{P}_0$  in Figs. 6 and 7) the MTL time delay increases, roughly twice (as reactive parameters are increased). In case of a nonuniform MTL, wave velocity is not constant, but it decreases gradually along the line leading to some intermediate time delay.

The method can easily be adapted to obtain responses to MTL initial voltage or current distributions, i.e. to  $v(x,0) \neq 0$  or  $i(x,0) \neq 0$ . Examples for the initial voltage distribution on the 1<sup>st</sup> wire of the form

$$v_1(x,0) = \sin^2(\pi(4x/l - 3/2)), \text{ if } 3l/8 \leq x \leq 5l/8, \quad (14)$$

$$v_1(x,0) = 0, \text{ otherwise,}$$

with all the other quantities equal zero, are shown in Fig. 8.

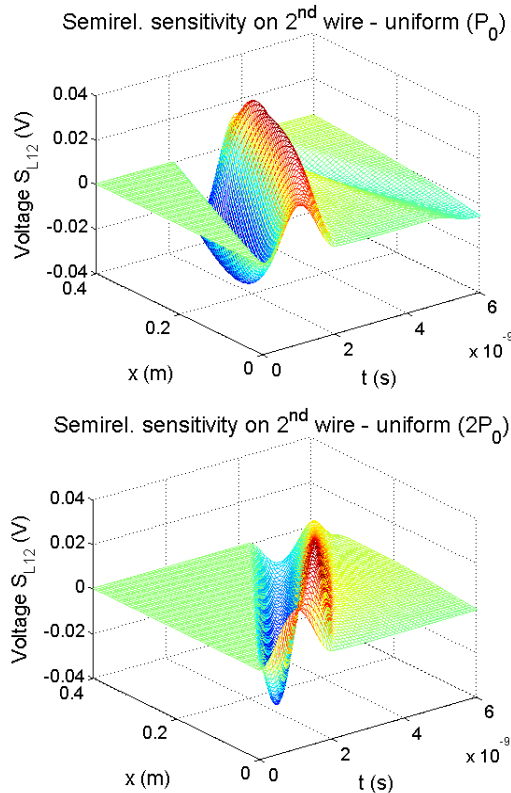


Fig. 7 Voltage semirelative sensitivity distribution w.r. to  $L_{12} \in \mathbf{L}_0(x)$  for (non)uniform MTLs

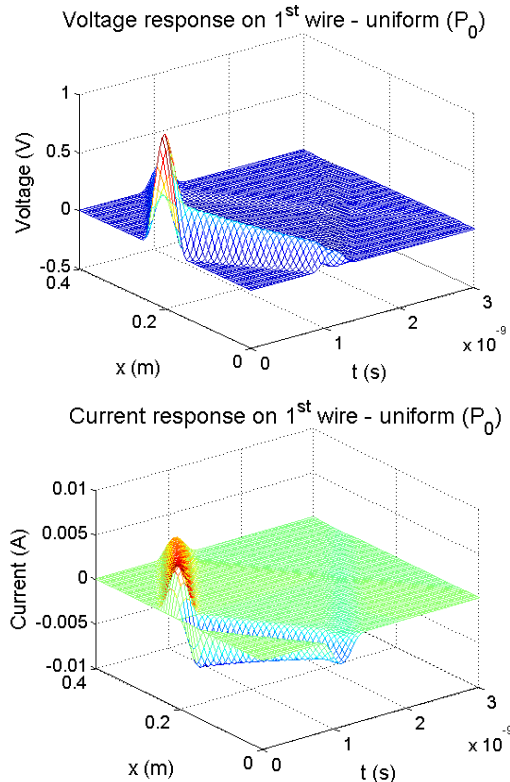


Fig. 8 Voltage and current responses to initial voltage  $v_1(x,0)$

In the end, some experiments with simulation of nonlinear MTLs are shown in Fig. 9. As in previous cases we will again consider both uniform and nonuniform MTLs to compare and assess correctness of the results.

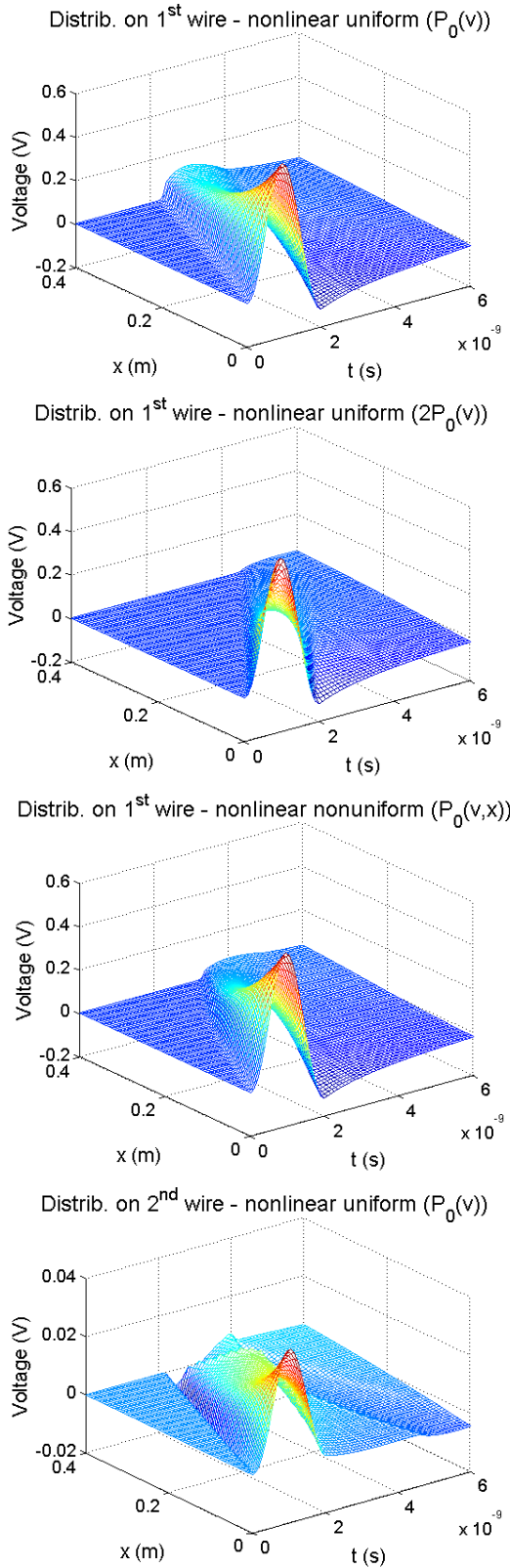


Fig. 9 Voltage distributions for nonlinear (non)uniform MTLs

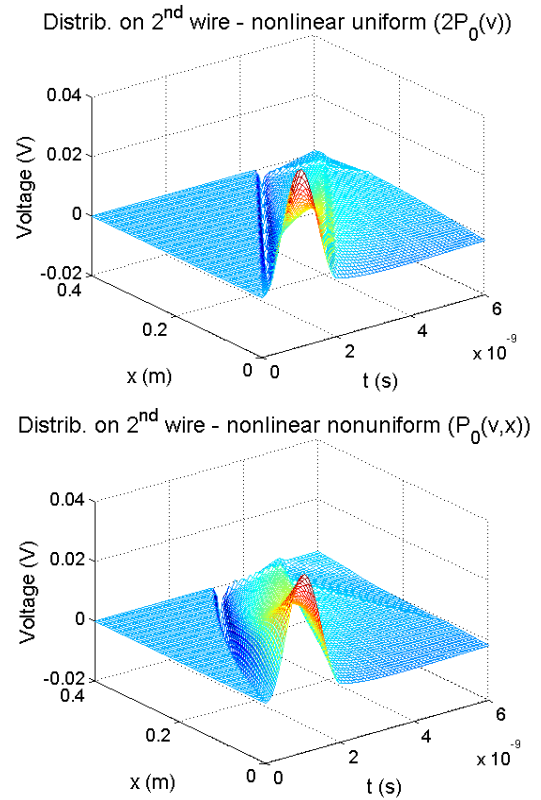


Fig. 9 Voltage distributions for nonlinear (non)uniform MTLs (continuation)

The nonlinearity is introduced through  $C_0$  matrix, in which the  $C_{11}$  and  $C_{22}$  p.-u.-l. capacitances are replaced by voltage-dependent ones, according to

$$C_{ii}(v_i) = \frac{C_{ii}}{(1 + |v_i|/V_p)^2}, \quad (15)$$

$i = 1, 2$ , for  $V_p = 0.75V$ . The MTL excitation corresponds to (11) used previously for the example in Fig. 6. In this case, however, (7) is modified into a form

$$\mathbf{x}^j = (\mathbf{A}^{j-1})^{-1} (\mathbf{B}^{j-1} \mathbf{x}^{j-1} + \mathbf{D}^j), \quad (16)$$

when  $\mathbf{A}^{j-1}$  and  $\mathbf{B}^{j-1}$  depend on actual  $\mathbf{x}^{j-1}$  and must be evaluated repeatedly in each time step.

All the results for linear MTLs were also compared with a state-variable method, e.g. [11], [12], and Laplace transform methods [13]–[17], with very good matching. In Appendix I, the relative errors of the state-variable method applied for the Thomson cable simulation in Fig. 2 are shown [6], [7].

#### V. MATLAB CPU TIMES EVALUATION

To evaluate the computational efficiency of the Wendroff method the CPU times have been evaluated with the help of a Matlab language. A test MTL system with varying number of active wires, from 1 to 10, is considered in Fig. 10.

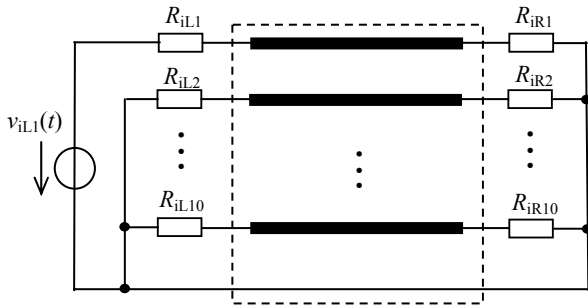


Fig. 10 Test  $(n+1)$ -conductor TL system,  $n = 1, 2, \dots, 10$

Some results following Fig. 10, computed as the function of MTL's wires, for the grid densities ranging from  $128 \times 128$  to  $512 \times 512$  points as the parameter, are shown in Fig. 11.

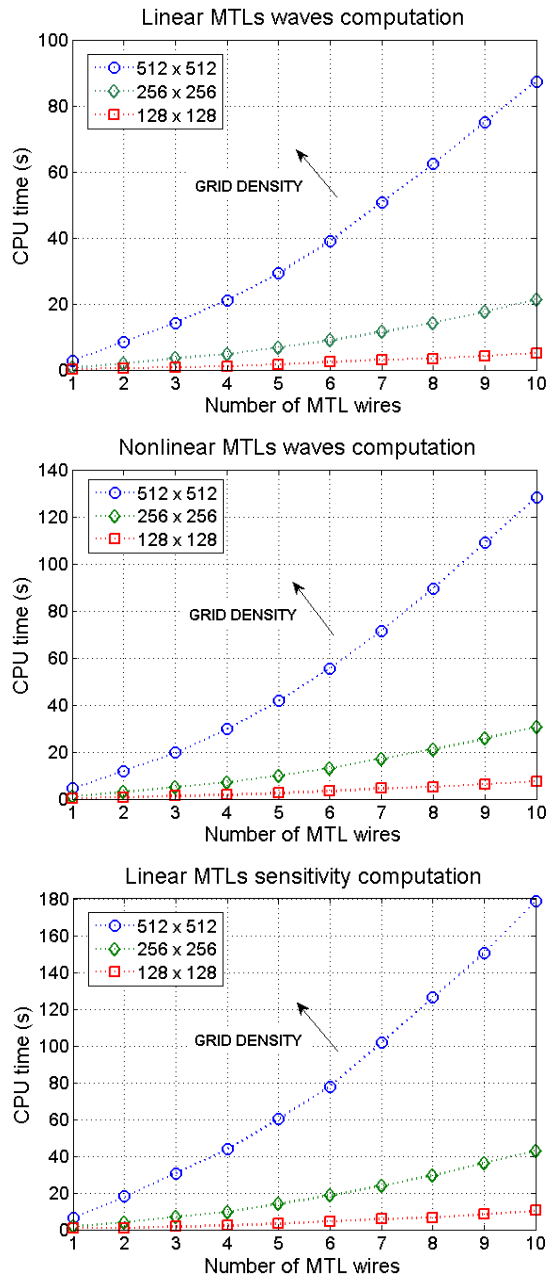


Fig. 11 Matlab CPU times of Wendroff method

All the computations have been performed on a common PC with the 2GHz processor and the 2GB RAM. To ensure fast computation and to prevent RAM to overflow, the code is based on the sparse matrices notation, as can be observed in Appendix II, where the Matlab code listing for the solution of linear nonuniform MTLs' voltage and current distributions is presented as an example.

VI. CONCLUSION

The paper has dealt with a technique for the time-domain simulation of multiconductor transmission lines based on an implicit Wendroff method. Namely, the technique was further generalized towards nonuniform MTLs and first experiments pertaining to nonlinear MTLs were done. In case of the linear MTLs, the method is stable and accurate enough to compute both voltage/current distributions and their sensitivities for practice. In case of nonlinear MTLs, further studies and experiments will be performed, especially in light of a method stability and an accuracy evaluation.

A few Matlab programme scripts and functions have been developed, see Appendix II as an example. As is obvious from Fig. 11, the CPU times are relatively favourable for practical applications, especially when comparing with other methods. That is why the method is planned to be further adapted for more complex systems containing the MTLs in future.

APPENDIX I

The relative errors for the Thomson cable solved via a state-variable method in the time domain [6], [7], compare Fig. 3.

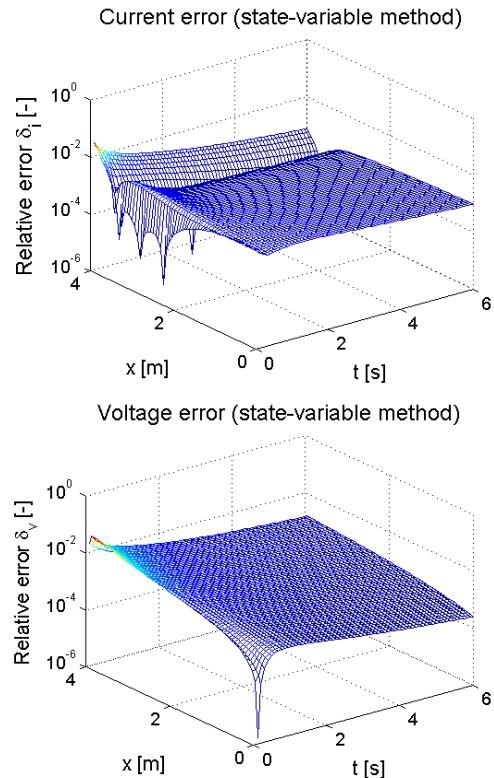


Fig. 12 Thomson cable relative errors for state-variable method

## WENDROFF METHOD MATLAB CODE LISTING

```

% ** Wendroff method Matlab code, Sept. 1, 2010 **
% Number of MTL wires
n=2;
% MTL length
len=0.4;
% Time interval limit
tmax=6e-9;
% Spatial division, k=1,...,K
K=2^9; dx=len/K;
% Time division, j=1,...,J
J=2^9; dt=tmax/J;
% MTL primary parameters
R0=[0.1,0.02;0.02,0.1];
L0=[494.6,63.3;63.3,494.6]*1e-9;
G0=[0.1,-0.01;-0.01,0.1];
C0=[62.8,-4.9;-4.9,62.8]*1e-12;
% Lumped-parameter circuits Thévenin matrices
RiL=[50,0;0,100]; RiR=[100,0;0,50];
% Inhomogeneity definition
RLx=exp(log(2)/len*(dx/2:dx:len)).'; GCx=RLx;
ord=2*n*(K+1);
% Matrices (4) definition
Av=-(R0/2+L0/dt)*dx; Bv=(R0/2-L0/dt)*dx;
Ai=-(G0/2+C0/dt)*dx; Bi=(G0/2-C0/dt)*dx;
% Matrices A and B of (7) definition
A1=kron(spdia([ones(K+1,1),...
-ones(K+1,1)], [0,1],K,K+1),eye(n));
A12=kron(spdia([RLx,RLx], [0,1],K,K+1),Av);
A21=kron(spdia([GCx,GCx], [0,1],K,K+1),Ai);
A=[A11,A12;A21,A11];
Ab1=sparse(2*n,(K+1)*n); Ab1(1:n,1:n)=speye(n);
Ab1(n+1:2*n,K*n+1:ord/2)=speye(n);
Ab2=sparse(2*n,(K+1)*n); Ab2(1:n,1:n)=RiL;
Ab2(n+1:2*n,K*n+1:ord/2)=-RiR;
Ab=[Ab1,Ab2];
A=[A;Ab];
B11=-A11;
B12=kron(spdia([RLx,RLx], [0,1],K,K+1),Bv);
B21=kron(spdia([GCx,GCx], [0,1],K,K+1),Bi);
B=[B11,B12;B21,B11];
Bb=sparse(2*n,ord);
B=[B;Bb];
% Exciting quantities definition
D=sparse(2*K*n,J);
ViL=sparse(n,J); ViR=sparse(n,J);
t=linspace(0,tmax,J+1);
i=1;
while t(i)<=0.2e-8
    ViL(1,i)=sin(pi/0.2e-8*t(i))^2; i=i+1;
end
D=[D;ViL;ViR];
% Zero initial conditions definition
V0=sparse(ord/2,J+1); I0=sparse(ord/2,J+1);
W=[V0;I0];
% Recursive equation (7) evaluation
for j=1:J
    W(:,j+1)=A\ (B*W(:,j)+D(:,j));
end
% Extracting voltages and currents to plot
V=W(1:ord/2,:); I=W(ord/2+1:ord,:);
Vxt=zeros(K+1,J+1,n); Ixt=zeros(K+1,J+1,n);
for i=1:n
    Vxt(:, :, i)=V(i:n*n*K+i, :);
    Ixt(:, :, i)=I(i:n*n*K+i, :);
end
% Plotting voltage and current distributions
xpl=64; tpl=64; rt=1:J/tp1:J+1; rx=1:K/xpl:K+1;
x=linspace(0,len,K+1);
for i=1:n
    figure; mesh(t(rt),x(rx),Vxt(rx,rt,i));
    xlabel('t(s)'); ylabel('x(m)');
    zlabel('Voltage (V)');
    figure; mesh(t(rt),x(rx),Ixt(rx,rt,i));
    xlabel('t(s)'); ylabel('x(m)');
    zlabel('Current (A)');
end

```

- [1] D. M. Sullivan, *Electromagnetic Simulation using the FDTD Method*. New York: IEEE Press, 2000.
- [2] W. Sui, *Time-Domain Computer Analysis of Nonlinear Hybrid Systems*. New York: CRC Press, 2002.
- [3] L. Dou, J. Dou, "Sensitivity analysis of lossy nonuniform multiconductor transmission lines based on Lax-Wendroff technique," *Int. J. Numer. Model.*, 2010, vol. 23, no. 3, pp. 165–182.
- [4] Z. Benešová, D. Mayer, "Overvoltage phenomena on transmission lines" in *Proceedings XI. International Symposium on Theoretical Electrical Engineering ISTE'2001*, Linz, 2001, pp. 1–4.
- [5] Z. Benešová, V. Kotlan, "Propagation of surge waves on interconnected transmission lines induced by lightning stroke," *Acta Technica CSAV*, 2006, vol. 51, no. 3, pp. 301–316.
- [6] L. Brančík, "Transient and sensitivity analysis of uniform multiconductor transmission lines via FDTD methods," in *Proceedings XV. International Symposium on Theoretical Electrical Engineering ISTE'2009*, Lübeck, 2009, pp. 109–113.
- [7] L. Brančík, B. Ševčík, "Modeling of nonuniform multiconductor transmission lines via Wendroff method," in *Proceedings the International Conference on Mathematical Models for Engineering Science MMES'2010*, Puerto De La Cruz, 2010, pp. 130–133.
- [8] C. R. Paul, *Analysis of Multiconductor Transmission Lines*. New York: John Wiley & Sons, 1994.
- [9] L. Brančík, "Sensitivity in multiconductor transmission line lumped-parameter models," in *Proceedings 31<sup>st</sup> International Conference Telecommunications and Signal Processing TSP'2008*, Paraffürdo, 2008, pp. 1–4.
- [10] J. R. Griffith, M. S. Nakhla, "Time-domain analysis of lossy coupled transmission lines," *IEEE Transactions on Microwave Theory and Techniques*, 1990, vol. 38, no. 10, pp. 1480–1487.
- [11] C. K. Cheng, J. Lillis, S. Lin, N. Chang, *Interconnect Analysis and Synthesis*. New York: John Wiley & Sons, 2000.
- [12] L. Brančík, "Transient and sensitivity analysis at semi-discrete multiconductor transmission line models," *Electroscope* [online]. Cited 2010-12-30. Available at: <http://www.electroscope.zcu.cz>.
- [13] L. Brančík, "Voltage/current waves sensitivity in hybrid circuits with nonuniform MTLs," in *Proceedings 10<sup>th</sup> IEEE Workshop on Signal Propagation on Interconnects SPI'2006*, Berlin, 2006, pp. 177–180.
- [14] L. Brančík, "Sensitivity analysis in multiconductor transmission line networks," in *Proceedings the 4<sup>th</sup> WSEAS International Conference Applications of Electrical Engineering AEE'2005*, Prague, 2005, pp. 170–175.
- [15] L. Brančík, "Novel techniques for sensitivity evaluation in multiconductor transmission line systems," *WSEAS Transactions on Communications*, 2005, vol. 4, no. 5, pp. 216–223.
- [16] J. Dědková, L. Brančík, "Laplace transform and FDTD approach applied to MTL simulation," *Piers Online*, 2008, vol. 4, no. 1, p. 16–20.
- [17] L. Brančík, "Matlab oriented matrix Laplace transforms inversion for distributed systems simulation," in *Proceedings of the 12<sup>th</sup> International Conference Radioelektronika'2002*. Bratislava, 2002, pp. 114–117.

**Lubomír Brančík** was born in Kyjov, Czech Republic, in 1961. He received his Ing. (MSc. equiv) degree in field of Microelectronics in 1985 from the Brno University of Technology, and CSc. (Ph.D. equiv) degree in field of Measurements in 1993 from the same university. He is currently a professor for theoretical electrical engineering at the Brno University of Technology, Dept. of Radio Electronics. He is a member of the IEEE and the IEICE. His research interest is in numerical methods for electronic circuits, computer-aided simulation methods and signal integrity analysis.

**Břetislav Ševčík** was born in Prostějov, Czech Republic, in 1985. He received his Ing. (MSc. equiv) degree in field of Electronics and Communication in 2009 from the Brno University of Technology. He is currently studying his Ph.D. study at the same university, Dept. of Radio Electronics. He is a student member of the IEEE. His research interest is in analysis and design of analog circuits, high-speed transmission lines design and signal integrity computer optimization methods.