GA-Based PIDA Control Design Optimization with an Application to AC Motor Speed Control

Sunisa Sornmuang and Sarawut Sujitjorn*

Abstract— PIDA controller has been proposed since 1996 as an extension to the conventional PID controller. The additional term “A” stands for acceleration. With this new term, a closed-loop system can respond faster with less overshoot. Originally, the design utilizes the dominant pole concept proceeded in the s-plane. As shown by simulation, this design approach is not suitable for high-order plants having delays and complex oscillatory modes. The article proposes an algebraic design approach which also utilizes the genetic algorithm (GA) to achieve design optimality. Comparison studies among the previous method, the gradient-search based method and the proposed approach are elaborated. Such studies were conducted against some benchmark plants defined by Astrom and Hagglund. As a result, the GA method with heuristically defined solution boundaries provides superior results. The proposed approach has been successfully applied to the speed control of an AC motor.

Keywords—Genetic algorithm, gradient search, optimization, PIDA controller, pole placement, speed control of ac motor.

I. Introduction

PID controller has been a worldwide solution for an effective control for many years. Researchers have paid a lot of attention to its design as evidenced by a great member of published patents and articles. For instance, some recent literatures include but not exhaustive (i) a collection of PIDA tuning rules [1], (ii) model-free design method based on experimental frequency response data [2], (iii) frequency-domain sensitivity-based design method [3]-[4], (iv) new auto tuning algorithms [5], an adaptive PID control using knowledge-based method [6], robust PID design via evolutionary computation [7], etc. Despite the existing knowledge, the PIDA controller was introduced in 1996 by Jung and Dorf [8], and claimed to deliver faster and smoother response than the PID could. The PIDA controller has been successfully applied to torsional resonance suppression [9], and control of an induction motor model [10], respectively.

So far, there have not been published works detailing the performance studies of the PIDA controller. Design details yet have not been proposed to achieve system optimality. This article offers contributions in the following issues: an investigation of the effectiveness of the Jung and Dorf’s design method, a new optimally algebraic design approach based on genetic algorithm (GA), a comparison study between the gradient search (GS) method and the proposed one, and application to speed control of a 3-phase motor.

Manuscript received September 1, 2010. This work was supported by Suranaree University of Technology, Thailand, under Grant SUT7-711-52-12-56.

The authors are with the School of Electrical Engineering, Suranaree University of Technology, Nakhon Ratchasima, 30000, Thailand.

*corresponding author: (sarawut@sut.ac.th).

II. Problem Formulation

To develop a method of control design, a simplified control system represented by the diagram in fig. 1 is considered. The plants for benchmarking the design methods have been proposed by Astrom and Hagglund [11] as follows:

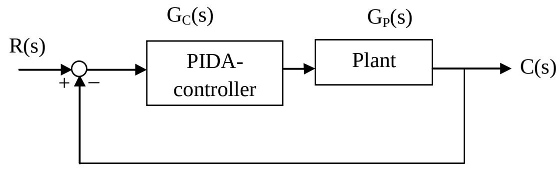


Fig. 1 A closed-loop system.

Plant 1 : $$G_P(s)=\frac{1}{(s+1)(\alpha s+1)(\alpha^2 s+1)(\alpha^3 s+1)},\ \alpha=0.5$$

Plant 2 : $$G_P(s)=\frac{1}{(s+1)^4}$$

Plant 3 : $$G_P(s)=\frac{-\alpha s+1}{(s+1)^3},\alpha=0.5$$

Plant 4 : $$G_P(s)=\frac{1}{(Ts+1)}e^{-s},T=10$$

Plant 5 : $$G_P(s)=\frac{1}{(Ts+1)^2}e^{-s},T=10$$

Plant 6 : $$G_P(s)=\frac{(s+6)^2}{s(s+1)^2(s+36)}$$

Plant 7 : $$G_P(s)=\frac{\omega_0^{\,2}}{(s+1)(s^2+2\zeta\omega_0 s+\omega_0^{\,2})},\omega_0=1,\zeta=0.1.$$

The PIDA controller has a practical form of (1) including the low-pass filters. Four parameters of the controller are $K_P\ K_I\ K_D$ and K_A, respectively, whilst d,e and f are filter parameters. An alternative representation in a polynomial form results in the following parameters: *k*, *a*, *b*, *z*, *d* and *e*, respectively. The design objective is to achieve a minimum response error due to a unit step input. Hence, the following performance specifications, i.e. rise-time, percent overshoot, settling time and steady-state errors, are considered.

$$G_C(s)=K_P+\frac{K_I}{s}+\frac{K_D s}{(s+f)}+\frac{K_A s^2}{(s+d)(s+e)} \qquad (1)$$
$$=\frac{K(s+a)(s+b)(s+z)}{s(s+d)(s+e)}\ \ ;f=d$$

III. Dominant Pole Based Design

The design method in [8] is based on a pair of dominant complex poles. Locations of the desired poles are arbitrary. Thus, to obtain a satisfactory result is a time consuming task if conducted manually. Even though with an aid of a computer, a few to several trials-and-errors to obtain a satisfactory design are inevitable. The readers may refer to [8] for the design procedures. The list below indicates the obtained PIDA controllers for the plants.

Plant 1 : $Gc(s) = \dfrac{14.17s^3 + 74.91s^2 + 185.7s + 175.8}{s}$

Plant 2 : $Gc(s) = \dfrac{3022s^3 + 27690s^2 + 106900s + 153600}{s}$

Plant 3 : $Gc(s) = \dfrac{1.96s^3 + 8.49s^2 + 14.81s + 10.48}{s}$

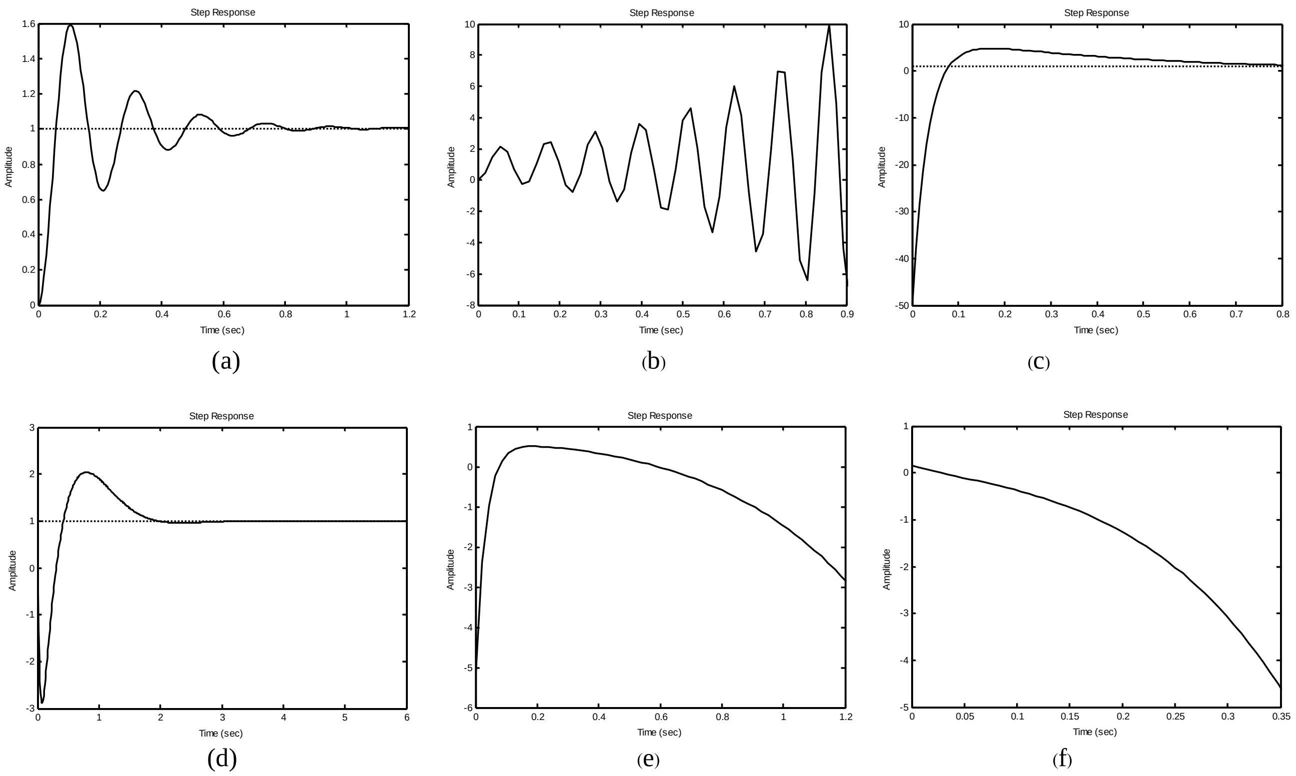
Plant 4 : $Gc(s) = \dfrac{-0.06s^3 + 7.64s^2 + 23.6s + 16.3}{s}$

Plant 5 : $Gc(s) = \dfrac{-83.57s^3 - 564.4s^2 - 1603s - 1540}{s}$

Plant 6 : $Gc(s) = \dfrac{0.19s^3 - 4.97s^2 - 60.83s - 116}{s}$

Plant 7 : $Gc(s) = \dfrac{35.1s^3 + 205s^2 + 533.6s + 529.8}{s}$.

Fig 2 illustrates the simulated step responses of the closed-loop systems. It can be noticed that some plants are not well stabilized, some are stable but unable to produce good responses. This is due to the difficulty to accomplish optimal design goals manually. Furthermore, only one pair of dominant poles may not satisfactorily govern these complex plants.



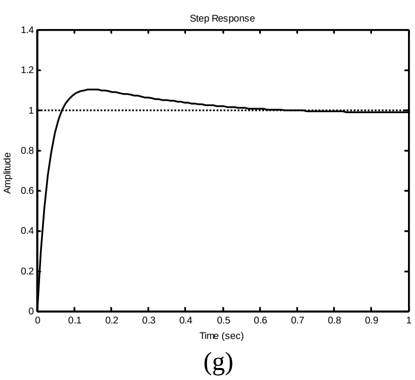


Fig. 2 Simulated step responses (dominant pole based design)
(a) Plant 1 (b) Plant 2 (c) Plant 3
(d) Plant 4 (e) Plant 5 (f) Plant 6
(g) Plant 7

IV. Optimal Algebraic Design

To reach an optimal pole-placement goal, some algebraic expressions have been derived. Firstly assume that the plant is described by a proper transfer function

$$G_P(s) = \frac{p(s)}{q(s)} = \frac{a_0 s^n + a_1 s^{n-1} + ... + a_n}{s^n + p_1 s^{n-1} + ... + p_n} \tag{2}$$

, and the closed-loop system possesses the characteristic equation

$$s^m + \alpha_1 s^{m-1} + ... + \alpha_m = 0 \tag{3}$$

, in which $m = n+3$ (n is the plant order). Closed-loop pole-placement is achieved by solving (4).

$$\begin{bmatrix} p_1 & 1 & 0 & 0 & 0 & a_0 & a_1 \\ p_2 & p_1 & 1 & 0 & a_0 & a_1 & a_2 \\ p_3 & p_2 & p_1 & a_0 & a_1 & a_2 & a_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ p_n & p_{n-1} & p_{n-2} & a_{n-3} & a_{n-2} & a_{n-1} & a_n \\ 0 & p_n & p_{n-1} & a_{n-2} & a_{n-1} & a_n & 0 \\ 0 & 0 & p_n & a_{n-1} & a_n & 0 & 0 \\ 0 & 0 & 0 & a_n & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \\ \alpha_{n+1} \\ \alpha_{n+2} \\ \alpha_{n+3} \end{bmatrix} \tag{4}$$

, where $[\mathbf{x}]^T = (1 + ka_0)^{-1}$

$$\begin{bmatrix} 1 & (d+e) & de & abzk & k(ab + z(a+b)) & k(a+b+z)) & k \end{bmatrix} \quad (5).$$

In this case, there are n+3 equations with 7 unknowns. Secondly, the plant is represented by a strictly proper transfer function

$$G_P(s) = \frac{p(s)}{q(s)} = \frac{a_1 s^l + a_2 s^{l-1} + ... + a_l}{s^n + p_1 s^{n-1} + ... + p_n} \; ; \; l < n \tag{6}$$

, and the closed-loop characteristic equation is also expressed by (3). Equation (7) has been derived for the pole-placement design

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & a_1 \\ p_1 & 1 & 0 & 0 & a_1 & a_2 \\ p_2 & p_1 & 0 & a_1 & a_2 & \vdots \\ \vdots & p_2 & a_1 & a_2 & \vdots & a_{n-1} \\ p_{n-1} & \vdots & a_2 & \vdots & a_{n-1} & a_n \\ p_n & p_{n-1} & \vdots & a_{n-1} & a_n & 0 \\ 0 & p_n & a_{n-1} & a_n & 0 & 0 \\ 0 & 0 & a_n & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \alpha_1 - p_1 \\ \alpha_2 - p_2 \\ \vdots \\ \alpha_n - p_n \\ \alpha_{n+1} \\ \alpha_{n+2} \\ \alpha_{n+3} \end{bmatrix} \tag{7}$$

,where $[\mathbf{x}]^T = \begin{bmatrix} (d+e) & de & abzk & k(ab + z(a+b)) & k(a+b+z)) & k \end{bmatrix}$.

In this case, there are n+3 equations with 6 unknowns. It is observed that some numerical approximations are necessary for solving such over/underdetermined cases. MATLAB has been used to solve these equations. Derivation of (4) and (7) appears in the appendix.

A. The gradient search

The gradient search (GS) method is a classic approach for optimization problems. The method has been applied for this particular control problem, and represented by the flow diagram in fig. 3. In the figure, the e_{th} stands for the acceptable threshold of the gradient $\|\nabla J\|$, in which the objective function J can be calculated using (8)

$$\begin{aligned} J = \int |e|^2 dt &+ \rho \left[\min \left\{ \left(P.O._{boundary} - P.O._{search} \right), 0 \right\} \right]^2 \\ &+ \rho \left[\min \left\{ \left(tr_{boundary} - tr_{search} \right), 0 \right\} \right]^2 \\ &+ \rho \left[\min \left\{ \left(ts_{boundary} - ts_{search} \right), 0 \right\} \right]^2 \\ &+ \rho \left[\min \left\{ \left(ess_{boundary} - ess_{search} \right), 0 \right\} \right]^2 \end{aligned} \tag{8}$$

, where $e(t)$ is the errors between the input and the output of the system, and ρ is the penalty factor set to 1,000. Noticeably, it is constructed from the time-domain performance specifications.

B. The genetic algorithm

The genetic algorithm (GA) is one of the efficient evolutionary algorithms. The algorithm has been applied to search for optimal solution for this control problem. The flow diagram in fig. 4 represents the work flow of the PIDA control design based on GA attempting to establish a set of optimal controller parameters minimizing the objective function J in (9), constructed under a similar concept to (8).

$$\begin{aligned} J = \int |e| dt &+ \left(P.O._{boundary} - P.O._{search} \right) + \left(tr_{boundary} - tr_{search} \right) \\ &+ \left(ts_{boundary} - ts_{search} \right) + \left(ess_{boundary} - ess_{search} \right) \end{aligned} \tag{9}$$

TABLE I
SYMBOLS FOR OBJECTIVE FUNCTIONS

Symbols	Meanings
$P.O._{boundary}$	percent overshoot allowance
$P.O._{search}$	percent overshoot from search
$tr_{boundary}$	rise-time allowance
tr_{search}	rise-time from search
$ts_{boundary}$	settling time allowance
ts_{search}	settling time from search
$ess_{boundary}$	steady-state error allowance
ess_{search}	steady-state error from search

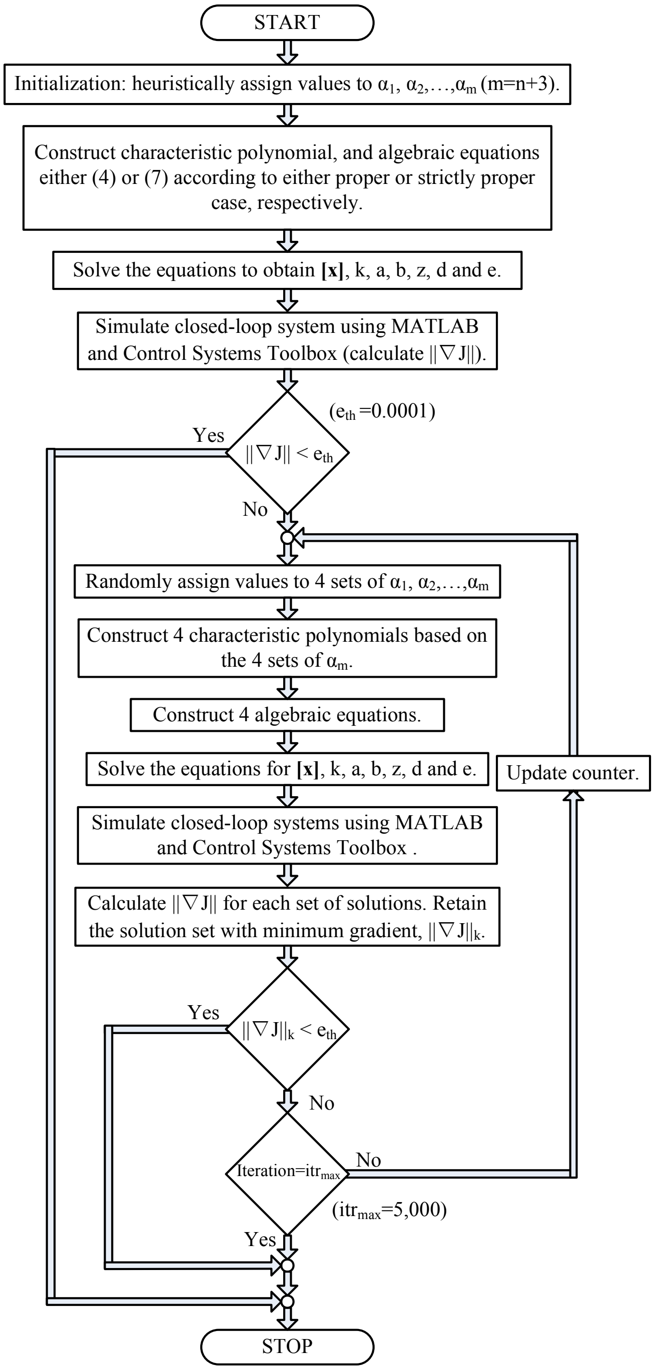


Fig. 3 Flow diagram of the GS method.

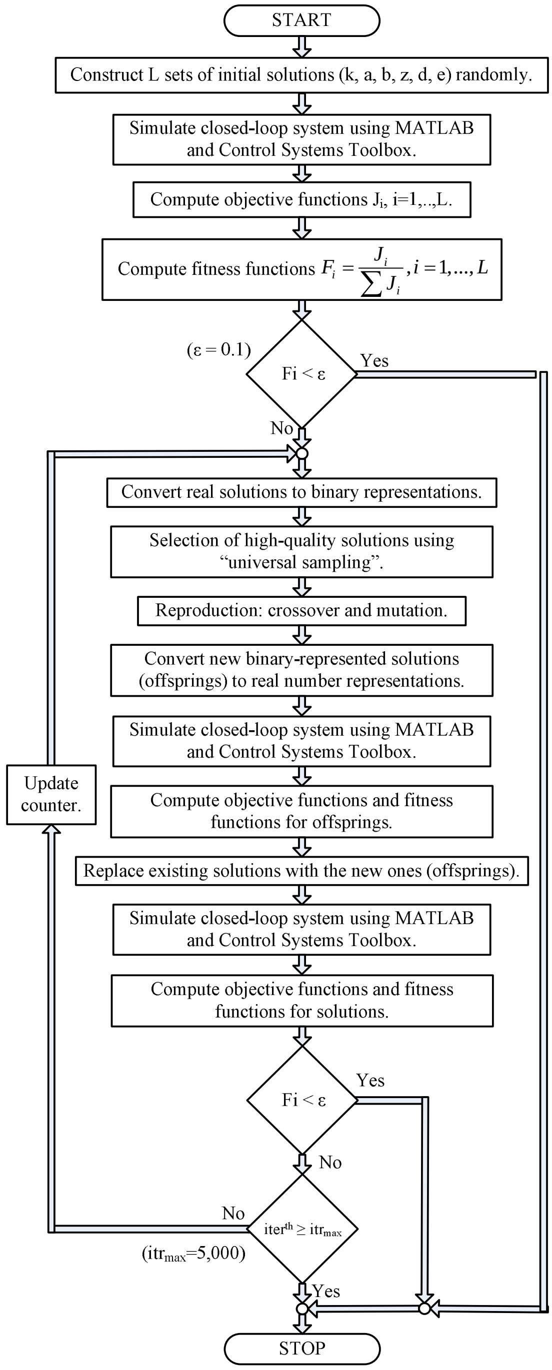
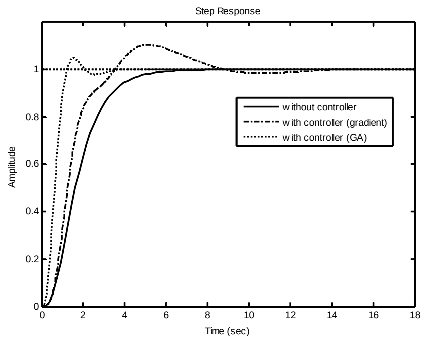


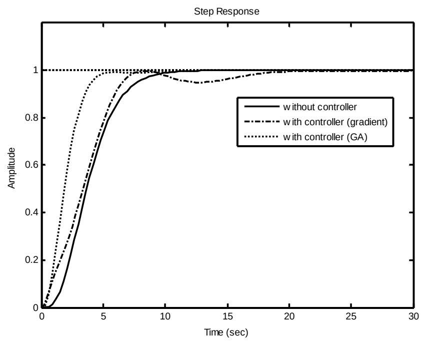
Fig. 4 Flow diagram of the GA method.

V. SIMULATION RESULTS

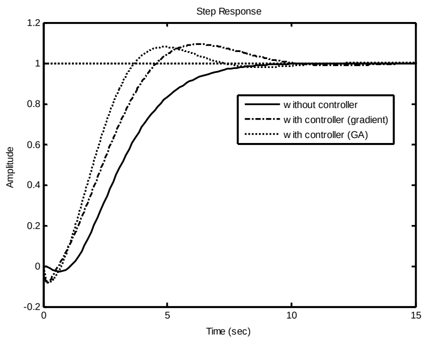
The optimal PIDA controllers resulted from the GS and GA based methods are summarized in Table II. Fig. 5 illustrates the closed-loop step responses. It can be observed that the GA based method results in the controllers of better performances for all cases in terms of rapid settling, low overshoot and stabilization. The numerical results concerning performances shown in Table II also confirm this. In addition, the results obtained from the GS method are used to aid the formation of solution boundaries for the GA method. In effect, the GA could converge to high-quality solutions more rapidly. Fig.6 illustrates the responses to step inputs and disturbances. Notice that, the systems with the controllers searched by the GA respond better to the input commands, and recover better from the external disturbances. The sensitivity (s_G^T) functions are shown in Fig.7. Both approaches provide very similar sensitivity characteristics in terms of rapid tracking response, and effective disturbance rejection, respectively.



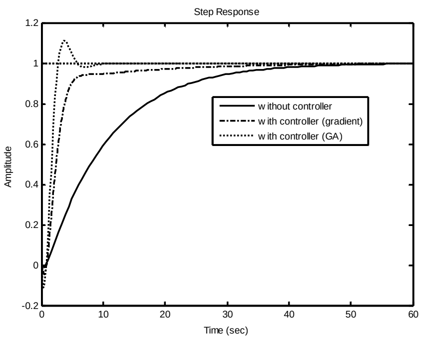
(a)



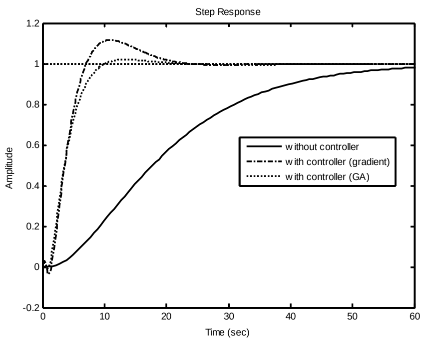
(b)



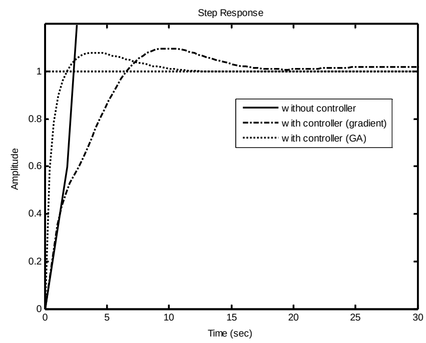
(c)



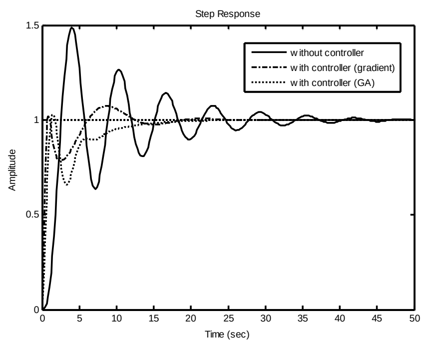
(d)



(e)



(f)



(g)

Fig. 5 Simulation results (GS and GA methods).
(a) Plant 1 (b) Plant 2 (c) Plant 3
(d) Plant 4 (e) Plant 5 (f) Plant 6
(g) Plant 7

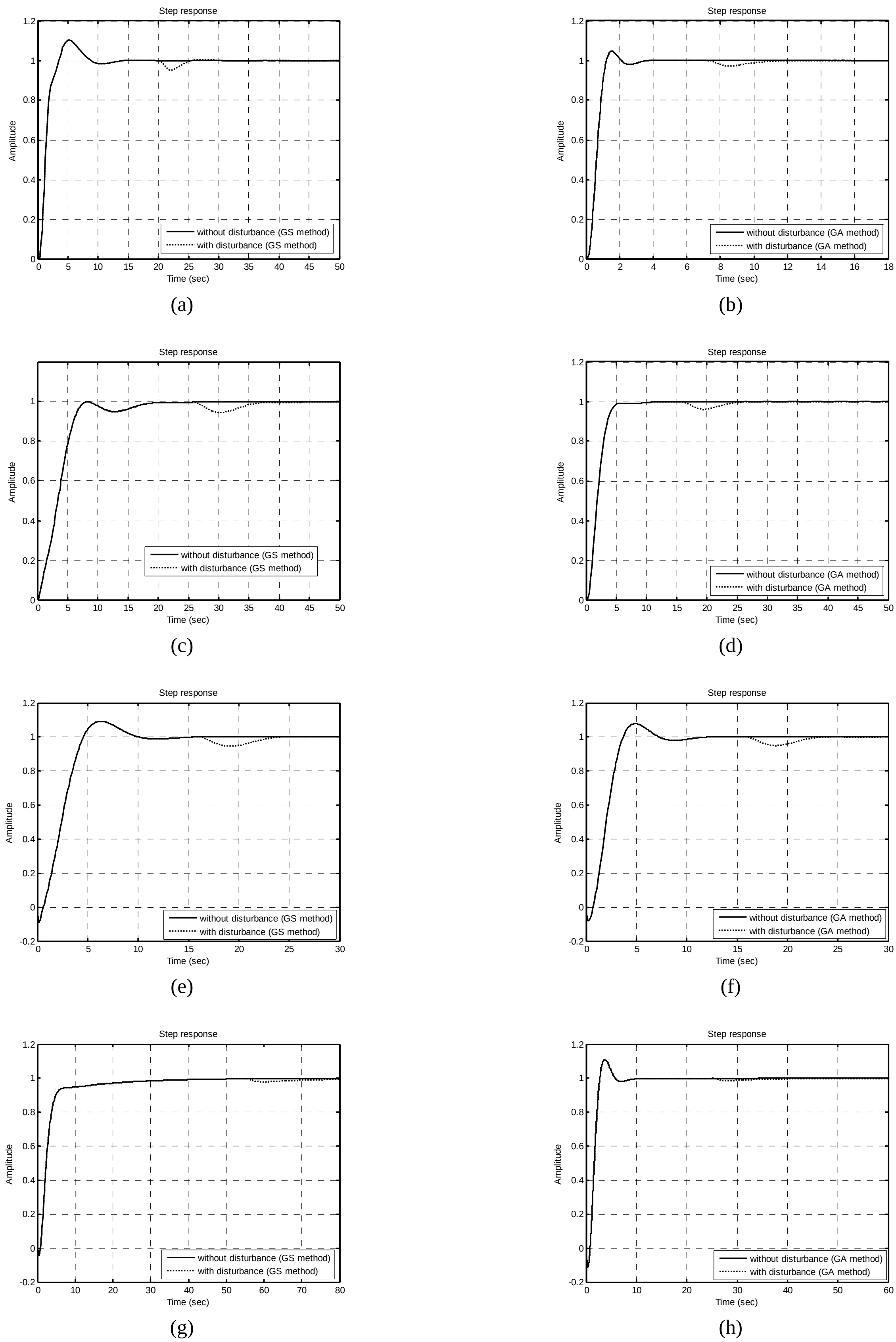
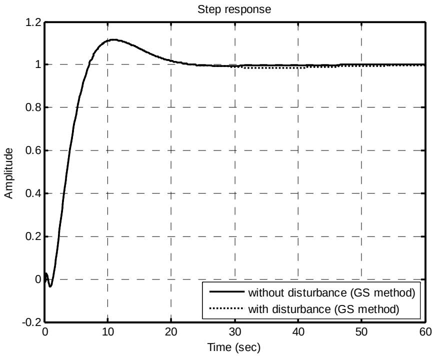
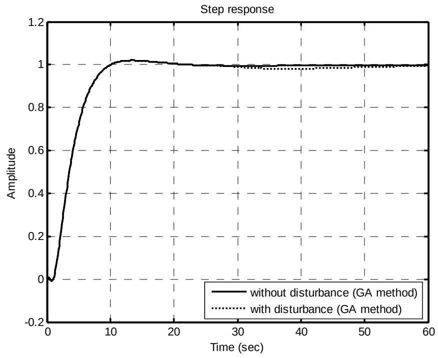


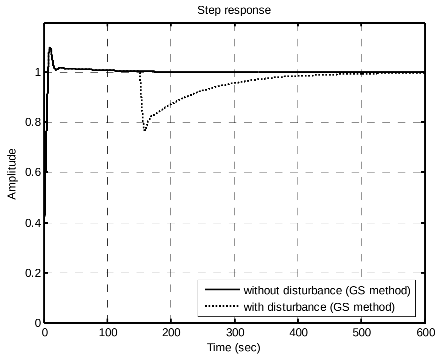
Fig. 6 Responses to step inputs and disturbances (GS and GA methods).
(a) Plant 1 (GS method) (b) Plant 1 (GA method) (c) Plant 2 (GS method) (d) Plant 2 (GA method)
(e) Plant 3 (GS method) (f) Plant 3 (GA method) (g) Plant 4 (GS method) (h) Plant 4 (GA method)



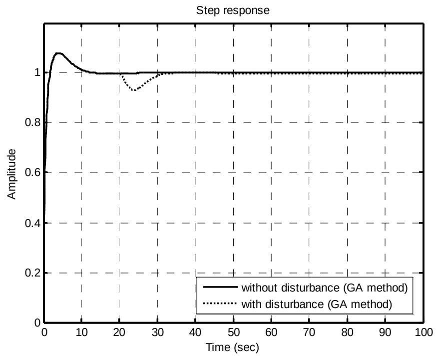
(i)



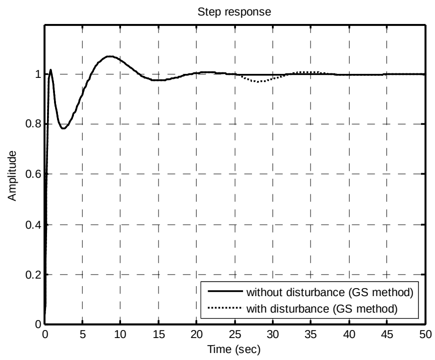
(j)



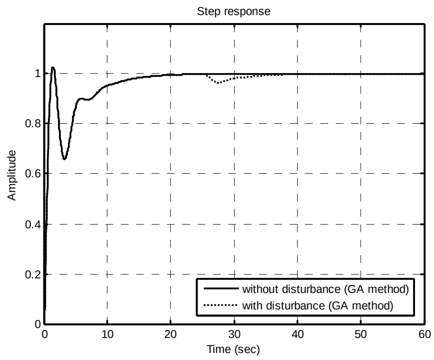
(k)



(l)

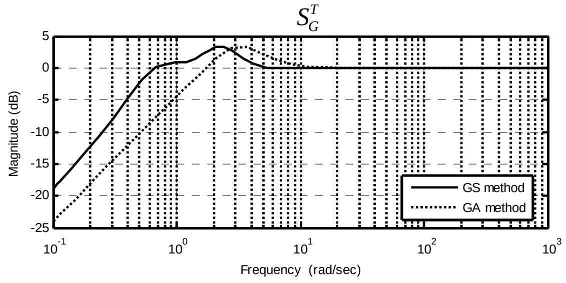


(m)

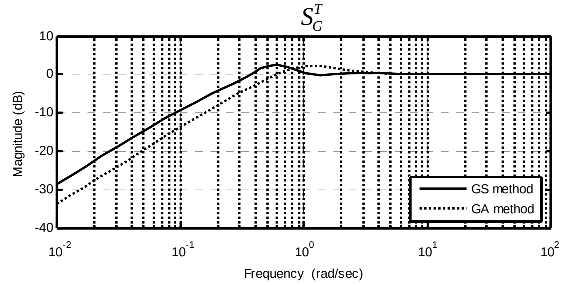


(n)

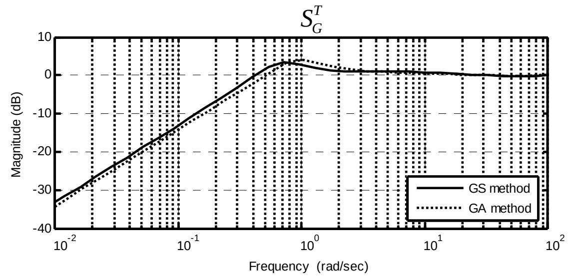
Fig. 6 (Cont.)
(i) Plant 5 (GS method) (j) Plant 5 (GA method)
(k) Plant 6 (GS method) (l) Plant 6(GA method)
(m) Plant 7 (GS method) (n) Plant 7 (GA method)



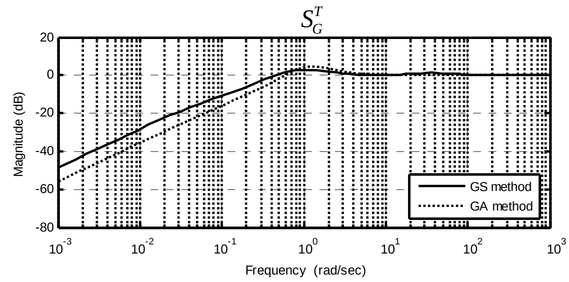
(a)



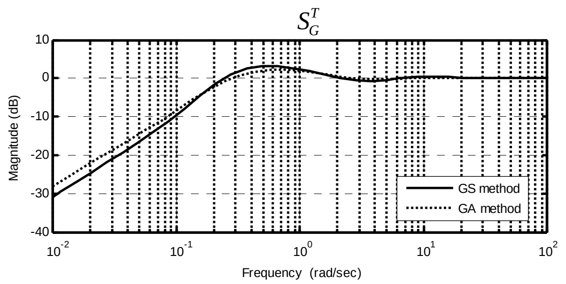
(b)



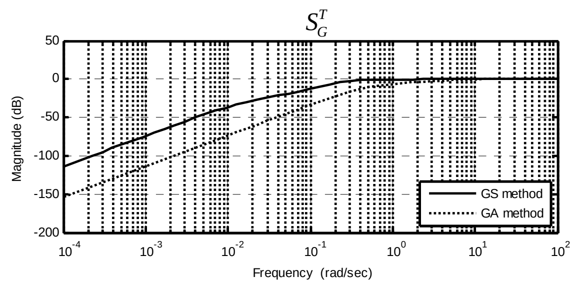
(c)



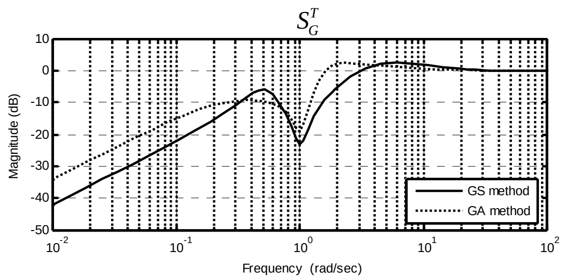
(d)



(e)



(f)



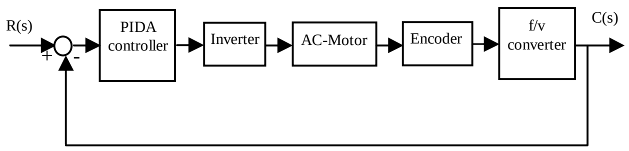
(g)

Fig. 7 Sensitivity functions (s_G^T).
(a) Plant 1 (b) Plant 2 (c) Plant 3
(d) Plant 4 (e) Plant 5 (f) Plant 6
(g) Plant 7

TABLE II. THE OBTAINED PIDA CONTROLLER

Plant	GS method						GA method					
	Controller	GM (dB)	PM (dg)	tr (s)	ts (s)	P.O. (%)	Controller	GM (dB)	PM (dg)	tr (s)	ts (s)	P.O. (%)
1	$\frac{-0.071s^3+15.92s^2+16.74s+14.6}{s^3+8.1s^2+16.38s}$	11.2	61	1.8	8.03	10.5	$\frac{-74.45s^3+2.09\times10^4s^2+5.12\times10^4s+3.11\times10^4}{s^3+288.9s^2+1.95\times10^4s}$	15	63.2	0.73	1.9	4.77
2	$\frac{673.8s^3+375.5s^2+854.9s+231}{s^3+58.96s^2+869.1s}$	37.4	72.5	5.12	16.8	0	$\frac{89.52s^3+236.2s^2+203.4s+56.93}{s^3+21.76s^2+118.4s}$	20.9	70.4	2.77	4.69	0
3	$\frac{174.3s^3+525.7s^2+848.3s+379}{s^3+57.91s^2+838.5s}$	19.8	57.9	2.9	9.04	9.26	$\frac{146.7s^3+790.2s^2+1251s+478.7}{s^3+63.28s^2+945.1s}$	14.2	58.1	2.14	6.63	7.91
4	$\frac{79.53s^3+3.31\times10^{-15}s^2+3887s+260}{s^3+63.14s^2+996.5s}$	14	71.7	3.71	40	25.4	$\frac{29.9s^3-1.66\times10^4s^2+2.15\times10^6s+1.99\times10^5}{s^3+1168s^2+3.27\times10^5s}$	9.49	53.8	1.42	5.35	11
5	$\frac{-2477s^3+6207s^2+1894s+107.6}{s^3+35.32s^2+311.8s}$	12.7	55.9	4.06	19.7	11.6	$\frac{-47.49s^3+2849s^2+655.2s+31.61}{s^3+33.76s^2+126.2s}$	14.9	67.7	5.36	9.07	2
6	$\frac{6.664s^3+3.197s^2+2.602s+0.02777}{s^3+4.69s^2+5.498s}$	Inf	70.6	5.01	16.3	9.91	$\frac{140s^3+253.7s^2+149.9s+29.06}{s^3+15.59s^2+60.74s}$	Inf	98.3	1.03	9.12	8.06
7	$\frac{714.1s^3+1281s^2+423.4s+331}{s^3+32.1s^2+257.6s}$	20.6	59.2	0.44	16.3	7.27	$\frac{676.1s^3+1532s^2+1146s+283.1}{s^3+71.71s^2+555.6s}$	34.1	47.1	0.81	14.1	2.8

VI. APPLICATION



(a)

(b)

Fig. 8 Closed-loop speed control of a 3-phase motor (a) block diagram, (b) experimental setup.

The GA-based optimal design has been applied to control the speed of a 3-phase induction motor (a Panasonic MAS04-90Y, 90W, 4P). A step-transient test was conducted for the identification of the motor model at the 600 rpm operating point. The identification using MATLAB and System Identification Toolbox results in a 3rd-order plant model expressed by (10). The GA search provides the controller

$$G_P(s) = \frac{3599s + 240000}{s^3 + 548.3s^2 + 77870s + 241900} \quad (10).$$

of the form (1), in which $K_P = 25.22, K_I = 79.62, K_D = 456.89,$ $K_A = 2070.9,\ d = 42549$ and $e = 42549,$ respectively. Driving the motor requires a PWM inverter and the controller implemented. The inverter used is a Toshiba VFS11-2002PM. Implementation of the controller uses op-amp RC networks to ensure real-time operation. The circuit diagram in Fig. 9 depicts the PIDA controller realizing the mentioned parameters.

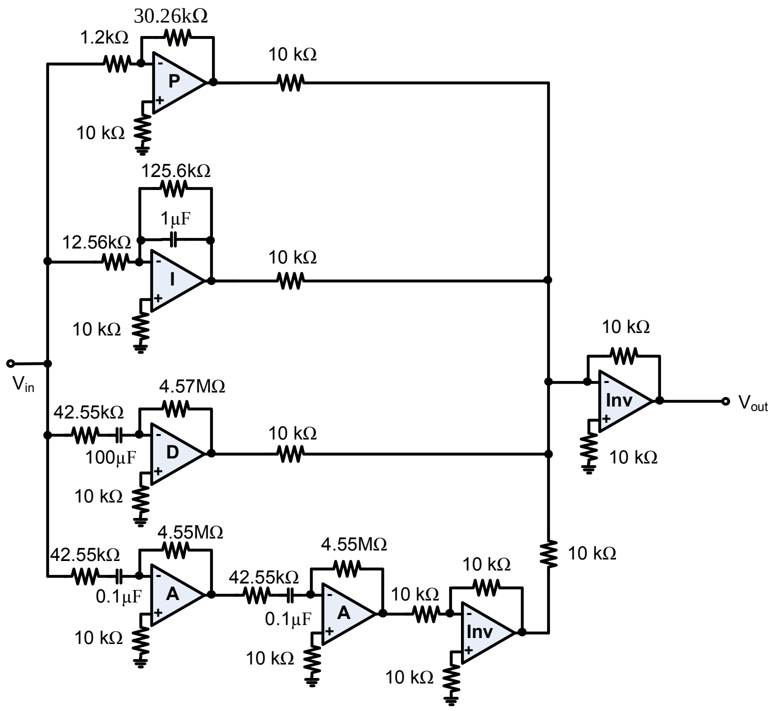


Fig. 9 Analog implementation of the controller.

It is necessary to realistically investigate the closed-loop system though simulation using PSIM in which the op-amp saturation limits can be taken in to account. Fig. 10 illustrates the PSIM simulation diagram in which the motor is represented by (10).

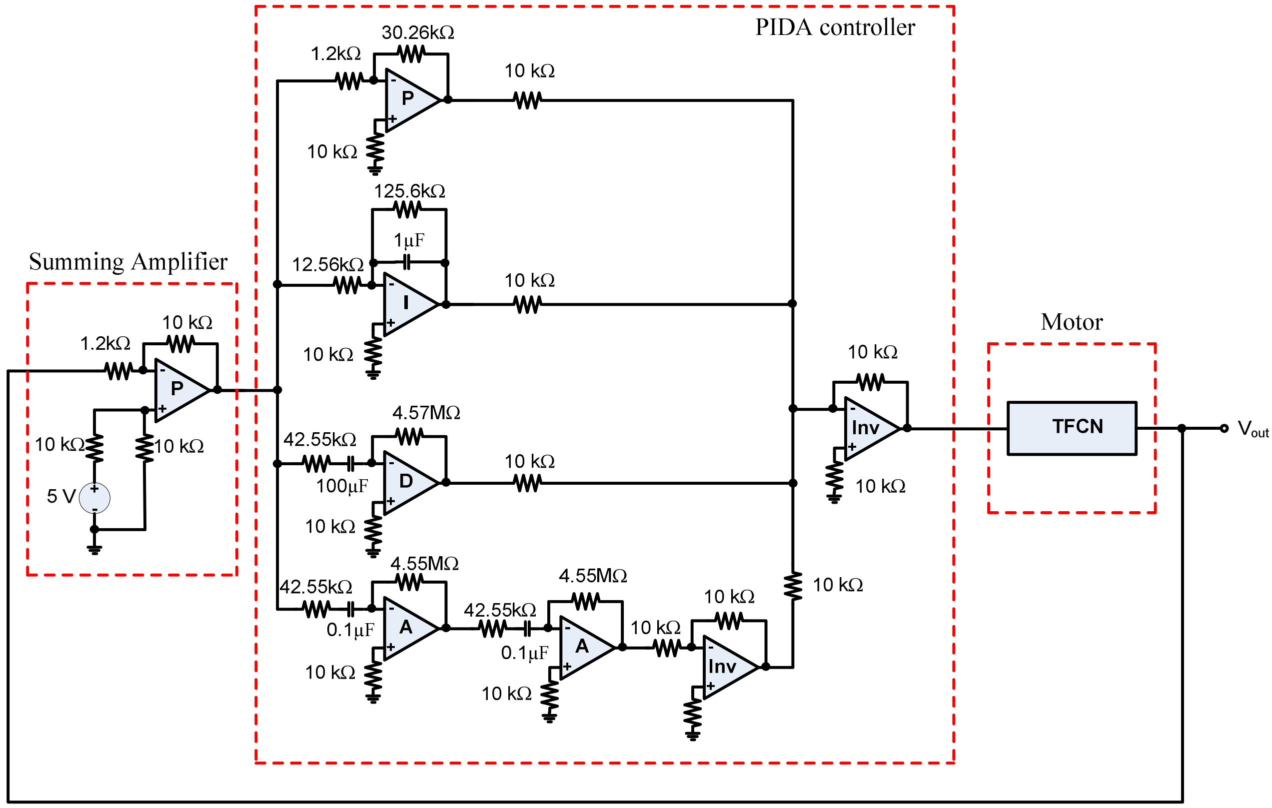


Fig. 10 Simulation diagram for PSIM.

The PSIM results appear in Fig. 11 including the realistic control signal (a) and the motor speed (b). The rise-time and settling time of the simulated speed are about 0.1 and 0.45 s, respectively. Fig. 12 illustrates the measured speed curves of the open-loop motor (a) and the PIDA controlled motor (b), respectively. Noticeably, the actual speed with the PIDA controller agree very well with the simulated result in Fig. 11(b).

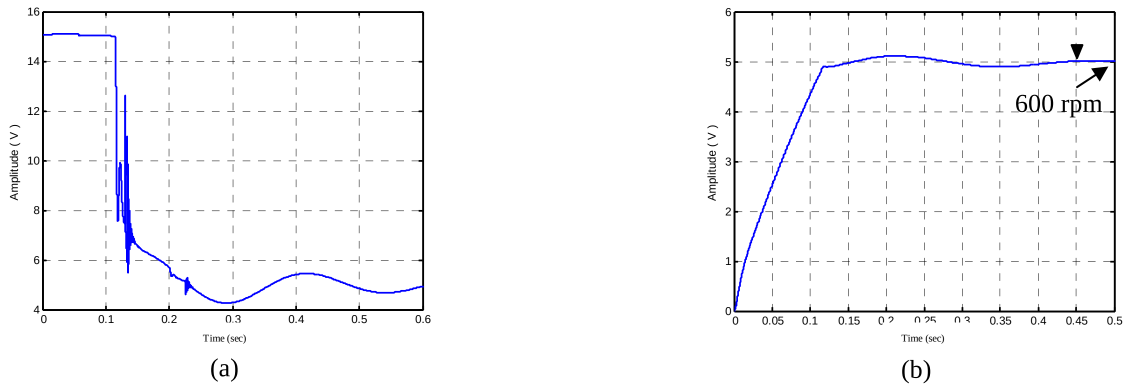
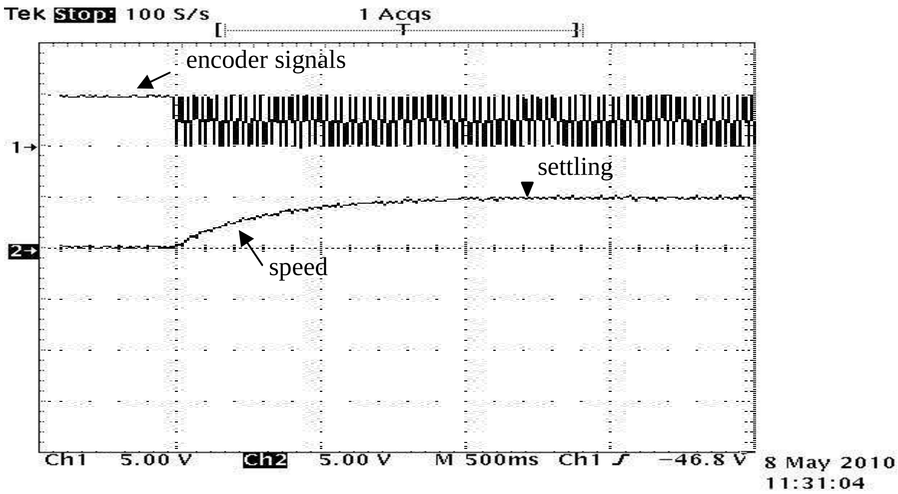
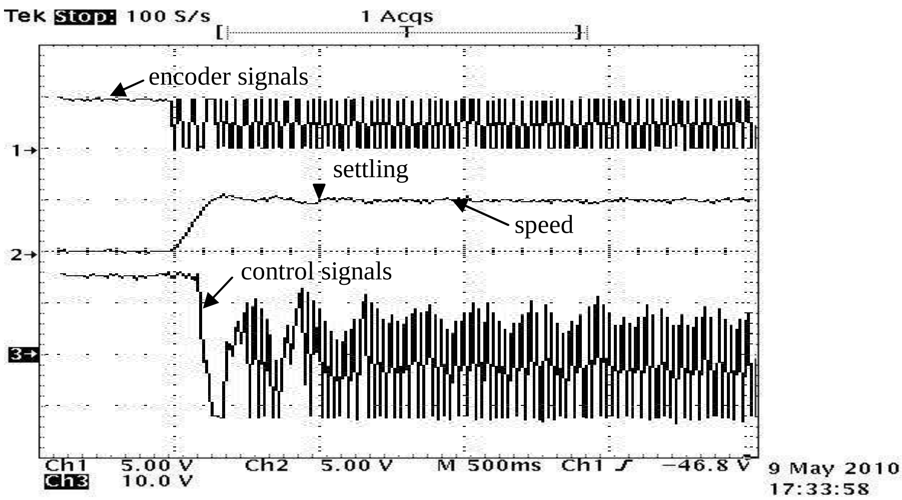


Fig. 11 PSIM results (a) control signal, (b) motor speed.



(a)



(b)

Fig. 12 Experimental speed responses of the motor. (a) without controller, (b) with controller

VII. Conclusion

This paper has briefly reviewed the conventional design of the PIDA controller based on the dominant pole concept. It provides the design results tested against some difficult to be controlled plants proposed by Astrom and Hagglund for benchmarking. It is evident that the conventional design approach fails in most cases due to the use of only one pair of dominant poles. This does not mean that the root-locus based method would fail. The authors believe that the method can be successfully applied if more complicated pole and zero assignments are considered with an aid of computing. The paper offers contributions to optimal design of the PIDA controller using search methods. The proposed method can be viewed as optimal pole placement with an aid of the gradient search (GS) and the genetic algorithm (GA). The algorithms are reviewed in the paper. Simulation results indicate that the GA-based method provide controllers with better closed-loop performances in comparison with the GS-based method. The paper also elaborates the control system implementation for an AC motor speed control. An analog controller has been used to ensure real-time operation. To obtain realistic simulation results, the closed-loop speed control of a 90 W-3ph induction motor is simulated on PSIM. Experimental results with a PWM inverter drive show a very good agreement with simulations, and indicate that the controlled motor responds about 2.4 times faster than the open-loop motor does.

APPENDIX

Derivations of (4) and (7).

Case I – Proper plant $(a_0 \neq 0)$

$$G_P(s) = \frac{p(s)}{q(s)} = \frac{a_0 s^n + a_1 s^{n-1} + ... + a_n}{s^n + p_1 s^{n-1} + ... + p_n} \quad \text{(a.1)}$$

Characteristic equation of the closed-loop system is

$$s^n + \left[\frac{p_1 + d + e + (a+b+z)ka_0 + ka_1}{1+ka_0}\right]s^{n-1}$$
$$+\left[\frac{\left\{\begin{matrix} p_2 + (d+e)p_1 + de + (ab+z(b+a))ka_0 \\ +(a+b+z)ka_1 + ka_2 \end{matrix}\right\}}{1+ka_0}\right]s^{n-2}$$
$$+\left[\frac{\left\{\begin{matrix} p_3 + (d+e)p_2 + dep_1 + abzka_0 \\ +(ab+z(b+a))ka_1 + (a+b+z)ka_2 + ka_3 \end{matrix}\right\}}{1+ka_0}\right]s^{n-3}$$

. . . .

. . . .

. . . .

$$+\left[\frac{\left\{\begin{matrix} p_n + (d+e)p_{n-1} + dep_{n-2} + abzka_{n-3} \\ +(ab+z(b+a))ka_{n-2} + (a+b+z)ka_{n-1} + ka_n \end{matrix}\right\}}{1+ka_0}\right]s^3$$
$$+\left[\frac{\left\{\begin{matrix} (d+e)p_n + dep_{n-1} + abzka_{n-2} \\ +(ab+z(b+a))ka_{n-1} + (a+b+z)ka_n \end{matrix}\right\}}{1+ka_0}\right]s^2$$
$$+\left[\frac{dep_n + abzka_{n-1} + (ab+z(b+a))ka_n}{1+ka_0}\right]s + \left[\frac{abzka_n}{1+ka_0}\right] = 0 \quad \text{(a.2)}$$

Equate (a.2) with $s^m + \alpha_1 s^{m-1} + ... + \alpha_m = 0$ (m=n+3), one can obtain (a.3)-(a.9)

$$\frac{p_1 + (d+e) + (a+b+z)ka_0 + ka_1}{1+ka_0} = \alpha_1 \quad \text{(a.3)}$$

$$\frac{\left\{\begin{matrix} p_2 + (d+e)p_1 + de + (ab+z(b+a))ka_0 \\ +(a+b+z)ka_1 + ka_2 \end{matrix}\right\}}{1+ka_0} = \alpha_2 \quad \text{(a.4)}$$

$$\frac{\left\{\begin{matrix} p_3 + (d+e)p_2 + dep_1 + abzka_0 \\ +(ab+z(b+a))ka_1 + (a+b+z)ka_2 + ka_3 \end{matrix}\right\}}{1+ka_0} = \alpha_3 \quad \text{(a.5)}$$

. . . .

. . . .

. . . .

$$\frac{\left\{\begin{matrix} p_n + (d+e)p_{n-1} + dep_{n-2} + (ab+z(b+a))ka_{n-2} \\ +abzka_{n-3} + (a+b+z)ka_{n-1} + ka_n \end{matrix}\right\}}{1+ka_0} = \alpha_n \quad \text{(a.6)}$$

$$\frac{\left\{\begin{matrix} (d+e)p_n + dep_{n-1} + abzka_{n-2} \\ +(ab+z(b+a))ka_{n-1} + (a+b+z)ka_n \end{matrix}\right\}}{1+ka_0} = \alpha_{n+1} \quad \text{(a.7)}$$

$$\frac{dep_n + abzka_{n-1} + (ab+z(b+a))ka_n}{1+ka_0} = \alpha_{n+2} \quad \text{(a.8)}$$

$$\frac{abzka_n}{1+ka_0} = \alpha_{n+3} \quad \text{(a.9)}$$

, and hence the following equation is obtained

$$\begin{bmatrix} p_1 & 1 & 0 & 0 & 0 & a_0 & a_1 \\ p_2 & p_1 & 1 & 0 & a_0 & a_1 & a_2 \\ p_3 & p_2 & p_1 & a_0 & a_1 & a_2 & a_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ p_n & p_{n-1} & p_{n-2} & a_{n-3} & a_{n-2} & a_{n-1} & a_n \\ 0 & p_n & p_{n-1} & a_{n-2} & a_{n-1} & a_n & 0 \\ 0 & 0 & p_n & a_{n-1} & a_n & 0 & 0 \\ 0 & 0 & 0 & a_n & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \\ \alpha_{n+1} \\ \alpha_{n+2} \\ \alpha_{n+3} \end{bmatrix} \quad \text{(a.10)}$$

, which is (4),

,where $[\mathbf{x}]^T = (1+ka_0)^{-1}$

$[1 \ (d+e) \ de \ abzk \ k(ab+z(a+b)) \ k(a+b+z) \ k]$.

Case II – Strictly proper plant $(a_0 = 0)$

$$G_P(s) = \frac{p(s)}{q(s)} = \frac{a_1 s^l + a_2 s^{l-1} + ... + a_l}{s^n + p_1 s^{n-1} + ... + p_n} \quad \text{(a.11)}$$

Characteristic equation of the closed-loop system is

$$s^n + \left(p_1 + d + e + ka_1\right)s^{n-1}$$
$$+\left(p_2 + (d+e)p_1 + de + (a+b+z)ka_1 + ka_2\right)s^{n-2}$$
$$+\left[\begin{matrix} p_3 + (d+e)p_2 + dep_1 + (ab+z(b+a))ka_1 \\ +(a+b+z)ka_2 + ka_3 \end{matrix}\right]s^{n-3}$$

.

.

.

$$+\begin{bmatrix} p_n+(d+e)p_{n-1}+dep_{n-2}+abzka_{n-3} \\ +(ab+z(b+a))ka_{n-2}+(a+b+z)ka_{n-1}+ka_n \end{bmatrix} s^3$$

$$+\begin{bmatrix} (d+e)p_n+dep_{n-1}+abzka_{n-2}+(ab+z(b+a))ka_{n-1} \\ +(a+b+z)ka_n \end{bmatrix} s^2$$

$$+\left(dep_n+abzka_{n-1}+(ab+z(b+a))ka_n\right)s+abzka_n=0 \quad \text{(a.12)}$$

Equate (a.12) with $s^m+\alpha_1 s^{m-1}+...+\alpha_m=0$ (m=n+3), one can obtain (a.13)-(a.19)

$$(d+e)+ka_1=\alpha_1-p_1 \quad \text{(a.13)}$$

$$(d+e)p_1+de+(a+b+z)ka_1+ka_2=\alpha_2-p_2 \quad \text{(a.14)}$$

$$\begin{aligned}&(d+e)p_2+dep_1+(ab+z(b+a))ka_1+(a+b+z)ka_2 \\ &+ka_3=\alpha_3-p_3\end{aligned} \quad \text{(a.15)}$$

.

.

.

$$\begin{aligned}&(d+e)p_{n-1}+dep_{n-2}+abzka_{n-3}+(ab+z(b+a))ka_{n-2} \\ &+(a+b+z)ka_{n-1}+ka_n=\alpha_n-p_n\end{aligned} \quad \text{(a.16)}$$

$$\begin{aligned}&(d+e)p_n+dep_{n-1}+abzka_{n-2}+(ab+z(b+a))ka_{n-1} \\ &+(a+b+z)ka_n=\alpha_{n+1}\end{aligned} \quad \text{(a.17)}$$

$$dep_n+abzka_{n-1}+(ab+z(b+a))ka_n=\alpha_{n+2} \quad \text{(a.18)}$$

$$abzka_n=\alpha_{n+3} \quad \text{(a.19)}$$

, and hence the following equation is obtained

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & a_1 \\ p_1 & 1 & 0 & 0 & a_1 & a_2 \\ p_2 & p_1 & 0 & a_1 & a_2 & \vdots \\ \vdots & p_2 & a_1 & a_2 & \vdots & a_{n-1} \\ p_{n-1} & \vdots & a_2 & \vdots & a_{n-1} & a_n \\ p_n & p_{n-1} & \vdots & a_{n-1} & a_n & 0 \\ 0 & p_n & a_{n-1} & a_n & 0 & 0 \\ 0 & 0 & a_n & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \alpha_1-p_1 \\ \alpha_2-p_2 \\ \vdots \\ \alpha_n-p_n \\ \alpha_{n+1} \\ \alpha_{n+2} \\ \alpha_{n+3} \end{bmatrix} \quad \text{(a.20)}$$

, which is (7).
,where $[\mathbf{x}]^T=\left[(d+e)\;\; de\;\; abzk\;\; k(ab+z(a+b))\;\; k(a+b+z))\;\; k\right]$.

ACKNOWLEDGMENT

The authors' thanks are due to Mr Supachai Chumwangwapee for his valuable suggestions, and Premier Automation Center Ltd., Bangkok, Thailand, for supporting the drive system.

REFERENCES

[1] K.H. Ang, G. Chong and Y. Li, "PID Control System Analysis, Design and Technology," *IEEE Trans. Control Systems Technology*, vol. 13, no. 4, pp. 559-576, 2005.

[2] L.H. Keel and S.P. Bhattacharyya, "Controller Synthesis Free of Analytical Models: Three Term Controllers," *IEEE Trans. Automatic Control*, vol. 53, no. 6, pp. 1353-1369, 2008.

[3] T. Emami and J.M. Watkins, "Robust Performance Characterization of PID Controllers in The Frequency Domain," *WSEAS Trans. Systems and Control*, vol. 4, no. 5, pp. 232-242, 2009.

[4] T. Emami and J.M. Watkins, "A Unified Approach for Sensitivity Design of PID Controllers in The Frequency Domain," *WSEAS Trans. Systems and Control*, vol. 4, no. 5, pp. 221-231, 2009.

[5] B. Sulc and S. Vrana, "Some Observations on Development and Testing of A Simple Tuning Algorithm for PID Controllers," *WSEAS Trans. Systems and Control*, vol. 2, no. 10, pp. 497-508, 2009.

[6] M. Foltin, J. Murgas and I. Sekaj, "A New Adaptive PID Control Approach Based on Closed-Loop Response Recognition," *WSEAS Trans. Systems*, vol. 5, no. 7, pp. 1738-1742, 2006.

[7] T. Kawabe, "Evolutionary Computation Method of Robust 2DOF PID Controller Design for Time-Delay Systems," *WSEAS Trans. Systems*, vol. 5, no. 2, pp. 340-346, 2006.

[8] S. Jung and R.C. Dorf, "Analytic PIDA Controller Design Technique for A Third Order System," *in Proc. 35th IEEE Conf. Decision and Control*, Kobe, Japan, 1996, pp. 2513-2518.

[9] U-Thaiwasin, S. Sujitjorn, Y. Prempraneerach and J. Ngamwiwit, "Torsional Resonance Suppression via PIDA Controller," *in Proc. IEEE Reg.-10 Conf.-TENCON 2000*, Kuala Lumpur, Malaysia, 2000, pp. 498-503.

[10] D-Y. Ha, I-Y. Lee, Y-S. Cho, Y-D. Lim and B-K. Choi, "The Design of PIDA Controller with Pre-compensator," *in Proc. IEEE Int. Symp. ISIE 2001*, Pusan, Korea, 2001, pp. 798-804.

[11] K.J. Astrom and T. Hagglund, "Benchmark Systems for PID Control," *in Proc. IFAC Int. Conf. Digital Control: Past, Present and Future of PID Control*, Terrassa, Spain, 2000, pp. 165-166.

Sunisa Sornmuang was born in 1984, Nakhonsawan, Thailand. She received her BEng (EE) in 2005 from Suranaree University of Technology (SUT), Thailand. She is currently a post-graduate student working towards her master's degree in electrical engineering at SUT. Her research interests include applied AI and control systems.

Sarawut Sujitjorn received his PhD (EE) in 1987 from the University of Birmingham, UK. He is a professor of electrical engineering at the School of Electrical Engineering, Suranaree University of Technology, Thailand. His research interests include control and intelligent systems, metaheuristics, power converters, signal processing and hybrid power filters.