The Efficiency of Value at Risk Models on Central and Eastern European Stock Markets

Simona Mutu, Peter Balogh, and Darie Moldovan

Abstract— In this paper we have analyzed the performance of some Value at Risk models through the quadratic loss function backtesting approach. In order to highlight the differences among VaR models we have calculated the risk measure through Historical Simulation, EWMA, GARCH and EVT models. VaR was calculated on daily data of five Eastern and Central European main indices: BET (Romania), PX50 (Czech Republic), BUX (Hungary), SOFIX (Bulgary) and WIG20 (Poland) from 30.09.2004 to 30.09.2010. In order to highlight different behaviors in the crisis period we have divided the data into two samples and found that only advanced VaR models such as Extreme Value Theory or GARCH models can adequately measure the risk of the capital markets and satisfy the requirements of the investors in periods characterized by extreme events.

***Keywords*—**Extreme Value Theory, EWMA models, GARCH models, Value at Risk.

I. Introduction

THIS paper analyzes the volatility of five stock exchange indices from Central and Eastern European financial markets in periods characterized by extreme events and propose some Value at Risk (VaR) models in order to quantify and manage the risk of portfolios exposed to these markets. The results could also be used by the investors in order to fundament their portfolio investment decision, through buying the titles that decrease their portfolio VaR and liquidating the positions held on titles that increase the portfolio VaR. Another object of the article is to show which of the VaR's models is better performing at high confidence levels of investors and which measure of risk performs better in periods characterized by extreme volatility market conditions.

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This paper is organized as follows. Section 2 presents the most important and recent studies in the literature regarding the subject of the paper. Section 3 mentions the Value at Risk methodology with emphasis on some of its models: Historical Simulation, EWMA models, GARCH models and Extreme Value Theory models. Section 4 presents the backtesting procedure. In section 5 is presented a case study which analyzes the volatility of five stock market indices form Central and Eastern Europe: BET (Romania), PX50 (Czech Republic), BUX (Hungary), SOFIX (Bulgary) and WIG20 (Poland) from 30.09.2004 to 30.09.2010, through the above models and its impact on the portfolio investment decision taken. Section 6 concludes our present research.

II. Literature Review

In order to fundament their investment decision, investors analyze the volatility of the financial titles by a variety of methods. One of the most used methodology is Value at Risk which expresses the maximum loss of a portfolio at a given confidence level [1]. Its accuracy depends crucially on the models used to estimate it.

Value at Risk is a very useful risk measure both for individual and institutional investors. The research on this methodology was initiated by Jorion [1], which was followed by Dowd [2], and Saunders [3], but it became a risk management tool for the financial institutions since July 1993 when the Group of Thirty Report was presented, when regulators, shareholders and management finally had a common measure to quantify the risk of a portfolio of different financial assets. This measure it is also recommended by the Basel Committee on Banking Supervision [4] in quantifying the banks' exposures to market risk. When computing VaR through different methods it should be taken into consideration that most of the financial returns' distributions aren't normally distributed at shorter and larger horizons. Moreover, at higher frequencies the return distributions tend to present non-normal features and are characterized by "fat-tails" [5]. These tails have a higher density than that which is predicted under the assumption of normality [6].

These have important consequences on estimating VaR, because the normal distribution could underestimate the risk and the capital requirements needed to cover the losses that results, putting the investor into a risky position [7]. The solution found was to use other distributions which allow for a better modeling of larger movements than the normal distribution (like Pareto distribution and Student t distribution). The recent studies proposed to analyze only the distribution of extreme returns, instead of describing the behavior of all of the returns [8] - [10].

Related to these studies is the Extreme Value Theory, introduced by Embrechts [11], [12], although the basics were initiated by Fisher and Tippett (1928) when proposing the Generalized Extreme Value (GEV) distribution [11]. The modeling of the financial variables through EVT was also studied by McNeil [13]-[15], by Danielsson and De Vries [16], [17] which computed a model for calculating the VaR, taking into account the inconsistency of extreme values and by Huisman et al. [18], [19] which proposed a new estimator for the tail index. Cott [20] demonstrated that traditional VaR methodologies tend to ignore extreme events. Bensalah [21] found that EVT is performing better than VaR when analyzing the evolution of daily exchange rates of Canadian/U.S. dollars. Applying the EVT on the overnight borrowing and interest rates on Turkey, Gencay and Selcuk [22] also came to this conclusion. Analyzing six major developed markets indices Gili and Kellezi [23] illustrated that EVT is better in modeling the tail related risk. LeBaron and Samanta [24] applied EVT to construct statistical tests in order to estimate the level of „fattness" in the tails of emerging and developed markets. Using Monte Carlo methods and bootstrapping tests of pooled returns they found that emerging markets have fatter negative tails than the developed ones. Schaumburg [25] proposed nonparametric quantile regression and EVT for computing Value at Risk.

The Value at Risk methodology has also been criticized in the last period by the financial authorities, because the majority of losses that investment companies have suffered originated in the trading book. Financial Service Authority highlighted that short-term observation periods and the assumption of normal distribution can lead to large underestimation of probability of extreme loss events [26]. Senior Supervisors Group (2010) also motivated the need for more adaptive risk management practices and the importance of supplementing VaR with other risk measures [27]. After a study on eleven banks (2007) that are significant competitors in the affected markets it was discovered that they identified weaknesses in their implementation of VaR and the calculations based on new market data ranged from about 30% to 80% higher compared with the calculations obtained using data sets reflecting earlier [27]. Also they reported between two and sixteen back-testing exceptions, generated by much higher volatility that the historical data series implied.

The biggest losses that banks have suffered during the financial crisis came from the trading portfolio because of the exposure to the securitized assets and collateralized debt obligations. As could be seen in Table 1, the capital requirements for marketable assets were much lower compared with banks' estimates.

TABLE I
TRADING ASSTES AND MARKET RISK CAPITAL REQUIREMENTS [36]

	Trading assets / Total assets		Market risk capital requirements / Total capital requirements	
	2009	2006	2009	2006
Bank of America	9	10	-	-
Citigroup	18	21	7	4
Goldman Sachs	40	40	-	-
JPMorgan Chase	20	27	-	-
Wells Fargo	3	1	-	-
Banco Santander	12	20	7	-
BNP Paribas	38	48	4	
Commerzbank	26	14	5	2
Credit Agricole	23	31	2	6
Deutsche Bank	16	32	9	4
ING Bank	13	22	2	-
Societe Generale	19	35	4	4
Barclays	14	29	14	10
HSBC Holdings	18	18	5	9
Royal Bank of Scotland	38	23	12	6
Standard Chartered	13	5	9	4
Credit Suisse	32	36	8	5
UBS	17	37	6	6
Nomura Holdings	45	36	47	-
Mitsubishi UFJ	14	6	2	2

(*Source:* BCBS, The Basel III Capital Framework: a decisive breakthrough, nov. 2010)

III. VALUE AT RISK MODELS

Institutional investors have developed models for quantifying, comparing and aggregating the risk connected with different positions and portfolios. One of the most used methods is Value at Risk, which is defined as the expected maximum loss of a portfolio over some time period and for some level of probability. From a statistical point of view, VaR entails the estimation of the quantile of the returns' distribution. In other words, Value at Risk is the probability that returns or losses (ζ) are smaller than –VaR over a period of time (T) [1]:

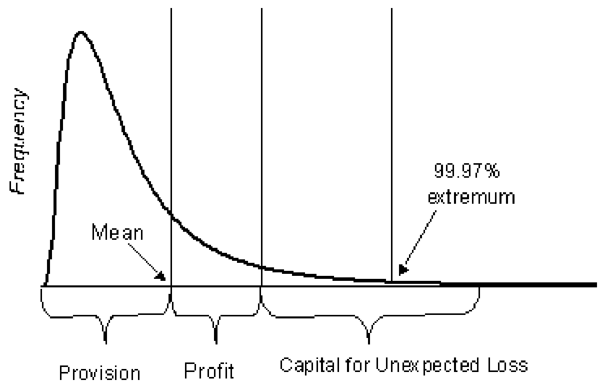
$$P_{VaR} = P(\xi < -VaR) = \int_{-\infty}^{-VaR} P_T \cdot \xi \cdot d\xi \qquad (1)$$

where P_T is the probability distribution of returns over the time horizon T.

In order to compute the VaR for a portfolio first we have to mark-to-market the portfolio and then to estimate the distribution of the portfolio's returns , which is a very delicate statistical problem. After this investors could apply three types of models for calculating VaR: parametric, non-parametric and semi-parametric [28]. What makes the difference among them is related to the way they solve the distribution problem. When the returns are normal, which is very rarely in practice, it is preferred the variance-covariance approach. When risk is recurrent VaR could be estimated by using historical time series and for new situations it should be modeled through EWMA and GARCH models, which confine more attention to the recent observations. When risk is sensitive to rare events it is used the Extreme Value Theory.

Graphically, Value at Risk could be represented like in Figure 1.

FIGURE I
THE VALUE AT RISK REPRESENTATION



The main limitation of the VaR methodology is that the assumption of normal distribution can lead to large underestimation of the probability of extreme events, which affects the capital requirements [26]. Also, the estimated distribution tends to fit only the central observations, falling in fitting the extreme observations.

Historical Simulation

Through historical simulation we could determine the risk of the portfolio by taking into consideration the past evolutions of the risk factors. The main advantage of using this method is the fact that we don't have to make any assumption about the risk factors' distribution, while the method is also easy to implement. With all these, the investors rarely use it in practice because of the imprecise predictions based on past events.

Disadvantages of the method are enumerated below:

• Estimates of future developments based on historical data, could lead to inaccurate forecasts if the past trend would not be repeated;

• Gives equal importance to historic data and recent data, which do not properly capture the impact of an increase in the volatility of recent data;

• Investment companies must have a historical database large enough to test the accuracy of the model, as shown by Vlaar (2000) in determining the Danish market interest rate risk;

• On the other hand, if the period of observation is too large, the most recent observations have the same impact as the most distant observations, since they are weighted equally (Brooks and Persand, 2000);

• It is difficult to apply if new risks appear in the stock trading because there is no historical data to capture their behavior;

• Based on the assumption of constant volatility and covariance of portfolio returns the model does not permit the incorporation of changes in the market;

• As most HS models calculates VaR on a sample collected over a period of 250 days, it appears the window effect. For example, if 250 days have elapsed from the occurrence of extreme market volatility, then it will not affect the calculation of VaR.

Given these drawbacks, some researchers have tried to remove them proposing various models based on the combination between historical simulation and parametric models.

EWMA models

The EWMA (Exponentially Weighted Moving Average) model was popularised by the RiskMetrics department from J.P. Morgan. The risk is modelled by applying an exponentially moving average on the variances and covariances of the financial series [29]. The main improvement is that the observations are given different weights, the most recent data getting the highest weight. The weights decline rapidly as we go back. Also, one of the most important advantages of the model is that it responds immediately to market crashes.

The volatility expressed by the model could be determined by the following equation [29]:

$$\sigma_t^2 = (1-\lambda)\sum_{t=1}^{n} \lambda^2 \left(R_t - \mu\right)^2 \qquad (2)$$

where σ_t^2 is the standard deviation, R_t is the return at moment t, μ is the mean value of the distribution, n is the time horizon, λ is the exponential factor that shows the persistency of volatility (it's value could change between 0 and 1), and 1- λ is a parameter that shows the speed with which a shock in the market is absorbed by the volatility.

By using a recursive substitution, we can rewrite the volatility as follows [29]:

$$\sigma_t^2 = \lambda \cdot \sigma_{t-1}^2 + \left(1-\lambda\right) \cdot R_t^2 \qquad (3)$$

This approach has two important advantages. One of the advantages is the fact that volatility reacts faster to shocks in the market because recent data carry more weight then the distant past data. The other advantage is obtained due to the fact that the volatility declines exponentially after any large shock, as the weight of the shock observation falls as λ^{t-1}.

One of the problems that this method has is choosing the exponential factor. RiskMetrics department recommends a 0.94 value for daily volatility estimations and 0.97 for monthly volatility estimations. If the exponential factor is large, the total variance would be affected poorly by the current variance. The closer the exponential factor λ is to unity, the smoother the data series become.

Taking these into consideration, the volatility for asset i at time t, could be written as follows [29]:

$$\sigma_{i,t} = \sqrt{\frac{1-\lambda}{1-\lambda^n} \sum_{j=0}^{n} \lambda^j \cdot r_{i,j}} \qquad (4)$$

The correlation between return forecasts could also be constructed in the same manner as performed for the volatility forecasts [29]:

$$\sigma_{12,t}^2 = (1-\lambda)\sum_{j=1}^{T} \lambda^{j-1} \left(r_{1t} - \overline{r_1}\right)\left(r_{2t} - \overline{r_2}\right) \qquad (5)$$

GARCH models

GARCH (Generalised Auto-Regressive Conditional Heteroscedasticity) models were proposed by Engle and introduced by Bollerslev [30], [31]. They are used in predicting the financial markets evolutions and have many versions. It encompasses a broad class of models that estimate and predict the volatility and the correlations between different assets. The GARCH (1,1) can be described as follows [31]:

$$\sigma_t^2 = \gamma + \beta \cdot \sigma_{t-1}^2 + \alpha \cdot X_{t-1}^2 \quad (6)$$

where γ, α and β are the predicted parameters. α and β values show the persistence of the volatility, and α+β must be greater than 1. If the parameters were higher than the average, volatility will also be high. The parameter β is the same as λ (the exponential factor from the EWMA model), and α is the same as 1-λ (from the EWMA model). When γ=0 then the EWMA equation would be a special version of the GARCH model.

Generally, the GARCH(p,q) model is [31]:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \cdot \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \cdot \sigma_{t-j}^2 \quad (7)$$

where ε_t is a white noise, which can be determined form the next equation [31]:

$$R_t = \beta \cdot X_i + \varepsilon_t \quad (8)$$

In order to estimate VaR using the GARCH model, we should create the distribution of daily returns for the portfolio and then fit the model to these returns. Next, it is necessary a large number of simulations as many days ahead as the maximum time horizon that is of interest. Once the simulations are done, we can find the selected percentiles of the distribution of portfolio values within each simulated day.

The model parameters can be estimated through the maximum likelihood method and must meet certain restrictions. They must be greater than 0, and for the process to be stationary it must be fulfilled the following condition:

$$0 \le \sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j \le 1 \quad (9)$$

In some cases there are aspects of the model which can be improved in order to capture better the characteristics and dynamics of the time series. First of all, the constraints imposed on the coefficients of the model are quite difficult to meet, so the solution would be the introduction of artificial conditions in order to force them to be within the range set. Secondly, in this form, the model does not take into account the effect of indebtedness (negative shocks have a greater impact on markets than the positive ones).

In order to remedy these drawbacks researchers have developed a series of extensions of the model that make it more flexible, responding in a different manner to the market shocks. In the paper "Glossary to ARCH(GARCH)" published in 2008, Bollerslev identified no less than 146 GARCH type models, which appeared in the literature. Next we turn attention to some of them that are used most frequently for econometric modeling of the market risk.

There are some extensions of the basic GARCH model that makes it more flexible: Exponential GARCH model, Integrated GARCH model, GARCH in Mean model, Threshold GARCH model which respond in a different manner to the shocks in the market.

•The GARCH-M (GARCH in Mean) considers the dependence of return's volatility. It was proposed by Engle, Lilien and Robins (1987) by studying the term structure of interest rates behavior in the U.S., then it was extended by Bollerslev, Engle and Wooldridge (1988). The GACRH in Mean formulae is the following:

$$y_t = c + \phi\, y_{t-1} + \delta\, h_t + u_t \quad (10)$$

•The T-GARCH model (Threshold GARCH) takes into account the effect of the debt by introducing a dummy variable (θ) to distinguish between the bad news (θ<0) and the good news (θ> 0). Zakoian model was introduced (1994), from the GJR-GARCH model of Glosten, Jaganathan and Runkle (1993). The Threshold GARCH model is the following:

$$h_t = \omega + \alpha_1 u_{t-1}^2 + \beta_1 h_{t-1} + \theta\, I_{t-1}\, u_{t-1}^2$$
$$I_{t-1} = 1 \text{ if } u_{t-1} < 0 \text{ and } I_{t-1} = 0 \text{ otherwise.} \quad (11)$$

•The E-GARCH model (Exponential GARCH) proposed by Nelson (1991) considers the asymmetric effects of positive and negative values of variables and uses the recorded log values to relax the constraints on the coefficients. The formulae for the Exponential GARCH model is:

$$\log h_t = = \omega + \beta_1 \log h_{t-1} + \alpha_1[\theta V_{t-1} + \gamma\{|V_{t-1}| - E|V_{t-1}|\}] \quad (12)$$

•The I-GARCH (Integrated GARCH) is actually a unit root GARCH model, which focuses on the persistence of past errors:

$$h_t = \omega + \alpha_1 u_{t-1}^2 + (1-\alpha_1)h_{t-1.} \quad (13)$$

Also, there have been developed a series of other heteroscedastic models like CHARMA (Tsay, 1987), RCA (Nicholls and Quinn, 1982), the stochastic volatility model (Melino and Turnbull, 1990; Jacquier, Polson and Rossi, 1994), stochastic volatility models with long memory and also multidimensional GARCH models: VECH (Bollerslev, Engle and Wooldridge, 1988), BEKK (Engle and Kroner, 1995), GO-GARCH (van der Wiede, 2002), GOF-GARCH (Lanne and Saikkonen, 2005).

With regard to accuracy, the investors should be concerned if the ex-post performance of the model is compatible with the theoretically desired level. Also, the capital-adequacy framework provides an incentive to develop efficient models that offer enough coverage in relation to the risk so that financial institutions could meet the supervisors' requirements with a minimum amount of capital.

Extreme Value Theory

Given the extreme variations that took place in the international financial markets, it was necessarily a more prudent approach for estimating the market risk in banks. Attempting to cover the shortcomings of previous models there have been developed a number of alternative methods for estimating VaR, through the combination of the parametric modeling with the non-parametric one. This has led to the development of models that incorporates extreme market volatilities, directly modeling the extreme values that appeared in the distributions of financial variables.

In order to remedy the problem of the fat-tails, EVT has become more and more used by financial institutions because it doesn't make any assumption about the form of the distribution of financial returns. This approach is focused on the extreme events and states that in the case of a very large sample it converges toward a limit distribution. There are two approaches in identifying extremes in the data [11]: block-maxima and excesses over a threshold. The first one, block-maxima approach, takes into consideration the maximum value the variables take in successive periods and the procedure consists in dividing the series of observations in blocks. The maximum value of each block constitutes the extreme value. This method is often used to analyze data with seasonality. The second approach, excess over a threshold, which has been used in the most recent applications, focuses on the values exceeding a given threshold.

The initial step in generating series by the **block-maxima method** is the Theorem of Fisher and Tippett (1928): Let (Xn) be a sequence of independent and identically distributed random variables. The maximum of the variables converges in law to the next distribution [11]:

$$H_{(\xi,\mu,\sigma)} = \begin{cases} \exp\left\{-\left[1+\xi(x-\mu)/\sigma\right]^{-1/\xi}\right\}, if \xi \neq 0 \\ \exp\left[-e^{-(x-\mu)/\sigma}\right], if \xi = 0 \end{cases} \quad (14)$$

where μ is a scalar, σ is the tendency and ξ indicates the thickness of the distribution's tail. The larger ξ, the thicker the tail. If ξ is negative, the distribution corresponds to a Weibull type; if ξ is zero the distribution corresponds to a Gumbel type; when ξ is positive it corresponds to a Frechet distribution. The most used in modeling the financial series is the last one, because performs better in capturing the fat tails.

For a given probability p, the expression of VaR will be the following [11]:

$$VaR_p = \begin{cases} \mu - \frac{\sigma}{\xi}\left(1-\left(-\log\left(1-\frac{1}{p}\right)\right)^{-\xi}\right), \xi \neq 0 \\ \mu - \sigma \log\left(-\log\left(1-\frac{1}{p}\right)\right), \xi = 0 \end{cases} \quad (15)$$

The second approach, **excess over a threshold**, consists in determine the distribution of excesses over a chosen threshold, which can be approximated by a Generalized Pareto Distribution [11]:

$$G_{(\xi,\beta)} = \begin{cases} 1-\left(1+\frac{\xi x}{\beta}\right)^{-1/\xi}, if \xi \neq 0 \\ 1-e^{-x/\beta}, if \xi = 0 \end{cases} \quad (16)$$

where β is the tendency and ξ is the threshold, which should be large enough in order to satisfy the condition that permits its application and at the same time it should leave sufficient observations for the estimation. This approach is considered to be more efficient and it is used by the most institutional investors.

So, the analytical expression of the VaR for a given probability p, could be defined as a function of GDP parameters [11]:

$$VaR_p = u + \frac{\sigma}{\xi}\left(\left(\frac{n}{N_u}p\right)^{-\xi} - 1\right) \quad (17)$$

where u is the threshold, n is the total number of observations and N_u is the number of observations above the threshold u.

IV. BACKTESTING PROCEDURE

Also, it is recommended to use the backtesting procedure, based on testing a sample of 250 days data behind the day VaR is calculated. Through this method it is tried to determine how many times the VaR limit has been exceeded. Financial institutions could use two approaches:

- The binary loss function approach, which is aimed to determine the factor k for capital dequacy. The test could be described as follows:

$$T_i = \begin{cases} 1, loss_i < VaR_i \\ 0, loss_i \geq VaR_i \end{cases} \quad (18)$$

The test result is:

$$T = \sum_i T_i .$$

- The quadratic loss function approach, which is used to compare different VaR models and consists in the following test:

$$T_i = \begin{cases} 1+(\Delta P_t - VaR_t)^2, \Delta P_t < VaR_t \\ 0, \Delta P_t \geq VaR_t \end{cases} \quad (19)$$

where P is the portfolio loss.

The test resut is:

$$T = \sum_i T_i .$$

One of the most representative studies on validation of the VaR is the Kupiec's (1997), who developed a methodology for determining the confidence intervals for the incidence of higher values than the estimated VaR. These confidence intervals are the quantiles of a distribution λ^2 with with one degree of freedom (relation 20):

$$l = -2\ln[(1-p)^{T-N} p^N] + 2\ln\left[\left(1-\frac{N}{T}\right)^{T-N}\left(\frac{N}{T}\right)^N\right] \quad (20)$$

where p is the probability used to estimate VaR, T is the number of observations in real activity and N is the number of observations that exceeded the VaR estimation.

Confidence intervals increase while decreasing the likelihood of trust and while increasing the number of observations (see Tabel II). For increased probability of failure, the assessment in this way of the methodology used for determining VaR can be useful, the number of actual values exceeding VaR in a year being significant. For low probabilities of failure, such as that proposed by the Basel Committee, this method of assessing the accuracy of the estimates is not significant. The number of observations in real activity should be very large in order to reach significant values and the phase shift time between the validation of the model and its implementation would be too high. This is one of the reasons why many financial institutions prefer to use a lower degree of confidence for the internal assessment of risk they are exposed to, making it easier to determine whether the model used underestimate or overestimate risk.

TABLE II
THE CONFIDENCE LEVELS

After obtaining the back-testing results it should be determined in which risk zone the bank is located, and on this basis it should be determined the value of "k" in the Basel's formulae for calculating the capital requirements for market risk. The more the bank will record multiple exceedances of the VaR the more it will move towards the major risk zone and accordingly will have to penalize the formulae for calculating capital requirements. The maximum number of exceedances of VaR on a 250 days horizon accepted by the Basel Committee is 4. If the exceedancies occurs more than four times than the VaR model is not appropriate. BCBS defined the following risk zones:

• the safe risk zone: no more than 4 exceedances of VaR;
• the medium risk zone: between 4 and 9 exceedances of VaR;
• the high risk area: more than 10 exceedances of VaR.

Probability (P)	The number of observations (T)		
	255 days	510 days	1000 days
0,010	0 < N < 7	1 < N < 11	4 < N < 17
0,025	2 < N < 12	6 < N < 21	15 < N < 36
0,050	6 < N < 21	16 < N < 36	37 < N < 65
0,075	11 < N < 28	27 < N < 51	59 < N < 92
0,100	16 < N < 36	38 < N < 65	81 < N < 120

V. ANALYZING THE VOLATILITY OF THE EASTERN AND CENTRAL EUROPEAN STOCK EXCHANGE MARKET

In order to highlight the differences among VaR models we have calculated the risk measure through Historical Simulation, EWMA, GARCH and EVT models. VaR was calculated on daily data of five Eastern and Central European main indices: BET (Romania), PX50 (Czech Republic), BUX (Hungary), SOFIX (Bulgary) and WIG20 (Poland) from 30.09.2004 to 30.09.2010. The daily closing prices were taken from http://www.bloomberg.com/markets/. Also, we have divided the data into two samples: the first sample is from 30.09.2004 to 30.09.2010 (1.647 observations) and the second sample is from 30.09.2008 to 30.09.2010 (548 observations), in order to highlight different behaviors in the crisis period.

The daily rentabilities were determined by the logarithmation of the indices' closing prices. Applying the Jarque Berra Test on the daily rentabilities series we have observed that the normal hypothesis is rejected. The distributions are leptokurtic, more sharpen than the normal ones, for all of the samples, a fact shown by the kurtosis coefficient. Analyzing the skewness coefficient we have observed that BET, SOFIX and WIG20 distributions present negative skewness, compared with the normal distribution, which indicates that the negative shocks are more frequent than de positive ones. On the opposite, PX50 and BUX present positive skewness (Table III and Table IV). Applying the ADF and the Philipe-Peron tests it has been seen that the series composed of the indices' closing prices have one unit roots were non stationary, but through first order differentiation they became stationary.

TABLE III
THE MOMENTS OF THE DISTRIBUTIONS (FIRST SAMPLE - 30.09.2004 TO 30.09.2010)

	R_{BET}	R_{PX50}	R_{BUX}	R_{SOFIX}	R_{WIG20}
Observations	1646	1646	1646	1646	1646
Mean	0.0017	0.0031	0.0047	0.0028	0.0037
Median	0.0005	0.0003	0.0002	0.0001	0.0001
Maximum	0.1876	0.2144	0.0318	0.2003	0.2455
Minimum	-0.1642	-0.1820	-0.1347	-0.1988	-0.2598
Std. Dev.	0.0201	0.0198	0.0183	0.0232	0.0190
Skewness	-0.1954	-0.0176	0.0245	-0.2118	-0.3958
Kurtosis	12.2005	21.1369	10.5432	22.8761	38.1524
Jarque-Bera	9148.75	29875.24	5432.67	27654.22	125432.21
Probability	0.0000	0.0000	0.0000	0.0000	0.0000

TABLE IV
THE MOMENTS OF THE DISTRIBUTIONS (SECOND SAMPLE - 30.09.2008 TO 30.09.2010)

	R_{BET}	R_{PX50}	R_{BUX}	R_{SOFIX}	R_{WIG20}
Observations	547	547	547	547	547
Mean	0.0013	0.0022	0.0038	0.0015	0.0026
Median	0.0003	0.0004	0.0001	0.0002	0.0003
Maximum	0.1534	0.2013	0.0298	0.1140	0.2106
Minimum	-0.2015	-0.1976	-0.1569	-0.2253	-0.2016
Std. Dev.	0.0321	0.0285	0.0216	0.0299	0.0201
Skewness	-0.2004	-0.1036	0.0764	-0.2735	-0.3873
Kurtosis	17.8243	26.4213	15.5190	28.7865	41.4382
Jarque-Bera	10023.43	32974.53	7653.85	30735.33	130872.37
Probability	0.0000	0.0000	0.0000	0.0000	0.0000

According to all these factors, the distribution of the rentabilities presents fat tails, which correspond to the extreme variations that took place on the stock exchange market. Using the Historical Simulation method it can lead to an overestimation of VaR, especially that the method describes the maximum expected loss. Here appears the "volatility clustering" phenomena, which can be remedied by the heteroscedasticity models EWMA and GARCH.

In order to eliminate the linear structure we propose some ARMA models studying the residuals' correlogram, for which the AIC and BIC criterions are minimum: ARMA(4) for the first sample and ARMA(5) for the second sample. The remained residuals have a non-linear structure which was detected by the BDS test elaborated in 1987 by Brock, Dechert and Scheinkman, in order to check the stochastic non-linearity. The BDS test's values are strong, which sustains the rejection of the normal hypothesis. This tendency reflects a degree of heteroscedasticity, which means that the present volatility depends on the previous volatility. Unless the data is filtered, this dependence will undermine the value of VaR. In order to eliminate the correlation between residuals we had to find some GARCH models. The best models identified were: GARCH(1,2), TGARCH and Orthogonal GARCH.

For expressing the current volatility as a function of the past volatility, we have also used EWMA models, giving more importance to the recent data. Estimating the volatilities and the correlations through EWMA(0.94), EWMA(0.92) and EWMA(0.90) we have calculated VaR for more levels of the persistence parameter λ. We have seen that the closer the value λ to unity, the higher the risk.

For a better characterization of the extreme values found at the upper and lower tails we have applied the Extreme Value Theory, using the excess over a threshold method. In our study we have used the Generalized Pareto Distribution to represent the extreme values, after sorting the values and choosing the following thresholds: 0.5% and 1% and 5%. We have observed that the higher the threshold, the higher the risk.

TABLE V
THE VAR ESTIMATIONS (FIRST SAMPLE - 30.09.2004 TO 30.09.2010)

VaR model	BET	PX50	BUX	SOFIX	WIG20
HS	0.018	0.021	0.016	0.022	0.019
EWMA (0.94)	0.021	0.025	0.017	0.024	0.022
EWMA (0.92)	0.019	0.022	0.015	0.021	0.018
EWMA (0.90)	0.015	0.018	0.012	0.020	0.015
GARCH(1,2)	0.023	0.020	0.018	0.025	0.024
TGARCH	0.019	0.020	0.020	0.023	0.023
Orthogonal Garch	0.017	0.019	0.022	0.025	0.019
EVT (0.5%)	0.020	0.020	0.019	0.020	0.020
EVT (1%)	0.016	0.018	0.017	0.019	0.018
EVT (5%)	0.015	0.017	0.013	0.016	0.011

Analyzing Table V it could be seen that on the 30.09.2004 – 30.09.2010 period the most accurate model was the GARCH class for BET, BUX and SOFIX and EWMA model for PX50 and WIG20. For the Romanian market the best models that reflect the risk was GARCH(1,2), followed by the EWMA (0.94) and EVT(0.5%). For the Czech market EWMA(0.025) model forecasts better the stock exchange market volatility, followed by EWMA(0.92) and Historical Simulation. For Hungary stock exchange market GARCH class is the most appropriate and within it the Orthogonal GARCH model is the best. On Bulgarian market GARCH class is on the first place, followed by EWMA and EVT. The Polish market risk could be better estimated by using EWMA models, followed by the GARCH class and by the EVT. As a common characteristic of these markets, on the period analyzed, it could be observed that EWMA, GARCH and EVT models perform better that the traditional Historical Simulation method.

Within the second sample (Table VI), that reflects the modeling of the most recent data, the situation changes. EVT is the most appropriate econometric model to forecast the risk, while the Historical Simulation method could lead to large underestimates of risk, putting the investors into a risky position. For all stock indices the EVT highlight better the risk at the highest level of the threshold. The best GARCH models for BET, PX50 and SOFIX is GARCH(1,2), while for BUX and WIG20 is the Orthogonal GARCH. Analyzing the EWMA class it could be seen that it tends to underestimate the volatility, especially when the persistence parameter takes lower values, because of its incapacity to takes into account the extreme movements that took place on the market during this period.

TABLE VI
VAR ESTIMATIONS (SECOND SAMPLE - 30.09.2008 TO 30.09.2010)

VaR model	BET	PX50	BUX	SOFIX	WIG20
HS	0.021	0.025	0.019	0.026	0.023
EWMA (0.94)	0.025	0.030	0.020	0.029	0.026
EWMA (0.92)	0.023	0.026	0.018	0.025	0.021
EWMA (0.90)	0.018	0.021	0.014	0.024	0.018
GARCH(1,2)	0.027	0.029	0.021	0.030	0.029
TGARCH	0.023	0.024	0.024	0.027	0.027
Orthogonal Garch	0.020	0.023	0.026	0.029	0.030
EVT (0.5%)	0.033	0.033	0.027	0.036	0.034
EVT (1%)	0.030	0.027	0.018	0.033	0.032
EVT (5%)	0.024	0.020	0.015	0.032	0.029

For testing the post efficiency of these methodologies we have used the back-testing, by simulating the stress scenarios for the least 245 days. We have applied the quadratic loss function approach, calculating how many times the VaR has been exceeded. For the first sample the best methods, which are in the minimum risk zone (VaR has been exceeded for no more that 4 times) are the GARCH models, followed by the EVT models and EWMA models. In the second sample the best methods from the post efficiency point of view are EVT and GARCH. EWMA models within the second sample is in the medium safety zone (between 4 and 9 violations of VaR). The method that failed mostly in accurately estimating the risk is the Historical Simulation method (between 11 and 15 violations of VaR for all of the indices), as shown in Table VII:

TABLE VII
BACK-TESTING RESULTS (QUADRATIC LOSS FUNCTION APPROACH)

	BET		PX50		BUX		SOFIX		WIG20	
	I	II	I	II	I	II	I	II	I	II
HS	8	12	6	13	10	11	8	15	14	15
EWMA (0.94)	3	4	1	4	2	5	3	5	2	10
EWMA (0.92)	2	4	1	4	3	5	4	6	4	7
EWMA (0.90)	2	8	4	6	4	9	6	9	5	2
GARCH(1.2)	1	2	1	2	2	3	1	2	3	4
TGARCH	2	2	3	4	4	2	2	3	2	3
Orthogonal Garch	3	3	2	2	3	2	3	1	2	1
EVT (0.5%)	2	1	3	2	2	1	3	1	3	2
EVT (1%)	2	2	3	2	3	3	3	2	4	2
EVT (5%)	2	2	4	4	3	3	3	2	4	3

VI. CONCLUSION

Analyzing some of the main indices of Central and Eastern European stock markets we wanted to highlight that only advanced VaR models such as Extreme Value Theory or GARCH models can adequately measure the risk of the capital markets and satisfy the requirements of the investors in periods characterized by extreme events. Also, in forecasting volatility on stock markets in crisis periods it should be used a shorter sample of data, the most recent one, in order to capture the large movements on the market. These estimations of risk could be taken into consideration by investors in order to fundament their portfolio investment decision regarding the titles listed on the Central and Eastern European stock markets.

REFERENCES

[1] P. Jorion, (2001) "Value at Risk: the new benchmark for managing financial risk", Ed. McGraw-Hill, 2001, pp. 107-111

[2] K. Dowd (1998), "Beyond Value at Risk. The New Science of Risk Management", Ed. John Wiley and Sons, London.

[3] A. Saunders (1999), "Credit Risk Measurement. New approaches to Value at Risk and other Paradigms", Ed. John Wiley&Inc, New York

[4] Basel Committee on Banking Supervision (2003), "The New Basel Acord", Available: http://www.bis.org/bcbs/qis/index.htm

[5] E. Fama (1965), "The Behavior of Stock-Market Prices", Journal of Business, 38(1), 34-105.

[6] B. Mandelbrot (1963), "The Variation of Certain Speculative Prices", Journal of Business, Vol. 36, pp. 394-419.32

[7] J. Berkowits, J. O'brien (2001), "Haw accurate are Value-at-Risk-Models at Commercial Banks?", FEDS Working Paper No. 2001-31, Available: http://www.bis.org/events/oslo/berkowitz.pdf

[8] M. Ferreira, J. Lopez (2003), "Forecasting volatility using historical data", Working Paper No. 13, New York University

[9] P. Burns (2002), "The quality of Value at Risk via Univariate GARCH", Available: http://www.burns-stat.com/pages/Working/varunigar.pdf

[10] J. Rombouts, M. Verbeek (2004), "Evaluating portfolio Value-at-Risk using semi-parametric GARCH models", Available: http://economics.ca/2005/papers/0040.pdf

[11] P. Embrechts (1999), "Extreme Value Theory as a Risk Management Tool. North American" Actuarial Journal , 3 (January)

[12] P. Embrechts, et al. (1997), "Modelling Extremal Events for Insurance and Finance", Ed. Springer-Verlag, Berlin

[13] A. McNEIL (1999), "Extreme Value Theory for Risk Managers", Working Paper,Department Mathematik, ETH Zentrüm, Zürich ethz.ch

[14] A. McNEIL (1998), "Calculating Quantile Risk Measures for Financial Return Series using Extreme Value Theory", Working Paper, Department Mathematik, ETH Zentrüm, Zürich

[15] A. McNEIL (1997), "Estimating the Tails of Loss Severity Distributions Using Extreme Value Theory", ASTIN Bulletin, Nº 27, pp. 117-137

[16] J. Daníelsson, C. de VRIES (1997), "Extreme Returns, Tail Estimation and Value-at-Risk. Manuscript" ,University of Iceland and Erasmus University, Rotterdam, http//www.hag.hi.is/~jond/research

[17] J. Daníelsson, C. de VRIES, G. Casper (1997), "Tail Index and Quantile Estimation with very High Frequency Data", Journal of Empirical Finance, Vol. 4, pp. 241-257

[18] R. Huisman et al. (1998), "VaR+: Fat Tails in Financial Risk Management", Journal of Risk, 1 (Fall), pp. 47-61

[19] R. Huisman et al. (1997), "Fat Tails in Small Samples", Working Paper Limburg Institute of Financial Economics (LIFE), Maastricht University, (September)

[20] J. Cott (2007), "Varying the VaR for unconditional and conditional environments", Journal of International Money and Finance, Kidlington, Vol. 26, Iss. 8,p. 1338

[21] Y. Bensalah (2000), "Steps in applying Extreme Value Theory to Finance: a review", Bank of Canada Working Paper 2000-20

[22] R. Gencay, F. Selcuk (2004), "Extreme Value Theory and Value-at-Risk: Relative performance in emerging markets", International Journal of Forecasting, 20, 287-303

[23] M. Gili, E. Kellezi (2001), "A global optimisation heuristic for portfolio choice with VaR and Exected Shortfall", Available: http://www.smartquant.com/references.php

[24] B. LeBaron, R. Samanta (2005) , "Extreme Value Theory and Fat Tails in Equity Markets", Available: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=873656

[25] J. Schaumburg (2010), "Predicting extreme VaR: Nonparametric quantile regression with refinements from extreme value theory", SFB 649 Discussion Paper 2010-009

[26] Financial Service Authority (2009), "The turner review: A regulatory response to the global banking crisis", Available: http://www.fsa.gov.uk/pubs/other/turner_review.pdf

[27] Senior Supervisors Group (2008), "Observations on Risk Management Practices during the Recent Market Turbulence", New York

[28] J. Arneric, E. Jurun, S. Pivac,(2007) "Theoretical Distributions in Risk Measuring on Stock Market", Proceedings of the 8th WSEAS Int. Conference on Mathematics and Computers in Business and Economics, Vancouver, Canada, June 19-21

[29] Riskmetrics (1996), Technical Document, Fourth edition, J.P. Morgan

[30] R. Engle, (2001), "The use of ARCH/GARCH Models in Applied Econometrics", Journal of Economic Perspectives, vol.12, no.5:23-33

[31] T. Bollerslev (1986), " Generalised Auto-Regressive Conditional Heteroscedasticity", Journal of Econometrics, 31, 307-327

[32] A. Jordi, T. Salvador (2010), "Solving Market Index Biases using Minimum Risk Indices", Proceedings of the 8th WSEAS Int. Conference on Mathematics and Computers in Business and Economics, Pcnang, Malaysia, March 23-26

[33] B. Racheva-Iotova, S. Stoyanov, (2009), "Extreme Events on a Univariate Level: Understanding the Fat-tails of a Risk Driver", *FinAnalytica Quantitative Series Modelling Risk in the Presence of Extreme Events* – Briefing 1, May 9, 2009, pp.1-9

[34] D. Mcmillan, A. Speight ,O. Apgwilym, (2000) "Forecasting UK stock market volatility", Applied Financial Economics, 2000, 10, pp. 435-448

[35] A. Assaf, (2006), "Extreme Observations in the MENA Stock Markets and Their Implication for VaR measures", 2006 Proceedings of the Middle East Economic Association, 26th annual meeting of MEEA in Boston, USA, January 6-8, 2006

[36] BCBS (2010), "The Basel III Capital Framework: a decisive breakthrough", November 2010

[37] G. Ulugad, K. Senel, A. S. Etaner-Uyar, H. Dag (2006), "Application of an evolutionary algorithm for VaR calculations", Proceedings of the 5th WSEAS Int. Conference on Artificial Intelligence, Knowledge Engineering and Data Bases, Madrid, Spain, February 15-17

[38] C. W. Cheong, A. Hassan, S. M. Nor, Z. Isa, "An Evaluation of Autoregressive Conditional Heteroskedasticity Modeling in Malaysian Stock Market", WSEAS Transactions on Business and Economics, Vol 3, 150-156