Prediction of the Price Cap under RPI-X Incentive Regulation of Network Industries Based on Productivity Benchmarking

Ondrej Machek, Ondřej Nowak, and Jiří Hnilica

***Abstract* —** Total factor productivity benchmarking has recently become an important tool of network industries regulation, especially in setting the price cap under incentive regulation. However, the calculation of productivity indexes is usually based on point estimates thus ignoring the inherent uncertainty of inputs, especially price level and market demand fluctuations. At the same time, there exist many methods for incorporating the uncertainty in financial models. In this article, we present a model which can be used in estimating future price caps based on Monte Carlo simulation. We demonstrate the use of the model on the example of Australian National Railways using real data from years 1979-1990.

***Keywords*—** Monte Carlo Simulation, Price Change Forecast, Total Factor Productivity, Uncertainty

I. Introduction

Network industries (energy and natural gas, water and sewerage, communications and transportation industries) are often considered to be "affected with public interest".[1] The protection and stability of network industries are of high importance for the government and its environmental, social and economic policy.

Network industries, often referred to as public utilities, often have the following properties [3]:

- inherent economies of scale and scope;
- essential product with a low price- and cross-elasticity of demand;
- capital-intensive production; the product represents an essential input for other industries of the entire economy;
- non-storable and non-transferable product, synchronous production and consumption, long periods of excess capacity, fixed connection with customers;
- variable and seasonal character of demand;
- obligation to serve all customers who are able and willing to pay for the service (public service obligation).

Because of the strategic importance of these industries and their reliability, and following the concepts of natural monopoly and destructive competition (see e.g. [12] for more detailed discussion) or for social reasons, the government may decide to regulate price level, quality-of-service, market entry and exit conditions and impose public service obligations.

The regulatory process traditionally devotes considerable attention to the price level. The main task of regulatory agencies is to set a price level that mimics the competitive price level, e.g. the price level established under general equilibrium in competitive markets. However, in a situation when competition is, for various reasons, not feasible, this tariff setting is a formidable task.

The situation of natural monopoly tariff setting is illustrated on Fig. 1. In the conditions of natural monopoly, the curve of average costs (AC) is decreasing due to the above-mentioned economies of scale. Monopoly power causes that the marginal revenues curve (MR) is steeper than the demand curve (D). The competitive output (Q) and price (P) is achieved at the intersection of demand (D) and marginal costs (MC). However, if the price was set at P, total revenues (P × Q) would not cover total costs (AC × Q) and the company would eventually go out of business, which is not desirable.

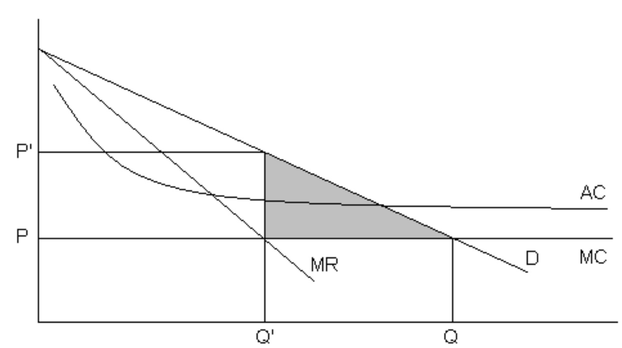

Fig. 1: The concept of natural monopoly

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[1] .The U.S. Supreme Court first used this term in the case Munn v. Illinois, 94 U.S.113 (1877).

It is evident that the firms operating in price-regulated industries do not face competitive pressures, but regulatory constraints. Their profits depend on the choice of the regulatory method and its parameters.

The total factor productivity approach (TFP) has recently become an important tool of regulation. Under this regulatory regime, the maximum price of services is set according to the relative performance of the firms. If a firm achieves to be more productive than other, comparable firms, it is rewarded by greater profits. In the opposite case, it is punished by lower profits.

For a regulated firm, forecasting future prices is an essential task, since the regulated prices directly influence its future cash flows, performance and growth. However, future prices are also determined by factor which may not be directly affected by the regulated firms, especially measurement errors, but also by fluctuation of price level and demand. Therefore, some degree of uncertainty in the calculations is practically inevitable.

Monte Carlo simulation has become a popular tool for decision-making under uncertainty for its simplicity and flexibility. In this paper, we propose a model based on the Monte Carlo simulation which can be used in the estimation of future price cap set by the regulator under incentive regulation based on productivity benchmarking. We demonstrate the forecast of future price changes on an empirical example based on a study on the Australian National Railways carried out by the Australian Industry Commission.

II. Methods of Price Regulation and X-factor

All methods of economic regulation are based on the idea that a company should be allowed to recover its costs and earn a commensurable return on the investments. The expenses which are not “prudent“, “known and measurable“ and “used and useful“ (see [13] for more detailed explication) are generally excluded and they are not allowed to be recovered. The revenue requirements (RR) can be calculated as

$$RR = O\,\&\,M + D + T + RB \times RoR \qquad (1)$$

where *O&M* denotes operating and maintenance costs, *D* denotes depreciation, *T* denotes taxes, *RB* is the regulatory asset base and *RoR* is the rate of return. The regulatory asset base consists of assets which are directly used in the relevant service provision (for example, passenger transport).

A. Cost-of-service regulation

The traditional cost-of-service (COS) regulation is based on simply summing allowed expenses, rate of return and calculating a required rate of return. The regulated firm is simply allowed to recover its expenses every year. Although this approach is simple and straightforward, it has several disadvantages – for example, information asymmetries between the regulator and regulated companies which may lead to gaming or data manipulation, the firms have an incentive to overinvest (A-J-W effect, [2]) or to invest imprudently (this kind of behavior is called gold-plating). Moreover, the tariff level has to be reviewed frequently (typically every year) which makes this method time-consuming and expensive.

B. Incentive regulation

The purpose of incentive regulation is to reduce the impact of the above-mentioned negative aspects of cost-of-service regulation and to motivate the company to behave efficiently. In general, we distinguish two basic alternatives of incentive regulation: price-cap and revenue-cap.

The price-cap method is based on setting maximum tariffs for services provided, whereas the revenue-cap method is based on capping total revenues. The formula is

$$P(t) = (1 + RPI - X) \times P(t-1) \qquad (2)$$

where *P(t)* is the price level in period *t* and *P(t–1)* denotes the price level in period *t–1*. Since the tariffs are capped according to the inflation rate (RPI-factor) and efficiency (X-factor), this method of regulation is also referred to as *RPI-X regulation.*

The principle of the revenue-cap regime is the same, but in this case, the firm’s yearly total revenues are capped, so the formula becomes

$$R(t) = (1 + RPI - X) \times R(t-1) \qquad (3)$$

where *R(t)* are the revenues in time *t* and *R(t–1)*denotes revenues in time *t–1*.

The price or revenue cap holds for a certain time period, until the cap is reset by the regulatory agency. If a company successes to improve its performance more than by the value given by the X-factor of efficiency, it will have positive profits during this period. This phenomenon is called *regulatory lag* and it represents the main incentive for the regulated firms to improve performance.

However, if the price or revenue cap is based solely on a company’s own costs, the negative effects of cost-of-service regulation (overinvestment, gold plating etc.) are not fully eliminated.

C. Regulatory benchmarking

In a competitive market, a firm must reduce its costs below the average level of its competitors in order to survive. In theory, the average costs (*AC*) of *i*-th firm on the market should satisfy the following formula:

$$AC_i \le \frac{\sum AC_j}{n-1} \qquad (4)$$

where AC_j denotes average costs of *j*-th company and *n* is the total number of firms operating on the market. Setting the price or revenue cap with respect to the performance of other, comparable companies is the main principle of regulatory benchmarking [20].

Regulatory benchmarking, if properly applied, strengthens the incentives for the regulated firms to behave efficiently. Benchmarking can either be used as a supplement of performance-based regulation or as a pure regulatory method, which is called *yardstick competition*. It can be based either on efficiency benchmarking (frontier methods, such as data envelopment analysis or regression methods) or productivity benchmarking (index methods). The level of incentive to behave competitively provided by a regulatory regime can be illustrated using Fig. 2 [14].

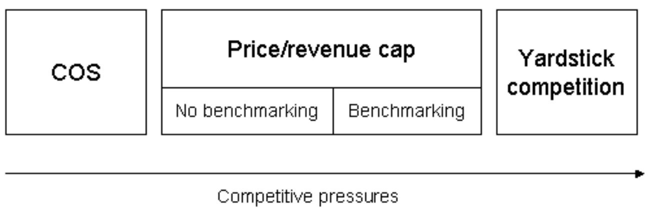

Fig. 2: The level of incentive of regulatory regimes

Besides these widely used regulatory methods, alternative approaches to tariff determination such as the application of fuzzy sets [22] are being developed, but still without significant empirical experience.

D. TFP benchmarking

This article deals with productivity (TFP) benchmarking regimes. These methods have been used in practice in many countries worldwide, for example in the US, in the United Kingdom, Australia, New Zealand or Latvia, among others.

Under TFP benchmarking, the regulated firm's productivity growth is compared with the productivity growth of the economy. When the firm succeeds to improve its productivity more than other firms, it is rewarded by greater profits.

In order to apply RPI-X incentive regulation, it is necessary to determine the above-mentioned X-factor, which can be calculated using total factor productivity (TFP) change. It is possible to derive the regulated price change as [15]

$$\Delta p = \Delta p_n - (\Delta TFP - \Delta TFP_n - (\Delta w - \Delta w_n)) \qquad (5)$$

where Δp_n is the inflation rate of economy outputs, Δw_n is the inflation rate of economy inputs, Δw is the inflation rate of the regulated industry inputs, ΔTFP is the change of productivity of the regulated industry and ΔTFP_n is the change of productivity of the economy.

The X-factor of efficiency can be rewritten as

$$X = (\Delta TFP - \Delta TFP_n) - (\Delta w - \Delta w_n) \qquad (6)$$

This expression is based on the assumption that all terms are deterministic. However, differences in productivity growth can also be caused by factors which may not be affected by the firms, especially measurement errors and random external influences. In the next section, we will introduce the TFP measurement and discuss possible random variables in the calculations.

III. Total Factor Productivity Measurement

Traditionally, productivity is defined as the ratio of output over input. In the case of only one output and one input, the situation is straightforward. In a more realistic situation when a firm produces multiple products and uses multiple inputs, it is necessary to aggregate the set of outputs and inputs so that the expression in numerator and denominator are scalar values. The total factor productivity (TFP) approach takes into account *all* possible inputs and outputs of the firm.

In economic theory, total factor productivity is measured indirectly. It is the output growth not explicable by changes in the amount of inputs (often referred to as Solow residual).

In economic practice, TFP change is measured by productivity indexes. Indexes are a common tool to measure price or quantity changes between two periods. Since in TFP calculations, we deal with the ratio of output and input quantities, we employ quantity indexes.

Indexes can be based on distance function or on price aggregation (for detailed discussion, see e.g. [5]). We will discuss two most frequently used representatives, the Malmquist and Törnqvist indexes.

A. Malmquist index of productivity

A produced can be defined as an economic agent transforming a set of inputs $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ into a set of outputs $\mathbf{y} = (y_1, y_2, \ldots, y_m)$. Generally, we consider these vectors to be strictly positive.

In order to define Malmquist index, we have first to introduce the notion of efficiency. The *efficiency* of a firm can be defined as a ratio of observed values of inputs and outputs to their their optimal values. The analysis of efficiency can be oriented either on minimizing inputs with given outputs or maximizing outputs with given inputs (these approaches are dual to each other). The production technology can be represented using a set of couples (input-output vectors)

$$T = \{(\mathbf{x}, \mathbf{y})\} \text{ where } \mathbf{x} \text{ is input to produce } \mathbf{y}. \qquad (7)$$

Another possible representation of production technology is the output requirement set $P(\mathbf{x})$, e.g.

$$P(\mathbf{x}) = \{\mathbf{y} : (\mathbf{x}, \mathbf{y}) \in T\} \qquad (8)$$

As a measure of efficiency, we can use the Debreu-Farrell approach ([6], [9]). Using the above-described notation, we can define the Debreu-Farell measure of technical efficiency as the maximum possible equiproportional increase of given output so that it still belongs to the output requirement set.

$$TE(\mathbf{x}, \mathbf{y}) = \max\{\Phi : \Phi\mathbf{y} \in P(\mathbf{x})\} \qquad (9)$$

The inverse value of technical efficiency is called distance function.

$$D(\mathbf{x},\mathbf{y}) = \min\{\lambda : \mathbf{y}/\lambda \in P(\mathbf{x})\} \quad (10)$$

When applying the output-maximizing approach, the lesser the distance from a production frontier, the better is the efficiency score. In the real world, the production frontier is unknown and has to be estimated using econometric methods (e.g. corrected ordinary least squares, COLS) or mathematical programming (e.g. data envelopment analysis, DEA). Using the above described definitions, we can define the Malmquist index of productivity. Consider a period during which the production has changed from $(\mathbf{x_t},\mathbf{y_t})$ to $(\mathbf{x_{t+1}},\mathbf{y_{t+1}})$. The Malmquist index of productivity for period t , respectively for period $t+1$, would be the ratio

$$M_t(\mathbf{x}_t,\mathbf{y}_t,\mathbf{x}_{t+1},\mathbf{y}_{t+1}) = \frac{D_t(\mathbf{x}_{t+1},\mathbf{y}_{t+1})}{D_t(\mathbf{x}_t,\mathbf{y}_t)}$$
$$M_{t+1}(\mathbf{x}_t,\mathbf{y}_t,\mathbf{x}_{t+1},\mathbf{y}_{t+1}) = \frac{D_{t+1}(\mathbf{x}_{t+1},\mathbf{y}_{t+1})}{D_{t+1}(\mathbf{x}_t,\mathbf{y}_t)}; \quad (11)$$

If the technology has changed during the period, these two indexes would result in different values. Therefore, it is common to employ the geometric mean of the two indexes and specify the Malmquist index of productivity as

$$M(\mathbf{x}_t,\mathbf{y}_t,\mathbf{x}_{t+1},\mathbf{y}_{t+1}) = \sqrt{\frac{D_t(\mathbf{x}_{t+1},\mathbf{y}_{t+1})}{D_t(\mathbf{x}_t,\mathbf{y}_t)} \times \frac{D_{t+1}(\mathbf{x}_{t+1},\mathbf{y}_{t+1})}{D_{t+1}(\mathbf{x}_t,\mathbf{y}_t)}} \quad (12)$$

Malmquist index is of great theoretical importance. However, it is necessary to estimate the real but unknown production frontier using econometric or mathematical programming methods. Often, it is more practical to employ the indexes based on price aggregation, which can be calculated only from two observations. We will deal with these indexes in the following text.

B. Törnqvist index of productivity

Amongst the most frequently used indexes based on price aggregation, we can cite the Törnqvist productivity index [21] or the Fisher productivity index [10]. These measures require data about input and output prices, but can be derived directly from empirical data. In this paper, we will use the Törnqvist index to measure productivity growth.

Törnqvist index is an example of the so-called superlative indexes [7] which, under certain conditions (see [8]), approach the Malmquist index. The calculation is based on observed or estimated prices of inputs and outputs.

As in the previous text, let $\mathbf{x} = (x_1,x_2,\ldots,x_n)$ denote the vector of input quantities, let $\mathbf{y} = (y_1,y_2,\ldots,y_m)$ denote the vector of output quantities, $\mathbf{w} = (w_1,w_2,\ldots,w_n)$ is the vector of input prices and $\mathbf{p} = (p_1,p_2,\ldots,p_m)$ is the vector of output prices, all vectors having strictly positive components.

The Törnqvist index of productivity is defined as a ratio of output quantity index Y_T and input quantity index X_T . Usually, the two quantity indexes are specified in their logarithmic form as

$$\ln Y_T(\mathbf{y},\mathbf{p}) = \frac{1}{2}\left(\sum\nolimits_m \left[\frac{p_{m,t}y_{m,t}}{\sum_m p_{m,t}y_{m,t}} + \frac{p_{m,t+1}y_{m,t+1}}{\sum_m p_{m,t++}y_{m,t+1}}\right]\ln\frac{y_{m,t+1}}{y_{m,t}}\right)$$
$$\ln X_T(\mathbf{x},\mathbf{w}) = \frac{1}{2}\left(\sum\nolimits_n \left[\frac{w_{n,t}x_{n,t}}{\sum_n w_{n,t}x_{n,t}} + \frac{w_{n,t+1}x_{n,t+1}}{\sum_n w_{n,t++}x_{n,t+1}}\right]\ln\frac{x_{n,t+1}}{x_{n,t}}\right) \quad (13)$$

And finally, the Törnqvist index of productivity can be specified as

$$\Pi_T(\mathbf{x},\mathbf{y},\mathbf{w},\mathbf{p}) = \frac{Y_T(\mathbf{y},\mathbf{p})}{X_T(\mathbf{x},\mathbf{w})} = e^{\ln Y_T(\mathbf{y},\mathbf{p}) - \ln X_T(\mathbf{x},\mathbf{w})} \quad (14)$$

So far, we considered decision makers who act in a world of absolute certainty. Clearly, economic subjects in the real world don’t operate under such favorable conditions.

The Törnqvist index depends on four vectors, out of which all can be represented by random variables; both input and output price fluctuations and input and output quantities which are dependent on the demand. The source data for the productivity calculations will most probably differ in years as the calculation for each year will be based on unique set of data.

For a regulated company, the estimation of the price or revenue cap in the following regulatory period is a challenging but important part of planning, since the above-defined revenue requirements (RR) directly influence future cash flows and future performance and growth.

The inherent uncertainty in the estimation of future productivity changes can be investigated quantitatively by the Monte Carlo simulation techniques.

IV. Uncertainty and the Monte Carlo Simulation

The real economic world is a place where many important decisions involve an element of risk and where economic agents face uncertainty. Consumers attempt to maximize their expected utility given budget constraints, whereas firms make uncertain output decisions given cost constraints. In fact, all agents deal with estimates instead of deterministic values.

Generally all the estimates are uncertain numbers with the occurrence rather on an interval with varying density of expectations than on a few discrete data points. Deterministic models working with a single point estimate are usually using only one number out of such an interval thus ignoring important and valuable information about the uncertainty [11]. Limiting ourselves only to single points however makes us lose information concerning not only the variance (degree of uncertainty or risk), but also the shape of the probability distribution, which might not be symmetrical or unimodal [19].

Furthermore, distinguishing the expected and most likely value is an integral part of business modeling, because the two values can differ significantly which can result in wrong model input. This is usually a case of asymmetric distributions where mean, modus and likeliest value do not overlap [11].

There are several methods for incorporating the uncertainty into the financial model, but due to its simplicity and flexibility, the Monte Carlo simulation is the most popular.

Monte Carlo simulation is an iterative computational algorithm which calculates numerous scenarios of a model by repeatedly drawing values from a user-predefined probability distributions and inserting them in the model. This imposes the basic requirement that the analyst is able to quantify input data. The output of the Monte Carlo simulation is a distribution of a monitored variable (in our case, the total factor productivity or price change), which however should not be understood as a probability distribution, but rather as a distribution of our ignorance (or uncertainty) concerning the model output.

The Monte Carlo simulation is used in many fields of economics where the analyst has to deal with risk and uncertainty, and can be used, among others, in business valuation [17], options pricing [16] or macroeconomic forecasting models (see e.g. [1]).

The structure of a simulation model is very similar to a deterministic model, with all functions and operations that link variables together, except that each variable is represented by a probability distribution instead of a single value. The objective is to calculate the combined impact of the variability in the model's parameters in order to determine a probability distribution of the possible model outcomes.

Replacing uncertain numbers with distributions allows for an integration of a wide range of improvements into financial modeling. Separately analyzing each uncertain variable, approximating its potential occurrence with a corresponding shape of the distribution and observing their joint influence on model outputs provides useful insight into what is beyond the average scenario.

A. Quantifying Uncertain Variables

There are essentially two sources of information used to quantify the variables within a risk analysis model: available data and expert opinions [23].

1) Determining Variability from Data

The observed data may come from a variety of sources: surveys, computer databases, history or research. Before making use of the data, the analyst should be satisfied with their reliability and representativeness. Anomalies in the data should be checked out where possible and proved outliers should be discarded. There are several techniques available to interpret observed data for a variable in order to derive a distribution that realistically models its true variability and our uncertainty about that true variability.

If there is not enough information about the analyzed data set, or for any other reason the assumption about the shape or type of probability distribution cannot be established, non-parametric distribution fitting methods may be used. In these cases, for each scenario the Monte Carlo simulation draws randomly from the underlying data set (an empirical distribution).

However, it can also be assumed that the analyzed data come from a known theoretical distribution. A typical example is the normal distribution, whose frequent occurrence is likely attributable to the central limit theorem, which predicts that the sum of a large number of independent random variables, each with finite mean and variance, will be approximately normally distributed. In these cases parametric distribution fitting methods are used. The empirical distribution of underlying data is then used only to determine the degree of fit to a theoretical (parametric) distribution. Various theoretical distributions can be analyzed to find the one that best fits the observed data. Compared to non-parametric fitting methods mentioned above this approach leads the simulation to abandon the original data set and draw data from the theoretical distribution. This however sometimes means ignoring gained empirical experience.

The shape of the probability distribution is closely related to the uncertainty regarding the estimation of values of its parameters, which is known as Second Order Distribution Fitting. With some simplification, methods for estimating the probability distribution of parameter values can be categorized into the three following groups: classical statistics, Bootstrap method and Bayesian statistics. All three methods are very useful, but require more effort and their applicability varies according to circumstances [23], [24].

2) Modeling Expert Opinions

In some situations proper data are not available and that is when expert opinions are often more suitable. When applying simulations, the potential of covering expert opinions is much larger as it is possible to create a distribution of all estimates. Thus no important information regarding collected estimates, including the uncertainty of them being correct, would be ignored. Each expert can further be assigned a weight of his or her estimate in order to distinguish between the qualities of various expert respondents. Such a weighting implies the probability of the estimate of the expert is correct, which may for example be derived from his or her reputation or an existing track record.

Furthermore it is easier to cooperate with each expert, because his or her subjective uncertainty concerning the estimate can be captured by a distribution. Thinking hard about the factors that could interfere with an expected base case scenario makes an expert consider both an upside and downside potential variability of the situation. Requiring an expert to also define the worst and the best case scenarios allows for an understanding of the range of potential outcomes. Only then is it possible to realize what can be expected if everything goes wrong and vice versa.

B. Modeling Dependencies

Often we are dealing with the question of to what extent within the model the behavior of one variable determines the expected occurrence of others. Our brain is able to work with similar relationships intuitively based on our empirical experience; however their proper implementation to a financial model may be complicated. This is not just an infamous problem of distinguishing correlation from causation, but also the issue of mathematical interpretation of the inner dynamics within the model.

Working with linear dependencies expressed by correlations is usually the easiest method. However, it is important to keep in mind that their application is correct only if there is a presumption that the dependence is "approximately" linear. It is inappropriate to use correlations otherwise.

In the model, which will be introduced in the following section of this paper, historical data will be the primary source for the uncertainty quantification.

V. Proposed Model

In this section, we propose a model which can be used in the estimation of future price cap set by the regulator which employs the above-described RPI-X incentive regulation based on productivity benchmarking. We use the notion of random vectors whose components are scalar-valued random variables on the same probability space.

Let $\mathbf{X} = (x_1,x_2,\ldots,x_n)$ denote a random vector of input quantities, let $\mathbf{Y} = (y_1,y_2,\ldots,y_m)$ denote a random vector of output quantities, $\mathbf{W} = (w_1,w_2,\ldots,w_n)$ is a random vector of input prices and $\mathbf{P} = (p_1,p_2,\ldots,p_m)$ is a random vector of output prices, with each component having a proper marginal probability distribution and strictly positive components.

We can use the Törnqvist index to estimate the productivity growth as in the previous sections, but in this case using random variables as the function arguments instead of deterministic values, i.e.

$$\Pi_T(\mathbf{X},\mathbf{Y},\mathbf{W},\mathbf{P}) = \frac{Y_T(\mathbf{Y},\mathbf{P})}{X_T(\mathbf{X},\mathbf{W})} \tag{15}$$

The productivity growth ΔTFP will be represented by a random variable with a specific probability distribution and its parameters, as well as the future price cap in the following regulatory period, which can be estimated as

$$\Delta p = \Delta p_n - (\Pi_T(\mathbf{X},\mathbf{Y},\mathbf{W},\mathbf{P}) - \Delta TFP_n - (\Delta w - \Delta w_n)) \tag{16}$$

And, similarly, the X-factor of efficiency will become a random variable which can be specified as

$$X = (\Pi_T(\mathbf{X},\mathbf{Y},\mathbf{W},\mathbf{P}) - \Delta TFP_n) - (\Delta w - \Delta w_n) \tag{17}$$

In order to apply this model in practice, the analyst has to estimate the probability distributions of the function arguments **X**, **Y**, **W** and **P**. This can be achieved either by using expert estimates or by employing numerical methods.

VI. Empirical Example

In this article, we will demonstrate the estimation of an uncertain price level change in the year 1991 using real data from years 1979-1990 based on a study on the Australian National Railways carried out by the Australian Industry Commission [5].

This Australian company is an example of firms operating in tariff-regulated network industries. It provided two basic services: freight (cargo) transport and public passenger transport. The freight transport was operated by railways in mainland (South Australia) and in Tasmania (Tasrail).

The TFP study is based on two non-capital inputs (labor and fuel), four capital inputs (land, building and perway, plant and equipment, rolling stock and “other inputs”) and three easily measurable outputs (mainlang freight, Tasrail freight and passenger services).

To sum up, three categories of output are considered:

- y_1: mainland freight, measured in net-tonne-kilometers;
- y_2: Tasrail freight (net-tonne-kilometers);
- y_3: passenger services (passenger-train-kilometers).

Further, six categories of input are considered:

- x_1: land, building and perway (1.000$);
- x_2: plant and equipment (1.000$);
- x_3: rolling stock (1.000$);
- x_4: labor (full-time staff);
- x_5: fuel (1.000 liters);
- x_6: other inputs (1.000$).

The more complex the relations between input variables are, the more difficult the analytical solution of the estimation of the price level change becomes. The estimation is the result of an aggregation of many random variables that are often mutually dependent and whose probability distributions are not usually known in advance and therefore must be estimated with their interdependencies included. At this point the employment of Monte Carlo simulation is convenient.

VII. Employment of Simulation Techniques

The model offers historical data set from years 1979-1990, which can be subjected to a regression analysis for each variable in order to fit the regression curve. The data, however, never fit the regression curve perfectly (i.e. usually, there is no functional relationship), but rather they are scattered along.

The regression curve enables to obtain the expected value of *Y* (i.e. dependent variable) for the year 1991. To make the process more realistic, a factor of an estimation error shall be added further to the expected value of *Y*. The estimation error should theoretically follow normal distribution with zero mean and standard deviation equal to the standard error of the regression (also known as the root mean square error or RMSE).

An example of this approach is shown at Fig. 3, which provides the historical quantity of mainland freight with its prediction for the year 1991. The historical data fit the regression line quite well (R^2 = 88%), with the standard error of regression equal to 371 091. After putting values into the regression equation, the calculation proceeds as

$$Y = 265454 * 1991 - 520517898 = 8002121 \quad (18)$$

The parameters of the normal distribution are therefore given by the estimation of Y (8 002 121) as the mean and RMSE (371 091) as the standard deviation.

With this procedure it is possible to combine regression analysis with Monte Carlo simulation in order to generate random predictions for all the variables within the model.

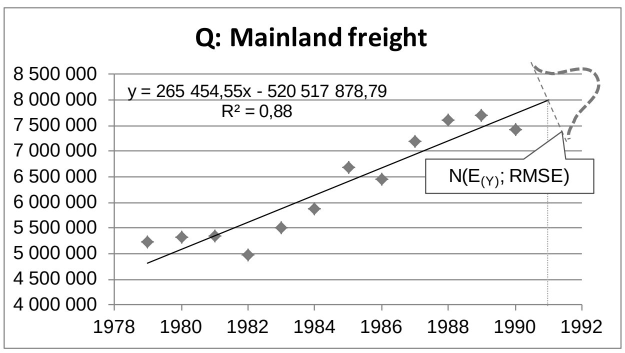

Fig. 3: Prediction for the year 1991 with regression

In business modeling it is also necessary to deal with the question of to what extent the behavior of one variable determines the expected occurrence of others. In this matter working with linear dependencies expressed by correlations is usually the easiest method. However, it is important to keep in mind that their application is correct only if there is a presumption that the dependence is "approximately" linear.

Two types of dependencies can be distinguished in this model. First, each variable is following a certain trend (i.e. some are increasing other are decreasing in time). This trend is being accounted for by the slope of the regression curve.

Second type of a possible dependency is the deviation from this trend between variables (i.e. correlations between regression residuals of variables). This dependency reflects situations when deviations from trend tend to occur simultaneously for some variables. To cover this phenomenon, the correlation matrix of regression residuals is used as a proxy for capturing dependencies between uncertain variables within the model. This correlation matrix defines correlations between the normal distributions introduced earlier.

To determine the price level change in the year 1991, we would also have to estimate the inflation rate of economy outputs vΔp_n, the inflation rate of economy inputs Δw_n, the inflation rate of the regulated industry inputs Δw and the total factor productivity change of the economy ΔTFP_n.

In the context of forecasting, these four parameters are uncertain and should also be the subjected to risk analysis, which will most likely result in their quantification by a probability distribution. This is however beyond the scope of this paper. Hence, this step will be excluded from the following analysis and these variables will be arbitrarily chosen as follows:

- Δp_n = 1.0728 (the inflation rate in Australia between 1990/1991);
- Δw - Δw_n = 0 (we do not suppose that input price inflation will exceed the overall input price inflation, following [18]);
- ΔTFP_n = 0.7 (the TFP growth between 1987-1992 calculated by OECD).

All the inputs for Monte Carlo simulation were entered to the spreadsheet using ModelRisk simulation software which is designed as an add-in for MS Excel. Its outputs are discussed in the following section of this paper.

VIII. Results

The distribution of the price level change in the year 1991, which is the output of Monte Carlo simulation, is shown at Fig. 4. The non-shaded area on the left side of the chart reflects the 55.3% probability of the decrease in the price level, which is higher than the 44.7% probability of its increase reflected by the shaded area on the right. Expected value of the change in the price level in the year 1991 is -1.8% with the standard deviation of 12.6%.

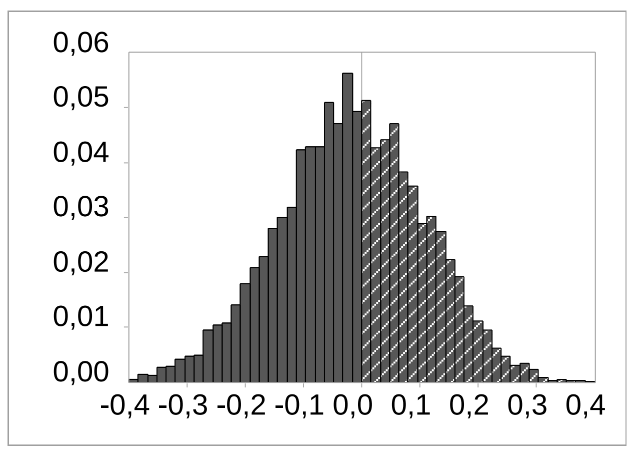

Fig. 4: Price level change distribution

A part of the interpretation of the result is often the identification of significant intervals. For example, the lower and upper fifth percentile can be cut off thus obtaining observed value at the 90% confidence level. In this case the price change is expected to be between -22.9% and +18.3% on the 90% level of confidence.

Another output of the simulation is the sensitivity analysis displayed at Fig. 5, which shows six variables (Q denotes quantity) with the highest impact on the expected price change in the year 1991.

Fig. 5: Price level change sensitivity analysis

Different models varying in structure and inputs shall have different sets of the most influential factors. These factors should be the primary concern of the management as they drive the forecasted value.

IX. Conclusion

The estimation of future revenues under government regulation based solely upon point estimates leads to many inaccuracies. This is caused not only by measurement errors, but also by random fluctuations of price level and demand in all model variables. These phenomena can be investigated quantitatively by simulation techniques.

In this paper, we proposed a simple model based on Monte Carlo simulation which can be used in the estimation of future price cap set by the regulator under incentive regulation based on total factor productivity benchmarking. We demonstrated the forecast of future price changes on an empirical example based on a study on the Australian National Railways carried out by the Australian Industry Commission.

The proposed model builds on a bivariate regression analysis, by which the time trend for each variable is estimated. The estimation error is added as a potential deviation from this trend in the forecasted period. The estimation error should follow a normal distribution with zero mean and standard deviation equal to RMSE of the regression analysis.

A possible interdependency between variables in deviations from this trend in partial years was also included in the model. To cover this phenomenon, the correlation matrix of regression residuals was used in order to define correlations between the above defined normal distributions.

Possible outputs of Monte Carlo simulation include probability distribution of the forecasted variable with relevant statistics. Sensitivity analysis further identifies variables with the highest influence on the forecasted variable, which can be represented for example by a tornado chart.

It is important to note that every model building is subjective in the choice of probability distribution and its parameters. However, practical decision-makers will acknowledge that every model has its assumptions.

Our model can be used in dealing with uncertainty under TFP-based incentive regulation regime. Under different regimes (such as efficiency benchmarking or data envelopment analysis), the principle would be similar – to avoid using single-point estimates, but rather to introduce random variables represented by probability distributions and to understand their interdependencies. In a dynamic and constantly changing environment, which is the case of network industries, all regulatory methods are affected by some degree of uncertainty and measurement errors, which have to be treated with attention.

References

[1] M. E. Andreica, A. Cristescu, and S. Pirciog, “Simulation scenarios of employment on the Romanian labor market,” in *Proc. of the 2nd WSEAS Int. Conf. on Applied informatics and Computing Theory*, Stevens Point, 2011, pp. 260–264.

[2] H. Averch H and L. L. Johnson, “Behavior of the Firm Under Regulatory Constraint,” *American Economic Review*, vol. 52, no. 5, pp. 1052–1069, 1962.

[3] J. Bonbright, *Principles of Public Utility Rates*. New York: Columbia University Press, 1961.

[4] D. W. Caves, L. R. Christensen and W. E. Diewert, “Multilateral Comparisons of Output, Input, and Productivity Using Superlative Index Numbers,” *Economic Journal*, vol. 92, pp. 73–86, 1982.

[5] T. J. Coelli, D. S. Prasada Rao, C. J. O’Donnell and G. E. Battese, *An introduction to Efficiency and Productivity Analysis*. New York: Springer, 2005.

[6] G. Debreu, “The Coefficient of Resource Utilization,” *Econometrica*, vol. 19, no. 3, pp. 273–292, 1951.

[7] W. E. Diewert, “Exact and Superlative Index Numbers,” *Journal of Econometrics*, vol. 4, no. 2, pp. 115–145, 1976.

[8] R. Färe, S. Grosskopf and D. Margaritis, “Efficiency and Productivity: Malmquist and More,“ in *The Measurement of Productive Efficiency and Productivity Growth*, H. O. Fried, C. A. K. Lovell and S. S. Schmidt Ed. New York: Oxford University Press, 2008, pp. 522–622.

[9] M. Farrell. “The Measurement of Productive Efficiency,” *Journal of the Royal Statistical Society*, vol. 120, no. 3, pp. 253–282, 1957.

[10] I. Fisher, *The Making of Index Numbers.* Boston: Houghton-Mifflin, 1922.

[11] D. Hertz, “Risk Analysis in Capital Investment,” *Harvard Business Review*, vol. 42, pp. 95–106, 1964.

[12] A. Kahn, *The Economics of Regulation: Principles and Institutions*. Cambridge: MIT Press, 1988.

[13] J. Lesser, *Fundamentals of Energy Regulation*. Vienna: Public Utilities Reports, 2007.

[14] O. Machek, “Performance-based regulation of energy utilities in Central Europe and Baltic states”, in *Proc. of the Int. Conf. on Applied Business and Economics*, Piraeus, 2011, p. 77.

[15] J. Makholm, „Elusive Efficiency and the X-Factor in Incentive Regulation: the Törnqvist v. DEA/Malmquist Dispute,“ *NERA Publications*, 2007.

[16] R. Mosca, L. Cassettari and P. G. Giribone, “Reliable Control of Convergence in Monte Carlo Pricing Methods for Options based on MSPE Technique,” in *Proc. of the WSEAS Int. Conf. on Mathematics and Computers in Business and Economics*, 2012, pp. 197–206.

[17] O. Nowak and J. Hnilica, “Business Valuation under Uncertainty,” in *Proc. of the 5th WSEAS Int. Conf. on Business Administration*, Harvard, 2012, pp. 87–92.

[18] Pacific Economics Group, *X-factor Recommendations for New Zealand Electricity Distribution Price Controls*, Pacific Economics Group, 2009.

[19] S. L. Savage and J. Danziger, *The Flaw of Averages: Why We Underestimate Risk in the Face of Uncertainty*. New York: John Wiley & Sons, 2009.

[20] A. Schleifer, “A theory of yardstick competition,” *Rand Journal of Economics*, vol. 16, no. 3, pp. 319–28, 1985.

[21] L. Törnqvist, “The Bank of Finland's Consumption Price Index,” *Bank of Finland Monthly Bulletin*, vol. 10, pp. 1–8, 1936.

[22] T. Tsabadze, “Application of fuzzy sets for the determination of electricity tariffs,” in *Proc. of the 2nd WSEAS Int. Conf. on Circuits, Systems, Communications & Computers*, Stevens Point, 2011, pp. 214–219.

[23] D. Vose, *Risk Analysis. A Quantitative Guide*. *Third Edition*. New York: John Wiley & Sons, 2008.

[24] J. Hnilica, and J. Fotr, *Applied Risk Analysis in financial management and investment decision making* (*Aplikovaná analýza rizika ve finančním managementu a investičním rozhodování*). Grada Publishing, 2009.