Computational information conservation theory (CICT): a natural framework for arbitrary-scale physical and biological system simulation

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Abstract**—**Human approach to experience is based on making decision in a natural uncertain environment by incomplete knowledge. Even Stochastic vs. Combinatorically Optimized Noise generation ambiguity emphasizes the major double-bind problem in current most advanced instrumentation systems, just at the inner core of human knowledge extraction by experimentation in science. To grasp a more reliable representation of reality and to get more effective physical and biological simulation techniques, researchers and scientists need two intelligently articulated hands: both stochastic and combinatorial approaches synergistically articulated by natural coupling. The first attempt to identify basic principles to get stronger simulation solution for scientific application has been developing at Politecnico di Milano University since the end of last century. The fundamental principles on computational information conservation theory (CICT), for arbitrary-scale system modeling and simulation from basic generator and relation through discrete paths denser and denser to one another, towards a never ending "blending quantum continuum," are recalled. Four examples are presented and discussed. This paper is a relevant contribute towards arbitrary-scale physical and biological systems modeling and simulation, to show how CICT can offer stronger and more effective system modeling algorithms for more reliable simulation.

Keywords**—**CICT, information geometry, arbitrary-scale system simulation, wellbeing.

I. INTRODUCTION

HUMAN approach to experience extraction is based on making decision in a natural uncertain environment by incomplete knowledge. The amount of information an individual can acquire in an instant or in a lifetime is finite, and minuscule compared with what the milieu presents [1]. The proliferation of new sciences extends our powers of sense and thought, but their rigorous techniques and technical language hamper communication; the common field of knowledge becomes a diminishing fraction of the total store, by the use of peculiar language and symbols, due to information segregation and dissipation. Even Stochastic vs. Combinatorically Optimized Noise generation ambiguity emphasizes the major information double-bind (IDB) problem in current most advanced instrumentation systems, just at the inner core of human knowledge extraction by experimentation in science [2]. In fact, even the most sophisticated instrumentation system is completely unable to reliably discriminate so called "random noise" (RN) from any combinatorically optimized encoded message, called "deterministic noise" (DN) [3]. Statistical and applied probabilistic theory is the core of classic scientific knowledge; it is the logic of "Science 1.0"; it is the traditional instrument of risk-taking. Unfortunately, the "probabilistic veil" can be very opaque computationally, in a continuum-discrete environment, and misplaced precision leads to information dissipation and confusion [4]. To develop resilient and antifragile application and simulation, we need stronger biological and physical system correlates; in other words, we need asymptotic exact global solution panoramas combined to deep local solution computational precision with information conservation and vice-versa.

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II. THE ROOT OF THE PROBLEM

The most fundamental concept of Mathematical Analysis is that of the function. Two sorts of functions are to be distinguished. First, functions in which the independent variable x may take every possible value in a given interval; that is, the variable is continuous. These functions belong to the domain of Infinitesimal Calculus (IC). Secondly, functions in which the independent variable x takes only given values; then the variable is discontinuous or discrete. In the same way, we talk of continuous probability distribution and discrete probability distribution. Unfortunately, to discrete variable the methods of IC are NOT applicable. To deal with discrete variables, we need the Finite Differences Calculus (FDC). The origin of this Calculus may be ascribed to Taylor [5], but the real founder of the theory was Jacob Stirling [6], who solved very advanced questions, and gave useful methods. Introducing the famous Stirling numbers, he paved the way even to an important part of modern combinatorics. To grasp a more reliable representation of reality and to get stronger biological and physical system correlates, researchers and scientists need two intelligently articulated hands: both stochastic and combinatorial approaches synergistically articulated by natural coupling [2]. The former, applied to all branches of human knowledge under the "continuum hypothesis" assumption, has reached highly sophistication level, and a worldwide audience. Many "Science 1.0" researchers and scientists up to scientific journals assume it is the ultimate language of science. The latter, less developed under the "discrete hypothesis" assumption in specific scientific disciplines, has been considered in peculiar application areas only. It has been further slowly developed by a few specialists and less understood by a wider audience. Let's say we need a fresh "Science 2.0" approach. Unfortunately, the above two mathematical research areas have followed separate development paths with no articulated synergic coupling. In the past, many attempts to arrive to a continuum-discrete unified mathematical approach have been proposed, all of them with big operational compromises, and we can go back at least to the introduction of the Riemann–Stieltjes integral, published in 1894 by Dutch mathematician Thomas Joannes Stieltjes (1856–1894) [7], which unifies sums and integrals.

Every approach that uses analytical function applies a top-down (TD) point-of-view (POV) implicitly. These functions belong to the domain of IC. From a system computational perspective, all approaches that use a TD scale-free POV allow for starting from an exact global solution panorama of analytic solution families, which offers a shallow local solution computational precision to real specific needs (in other words, from global to local POV overall system information is not conserved, as misplaced precision leads to information dissipation [3],[4]). In fact, usually further analysis and validation (by probabilistic and stochastic methods) is necessary to get localized computational solution of any practical value, in real application. A local discrete solution is worked out and computationally approximated as the last step in their line of reasoning, that started from an overall continuous system approach (from continuum to discrete ≡ TD POV). Unfortunately, the IC methods are NOT applicable to discrete variable. To deal with discrete variables, we need FDC. FDC deals especially with discrete functions, but it may be applied to continuous function too. As a matter of fact, it can deal with both discrete and continuous categories conveniently. In other words, if we want to achieve an overall system information conservation approach, we have to look for a convenient bottom-up (BU) scale-relative POV (from discrete to continuum view ≡ BU POV) to start from first, and NOT the other way around! Then, a TD POV can be applied, if needed. Current human made application and system can be quite fragile to unexpected perturbation because Statistics can fool you, unfortunately. Deep epistemic limitations reside in some parts of the areas covered in risk analysis and decision making applied to real problems [4]. We need tools able to manage ontological uncertainty more effectively [8],[9]. Current computational system modelling and simulation has to face and to overcome two orders of issues at least, immediately:

1- To develop stronger, more effective and reliable correlates by correct arbitrary multi-scale (AMS) modelling approach for complex system [10];

2- To minimize the traditional limitation of current digital computational resources that are unable to capture and to manage even the full information content of a single Rational Number $\boldsymbol{Q}$ leading to information dissipation and opacity [11],[12].

III. CICT

The first attempt to identify basic principles, to synergistically articulate Computational Information Conservation Theory (CICT) by natural coupling to Geometric Theory of Information (GTI) [13] and Information Geometry (IG) [14], for scientific research and application, has been developing at "Politecnico di Milano University" since the end of last century. In 2013, the basic principles on CICT, from discrete system parameter and generator, appeared in literature and a brief introduction to CICT was given in 2014 [2].

Traditional Number Theory and modern Numeric Analysis use mono-directional interpretation (left-to-right, LTR) for $\boldsymbol{Q}$ Arithmetic single numeric group generator, so information entropy generation cannot be avoided in contemporary computational algorithm and application. On the contrary, according to CICT, it is quite simple to show information conservation and generator reversibility (right-to-left, RTL), by using basic considerations only.

To better understand the CICT fundamental relationship that tie together numeric body information of divergent and convergent monotonic power series in any base (in this case decimal, with no loss of generality) with D ending by digit 9 is given by the following correspondence equation [3]:

$$\frac{1}{D} = \sum_{k=0}^{\infty} \frac{1}{10^W} \left(\frac{\overline{D}}{10^W} \right)^k \Rightarrow \cdots \Leftarrow \quad Div\left(\frac{1}{D}\right) = \sum_{k=0}^{\infty} (D+1)^k \qquad (01)$$

where $\overline{D}$ is the additive 10^W complement of D, i.e. $\overline{D} = (10^W - D)$, W is the word representation precision length of the denominator D and "Div" means "Divergence of". Further generalizations related to D ending by digit 1 or 3 or 7 are straightforward. Furthermore, When $\overline{D} > D$ the formal power series on the left of (01) can be rescaled mod$D$, to give multiple convergence paths to $1/D$, but with different "convergence speeds." The total number of allowed convergent paths, as monotonic power series, is given by the corresponding Q_L value, at the considered accuracy level L [11]. So, increasing the level of representation accuracy, the total number of allowed convergent paths to $1/D$, as monotonic power series (as allowed conservative paths), increases accordingly and can be counted exactly, and so on, till maximum machine word length and beyond: like discrete quantum paths denser and denser to one another, towards a never ending "blending quantum continuum," by a TD perspective. Rational representations are able to capture two different type of information at the same time, modulus (usual quotient information) and associated inner or intrinsic period information which an inner phase can be computed from. So, rational information can be better thought to be isomorphic to vector information rather than to usual scalar one, at least. Furthermore, our knowledge of RFD Q_L and corresponding RFD R_L can allow reversing numeric power convergent sequence to its corresponding numeric power divergent sequence uniquely. Reversing a convergence to a divergence and vice-versa is the basic property to reach information conservation, i.e. information reversibility, as from (01). CICT results have been presented in term of classical power series to show the close relationships to classical and modern control theory approaches for causal continuous-time and discrete-time linear systems [12].

Traditional digital computational resources are unable to capture and to manage not only the full information content of a single Real Number $\boldsymbol{R}$, but even Rational Number $\boldsymbol{Q}$ is managed by information dissipation. In numeric representation of Rational Number $\boldsymbol{Q}$, rational proper quotient is represented by infinite repetition of a basic digit cycle, called “reptend” (the repeating decimal part). Let us consider fraction $1/D$, where D in $\boldsymbol{Z}$, called Egyptian fraction, with no loss of generality for common fraction. According to CICT, the first repetition of basic digit cycle of max length L corresponds to the first full scale interval where number information can be conserved completely, and is called "Representation Fundamental Domain" (RFD) [11]. Elementary number theory considerations give us worst case RFD word length $L = D – 1$ digits, if and only if 10 (by decimal base representation system, with no loss of generality) is a primitive root modulo D. Otherwise L is a factor of "D - 1." If the period of the corresponding repeating decimal to $1/D$ for prime D is equal to "D – 1", then the repeating decimal part is called "cyclic number" and D can be referred as "primitive number" or solid number (SN) [11] or "full reptend prime" elsewhere. Thus a SN is necessarily prime. It is a sufficient qualification. Conversely a prime number may not be a SN. In classical arithmetic long division algorithm (the one you learn to divide at elementary school), usual dominant result (quotient, Q) is important, and traditionally minority components (remainders, R) are always discarded. What a waste! In fact, CICT shows that Remainder R_L, at any division computation evolutive stage L, is the fixed multiplicative ratio of a formal power series associated to optimal decimal representations of $1/D$, at increasing arbitrary accuracy levels.

In 2013, CICT showed that long arithmetic division minority components (Remainders, R), for long time concealed relational knowledge to their dominant result (Quotient, Q), not only can always allow quotient regeneration from their remainder information to any arbitrary precision, but even to achieve information conservation and entropy minimization, in systems modelling and post-human cybernetic approaches [11],[15]. According to CICT optimized infocentric worldview, symmetry properties play a fundamental role and affect word level structures and properties in analogous way to phoneme level and syllable level properties which create "double articulation" in human language, at least [16],[17],[18]. Therefore, traditional $\boldsymbol{Q}$ Arithmetic can be regarded as a highly sophisticated open logic, powerful and flexible optimized "OpeRational" (OR) LTR and RTL formal numeric language of languages, with self-defining consistent words and rules, starting from self-defined elementary generator and relation, based on recursively self-defining atom [11]. For instance, at any LTR computation stage, with remainder knowledge only, it is always possible to regenerate exact quotient and new remainder information at any arbitrary accuracy, with full information conservation. It is like to process tail information to regenerate the associated body information. Thanks to the above properties, the division algorithm can become free from trial and error like in Finite Segment P-adic representation systems, but with no usually associated coding burden [11]. The rich operative scenario offered by combinatorial modular group theory is full of articulated solutions to information processing problems. One of the earliest presentations of a group by generator and relation was given by the Irish mathematician William Rowan Hamilton in 1856, in his Icosian Calculus, a presentation of the icosahedral group [19],[20]. Every group has a presentation, and in fact many different presentations. A presentation is often the most compact way of describing the structure of the group. In abstract algebra, the "fundamental theorem of cyclic groups" states that every subgroup of a cyclic group G is cyclic. Moreover, the order k of any subgroup of a cyclic group G of order n is a divisor of n, and for each positive divisor k of n, the group G has exactly one subgroup of order k. This is just the first step to start an intriguing voyage from the concept of "presentation of a group" to the concept of "representation theory" for combinatorial modular group theory [21].

Furthermore, CICT sees rational geometric series as simple recursion sequences in a wider recursive operative framework where all algebraic recursion sequences of any countable higher order include all the lower order ones and they can be optimally mapped to rational number system $\boldsymbol{Q}$ OR representations and generating functions. For instance, arithmetic progression and Lucas sequences are recursion sequences of the second order. Lucas sequences are certain integer sequences that satisfy Lucas recurrence relation defined by polynomials $Un(P,Q)$ and $Vn(P,Q)$, where Un, Vn are specific polynomials and P, Q are fixed integer coefficients. Any other sequence satisfying this recurrence relation can be represented as a linear combination of the Lucas sequences $Un(P,Q)$ and $Vn(P,Q)$. Famous examples of Lucas sequences include the Fibonacci numbers, Mersenne numbers, Pell numbers, Lucas numbers, Jacobsthal numbers, and a superset of Fermat numbers. CICT is able to fold any recursion sequence of the first order into one digit number D_1, any recursion sequence of second order into a two digit number D_2, any recursion sequence of the third order into a three digit number D_3 and so on to higher orders. Then, you can interpret their asymptotic convergence ratios as increasing accuracy approximations to related asymptotic roots from corresponding first, second, third, ..., *n-th* order equations respectively [8].

Thanks to this brand new knowledge and following this line of generative thinking, it is possible immediately to realize that traditional $\boldsymbol{Q}$ Arithmetic can be even interpreted, by new eyes, as a highly sophisticated open logic, powerful and flexible LTR and RTL evolutionary, generative, formal numeric language of languages, with self-defining consistent numeric words and rules, starting from elementary generator and relation (you get your specific formal numeric language by just simply choosing your most convenient numeric base to polynomially structure your information). American linguist, philosopher, cognitive scientist, logician, Avram Noam Chomsky's (1928-) Theory of Syntax came after his criticism of probabilistic associative models of word order in sentences by Markov process approaches, in 1957 [22],[23]. As a matter of fact, since 1951, the inadequacy of probabilistic LTR models (Markov process) had already been noticed by American psychologist and behaviorist Karl Spencer Lashley (1890-1958), who anticipated Chomsky's arguments [24], by observing that probabilities between adjacent words in a sentence have little relation to grammaticality of the string. Ambiguity too provides a strong indication that sentences carry a structure [24]. Eventually, CICT defines an arbitrary-scaling discrete Riemannian manifold uniquely, under hyperbolic geometry (HG) metric, that, for arbitrary finite point accuracy level L going to infinity under the criterion of scale relativity invariance, is isomorphic (even better, homeomorphic) to classic IG Riemannian manifold (exact solution theoretically).

IV. Operative Considerations

The classical experimental noise discrimination problem is still faced by the single domain channel transfer function concept (Shannon's noisy channel, Fig.1 top diagram), starting from classic Shannon's information theory concept [24], and then applying traditional perturbation computational model under either additive or multiplicative perturbation hypothesis [25]. Our main idea is that an assessment of system fragility (and control of such fragility) is more useful, and more reliable, than probability risk management and data-based methods of risk detection. Main attention focus should not be to attempt to predict black swan events, but to build system robustness against negative ones that occur and be able to exploit positive ones. In the past five decades, trend in Systems Theory, in specialized research area, has slowly shifted from classic single domain information channel transfer function approach (Fig.1 top diagram) to the more structured ODR Functional Sub-domain Transfer Function approach (by Observation, Description and Representation Functional Blocks, Fig.1 middle diagram) [12]. Shortly, the ODR approach allows for fitting theoretical system design consideration to practical implementation needs much better (according to information "Input, Processing, Output" paradigm, respectively) than classic single domain channel approach, as shown by Fig.1 middle diagram. As a matter of fact, by iterating full process over repeated scale-related controlled "Observations," it is possible to improve the accuracy level of any associated "Description," validated by a related and endorsed scale related "Representation," and therefore to better the overall system knowledge extraction process under test: human beings call this process "learning by experience." Thanks to the ODR approach, a deeper awareness about information acquisition and generation limitations by classical experimental observation process has been grown. In fact, usual elementary arithmetic long division remainder sequences can be even interpreted as combinatorically optimized coding sequences for hyperbolic geometric structures, as points on a discrete Riemannian manifold, under HG metric, indistinguishable from traditional random noise sources by classical Shannon entropy computation, and current, most advanced instrumentation system [3]. Specifically, CICT showed that classic Shannon entropy is completely unable to reliably discriminate so called computational "random noise" (RN) from any combinatorically optimized encoded message by OECS, called "deterministic noise" (DN) in [3]. Paradoxically if you don't know the generating process for the folded information, you can't tell the difference between an information-rich message and a random jumble of letters. This is "the information double-bind" (IDB) problem in contemporary classic information and algorithmic theory [3]. Therefore, one of the first practical result has been to realize that classical experimental observation process, even in highly ideal operative controlled condition, like the one achieved in current, most sophisticated and advanced experimental laboratories like CERN [27], can capture just a small fraction only of overall ideally available information from unique experiment. The remaining part is lost and inevitably dispersed through environment into something we call "background noise" or "random noise" usually, in every scientific experimental endeavor. The amount of information an individual can acquire in an instant or in a lifetime is finite, and minuscule compared with what the milieu presents; many questions are too complex to describe, let alone solve, in a practicable length of time [1]. The same is true for all other cascading functional blocks in the ODR transmission channel from source to destination, if careful information conservation countermeasure is not provided at each step. Traditionally, the horizons of accumulating ignorance are expanding faster than any person can keep up with. The proliferation of new sciences extends our powers of sense and thought, but their rigorous techniques and technical language hamper communication; the common field of knowledge becomes a diminishing fraction of the total store. By biomedical cybernetics point of view, to get closer to real computational information conservation, ODR Functional Sub-domain Transfer Function block diagram (Fig.1 middle diagram) must be completed by a corresponding irreducible complementary "ODR Information Channel Co-domain Diagram" to get reliable strategic overall information functional closure (Fig.1 bottom diagram) [12]. Starting at the Observation step, interaction between an Experimental Field with a scale related Action Domain is established and discrete data are captured. Observation is properly described as a fact-finding rather than a fact-collecting procedure, because the idea of finding includes both selection by controlled perturbation and efficient structured collection. The quality of Observation does then depend on the degree of completeness by which experimental folded information is allowed to be efficiently captured from our experimental field into our subjective structured Action Domain and properly formatted, according to observation experience and shared rules (System Input Transformation), to be passed to next processing block. Then the second step, Description, can format and formalize folded subjective observation into an unfolded systemic minimal insured scale precision and/or accuracy Representation Domain, to be shared by the majority of interacting entities which use the same formal language to communicate (Overall System State), to be passed to the last step. Finally, the quality of the Representation stage does depend on the degree of scale related completeness by which unfolded information is allowed to be focused and re-folded to be efficiently presented to specific shared, human knowledge (System Output Transformation). Then a validation process can start and an endorsement can be assigned eventually, according to convenient Representation support quality level for scientific knowledge synthesis, cultural analytics, information/perceptual aesthetics, etc. [11].

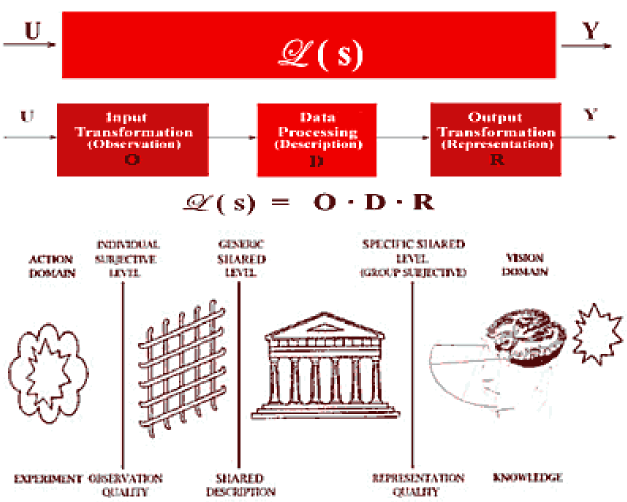

Fig. 1 Top diagram: Traditional Single Domain Channel (SDC) Transfer Function. Middle diagram: Decomposition of SDC Transfer Function into more structured ODR Functional Sub-domain Transfer Function (Observation, Description and Representation Functional Blocks). Bottom diagram: ODR Information Channel Co-domain Diagram for System Information Conservation.

The ODR approach has contributed to create deeper awareness about traditional information acquisition, formalization and reproduction process limitations, constrained by classical experimental observation scale finiteness and new multimedia data acquisition and reproduction implementation. As a matter of fact, traditional rational number system ***Q*** properties allow to generate an irreducible co-domain for every computational operative domain used. Then, all computational information usually lost by using the classic information approach, based on the traditional noise-affected data model stochastic representation, can be captured and fully recovered to arbitrary precision by a corresponding evolutive irreducible complementary co-domain, step-by-step. Co-domain information can be used to correct any computed result, achieving computational information conservation (theoretically, virtually noise-free data), according to CICT Infocentric Worldview [3]. A further detailed description of the diagrams of Fig.1 far exceeds the scope of present discussion and the interested reader is referred to [28]. Next section will show four computational examples for our CICT approach.

V. Computational Examples

CICT is a Natural Framework for arbitrary-scale Computer Science and Systems Biology Modeling in the current landscape of modern Geometric Science of Information (GSI), Geometric Algebra (GA) and Geometric Calculus (GC) [29-32]. We have selected four examples to offer a quick panorama of a few CICT fundamental properties to take advantage from scale related self-similarity modeling (Example No.1) and from scale related coherent polynomial closure of power series for information conservation (Example No.2). Example No.3 shows how leading zeros in positional notation representation system for CICT ***Q*** Arithmetic do count effectively, and can even model the quantum-classical system transition quite efficiently. Finally, Example No.4 shows that any natural number *D* in ***N*** has associated a specific, non-arbitrary external or exterior phase relationship to take into account full information conservation. With no scale related coherent inner phase information, we get system decoherence, entropy generation, information dissipation and algorithmic quantum incomputability on real macroscopic machines.

A. Example No.1

Looking back to geometric series one can see a remarkable correspondence to the self-similarity concept. If we formally scale the following series *S*:

$$S=\sum_{k=0}^{\infty} q^k = 1+q^1+q^2+q^3+\dots \qquad (02)$$

with the factor q, we obtain:

$$qS = q\sum_{k=0}^{\infty} q^k = q^1+q^2+q^3+q^4\dots, \qquad (03)$$

therefore

$$S=\sum_{k=0}^{\infty} q^k = 1+q\sum_{k=0}^{\infty} q^k \quad . \qquad (04)$$

This is the well-known "self-similarity" property of geometric series. The value of the sum *S* is $\bar{0}1.\bar{0}$ plus the scaled down version of the whole series. Naturally, self similarity only holds for the limit, but not for any finite stage. For example, let us suppose finite stage $k = 2$:

$S_2 = 1+q+q^2$, then:

$$S_3 = 1+qS_2 = 1+q+q^2+q^3 \neq S_2 \quad . \qquad (05)$$

As a simple example, for the sake of simplicity, let us use decimal base representation system, with no loss of generality, and let us consider $q = 3/10$ as LTR elementary generator. Then, we obtain the following convergent series:

$$S = \sum_{k=0}^{\infty} (3/10)^k = 1 + (3/10^1) + (9/10^2) + (27/10^3) + \ldots = = 10/7 \quad (06)$$

and

$$(3/10)\, S = \sum_{k=0}^{\infty} (3/10)^k = (3/10^1) + (9/10^2) + (27/10^3) + \ldots , \quad (07)$$

therefore

$$\sum_{k=0}^{\infty} (3/10)^k = 1 + (3/10) \sum_{k=0}^{\infty} (3/10)^k$$

as stated previously.

For finite stage, as we already stated, let us suppose $k = 2$, then:

$$S_2 = 1 + 3/10 + 9/100 = 139/100$$

and

$$1 + (3/10) S_2 = 1 + (3/10^1) + (9/10^2) + (27/10^3) = = 1417/1000 \neq S_2 \ . \quad (08)$$

Apparently by finite step we lost self-similarity, the fundamental property of geometric series, and this fact may seem, at first sight, a strong limitation to proceed further. But, self-similarity is still there, just a little less manifest. We can turn an apparent limitation into a striking computational advantage. In fact, it is possible to conceive an evolutive self-similar arithmetic correspondence (called complementary series or co-series) to original geometric series, step by step, which can act as a continuous connection from finite geometric power increment to its asymptotic limit to conserve characteristic computational information in a coherent way. To compute the corresponding LTR (Left To Right) evolutive complementary arithmetic co-series (additive complement series), we introduce the fundamental concept of "coherent correspondence". Therefore, given any single term of original geometric series S_k, as s_k with an operational representation N_k/D_k , its "coherent correspondent term" m_c for correspondent complementary co-series M_1 is given, in this case, by $(D_k - N_k)/(D_k)^2$. So, the correspondent LTR complementary co-series M_1 of our example is given by:

$$\begin{aligned} M_1 &= \sum_{c=0}^{\infty} m^c = m_0 + m_1 + m_2 + m_3 + \ldots = \\ &= 0 + (7/10^2) + (91/10^4) + (973/10^6) + \ldots = \\ &= 7\{(0/10^0) + (1/10^2) + (13/10^4) + (139/10^6) + \ldots\} = \\ &= 7\{S_0/(10^2) + S_1/(10^3) + S_2/(10^4) + \ldots\} = \\ &= 7\{10/873\} = 7\{10/(9*97)\} \quad . \end{aligned} \quad (09)$$

As a matter of fact, M_1 is just the first co-series of a countable family M_r of complementary co-series to original series S, at different accuracy level r, given by:

$$M_r = D \sum_{c=0}^{\infty} S_c / (10)^{c+r+1} \quad r = 1, 2, \ldots, \infty \quad (10)$$

where D is the reduced denominator of the limit of original series S and S_c is its finite stage sum at stage c. So, in other words, each M_r is an irreducible co-domain, at different accuracy level r, for the original domain S. It is immediate to verify the following co-series limiting values:

$$\begin{aligned} M_2 &= 7\{10^2/98703\} = 7\{10^2/(99*997)\} \ , \\ M_3 &= 7\{10^3/9987003\} = 7\{10^3(999*9997)\} \ , \\ &\ldots \\ M_7 &= 7\{10^7/999999870000003\} \\ &= 7\{10^7/(9999999*99999997)\} \ , \end{aligned} \quad (11)$$

and so on. So, in other words, each M_r represents an irreducible co-domain, at accuracy level r, for the original domain S, with $q = 3/10$ as LTR elementary generator, in this case. Then, co-domain multiscale evolutive structured information, synthesized by its limiting value, can be used for deterministic noise source coherent tuning or checking for the presence of such specific generator in system "background noise." In fact, their numeric limiting values, by elementary arithmetic long division algorithm, supply us with cyclic remainder sequences perfectly tuned to deterministic source generators. By this kind of operational flexibility, a machine can generate autonomously, either on-the-fly or in advance stored in a-priori knowledge-base, combinatorically optimized sequences to check for the presence of suspect "deterministic noise sources" in its probing field and then acting accordingly to obtain a virtually homogeneous and uniform machine experimental reference domain. Following this line of thought, it is possible to overcome the dreadful ambiguity and limitations of the traditional Shannon entropy concept [25]. Our results are presented in term of classical power series to show the close relationships to classical and modern control theory approaches for causal continuous-time and discrete-time linear systems. Usually, the continuous Laplace transform is in Cartesian coordinates where the x-axis is the Real axis [33] and the discrete Z-transform is in circular coordinates, where the Rho-axis is mapping the Real axis [34].

B. *Example No.2*

We use an arbitrary-scalable system top-down approach, i.e. from overall system to system components, an so on, arriving to single block, single digit computational information conservation. In this case, we start with Natural numbers as generators, and their geometric powers, to compute their coherent functional closures, by using decimal system operative representation ($r = 10$), with no loss of generality. To get a coherent functional closure our rule is simple. One digit word number to the second power gives two digit number word, to the third power gives a three digit number word, to the fourth power gives four digit number word, and so on. Leading zeroes do count, so you have to fill in all word digits. We start with Natural number $D = 3$ as a generator, and $W = 1$, where W is the word representation precision length of number D and k its power exponent. We have:

$$
\begin{array}{ll}
3^1 = 3 & 10^1 - 3^1 = \overline{D}\cdot(1)_1 = 7 \\
3^2 = 09 & 10^2 - 3^2 = \overline{D}\cdot(13)_2 = 91 \\
3^3 = 027 & 10^3 - 3^3 = \overline{D}\cdot(139)_3 = 973 \\
\bullet\bullet\bullet & \bullet\bullet\bullet \\
3^k & 10^k - 3^k = \overline{D}\cdot\left(3^0\cdot 10^{k-1} + 3^1\cdot 10^{k-2} + \ldots + 3^{k-2}\cdot 10^1 + 3^{k-1}\cdot 10^0\right)_k \equiv PC,
\end{array} \quad (12)
$$

where $\overline{D}$ is the additive 10^W complement of D, i.e. $\overline{D} = (10^W - D)$. On the left column we have the powers of 3 and on the right side their corresponding coherent functional closures. It is simple to see that for k going to infinity even the asymptotic expression in round bracket $(\ldots)_k \equiv PC$ from (12) becomes an infinite polynomial and therefore an incomputable expression. Nevertheless it has quite a definite and unique evolutive polynomial structure, easily to be computed exactly to any arbitrary precision by CICT [11]:

$$
\lim_{k\to\infty}(\ldots)_k = \left(\frac{10^\infty}{\overline{D}}\right)_\infty = \left(\frac{10^\infty}{7}\right)_\infty = \left(\overline{142857}\right)_\infty . \quad (13)
$$

In fact, from (13), we see that our final result is an infinite string of digits, impossible to handle in full by any traditional macroscopic real computational machine. Furthermore, the overall asymptotic limit of the coherent functional closure is:

$$
\overline{D}\cdot\lim_{k\to\infty}(\ldots)_k = \overline{D}\cdot\left(\frac{10^\infty}{\overline{D}}\right) = 7\cdot\left(\frac{10^\infty}{7}\right) = 7\cdot\left(\overline{142857}\right)_\infty = \overline{999999} . \quad (14)
$$

Now, it is interesting to appreciate the difference of the asymptotic numeric value given by (14), and its corresponding value obtained by traditional symbolic computation (i.e. 10^∞) by using the IC scale-free POV. It is an infinitesimal difference, but this time we know its evolutive growing law precisely, from the discrete to the continuum POV, exactly. It is straightforward to arrive to the general relationship for any power k of any Natural number D represented by decimal word precision length W, as:

$$
10^{W\cdot k} - D^k = \overline{D}\left(D^0\cdot 10^{W\cdot(k-1)} + D^1\cdot 10^{W\cdot(k-2)} + \ldots + D^{k-2}\cdot 10^{W\cdot 1} + D^{k-1}\cdot 10^{W\cdot 0}\right)_k , \quad (15)
$$

where $\overline{D}$ is the usual additive 10^W complement of D, i.e. $\overline{D} = (10^W - D)$. As a matter of fact, CICT rational number system ***Q*** numeric properties allow to generate an irreducible co-domain for every computational operative domain used. Then, all computational information usually lost by using classic information approach, based on the traditional noise-affected data stochastic model only, can be captured and fully recovered to arbitrary precision by a corresponding complementary co-domain, step-by-step, to obtain a Resilient ODR system (RODR, for short), according to CICT Infocentric Worldview.[3]

C. Example No.3

Usual knowledge on significant figures of a number teaches that any 0 digit that comes before the first nonzero digit (leading zeros) can be omitted in a number string in positional notation representation system [35]. When leading zeros occupy the most significant digits of an integer, they could be left blank or omitted for the same numeric value [36]. Therefore, the usual decimal notation of integers does not use leading zeros except for the zero itself, which would be denoted as an empty string otherwise [37]. However, in decimal fractions between 0.0 and 1.0, the leading zeros digits between the decimal point and the first nonzero digit are necessary for conveying the magnitude of a number and cannot be omitted [35]. Let us introduce a convenient LTR symbolic compression operator as SCO ≡ <M│DS>, where DS is a finite digit string of length L and M is the number of times DS is repeated to get our unfolded digit string in full (e.g. (4│1) ≡ 1111 or (2│123) ≡ 123123). Usual symbolic string operations can be applied to SCO. Then, we can write usual rational number OpeRational Representation (OR) corresponding to their Symbolic Representation (SR) as [11]:

$$
\begin{aligned}
Q1 &= \frac{1}{D1} = \frac{1}{9} = 0.11111111111\ldots \\
Q2 &= \frac{1}{D2} = \frac{1}{99} = 0.01010101010101010101\ldots \\
Q3 &= \frac{1}{D3} = \frac{1}{999} = 0.001001001001001001001001001001\ldots
\end{aligned} \quad (16)
$$

in a more compact RFD Q_L format as:

$$
\begin{aligned}
Q1 &= \frac{1}{D1} = \frac{1}{9} \equiv 0.<\infty \mathrm{I} 1> \\
Q2 &= \frac{1}{D2} = \frac{1}{99} \equiv 0.<\infty \mathrm{I} 01> \\
Q3 &= \frac{1}{D3} = \frac{1}{999} \equiv 0.<\infty \mathrm{I} 001> .
\end{aligned} \quad (17)
$$

In the same way, we can write for Solid Number (SN) $D4 = 7$ [11]:

$$
\begin{aligned}
Q4 &= \frac{1}{D4} = \frac{1}{7} = 0.142857142857142857142857142857\ldots \quad \text{as} \\
Q4 &= \frac{1}{D4} = \frac{1}{7} \equiv 0.<\infty \mathrm{I} 142857> .
\end{aligned} \quad (18)
$$

On the other hand, we have:

$$
\begin{aligned}
Q5 &= \frac{1}{D5} = \frac{1}{142857} = 0.000007000007000007000007000007\ldots \quad \text{written as} \\
Q5 &= \frac{1}{D5} = \frac{1}{142857} \equiv 0.<\infty \mathrm{I} 000007> \quad \text{either (first compressed form)} \\
Q5 &= \frac{1}{D5} = \frac{1}{142857} \equiv 0.<\infty \mathrm{I}(<5\mathrm{I}0><1\mathrm{I}7>)> \quad \text{or (second compressed form)} .
\end{aligned} \quad (19)
$$

Now, we can realize that $Q4$ RFD is related by $Q5$ RFD and vice-versa by periodic scale relativity (precision length) $L = 6$. So, to conserve the full information content of rational correspondence between $Q4$ and $Q5$, we realize that we have to take into account not only the usual $Q4$ and $Q5$ modulus information, but even their related periodic precision length information $L = 6$ (external world representation phase). As far as it concerns *D*5 it comes almost automatically from previous example (see (19)), but the same it is not true for *D*4 (see (18)) because we wrote digit 7 only as denominator, without its five leading zeros, according to traditional knowledge on significant figures of a number. In that way, we lose the rational correspondence intrinsic period information (coherence) which an inner relative phase for each RTL string generator can be computed from (i.e. from their optimized exponential cyclic sequences (OECS) of R_L [11]). With no scale related coherent inner phase information, we get system decoherence, entropy generation and information dissipation. In fact, misplaced precision leads to information opacity, fuzziness, irreversibility, chaos, complexity and confusion. Therefore, rational information can be better thought to be isomorphic to vector information rather than to usual scalar one, at least. Now, from (19) second compressed form, it is immediate to verify the following phase relations:

$$\begin{aligned}
QQ1 &= \frac{1}{DD1} = \frac{1}{142857} \equiv 0.<\infty \mathrm{I}(<5\mathrm{I}0><1\mathrm{I}7>)> \\
QQ2 &= \frac{1}{DD2} = \frac{1}{142857142857} \equiv 0.<\infty \mathrm{I}(<11\mathrm{I}0><1\mathrm{I}7>)> \\
QQ3 &= \frac{1}{DD3} = \frac{1}{142857142857142857} \equiv 0.<\infty \mathrm{I}(<17\mathrm{I}0><1\mathrm{I}7>)> \\
&\bullet\bullet\bullet \quad \bullet\bullet\bullet \\
QQN &= \frac{1}{DDN} = \frac{1}{<\mathrm{N}\,\mathrm{I}142857>} \equiv 0.<\infty \mathrm{I}(<5+6(\mathrm{N}-1)\mathrm{I}0><1\mathrm{I}7>)> \quad \text{for } \mathrm{N}=1,2,3,\dots,\infty
\end{aligned} \tag{20}$$

Therefore, we can write the following relation:

$$CQ4 = \frac{1}{CD4} \equiv \frac{1}{<\infty \mathrm{I}(<\infty \mathrm{I}0><1\mathrm{I}7>)>} \equiv 0.<\infty \mathrm{I}142857> \tag{21}$$

According to our SCO approach, the coherent representation *CD*4 emerges out of an LTR infinity of symbolic structured infinite length sequences as in (21). By this point of view, traditional natural numbers, according to human common knowledge, appear as just the rightmost approximated part of those sequences. So, *CD*4 in (21) is the correct coherent relation representation of traditional scalar modulus *D*4 in (18) as denominator, while scalar modulus *D*4 in (17) can be interpreted as the decoherenced relation representation of *CD*4 denominator in (21). Leading zeros in positional notation representation system for CICT ***Q*** Arithmetic do count effectively, and can model the quantum-classical system transition quite efficiently. Finally, our knowledge of RFD Q_L and corresponding RFD R_L can allow reversing LTR numeric power convergent sequence to its corresponding RTL numeric power divergent sequence uniquely [11]. Reversing a convergent sequence into a divergent one and vice-versa is the fundamental property to reach information conservation, i.e. information reversibility. Eventually, OECS have strong connection even to classic DFT algorithmic structure for discrete data, Number-Theoretic Transform (NTT), Laplace and Mellin Transforms [3].

D. Example No.4

For SN = 7.0 = *D* [11], to conserve the full information content of rational correspondence at higher level, we realize that we have to take into account not only the usual modulus information, but even the related external or extrinsic RFD periodic precision length information W = 6 (numeric period or external phase representation) in this case (i.e. D5 ≡ 000007 as base RFD, and yes for CICT leading zeros do count [31]!). We can use Euler's formula to establish the usual fundamental relationship between trigonometric functions and the complex exponential function:

$$e^{ix} = \cos x + i \sin x \,, \tag{22}$$

where *e* is the base of the natural logarithm and $i = \sqrt{-1}$. It establishes the fundamental relationship between the trigonometric functions and the complex exponential function. We obtain:

$$CQ1 = \frac{1}{7} e^{i\frac{\pi(2n+1)}{3}} = \frac{1}{7}\left(\cos\left(\frac{2\pi(n+1)}{6}\right) + i \sin\left(\frac{2\pi(n+1)}{6}\right)\right) \tag{23}$$

and

$$CD1 = \frac{1}{CQ1} = 7e^{-i\frac{\pi(2n+1)}{3}} = 7\left(\cos\left(-\frac{\pi(2n-1)}{3}\right) + i \sin\left(-\frac{\pi(2n-1)}{3}\right)\right) = 7\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) \ \text{p.v.} \tag{24}$$

for n = 1, 2, 3, …in ***N***, where p.v. means principal value. CICT shows that any natural number *D* in ***N*** has associated a specific, non-arbitrary external or exterior phase relationship [3] that we must take into account to full conserve its information content by computation in direct Euclidean space [2]. The interested reader will have already guessed the relationship of our result to de Moivre number or root of unity (i.e. any complex number that gives 1.0 when raised to some integer power of *n*. In this way, we can exploit Rational numbers ***Q*** full information content to get effective and stronger solutions to current system modelling problems. We have shown how to unfold the full information content hardwired into Rational OR representation [11] (nano-microscale discrete representation) and to relate it to an assumed continuum framework (meso-macroscale) with no information dissipation. CICT phased generator (PG) approach [38] combined to GA and GC unified mathematical language can offer an effective and convenient "Science 2.0" universal framework, by considering information not only on the statistical manifold of model states but also on the combinatorial manifold of low-level discrete, phased generators and empirical measures of noise sources, related to experimental high-level overall perturbation. A synergic coupling between GA, GC and CICT [38-40] offers stronger arbitrary-scale computational solutions which unify, simplify, and generalize many areas of mathematics that involve geometric ideas.

Scale related, coherent precision correspondence leads to transparency, ordering, reversibility, kosmos, simplicity, clarity, and, as you saw from previous discussion, to algorithmic quantum incomputability on real macroscopic machines [9]. CICT fundamental relation (see (1)) allows to focus our attention on combinatorically optimized number pattern generated by LTR or RTL phased generators and by convergent or divergent power series with no further arbitrary constraints on elementary generator and relation. Thanks to subgroup interplay and intrinsic phase specification through polycyclic relations in each SN remainder sequence, word inner generator combinatorial structure can be arranged for "pairing" and "fixed point" properties for digit group with the same word length [3]. As a matter of fact, those properties ("pairing" and "fixed point") are just the operational manifestation of universal categorical irreducible dichotomy hard-wired into integer digit and digit group themselves (i.e. "evenness" and "oddness") and to higher level structures (i.e. "correspondence" and "incidence"). Actually, since space is limited, the discussion here will not be extended further to the subgroup interplay of the family group and polycyclic groups. We refer the interested reader to good general references on polycyclic groups [38],[39].

VI. Conclusion

The final result is CICT new awareness of a hyperbolic framework of coded heterogeneous hyperbolic structures, underlying the familiar Euclidean surface representation system [40]. CICT emerged from the study of the geometrical structure of a discrete manifold of ordered hyperbolic substructures, coded by formal power series, under the criterion of evolutive structural invariance at arbitrary precision. It defines an arbitrary-scaling discrete Riemannian manifold uniquely, under hyperbolic geometry (HG) metric, that, for arbitrary finite point accuracy level L going to infinity under scale relativity invariance, is isomorphic (even better, homeomorphic) to classic IG Riemannian manifold (exact solution theoretically). In other words, HG can describe a projective relativistic geometry directly hardwired into elementary arithmetic long division remainder sequences, offering many competitive computational advantages over traditional Euclidean approach. It turns out that, while free generator exponentially growing sequences can be divergent or convergent, their closures can be defined in terms of polynomials (see Example No.2). Furthermore, combinatorically OECS have strong connection even to classic DFT algorithmic structure for discrete data, Number-Theoretic Transform (NTT), Laplace and Mellin Transforms [3]. In this way, even simple scalar moduli can emerge out from sequences of phased generators.

CICT can help to reach a unified vision to many current biophysics and physics problems and to find their optimized solutions quite easily. Expected impacts are multifarious and quite articulated at different system scale level. One of the first practical result was that usual elementary arithmetic long division remainder sequences can be even interpreted as combinatorically optimized coding sequences for hyperbolic geometric structures, as point on a discrete Riemannian manifold, under HG metric, indistinguishable from traditional random noise sources by classical Shannon entropy, and contemporary most advanced instrumentation systems. Specifically, CICT showed that classical Shannon entropy computation is completely unable to reliably discriminate so called computational "random noise" from any combinatorically optimized encoded message by OECS, called "deterministic noise" (DN) in [3]. As a matter of fact, for any free generator, CICT can provide us with an "ecoco-domain" multiscale evolutive structured family of sequences that can be used for checking for the presence of a specific generator in laboratory or system "background noise" [3]. Following CICT approach, it is possible even to extend the classic Shannon entropy concept to arrive to a stronger and specific "Coherent Shannon entropy" (CSE) approach. Second result was to realize that classical experimental observation process, even in highly ideal operative controlled condition, like the one achieved in contemporary most sophisticated and advanced experimental laboratories like CERN, can capture just a small fraction only, with misplaced precision, of overall ideally available information from unique experiment. The remaining part is lost and inevitably added to something we call "background noise" or "random noise" usually, in every scientific experimental endeavor. CICT can help us to develop strategies to gather much more reliable experimental information from single experiment and to conserve overall system information. In this way, coherent representation precision leads to information conservation and clarity. The latest CICT claim has been that the "external" world real system physical manifestation properties and related human perception are HG scale related representation based, while Euclidean approximated locally.

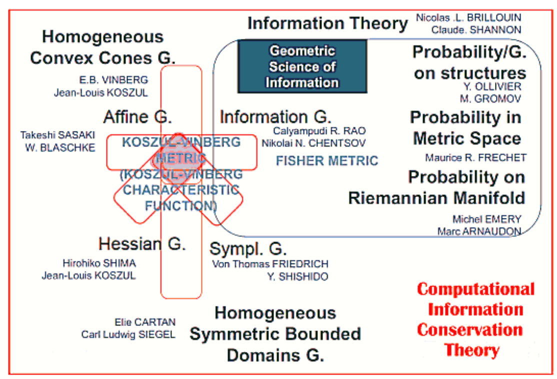

Fig. 2 Computational Information Conservation Theory (CICT) is a Natural Framework for Arbitrary Scale Computer Science and Systems Biology Modeling in the current landscape of modern Geometric Science of Information (GSI), Geometric Algebra (GA) and Geometric Calculus (GC).

Furthermore, the fundamental play of human information observation interaction with an external world representation is related by the different manifestation and representation properties of a unique fundamental computational information structuring principle: the Kelvin Transform (KT) [3]. KT is key to efficient scale related information representation, structuring "external space" information to an "internal representation" and vice-versa by inversive geometry.

More generally, CICT is a natural framework for arbitrary-scale computer science and systems biology modeling in the current landscape of modern GSI, GA and GC as depicted in Fig.2. Specifically, high reliability organization (HRO), mission critical project (MCP) system, very low technological risk (VLTR) and crisis management (CM) system will be highly benefitted mostly by these new techniques. This paper is a relevant contribute towards arbitrary-scale computer science and systems biology modeling, to show how computational information conservation can offer stronger and more effective system modeling algorithms for more reliable simulation to support and enhance even system creativity [44].

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