Default contagion in financial networks

Chiara Benazzoli and Luca Di Persio

Abstract—**The preset work aims at giving insights about how the theory behind the study of complex networks can be profitably used to analyse the increasing complexity characterizing a wide number of current financial frameworks. In particular we exploit some well known approaches developed within the setting of the *graph theory*, such as, e.g., the Erdős and Rény model, and the Barabási-Albert model, as well as producing an analysis based on the evolving network theory. Numerical simulations are performed to study the spread of financial peak events, as in the case of the default of a single bank belonging to a net of interconnected monetary institutions, showing how the knowledge about the underlying graph theory can be effectively used to withstand a financial default contagion.**

Keywords—**Financial networks, default spread, graph theory, random graphs**

I. INTRODUCTION

Modern monetary systems are characterized by a high grade of interconnections between players that share financial products of heterogeneous type with high frequency also exploiting fast communication channels. This leads to graph-structures with a rather complicated topologies where each node can be linked to a wide number of other ones by mean of hedges with weights of quite different magnitude scale. The latter implies that isolated peak events, that may occur at a certain node in a given financial network, could spread their effects along the whole structure. As an example, the failure of a key bank may cause a big backlash to the other institutions which are linked to it, hence leading to a cascading failures and global financial crisis. Recently, concrete examples of such a cascading phenomena have been observed during the worldwide financial crisis of 2007-2008, e.g., concerning the high negative impact caused by the Lehman Brothers bankruptcy.

The present work is subdivided into the three following sections: in Section II we show the reasons why complex network theory can be usefully and naturally applied in order to study financial nets of interconnected *agents*. Moreover, following [1] and [2], some considerations about systemic risk and risk contagion in banking network are discussed; in Section III, the major mathematical results related to the Erdős-Rény model, the Barabási-Albert model, and those concerning the evolving network theory, are collected, mainly following [3] and [4]; in Section IV, we generalise the approaches proposed in [5] and [6], where the authors analysed a *static* interbank network model to describe contagion effects, by allowing to new banks to enter in the network, establishing links with the ones already present in the system. The latter dynamic will be shown to produce a reduction in the number of the defaulting banks.

C. Benazzoli is with the Department of Mathematics, University of Trento, Via Sommarive, 14 - 38123 Povo, Italy (e-mail: chiara.benazzoli@unitn.it).

L. Di Persio is with the Department of Computer Science, University of Verona, Strada le Grazie, 15 - 37134 Verona, Italy, (e-mail: luca.dipersio@univr.it).

II. BANKING NETWORK AND SYSTEMIC RISK

The increasing complexity of the banking systems has suggested the use of approaches belonging to the graph theory in general, with particular respect to the possibility of exploiting results coming from the theory of random graphs, see, e.g., [7, Chapt.5], and references therein.

The main idea is to represent a banking network by associating to each financial institution composing it, e.g., insurance companies, banks, pension funds, etc., a vertex of a graph whose connecting edges represent the liability or the financial exposure financial player with respect to the others. It follows that latter links can potentially become the tie through which the distress of an individual component of the net propagate to the others ones.

Our main goal is to understand how a bank's failure may damage the whole financial system or, in other words, to analyse how the banking network reacts to the default of one participant. The impact that the failure of a financial institution has with respect to the stability of the whole market of which it is a component is the so called *systemic risk*. An unambiguous definition of the systemic risk is rather difficult to give, since it depends on multiple factors which vary in time as, e.g., the reflection of the market movements and cycle trends caused by new regulatory constraints. Moreover the systemic risk also varies in space, according to specific regional, national or even international, economic policies. Last but not least systemic risk depends on a wide number of heterogeneous type of financial agents which are interconnected by different types of links it can be transmitted through. However, some key factors are acknowledged as measures of how an institution represents a possible source, or transmission vehicle, of systemic risk. First of all, the size of the entity: the bigger is its the size of the particular financial institution we are interested in, the higher is the impact of its possible default. Secondly, its degree of interconnection substitutability, namely its *core business* function, e.g., banking, custody, fund management, brokerage, clearing, etc. It is worth to mention that the central bank and a small private bank play a significantly different role in the banking network to which they contribute.

Every financial collapse begins with the downfall of a single *player*. The causes of this initial failure are many. In particular they can be of exogenous type as in the case, e.g., of recessions, wars, political crisis, etc., or they can be originated within the financial system itself, as frauds or misapplication of mathematical models. Analogously, there are several ways through which these shocks can be spread over the whole network. They can be divided into three different classes:

i propagation due to a direct counterpart exposures: this takes into account all the losses which may occur if banks default in their obligations to other bank in the interbank market;

ii propagation due to asset price contagion: a failing bank is forced to sell its assets at fire price, leading to a fall in market prices and then affect the capital position of the others banks which hold the same assets;

iii propagation due to liquidity hoarding: banks may reduce the maturity of their loans or completely cut them, as a defensive measure against future liquidity shortfall and/or future defaults.

III. Mathematical Models

The way in which we build the topology of the graph models reflects the different possible scenarios for the specific banking system considered. In what follows, we will focus on the use of some random graph model from the financial network perspective. Namely we will exploit the Erdős-Rényi model, the Barabási-Albert model and a modification of the latter which allows us to better understand the random dynamics governed by financial failures and their related spread through the network they are part of.

A. The Erdős-Rényi model

The random graph model proposed by Erdős and Rényi, see [8], [4], defines an undirected random graph with a fixed number $N > 0$ of nodes. The number $n > 0$ of connections, or edges, is fixed a priori, while the configuration of the network is chosen uniformly randomly from the possible $\frac{N(N-1)}{2}$ alternatives.

In such a model the degree k_b of a given node b is typically modeled as a binomial random variable $Bin(N-1, p)$, $p \in (0,1)$ in order to avoid trivial settings, therefore the mean network's degree is given by $\mathbb{E}[k] = (N-1)p$. Moreover the expected number of nodes with degree k, denoted by $\#k$, turns out to be

$$\mathbb{E}[\#k] = N\binom{N-1}{k}p^k(1-p)^{N-1-k} = \lambda_k\,.$$

As it is well known, if we consider the limit $N \to \infty$, the distribution of $\#k$ approaches a Poisson distribution with parameter λ_K, and therefore the degree distribution reads as follow

$$P(k) \simeq e^{-pN}\frac{(pN)^k}{k!} = e^{-\langle k\rangle}\frac{\langle k\rangle^k}{k!}\,,$$

where $\langle k\rangle$ denotes the average degree of the graph.

Latter model can be used to represent networks of homogeneous institutions in a closed sector. Indeed since the number of nodes N is fixed, the order of the graph do not vary in time. The word homogeneous draws attention to the similarity of the nodes because in this model they all are treated equally. The latter follows from the fact that the *connection probability* is equally chosen for all the nodes. Latter scenario is rather reasonable if all the institutions can be considered similar to each other in size, history and budgets. In this context, an edge which is symmetric and unweighted, may imply that the two banks share the same assets,at least from a mere quantitative point of view, in their respective portfolio.

If we consider a random graph structure as before, we are left with considering the so called associated *network probability matrix* which gives us, for every couple of players acting in the financial network, the probability that they can be connected. Of course the *vice versa* also holds, namely a random graph can be built from a probability matrix through its entries representing the associated edge probabilities. By slightly modify such a matrix, the Erdős-Rényi model can be extended to directed and/or weighted graphs. We would like to underline that the just mentioned two features are crucial for modeling banking network since it is not always the case that the relation of two financial institutions is symmetric. The latter implies that, exploiting directed edges, we can distinguish lending from borrowing banks. Moreover, by made use og weighted networks, we can relate the weight of each edge to the respective loan amount.

Examples of weighted and directed random graph are considered in, e.g., [6] and [5], where the authors consider each node, namely each bank, with a simplified balance sheet summarizing related financial conditions. The assets of each bank are divided into two main categories, the interbank loans and the external assets, whereas the liabilities are split into the interbank borrowing and the customer deposits. Then an institution in the network is solvent as long as its net worth, or capital, stays positive. The weights of the directed edges represent the volume of the loans. A failing bank transmits a shock which is supposed to be equally subdivided to all its creditors. This may lead to a domino cascade if a creditor bank, which cannot lose more than it lent to the failing bank, cannot withstand the loss. We study a generalisation of such a model in Section IV, also providing related insightful numerical simulations in Subsection IV-C.

B. The Barabási-Albert model

It is worth to mention that, from the financial modeling point of view, one of the major drawback affecting the random graph theory, concerns its inability to represent the scale-free nature, i.e. the so called *power law behaviour*, of the degree distribution, which is known to characterize, in particular, real-world, financial networks, see, e.g., [9, Chapt.5].

The two main causes leading to a power law degree distribution, are the ability of a network to growth by adding new nodes, and the so-called *preferential attachment* characteristic. The preferential attachment, in contrast to the random one, models the *growth behaviour* of the graph by letting the connection (between vertices) probabilities $\Pi(k)$ to depend on the degree of the nodes themselves. The latter implies that, depending on the particular network we are dealing with, a new node may be more likely to link with a highly connected node, resp. to a lowly connected one. From a modeling point of view, the understanding of what are the conditions that lead $\Pi(k)$ to be an increasing or a decreasing function in k, is a rather delicate question. Concretely, it is reasonable to suppose that Π should be increasing with respect to a bank with a high level of exposure since it is more likely that it continues to raise is relevance compared with other banks characterized by lower degrees. The latter aspect reflects the fact that an elevated number of edges stand for a high level of trust in the bank soundness by almost all the institutions acting in the same net. Previous approach is empirically witnessed by collective aggregation phenomena by imitation, namely since a large number of players simultaneously trust in the financial capabilities of another one, then suddenly others players add to the former, hence amplifying such a phenomena during time, in a sort of avalanche dynamic which also produces *heavy tails phenomena*, how it has been pointed out in, e.g., [10], [11], [12], etc .

Previously described phenomena can be studied by mean of the Barabási-Albert model which is particularly well suited for the representation of the banking system of developing economies. New institutes are free to enter and they are more likely to clinch deals with *older* and highly connected banks. At time $t = 0$, few financial institutions, let us indicate them by a the constant $N_0 > 0$, are already present in the system. At every time step, e.g. at every month or trimester, a new node enters in the system and it is connected to $0 < n < N_0$ different nodes. The preferential attachment is modeled by an increasing linear function in the degree distribution, i.e. $\Pi(k_b) = \frac{k_b}{\sum_{i \in B} k_i}$. Since at every time the degree of a node b increases if a new node enters the system and links to it and this happens with probability $\Pi(k_b)$, then k_b has to satisfies

$$\frac{\partial k_b}{\partial t} = n\Pi(k_b) = n\frac{k_b}{\sum_{i=1}^{N-1} k_i} \ ,$$

where k is thought as a continuous variable. Assuming that the node b is introduced at time t_b, then its initial condition is given by $k_b(t_b) = n$, therefore, by solving the related Cauchy problem, we have that

$$k_b = n\sqrt{\frac{2t + 2\frac{e_0}{n} - 1}{2t_b + 2\frac{e_0}{n} - 1}}$$

and the degree distribution is given by

$$P(k) = \frac{\partial \mathbb{P}(k_b(t) < k)}{\partial k} = \frac{n^2}{k^3}\left(2t + 2\frac{e_0}{n} - 1\right) \ ,$$

so that, taking the limit $t \to \infty$, we asymptotically have $P(k) \sim 2n^2k^{-3}$. Therefore, the Barabási-Albert model leads to an asymptotic degree distribution that is independent on time, as well as on the system size, implying that the network reaches a stationary scale-free state. It is worth to mention that also the Barabási-Alber model shows clear limits in representing banking systems. As an example, financial networks are of finite size and, in addition, we can not expect that the *attachment mechanism* is time invariant, while the whole financial structure does. A second major lack concerns the properties of the power-law distribution implied by the model itself, which has lead us to formulate the new approach presented in the next section.

C. The theory of evolving network

One limitation of the Barabási-Albert model is the fact that exponent of the power-law degree distribution is a fixed constant (equal to 3) while real networks shows different values for this parameter, usually between 1 and 3. One way to modify this feature consists in modeling the *preferential attachment mechanism* in a different way, namely exploiting a power law probability distribution of the form: $\Pi(k) \sim k^{\alpha}$. However in order to preserve the scale-free nature of the network the preferential attachment has to be asymptotically linear, i.e. $\pi(k_b) \sim ak_b$ as $k_b \to \infty$. The latter implies that the rate equation leads to $P(k) \sim k^{-\gamma}$ where $\gamma = 1 + \frac{\mu}{a}$, where the α-th moment of $N_k(t)$ satisfies $\sum_k k^{\alpha} N_k(t) = \mu t$, as $t \to \infty$.

We would like to underline that there are other interesting features that a financial network may share, which we have not examined so far, namely the *initial attachment* and the *growing rate* property. Notice that in a real network a new node may link to an isolated node, which is not the case in the Barabási-Albert approach, since $\Pi(0) = 0$. The latter could be, e.g., the preferential behaviour followed by new players which enter the system, and are characterized by a high propensity to risk, hence supposing that isolated financial agents already in the net, can offer them risky investments, but with a potential high remuneration. Moreover real networks grow differently from one another, while the Barabási-Albert model is characterized by a constant rate of growth. As an example, accelerated growth means that the average degree of the network increases in time, equivalently that the number of edges increases at a faster rate than the number of nodes. In a banking network the latter represents the possibility for a bank, which is already in the system, to add new links, hence to start financial interactions with new institutions in addition to those already in place. Besides personal goals of each bank, as well as unpredictable changes in the market regulation, may suggest to some institution to change their deals, to find new commercial partners or to close some of the existing ones, moreover we have also to take into account that most of the interbank transactions happened relatively quickly compared to more standard contracts as in the case of those stipulated between an institution and a private for an house loan. From the point of view of the graph topology, the aforementioned changes imply additions, removals and rewiring of the edges. In what follows we will show how to modify the standard Barabási-Albert model in order to taken into account previous dynamics. At the initial time $t = 0$, the network system is composed by $N_0 > 0$ nodes and 0 edges. At every time step one of the following three possibilities is realized:

i $0 \leq n \leq N_0$ edges are added with probability $p \in (0, 1)$, moreover each of them is linked to a node randomly selected using a uniform distribution, while a node is chosen with probability depending on its degree $\Pi(k_b) := \frac{k_b+1}{\sum_i (k_i+1)}$;

ii with probability $q \in (0, 1 - p)$, $n \leq N_0$ contracts are rewritten, i.e. a uniformly chosen financial institution b rescinds a contract, and then the correspondent edge is removed, from a randomly selected counterpart, replacing it with a new contract, therefore a new edge is added, with a different party chosen with probability $\Pi(k_b)$;

iii with probability $1-p-q$ a new financial institution enters in the system, underwriting $0 \leq n \leq N_0$ contracts with players, already in the system, which are are selected at random with probability $\Pi(k_i)$, according on their degree.

The dynamic characterized by the aforementioned rules (i)-(iii), leads to the following equation

$$\frac{\partial k_b}{\partial t} = (p - q)\frac{n}{N} + n\Pi(k_b) \, ,$$

whose solution is given by

$$k_b(t) = (A(p, q, n) + n + 1)\left(\frac{t}{t_b}\right)^{\frac{1}{B(p,q,n)}} - A(p, q, n) - 1 \ ,$$

where

$$A(p,q,n) = (p-q)\left(\frac{2n(1-q)}{1-p-q}+1\right) ,$$

and

$$B(p,q,n) = \frac{2n(1-q)+1-p-q}{n} ,$$

so that the corresponding degree distribution is given by

$$P(k) \sim (k + A(p,q,n)+1)^{-B(p,q,n)-1} ,$$

see [3] for further details.

As a final remark, let us note that in the Barabási-Albert model, one fundamental hypothesis is the monotonic increasing development of the network, which is not a realistic assumption for a financial network in general, and for a interconnected set of banks in particular. In fact, from a realistic financial perspective, every private institution is subjected to potential failure which may substantially modify its contracts structure, even in the case when such a failure does not cause the permanent default of the associated bank. The latter implies the possible cut of a certain number of edges, namely of links between such a bank and institutions previously linked to it. Concerning such an approach, in [13], the authors studied some examples of undirected networks where existing edges are withdrawn and they obtained that, in the limit, the considered model still has a scale-free nature where the degree exponent depends on how the removing rate has been chosen.

1) Competition: In the interbank lending market, banks borrow and lend money between each other in order to manage liquidity and satisfy regulations such as reserve requirements. As long as banks lend money to each other, the interbank market can be considered rather stable. Nevertheless such an *equilibrium point* should be considered as an *unstable equilibrium*, in fact its stability greatly depends on a number of heterogeneous factors, both exogeneous and endogeneous. If the combined action of such factors determines a sufficiently large perturbation, then market crashes, banks reduce their loans and start *competing* with each other to find the necessary liquidity. In such a scenario, following, e.g., the work by Bianconi and Barabási, [14], each bank should be characterized by a so called *fitness parameter*, tipically indicated by η, namely a credit rating, depending on qualitative information as, for example, the institute history, its type of governance, its multiannual planning, and on quantitative information, such, e.g., its financial statements or its ability to compete with its counterparts. The introduction of this parameter embodies another important feature: it allows to young but *solid* banks, namely nodes which have been later introduced in the network but characterized by high financial performances, to quickly acquire new edges.

According with latter idea, at every time step a new institution b enters the network embedded with a fitness parameter η_b, which is chosen from a priori stated distribution $\rho(\eta)$.

Such a new entry is then linked to $n > 0$ different institutes that are already part of the system, with probability $\Pi(i) = \frac{\eta_i k_i}{\sum_j \eta_j k_j}$.

The continuum theory for the rate of the b's degree implies

$$\frac{\partial k_b}{\partial t} = n \frac{\eta_b k_b}{\sum_j \eta_j k_j} .$$

Denoted with t_b its entering time and then considering the initial condition $k_b(t_b) = n$, the unique solution for k_b is given by

$$k_b(t) = n\left(\frac{t}{t_b}\right)^{\beta(\eta_b)} ,$$

where

$$\beta(\eta) = \frac{\eta}{\rho(\eta)\dfrac{\eta}{1-\beta(\eta)}\, d\eta} .$$

IV. BANKING SYSTEM AND SIMULATION MODELING

In what follows we provide an improved model of banking network in which, as the main difference with respect to the work cited so far, we allow for new banks to enter in the network, for a total number that it is upper bounded and strictly depends on the state of the system.

It follows that previous mathematical results cannot be applied to our model, and we underline that considering entering nodes and preferential attachment mechanisms, allow to obtain new insights concerning more realistic banking network dynamics.

A. Banking system and balance sheets

We consider a fixed number $N_0 \in \mathbb{N}^+$ of financial institutions, which are the nodes of the financial network. For simplicity, we will call them banks in what follows. Therefore, as explained in previous sections, each edge represents a weighted directional lending between the two banks. We assume that the probability p_{ij} that two ordered banks i and j close a contract in which i lends money to j, is constant across all pairs (i, j) and therefore, it will simply denoted by $p \in (0, 1)$. The latter implies that we are considering homogeneous banks.

The balance sheet of each bank depends on three exogenously imposed parameters, namely θ, γ and E, as explained in the following paragraphs.

The assets of each bank i, denoted by a_i, are composed of the external assets e_i and of the interbank assets i_i, that are the borrowing from the other banks, whereas its liabilities l_i consists of the customer deposits c_i and the interbank borrowing b_i. The capital buffer, or net worth c_i, is the excess of assets over liabilities. By the balance sheet identity $a_i = l_i + c_i$. The solvency condition for bank i means that the capital buffer has to be positive.

The total external assets of the banking system and their percentage in total assets are fixed, and denoted by E and $1 - \theta$ respectively. Naturally it follows that θ represents the percentage of interbank assets in total assets. Moreover we assume that the borrowings/loans between one and one another are all of the same size w. This magnitude is computed as the ratio between the total interbank exposure and the number of links in the graph.

By knowing w and the structure of the network we can compute the interbank assets and borrowings of each bank i, in fact they are proportional by w to its out-degree and its in-degree, respectively.

The external assets of each bank i are defined as follow

$$e_i := b_i - i_i + \frac{E - \sum_{i=1}^{N_0} b_i - i_i}{N_0} ,$$

which means that e_i equals the difference of its borrowing and its loans, plus an amount which is equal for all the banks, so as to meet the constraint $E = \sum_{i_1}^{N} e_i$.

Eventually, the net worth of each bank is set as a fixed proportion γ of total assets at bank level, i.e. $c_i = \gamma a_i$, and their consumer deposits are computed to meet the balance sheet identity, i.e. $d_i = a_i - c_i - b_i$.

B. Transmitting and withstanding shocks

In the first phase, e.g. as result of a fraud, an initial shock hits one (evenly randomly chosen) bank, say the node i, wiping out all the value associated to its external assets. The volume of this shock is denoted by s_i. This loss is absorbed by the net worth of the bank, c_i. If this is not enough to cover the shock, namely if $c_i < s_i$, the bank i defaults and the residual shock $s_i - c_i$ is transmitted to its creditors.

In the second phase, the residual shock is distributed equally between the defaulting bank's creditors, then its k creditor banks experience a shock s_j given by

$$s_j = \min\left(\frac{s_i - c_i}{k}, w\right) .$$

Note that a creditor bank cannot lose more than it has lent at first. As before, the system suffers further failures if any of the k creditors cannot withstand the transmitted shock, which happens if $s_j > c_j$, for the same j. If the latter happens, than the correspondent, *insolvent*, banks transmit their residual shock in a third phase and so on and so forth. As a consequence, such a mechanism may lead to a *domino falling*.

Nevertheless, from the second phase on, we assume that when, and even if, a bank in the network defaults, new banks are willing to enter in the system. The random number of entering banks is distributed according with a Gamma distribution, with parameters $(\hat{k}, \hat{\theta})$, where

$$\hat{k} = \left(\frac{\text{Number of defaults in this phase}}{\text{Initial number of banks in the network}}\right)^{-1} \times 10 ,$$

and $\hat{\theta} = 1$.

The balance sheet of the new entering banks is built as follows. The current state of the network and the average balance sheet of the still working banks are computed and used as a benchmark. Since the network is facing a period of crisis, we assume that the banks which enter in the system bring new cash in the interbank lending market. Therefore they lend more money than they borrow: in particular we assume that its in-degree and the out-degree are the average degree of the node, rounded down and up respectively. The volume of each loan/borrowing is still w. We assume that the probability for these bank of closing a deal with a counterpart i depends on the state of bank i, according to a fitness parameter, i.e. a credit rating, $\Pi(i)$. In our model, the fitness parameter $\Pi(i)$ is defined by

$$\Pi(i) = q\frac{c_i}{\sum_k c_k} + (1-q)\frac{b_i + i_i}{\sum_k b_k + c_k} ,$$

where q is a positive value $q \in [0, 1]$. Note that for a bank i is easier to close a deal if its net worth is high and if it has already a lot of loan agreements with the other banks in the network. Indeed the latter denotes a good state of i as counterparty.

TABLE I
SUMMARY OF THE PARAMETERS' VALUES

Parameter	Benchmark Value
N_0	25
p	0.2
E	100000
γ	5%
θ	20%

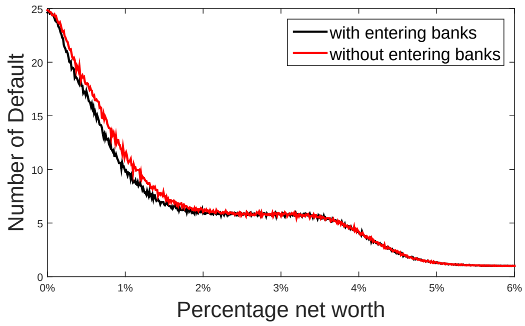


Fig. 1. Number of defaults for $\gamma \in [0, 0.06]$

Lastly, the volume of the external assets of the new banks equals the average volume of the external assets of the initial N_0 banks, namely it equals $\frac{E}{N_0}$ moreover, their net worth and their customer deposits are computed as before.

C. Simulations

Summing up the peculiarities of the proposed model, it follows that its characteristics can be summarized by five parameters:

- N_0, the initial number of banks in the network;
- p, the connection probability between any two nodes;
- E, the total external assets;
- γ, the percentage of net worth to total assets;
- θ, the percentage of interbank assets to total assets.

As a benchmark we consider the case when the aforementioned parameters take values as in Table I. The value of q in the fitness parameter is set equal to 0.5. We perform 500 simulations for each set of parameters $(N_0, p, E, \gamma, \theta)$ and we report the average number of defaults among the initial N_0 banks. The obtained result is compared to the number of defaults when no new banks enter in the network.

1) Impact of bank capitalization: Figure 1 shows how many banks, among the initial N_0 banks, default during the crisis as a function of the bank capitalization, i.e. γ, both when new banks enter in the system and when they do not. It can be seen that mechanism which allows new banks to enter the system, slows down the contagion transmission.

First, in our model the entering banks in each phase are few and they are not enough to completely stop the cascade falling. Second, the new banks enter in the system from phase II on, therefore they simply can not avoid the phase II defaults. It follows that, when the initial shock is significantly hight and leads to the completely default of the system, i.e. when the number of defaults equals the initial number of nodes N_0, or when the initial shock is as irrelevant that even the first hit bank is able to withstand it, the liquidity which the entering banks take into the system has no impact on the number of defaulting banks.

Fig. 2. Number of defaults as function of p when$\gamma = 1\%$

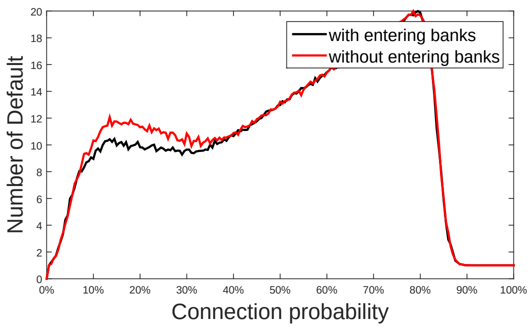
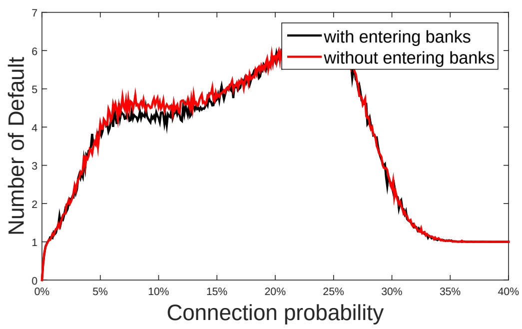


Fig. 3. Number of defaults as function of p when $\gamma = 3\%$



2) Impact of connectivity: Figures 2 and 3 shows the number of defaulting banks, between the N_0 that are in the network at time $t = 0$, as a function of the connection probability p. Also in this case, both when $\gamma = 0.03$ and when $\gamma = 0.01$, the entry of new banks decreases the average number of defaulting banks.

All the previous considerations concerning the role of the entering banks are still valid here. The mitigation of contagion effect of the entering banks is evident especially for $p \in [0.08, 0.40]$ and $p \in [0.05, 0.15]$ when γ equals 1% and 3%, respectively.

V. Conclusions

We have exploited the theory behind the approaches proposed by Erdős and Rény, and by Albert, and Barabási, in the context of random graph analysis, mainly with the aim to study the spread of default in highly interconnected scenarios. Latter setting can be widely observed when studying modern financial networks, in particular with respect to the description of the complexity of their interconnections. Nevertheless the aforementioned approaches are lacking of basic properties that characterize real financial markets, in particular with respect to the possibility to allow for the non deterministic entering of new *players* establishing a randomly chosen number of links. Such a generalisation, allow to better describe the dynamic originated by the failure of a big financial institution, as in the case of a central bank, within the financial network of which it is part of.

Bibliography

References

[1] A. G. Haldane, R. M. May, Systemic risk in banking ecosystems, Nature 469 (7330) (2011) 351–355.
[2] J.-C. Rochet, J. Tirole, Interbank lending and systemic risk, Journal of Money, credit and Banking (1996) 733–762.
[3] R. Albert, A.-L. Barabási, Statistical mechanics of complex networks, Reviews of modern physics 74 (1) (2002) 47.
[4] E. N. Gilbert, Random graphs, The Annals of Mathematical Statistics (1959) 1141–1144.
[5] E. Nier, J. Yang, T. Yorulmazer, A. Alentorn, Network models and financial stability, Journal of Economic Dynamics and Control 31 (6) (2007) 2033–2060.
[6] R. M. May, N. Arinaminpathy, Systemic risk: the dynamics of model banking systems, Journal of the Royal Society Interface 7 (46) (2010) 823–838.
[7] W. Paul, J. Baschnagel, Stochastic Processes: From Physics to Finance, 2nd edition, Springer, 2013.
[8] A. Renyi, P. Erdos, On random graphs, Publicationes Mathematicae 6 (5) (1959) 290–297.
[9] G. Caldarelli, Scale-Free Networks Complex Webs in Nature and Technology, Oxford University Press, Oxford Finance Series, 2007.
[10] R. Cont, J.-P. Bouchaud, Herd behavior and aggregate fluctuations in financial markets, Macroeconomic Dynamics 4 (2) (2000) 27.
[11] D. Hirshleifer, S. Hong Teoh, Herd behaviour and cascading in capital markets: a review and synthesis, European Financial Management (2003) 25–66.
[12] T.-C. Huang, B.-H. Lin, T.-H. Yang, Herd behavior and idiosyncratic volatility, Journal of Business Research 68 (4) (2015) 763–770.
[13] S. N. Dorogovtsev, J. F. Mendes, Evolution of networks: From biological nets to the Internet and WWW, Oxford University Press, 2013.
[14] G. Bianconi, A.-L. Barabsi, Competition and multiscaling in evolving networks, Europhysics Letters 54.
[15] P. Gai, S. Kapadia, Contagion in financial networks, in: Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, The Royal Society, 2010, p. rspa20090410.