# Dynamic Output Feedback Fault Tolerant Controller for an Exploited Polynomial Fish Population Model

A. Ait Kaddour, N. Elalami, E. Elmazoudi

***Abstract—*** This paper deals with the design of robust fault estimation and fault tolerant control of a continuous age structured model of a harvested fishery system. Takagi-Sugeno approach is used to represent the harvested fish population model, where the effort is used as control term, the age classes as states and the quantity of captured fish per unit of effort as measured output. Actuators faults are initially estimated using a fuzzy observer. Then, based on the fault estimation, an observer based dynamic output feedback fault tolerant controller is designed to reject the effect of faults by stabilizing the closed loop system. The synthesis of the control and the observer are independently made. The conditions of the observer convergence and of the control existence are formulated in terms of Linear Matrix Inequalities. The simulation results demonstrate the effectiveness and applicability of the proposed design method.

***Keywords—*** fault-tolerant control (FTC), harvested fish population, Takagi-Sugeno multimodel, Linear Matrix Inequalities (LMI).

## I. Introduction

HEAVY use of the world's marine fisheries is a severe concern. Significant increases in human population and rapid technological advances during the past decades have resulted in a vast increase in global production of marine capture fisheries. Today, these fisheries are in trouble as their populations are being depleted to dangerously low levels, which are affecting national economies and local communities' socio-economic well-being. The optimal management of renewable resources, which has a direct relationship to sustainable development, should receive much attention. It requires further discussion in order to understand short and long term exploitation patterns [1]. Therefore, it is very important for key stakeholders to plan a suitable pattern that sustains fisheries at a good level of productivity and meets economic goals.

Nonlinear control techniques are useful for studying and controlling complex systems. Although they have been initially developed for mechanical and electrical systems their applications to biological and environmental problems are growing [2]-[4].

Recently, there has been a growing interest in the Takagi–Sugeno T-S fuzzy system since it is a powerful solution that bridges the gap between linear and nonlinear control systems. Many important results on analysis and synthesis for the T–S fuzzy system have been reported [9]-[12]. Considering the advantage of the T–S fuzzy system to approximating complex nonlinear systems, we will use it to describe the harvested fish population dynamics in this paper.

Because of their nonlinearity and their complexity that are associated with biological phenomena (birth, death, growth, cannibalism, intra-stage competition for food and space, etc.), and because also of massive over exploitation, breakdown of regulatory and enforcement systems, harvested fish populations dynamics are inevitably subjected to disturbances which have as origin the noises due to the environment and the model uncertainties. Moreover, sensors and/or actuators can be corrupted by different faults or failures.

To improve fish resources sustainability, FTC scheme must be considered in designing good harvesting policies. FTC allows controlling system in such a way that it fulfills desired objectives in the presence of non-critical faults in components of the system [5]-[8].

The existing FTC design approaches are classified into two main groups [8]. The first group is called passive fault tolerant control or robust control. This kind of control is described in [16]-[19]. In this approach, the faults are considered as uncertainties. Therefore, the control is designed to be robust only to the specified faults. The second group is called active fault tolerant control. Contrarily to the passive FTC, active FTC requires a FDI block to detect, isolate and estimate the faults [20]-[22]. The FTC module uses the information issued from the FDI block to reconfigure the control law in order to compensate the fault and ensure acceptable system performances. In general, the active approach is less conservative than the passive one, which has increasingly been the main methodology in designing FTC systems [23].

Based on the idea given in [5], this paper addresses the problem of controlling exploited fish population systems subject to faults and unknown bounded disturbances, trough dynamic output feedback fault tolerant controller and using T-S fuzzy models. To the best of our knowledge, this problem has not been studied in the literature.

Achraf Ait Kaddour is with the Electrical Engineering Department, Ecole Mohammadia d'Ingénieurs, Rabat, Morocco (phone: +212661117220; e-mail: achraff@gmail.com).

Noureddine Elalami is with the Electrical Engineering Department, Ecole Mohammadia d'Ingénieurs, Rabat, Morocco (e-mail: prelalami@gmail.com).

Elhoussine Elmazoudi is with Laboratoire de Recherche en Economie de l'Energie, Environnement et Ressources, Département d'Economie, Cadi Ayyad University, Marrakech, Morocco (e-mail: h_mazoudi@yahoo.fr).

The rest of this paper is organized as follows. In the second section, a fuzzy-augmented fault estimation observer design is proposed to achieve fault estimation of T–S models with actuator faults. Based on the information of online fault estimation, an observer based dynamic output feedback fault tolerant controller is designed to compensate the effect of faults by stabilizing the closed-loop system. Sufficient conditions for the existence of both observer and controller are given in terms of linear matrix inequalities (LMIs). The third section is dedicated to the description of the continuous stage structured model, which is transformed to a T-S fuzzy model. Finally, simulation results are presented to demonstrate the effectiveness of the proposed technique.

## II. Observer And Controller Design And Analysis

In this section, we propose a multi objective fuzzy augmented fault-estimation observer, including a regional pole placement and a H∞ performance level, in order to reject the effect of disturbances on the system as much as possible and also guarantee the convergence speed of fault estimation as well. Then, using the obtained fault estimate, a fuzzy observer-based dynamic output feedback FTC is designed to guarantee the system stability in the presence of actuator faults. In the design process, the synthesis of the control and the observer are independently made, and their performances are considered simultaneously, which is convenient for calculating the design parameters.

### A. Model Representation

The design procedure described in this section begins with representing a given nonlinear plant by the so-called T-S fuzzy model. The fuzzy model proposed by Takagi and Sugeno [12] is described by fuzzy IF-THEN rules that represent local linear input-output relations of a nonlinear system. The main feature of a T-S fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model. The overall fuzzy model of the system is achieved by fuzzy ‘‘blending’’ of the linear system models.

A dynamic T-S fuzzy model is described by a set of fuzzy “IF … THEN” rules with fuzzy sets in the antecedents and dynamic linear time-invariant systems in the consequents. A generic T-S plant rule can be written as follows [10]:

**Model Rule i:**

***IF*** *$z_1(t)$ is $M_{i1}$ and … and $z_p(t)$ is $M_{ip}$,*

***THEN***

$$\dot{x}(t) = A_i x(t) + B_i\big(u(t)+f(t)\big) + D_{1i} w(t)$$
$$\dot{y}(t) = C_i x(t) + D_{2i} w(t)$$

Where, $M_{ij}$ is the fuzzy set and $r$ is the number of model rules; $x(t) \in \boldsymbol{R}^n$ is the state vector, $u(t) \in \boldsymbol{R}^m$ is the input vector, $y(t) \in \boldsymbol{R}^q$ is the output vector, f(t) ∈ Rm represents the additive actuator fault, and $\omega(t) \in \boldsymbol{R}^d$ is the disturbance, which is assumed to belong to $L_2[0,\infty)$. $A_i \in \boldsymbol{R}^{nxn}$ , $B_i \in \boldsymbol{R}^{nxm}$, and $C_i \in \boldsymbol{R}^{qxn}$ ; $D_{1i}$ and $D_{2i}$ are constant real matrices of appropriate dimensions. It is supposed that matrices $B_i$ are of full column rank, i.e., rank($B_i$) = m, the pairs ($A_i$ , $B_i$ ) are controllable, and the pairs ($A_i$ , $C_i$) are observable. $z_1(t)$,… $z_p(t)$ are known premise variables that may be functions of the state variables, external disturbances, and / or time.

Each linear consequent equation represented by $A_i x(t) + B_i u(t)$ is called a ‘‘subsystem.’’

We will use $z(t)$ to denote the vector containing all the individual elements $z_1(t)$,… $z_p(t)$.

Given a pair of (x(t), u(t)), and using singleton fuzzifier, max-product inference and center average defuzzifier, we can write the aggregated fuzzy model as:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i(z(t))\{A_i x(t) + B_i(u(t)+f(t)) + D_{1i} w(t)\}}{\sum_{i=1}^{r} w_i(z(t))} \quad \textbf{(1)}$$

Where z(t) = [$z_1(t)$ $z_2(t)$… $z_p(t)$],

and $w_i\big(z(t)\big) = \prod_{j=1}^{p} M_{ij}(z_j(t))$

The term $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in $M_{ij}$.

(1) Can be written as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t))(A_i x(t) + B_i\big(u(t)+f(t)\big) + D_{1i} w(t)) \quad \textbf{(2)}$$

where : $\mu_i\big(z(t)\big) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}$

Since $\sum_{i=1}^{r} w_i(z(t)) > 0$

and $w_i\big(z(t)\big) \geq 0, \quad i = 1, 2, \dots, r,$

we have : $\sum_{i=1}^{r} \mu_i(z(t)) = 1$

and $\mu_i\big(z(t)\big) \geq 0, \quad i = 1, 2, \dots, r,$ for all t.

The global output of T-S model is interpolated as follows:

$$y(t) = \sum_{i=1}^{r} \mu_i\big(z(t)\big)\,[C_i x(t) + D_{2i} w(t)] \quad \textbf{(3)}$$

For simplicity, we introduce the following notations:

$\mu_i = \mu_i\big(z(t)\big)$; $A(\mu) = \sum_{i=1}^{r} \mu_i A_i$; $B(\mu) = \sum_{i=1}^{r} \mu_i B_i$

$C(\mu) = \sum_{i=1}^{r} \mu_i C_i$ ; $D_1(\mu) = \sum_{i=1}^{r} \mu_i D_{1i}$

and $D_2(\mu) = \sum_{i=1}^{r} \mu_i D_{2i}$

Then, the T–S fuzzy model (2) and (3) can be rewritten as:

$$\dot{x}(t) = A(\mu)x(t) + B(\mu)\big(u(t)+f(t)\big) + D_1(\mu)w(t) \quad \textbf{(4)}$$

$$y(t) = C(\mu)x(t) + D_2(\mu)w(t) \quad \textbf{(5)}$$

### B. Fuzzy augmented fault-estimation observer design

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In order to detect and estimate faults, the following fault estimation observer is constructed:

$$\hat{x}' = A(\mu)\hat{x}(t) + B(\mu)\left(u(t) + \hat{f}(t)\right) - L(\mu)\big(y(t) - \hat{y}(t)\big) \quad \textbf{(6)}$$

$$\hat{y}(t) = C\hat{x}(t) \quad \textbf{(7)}$$

$$\hat{f}' = -F(\mu)\big(\hat{y}(t) - y(t)\big) \quad \textbf{(8)}$$

where $\hat{x}(t)$ is the observer state, $\hat{y}$ is the observer output, and $\hat{f}(t)$ is an estimate of the fault $f(t)$. $L(\mu) = \sum_{i=1}^{r} \mu_i L_i$ and $F(\mu) = \sum_{i=1}^{r} \mu_i F_i$ are the gain matrices to be determined.

Let us define:

$e_x(t) = \hat{x}(t) - x(t)$ ; $e_y(t) = \hat{y}(t) - y(t)$

and $e_f(t) = \hat{f}(t) - f(t)$

Then, the error dynamics is ruled by :

$$\dot{\bar{e}}(t) = \big(\bar{A}(\mu) - \bar{L}(\mu)\bar{C}(\mu)\big)\bar{e}(t) + (\bar{L}(\mu)\overline{D_2}(\mu) - \overline{D_1}(\mu))v(t) \quad \textbf{(9)}$$

$$e_y(t) = \bar{C}(\mu)\bar{e}(t) - \overline{D_2}(\mu)v(t) \quad \textbf{(10)}$$

where:

$\bar{e}(t)=\begin{pmatrix} e_x(t) \\ e_f(t)\end{pmatrix}$ ; $\nu(t)=\begin{pmatrix} w(t) \\ \dot{f}(t)\end{pmatrix}$ ; $\bar{A}(\mu)=\begin{pmatrix} A(\mu) & B(\mu) \\ 0 & 0\end{pmatrix}$ $\bar{L}(t)=\begin{pmatrix} L(\mu) \\ F(\mu)\end{pmatrix}$ ; $\bar{C}(t)=(C(\mu)\ \ 0)$ ; $\overline{D_1}(\mu)=\begin{pmatrix} D_1(\mu) & 0 \\ 0 & I_m\end{pmatrix}$ $\overline{D_2}(t)=(D_2(\mu)\ \ 0)$

We assume that $\dot{f}(t)\ \in L_2[0\ \infty[$.

In [5], a multi-objective fuzzy augmented fault estimation observer design method under regional pole placement and $H_\infty$ performance specifications is proposed to achieve robust fault estimation.

*Theorem 1: For two given positive scalars α and r, the eigenvalues of $\left(\bar{A}(\mu)-\bar{L}(\mu)\bar{C}(\mu)\right)$ belong to D(α,r), and the error dynamics (13) satisfy the $H_\infty$ performance index $\|e_f(t)\|_2 < \gamma\|\nu(t)\|_2$, if there exists a symmetric positive definite matrix $\bar{P}$ and matrices $\bar{Y}_i$ such that :*

*min γ subject to*

$$\Psi_{ii}<0\ ,\ i=1,\dots,q \tag{11}$$
$$\Psi_{ij}+\Psi_{ji}<0\ ,\ 1\le i<j\le q \tag{12}$$
$$\Phi_{ii}<0\ ,\ i=1,\dots,q \tag{13}$$
$$\Phi_{ij}+\Phi_{ji}<0\ ,\ 1\le i<j\le q \tag{14}$$

*where :*

** denotes the symmetric elements in a symmetric matrix.*

$$\Psi_{ij}=\begin{bmatrix}-\bar{P} & \bar{P}\bar{A}_i-\bar{Y}_i\bar{C}_j-\alpha\bar{P} \\ * & -r^2\bar{P}\end{bmatrix},\ \ \overline{I_m}=\begin{bmatrix}0\\ I_m\end{bmatrix}$$

$$\Phi_{ij}=\begin{bmatrix}\phi_{11} & \overline{Y_i D_{2j}}-\overline{PD_{1i}} & \overline{I_m} \\ * & -\gamma I_{d+m} & 0 \\ * & * & -\gamma I_m\end{bmatrix},\ Y_i=PL_i$$

$$\phi_{11}=\bar{P}\bar{A}_i+\bar{A}_i^T\bar{P}-\bar{Y}_i\bar{C}_j-\bar{C}_j^T\bar{Y}_i^T.$$

Proof: See [5]

## C. Fuzzy dynamic output feedback-fault tolerant controller design

On the basis of the obtained online fault-estimation information, we design a fault-tolerant controller to guarantee stability in the presence of faults. Since the state x(t) is unmeasurable, we use the fuzzy dynamical output feedback-controller scheme, to construct the dynamic output feedback-fault tolerant controller for T–S fuzzy models as:

$$\dot{\xi}(t)=A_k(\mu,\mu)\xi(t)+B_k(\mu)y(t) \tag{16}$$
$$u(t)=C_K(\mu)\xi(t)+D_K y(t)-\hat{f}(t) \tag{17}$$

where $\xi(t)$ is the state, $A_k(\mu,\mu)=\sum_{i=1}^r\sum_{j=1}^r\mu_i\mu_j A_{K_{ij}}$, $B_k(\mu)=\sum_{i=1}^r\mu_i B_{K_i}$, $C_K(\mu)=\sum_{i=1}^r\mu_i C_{K_i}$ and $D_K$ are the designed controller matrices.

Substituting (5) into (16) and (17), one obtains

$$\dot{\xi}(t)=\dot{A}_k(\mu,\mu)\xi(t)+B_k(\mu)C(\mu)x(t)+B_k(\mu)D_2(\mu)\omega(t) \tag{18}$$

$$u(t)=C_K(\mu)\xi(t)+D_K C(\mu)x(t)+D_K D_2(h)\omega(t)-\hat{f}(t) \tag{19}$$

Then, one gets

$$\dot{\tilde{x}}(t)=\tilde{A}(\mu,\mu)\tilde{x}(t)+\tilde{D}_1(\mu,\mu)\pi(t) \tag{20}$$
$$y(t)=\tilde{C}(\mu)\tilde{x}(t)+\tilde{D}_2(\mu)\pi(t) \tag{21}$$

where :

$\tilde{x}(t)=\begin{bmatrix}x(t)\\ \xi(t)\end{bmatrix}$, $\pi(t)=\begin{bmatrix}\omega(t)\\ e_f(t)\end{bmatrix}$,

$$\tilde{A}(\mu,\mu)=\begin{bmatrix}A(\mu)+B(\mu)\dot{D}_K C(\mu) & B(\mu)C_K(\mu)\\ B_K(\mu)C(\mu) & A_K(\mu,\mu)\end{bmatrix}$$

$$\widetilde{D_1}(\mu,\mu)=\begin{bmatrix}D_1(\mu)+B(\mu)\dot{D}_K D_2(\mu) & B(\mu)C_K(\mu)\\ B_K(\mu)D_2(\mu) & 0\end{bmatrix}$$

$\tilde{C}(\mu)=\bar{C}(\mu)=[C(h)\quad 0]$

$\widetilde{D_2}(\mu)=\overline{D_2}(\mu)=[D_2(h)\quad 0]$

*Theorem 2: Assume a fault f(t) occurs at some unknown time $t_f$ , and let α and r be two positive scalars. The eigenvalues of $\tilde{A}(\mu,\mu)$ belong to D(α,r), and the system dynamics (20) and (21) satisfy the H∞ performance index $\|y(t)\|_2 < \gamma\|\mu(t)\|_2$ , if there exist two symmetric positive definite matrices X, Y and matrices $\hat{A}_{ij}$, $\hat{B}_i$, $\hat{C}_i$ and $\hat{D}$ such that*

*min γ subject to*

$$\Pi_{ii}<0,\qquad i=1,\dots,q \tag{22}$$
$$\Pi_{ij}+\Pi_{ji}<0,\ 1\le\ i<j\le\ q \tag{23}$$
$$\Xi_{ii}<0,\qquad i=1,\dots,q \tag{24}$$
$$\Xi_{ij}+\Xi_{ji}<0,\ 1\le\ i<j\le\ q \tag{25}$$

*where*

$$\Pi_{ij}=\begin{bmatrix}-X & -I_n & \pi_{13} & A_i+B_i\hat{D}C_j-\alpha I_n\\ * & -Y & \hat{A}_{ij}-\alpha I_n & YA_i+\hat{B}_j C_i-\alpha Y\\ * & * & -r^2X & -r^2I_n\\ * & * & * & -r^2Y\end{bmatrix}$$

*and* $$\Xi_{ij}=\begin{bmatrix}\chi_{11} & \chi_{12} & D_{1i}+B_i\hat{D}D_{2j} & -B_i & XC_i^T\\ * & \chi_{22} & YD_{1i}+\hat{B}_j D_{2i} & -YB_i & C_i^T\\ * & * & -\gamma I_d & 0 & D_{2i}^T\\ * & * & * & -\gamma I_m & 0\\ * & * & * & * & -\gamma I_p\end{bmatrix}$$

*with*

$$\pi_{13}=A_iX+B_i\hat{C}_j-\alpha X$$
$$\chi_{11}=A_iX+XA_i^T+B_i\hat{C}_j+\hat{C}_j^TB_i^T$$
$$\chi_{12=}\hat{A}_{ij}^T+A_i+B_i\hat{D}C_j$$

*and* $$\chi_{22=}YA_i+A_i^TY^T+\hat{B}_jC_i+C_i^T\hat{B}_j^T$$

The parameter matrices of the controller are given by:

$D_K=\hat{D}$

$C_{Ki}=(\hat{C}_i-D_KC_iX)M^{-T}$

$B_{Ki}=N^{-1}(\hat{B}_i-YB_iD_K)$

$$C_{Kij}=N^{-1}(\hat{A}_{ij}-Y\left(A_i+B_iD_KC_j\right)X)M^{-T}-B_{Kj}C_iXM^{-T}-N^{-1}YB_iC_{Kj}$$

where M, N satisfy $MN^T=I_n-XY$.

Proof: See [5].

# III. Fish Population System and T-S Modeling

## A. Problem Formulation and Assumptions

We consider a population of exploited fish which is structured in n age classes (n ≥ 2), where every stage i is described by the evolution of its biomass $X_i$ for 0 ≤ i ≤ n. Each stage in the stock (i = 1... n) is characterized by its fecundity, mortality and predation rates. In addition, a fishing effort is included in the global mortality term. The dynamic of the fish population can be represented by the following system of ordinary differentials equations [14,15]:

$$\begin{cases} \dot{X}_0 = -\alpha_0 X_0 + \sum_{i=1}^{n} f_i l_i X_i - \sum_{i=0}^{n} p_i X_i X_0 \\ \dot{X}_1 = \alpha X_0 - (\alpha_1 + q_1 E) X_1 \\ \quad \vdots \\ \quad \vdots \\ \dot{X}_n = \alpha X_{n-1} - (\alpha_n + q_n E) X_n \\ Y = q_1 X_1 + q_2 X_2 + \dots + q_n X_n \end{cases} \quad \textbf{(26)}$$

Where:

$\alpha_i = \alpha + M_i$

$M_i$ : is the natural mortality of the individuals of the $i^{th}$ age class;

$\alpha$ : is the linear aging coefficient;

$p_0$ : is the juvenile competition parameter;

$p_i$ : is the predation parameter of class i on class 0;

$f_i$: is the fecundity rate of class i ;

$l_i$ : is the reproduction efficiency of class i ;

$q_i$ : is the catchability of the individuals of the $i^{th}$ age class;

$X_i$ : is the biomass of class i;

$E$ : is the fishing effort at time t and is regarded as an input;

$Y$ : is the total catch per unit of effort and is regarded as output;

Let us note that all the parameters of the model are positive. The recruitment from one class to another can be represented by a strictly positive coefficient of passage. The passage rate α from the juvenile class to the adult stages is supposed to be constant with respect to time and stages. This means that the time of residence is equal to 1/α. The laying eggs are considered continuous with respect to time. The total number of eggs introduced in the juvenile stage is given by $\sum_{i=1}^{n} f_i l_i X_i$. The cannibalism term $\sum_{i=0}^{n} p_i X_i X_0$ is based on the Lotka-Volterra predating term between class i and class 0. The intra-stage competition for food and space is expressed as $p_0 X_0^2$. The mortality of each stage i is caused by the fishing and natural mortality which is supposed linear [14].

One supposes that the system (26) satisfies the following assumptions:

*Assumption 1 :*

One non-linearity at least must be considered.

$\sum_{i=0}^{n} p_i \neq 0$

*Assumption 2 :*

The spawning coefficient must be big enough so as to avoid extinction.

$\sum_{i=1}^{n} f_i l_i \pi_i > \alpha_0$

where : $\pi_i = \frac{\alpha^i}{\prod_{j=1}^{i} (\alpha_j + q_j \bar{E})}$ and $\bar{E}$ is a constant fishing effort.

Under the assumptions 1 and 2 the system (26) has two equilibrium points [19] :

The first one is the origin X = 0 which corresponds to an extinct population and is therefore not very interesting. The second one is the nontrivial equilibrium $X^*$ defined as :

$X_i^* = \pi_i X_0^*$ , and $X_0^* = \frac{\sum_{i=1}^{n} f_i l_i \pi_i - \alpha_0}{\sum_{i=1}^{n} p_i \pi_i + p_0}$

*Assumption 3 :*

All age classes are subject to catch and the oldest one yield eggs.$\forall i = 1 \dots n \; q_i > 0$ and $f_i l_i \neq 0$

### *B. State Transformation*

Let $\bar{E}$ a constant fishing effort. Using the change of coordinate $x_i = X_i - X_i^*$ and $u = E - \bar{E}$ the system (26) can be transformed into:

$$\dot{x} = A(x)x + B(x)u \quad \textbf{(27)}$$

Where:

$$A = \begin{bmatrix} k_0 - p_0 x_0 & k_1 - p_1 x_1 & k_2 - p_2 x_2 & & k_n - p_n x_n \\ \alpha & -(\alpha_1 + q_1 \bar{E}) & 0 & \cdots & 0 \\ 0 & \alpha & -(\alpha_2 + q_2 \bar{E}) & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & & -(\alpha_n + q_n \bar{E}) \end{bmatrix}$$

with $k_0 = -(\alpha_0 + 2p_0 X_0^* + \sum_{i=1}^{n} p_i X_i^*)$ ,

and $k_i = l_i f_i - p_i X_i^*$; i=1...n

$$B = \begin{bmatrix} 0 \\ -q_1 X_1^* - q_1 x_1 \\ -q_2 X_2^* - q_2 x_2 \\ -q_3 X_3^* - q_3 x_3 \\ \vdots \\ -q_n X_n^* - q_n x_n \end{bmatrix}$$

### *C. Construction of TS Fuzzy Model*

For simplicity, we consider that n=2, and $x_i \in [-a, a], a \in \mathbb{R}^{*+}$. The modeling approach used in this paragraph is the sector non-linearity procedure [10].

The system (27) has two non constant terms: $x_0$, and $x_1$ . For non-constant terms, define:

$Z_i(t) = x_i$ , i=0,1.

Next, calculate the minimum and maximum values of $z_i(t)$ under $x_i \in [-a, a]$. They are obtained as follow:

max $z_i(t)$ = a , min $z_i(t)$ = -a

From the maximum and minimum values, $z_i(t)$ can be represented by :

$z_i(t) = M_i^1(z_i(t))$.a+$M_i^2(z_i(t))$.(-a)

where $M_i^1(z_i(t)) + M_i^1(z_i(t)) = 1$

Therefore the membership functions can be calculated as

$M_i^1(z_i(t)) = \frac{z_i(t)+a}{2a}$ ; $M_i^2(z_i(t)) = \frac{a - z_i(t)}{2a}$

We name the membership functions ‘‘Positive,’’ ‘‘Negative,’’ ‘‘Zero,’’ ‘‘Not zero,’’ ‘‘Big,’’ and ‘‘Small,’’ respectively.

Then, the nonlinear system (26) is represented by the following fuzzy model:

*Model Rule 1:*

*IF* $z_0(t)$ is ‘‘Negative’’ and $z_1(t)$ is ‘‘Not Zero’’ and $z_2(t)$ is ‘‘Big’’

*THEN* $\dot{x}(t) = A_1 x(t) + B_1 u(t)$

*Model Rule 2:*

*IF* $z_0(t)$ is ‘‘Negative’’ and $z_1(t)$ is ‘‘Not Zero’’ and $z_2(t)$ is ‘‘Small’’

*THEN* $\dot{x}(t) = A_2 x(t) + B_2 u(t)$

*Model Rule 3:*

*IF* $z_0(t)$ is ‘‘Negative’’ and $z_1(t)$ is ‘‘Zero’’ and $z_2(t)$ is ‘‘Big’’

*THEN* $\dot{x}(t) = A_3 x(t) + B_3 u(t)$

*Model Rule 4:*
*IF* $z_0(t)$ is ‘‘Negative’’ and $z_1(t)$ is ‘‘Zero’’ and $z_2(t)$ is ‘‘Small’’

*THEN* $\dot{x}(t) = A_4x(t) + B_4u(t)$

*Model Rule 5:*
*IF* $z_0(t)$ is ‘‘Positive’’ and $z_1(t)$ is ‘‘Not Zero’’ and $z_2(t)$ is ‘‘Big’’

*THEN* $\dot{x}(t) = A_5x(t) + B_5u(t)$

*Model Rule 6:*
*IF* $z_0(t)$ is ‘‘Positive’’ and $z_1(t)$ is ‘‘Not Zero’’ and $z_2(t)$ is ‘‘Small’’

*THEN* $\dot{x}(t) = A_6x(t) + B_6u(t)$

*Model Rule 7:*
*IF* $z_0(t)$ is ‘‘Positive’’ and $z_1(t)$ is ‘‘Zero’’ and $z_2(t)$ is ‘‘Big’’

*THEN* $\dot{x}(t) = A_7x(t) + B_7u(t)$

*Model Rule 8:*
*IF* $z_0(t)$ is ‘‘Positive’’ and $z_1(t)$ is ‘‘Zero’’ and $z_2(t)$ is ‘‘Small’’

*THEN* $\dot{x}(t) = A_8x(t) + B_8u(t)$

Here, $z_0(t)$, $z_1(t)$ and $z_2(t)$ are premise variables and

$$A_1 = \begin{bmatrix} k_0 + p_0a & k_1 + p_1a & k_2 + p_2a \\ \alpha & -(\alpha_1 + q_1\bar{E}) & 0 \\ 0 & \alpha & -(\alpha_2 + q_2\bar{E}) \end{bmatrix}$$
$$A_2 = \begin{bmatrix} k_0 + p_0a & k_1 + p_1a & k_2 + p_2a \\ \alpha & -(\alpha_1 + q_1\bar{E}) & 0 \\ 0 & \alpha & -(\alpha_2 + q_2\bar{E}) \end{bmatrix}$$
$$A_3 = \begin{bmatrix} k_0 + p_0a & k_1 + p_1a & k_2 + p_2a \\ \alpha & -(\alpha_1 + q_1\bar{E}) & 0 \\ 0 & \alpha & -(\alpha_2 + q_2\bar{E}) \end{bmatrix}$$
$$A_4 = \begin{bmatrix} k_0 + p_0a & k_1 + p_1a & k_2 + p_2a \\ \alpha & -(\alpha_1 + q_1\bar{E}) & 0 \\ 0 & \alpha & -(\alpha_2 + q_2\bar{E}) \end{bmatrix}$$
$$A_5 = \begin{bmatrix} k_0 - p_0a & k_1 - p_1a & k_2 - p_2a \\ \alpha & -(\alpha_1 + q_1\bar{E}) & 0 \\ 0 & \alpha & -(\alpha_2 + q_2\bar{E}) \end{bmatrix}$$
$$A_6 = \begin{bmatrix} k_0 - p_0a & k_1 - p_1a & k_2 - p_2a \\ \alpha & -(\alpha_1 + q_1\bar{E}) & 0 \\ 0 & \alpha & -(\alpha_2 + q_2\bar{E}) \end{bmatrix}$$
$$A_7 = \begin{bmatrix} k_0 - p_0a & k_1 - p_1a & k_2 - p_2a \\ \alpha & -(\alpha_1 + q_1\bar{E}) & 0 \\ 0 & \alpha & -(\alpha_2 + q_2\bar{E}) \end{bmatrix}$$
$$A_8 = \begin{bmatrix} k_0 - p_0a & k_1 - p_1a & k_2 - p_2a \\ \alpha & -(\alpha_1 + q_1\bar{E}) & 0 \\ 0 & \alpha & -(\alpha_2 + q_2\bar{E}) \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 0 \\ -q_1X_1^* - q_1a \\ -q_2X_2^* - q_2a \end{bmatrix}; B_2 = \begin{bmatrix} 0 \\ -q_1X_1^* - q_1a \\ -q_2X_2^* + q_2a \end{bmatrix}$$
$$B_3 = \begin{bmatrix} 0 \\ -q_1X_1^* + q_1a \\ -q_2X_2^* - q_2a \end{bmatrix}; B_4 = \begin{bmatrix} 0 \\ -q_1X_1^* + q_1a \\ -q_2X_2^* + q_2a \end{bmatrix}$$
$$B_5 = \begin{bmatrix} 0 \\ -q_1X_1^* - q_1a \\ -q_2X_2^* - q_2a \end{bmatrix}; B_6 = \begin{bmatrix} 0 \\ -q_1X_1^* - q_1a \\ -q_2X_2^* + q_2a \end{bmatrix}$$
$$B_7 = \begin{bmatrix} 0 \\ -q_1X_1^* + q_1a \\ -q_2X_2^* - q_2a \end{bmatrix}; B_8 = \begin{bmatrix} 0 \\ -q_1X_1^* + q_1a \\ -q_2X_2^* + q_2a \end{bmatrix}$$
$$\mu_1(z(t)) = M_0^2(z_0(t)) \times M_1^1(z_1(t)) \times M_2^1(z_2(t))$$
$$\mu_2(z(t)) = M_0^2(z_0(t)) \times M_1^1(z_1(t)) \times M_2^2(z_2(t))$$
$$\mu_3(z(t)) = M_0^2(z_0(t)) \times M_1^2(z_1(t)) \times M_2^1(z_2(t))$$
$$\mu_4(z(t)) = M_0^2(z_0(t)) \times M_1^2(z_1(t)) \times M_2^2(z_2(t))$$
$$\mu_5(z(t)) = M_0^1(z_0(t)) \times M_1^1(z_1(t)) \times M_2^1(z_2(t))$$
$$\mu_6(z(t)) = M_0^1(z_0(t)) \times M_1^1(z_1(t)) \times M_2^2(z_2(t))$$
$$\mu_7(z(t)) = M_0^1(z_0(t)) \times M_1^2(z_1(t)) \times M_2^1(z_2(t))$$
$$\mu_8(z(t)) = M_0^1(z_0(t)) \times M_1^2(z_1(t)) \times M_2^2(z_2(t))$$

$z_0(t)$, $z_1(t)$ and $z_2(t)$ are premise variables.

## IV. SIMULATION RESULTS AND DISCUSSION

To demonstrate the effectiveness of the proposed observer and controller, we consider a numerical example obtained from the stabilization of a fishery characterized by the parameter values given in Table 1, which are retained from the literature [4, 14,15]. Here we have employed for the simulation a constant fishing effort $E(t) = \bar{E}$ and arbitrary initial states: $X(0) = (50, 30,15)$ and $\hat{X}(0) = (20,9,50)$ .

| ***Stage i*** | 0 | 1 | 2 |
|---|---|---|---|
| $p_i$ | 0.2 | 0 | 0.1 |
| $f_i$ | | 0 | 0.5 |
| $l_i$ | | 0 | 10 |
| $m_i$ | 0.5 | 0.2 | 0.2 |
| $q_i$ | 0 | 0.1 | 0.15 |
| $a$ | | 5.6 | |
| $M_i$ | 0.5 | 0.2 | 0.2 |
| $\alpha$ | | | 0.8 |
| $\alpha_i$ | 1.3 | 1 | 1 |
| $\bar{E}$ | | 1 | |
| $x_{ini}$ | 0.9 | 3.1 | 5 |
| $x^*$ | 4.9 | 3.56 | 2.48 |

TABLE I : PARAMETER VALUES USED FOR SIMULATION

It is clear that the parameters satisfy assumptions (1), (2), (3).

The activation functions depend on the states of the system. Figure 1, 2, 3 and 4 give their time-evolution showing that the system is clearly nonlinear since $\mu_i$ are not constant functions.

By using these activation functions and the parameters given in table 1, the nonlinear system (26) can be represented exactly by the following T-S fuzzy model:

$$\dot{x}(t) = \sum_{i=1}^{8} \mu_i(x(t)) \{A_ix(t) + B_iu(t)\}$$

Here :

$x(t) = [X_0 \quad X_1 \quad X_2]^T$ ,

$$A_1 = \begin{bmatrix} 6.4890 & 0 & 9.7517 \\ 0.8000 & -1.1000 & 0 \\ 0 & 0.8000 & -1.1500 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 6.4890 & 0 & -0.2483 \\ 0.8000 & -1.1000 & 0 \\ 0 & 0.8000 & -1.1500 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 6.4890 & 0 & 9.7517 \\ 0.8000 & -1.1000 & 0 \\ 0 & 0.8000 & -1.1500 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 6.4890 & 0 & -0.2483 \\ 0.8000 & -1.1000 & 0 \\ 0 & 0.8000 & -1.1500 \end{bmatrix},$$

$$A_5 = \begin{bmatrix} -13.5110 & 0 & 9.7517 \\ 0.8000 & -1.1000 & 0 \\ 0 & 0.8000 & -1.1500 \end{bmatrix},$$

$$A_6 = \begin{bmatrix} -13.5110 & 0 & -0.2483 \\ 0.8000 & -1.1000 & 0 \\ 0 & 0.8000 & -1.1500 \end{bmatrix},$$

$$A_7 = \begin{bmatrix} -13.5110 & 0 & 9.7517 \\ 0.8000 & -1.1000 & 0 \\ 0 & 0.8000 & -1.1500 \end{bmatrix},$$

$$A_8 = \begin{bmatrix} -13.5110 & 0 & -0.2483 \\ 0.8000 & -1.1000 & 0 \\ 0 & 0.8000 & -1.1500 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ -5.3569 \\ -7.8724 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ -5.3569 \\ 7.1267 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 0 \\ 4.6431 \\ -7.8724 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 0 \\ 4.6431 \\ 7.1267 \end{bmatrix}$$

$$B_5 = \begin{bmatrix} 0 \\ -5.3569 \\ -7.8724 \end{bmatrix}, \quad B_6 = \begin{bmatrix} 0 \\ -5.3569 \\ 7.1267 \end{bmatrix}$$

$$B_7 = \begin{bmatrix} 0 \\ 4.6431 \\ -7.8724 \end{bmatrix}, \quad B_8 = \begin{bmatrix} 0 \\ 4.6431 \\ 7.1267 \end{bmatrix}.$$

Using an LMI solver, and from conditions of theorem 1 we first obtain the observer parameters. With the circle region D(5,15), the minimum attenuation value is γ = 7.8743.10-4.

Next, we design the controller. By solving the conditions in Theorem 2 with the circle region D(5, 15), one obtains the minimum attenuation value γ = 0.5964.

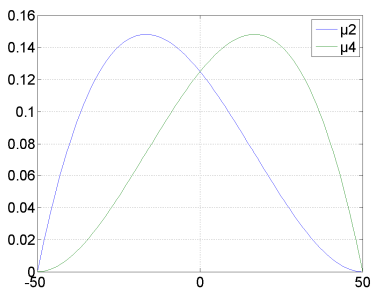


Fig. 1 Activation functions $\mu_2$ and $\mu_4$

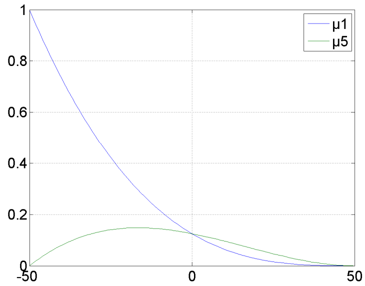


Fig. 2 Activation functions $\mu_1$ and $\mu_5$

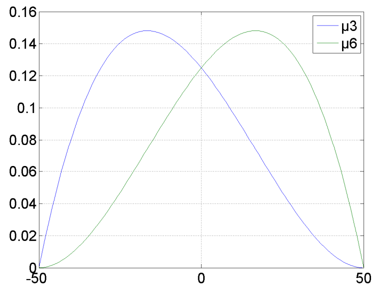


Fig. 3 Activation functions $\mu_3$ and $\mu_6$

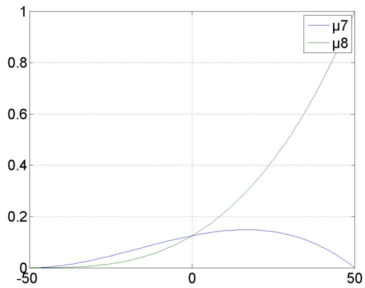


Fig. 4 Activation functions $\mu_7$ and $\mu_8$

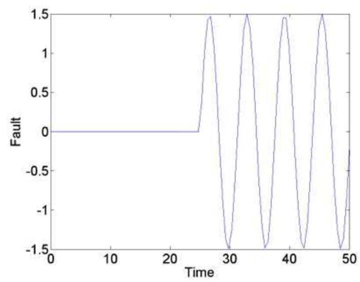


Fig. 5 Fault f(t)

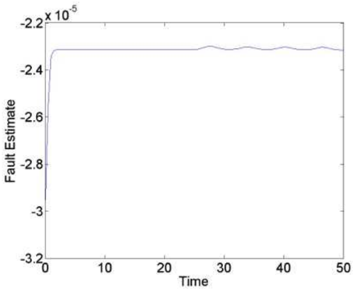


Fig. 6 Fault estimate $\hat{f}(t)$

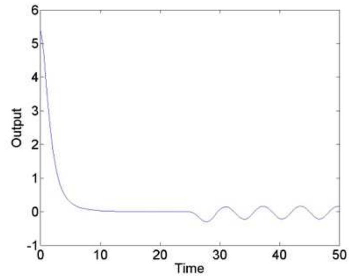


Fig. 7 Output

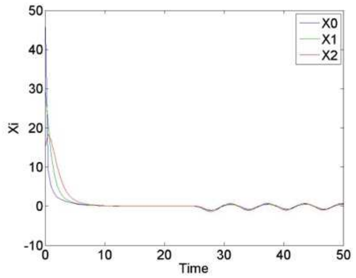


Fig. 8 States

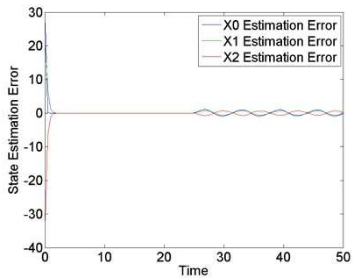


Fig. 9 State estimation error

The obtained numerical values of those matrixes: $L_i$, $F_i$, $\hat{A}_{ij}$, $B_{Ki}$ , $C_{Ki}$ and $D_{Ki}$ are not given here to limit the length of this paper.

It is supposed that ω(t) is band-limited white noise. Fig. 5 illustrates fault and Fig.6 illustrates fault estimation's simulation results, while the system output response and the system states are shown in Fig. 7 and 8 respectively. The states estimation error is given in Fig. 9.

The simulation results show the good estimation of the state of the system. Indeed, Fig. 8 shows the trajectories of the system states $x_0(t)$, $x_1(t)$ and $x_2(t)$ and their good estimation (Fig. 9) despite the presence of disturbance.

From the above simulation results, the proposed design still achieves the performance under actuator faults, and the fuzzy controller guarantees the stability of the closed-loop system. Note that, the proposed fuzzy observer can almost realize accurate fault estimation.

## V. Conclusion

In this paper, an observer-based robust fault estimation and FTC is developed for a harvested fish population system, using the T-S approach. The proposed design guarantees given stability requirements, while limiting the influence of disturbances, in the presence of actuator faults. Simulation results showed the effectiveness of the proposed design. The present paper shows that the control and estimation problems in fisheries management can also be investigated from the point of view of engineers, by combining modern control theory, computer science and mathematics.

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**Achraf Ait Kaddour** obtained the electrical engineer degree in 2002 and M.Sc degree in Control of industrial processes in 2008, from Ecole Mohammadia d’Ingénieurs, Mohammed V University, Morocco. He is a PhD student at Laboratory of Automatic Control and Industrial Computing LA2I, Department of Electrical Engineering at Ecole Mohammadia d’Ingénieurs, Mohammed V University, Morocco. His main research areas include nonlinear system control, dynamical systems, and applications of intelligent algorithms. Email:achraff@gmail.com.

**El Houssine El Mazoudi** received his Masters in Automatic Control from the Department of Physics, Faculty of Science, University of Marrakech, Morocco. In April 2008, he obtained a doctorate in Automatic Control and Computing from the Faculty of Sciences and Techniques, Fes, Morocco. Currently, he is an Assistant Professor at Cadi Ayad University, Morocco. His teachings include computing, linear algebra, calculus and mathematics for business. He is a dynamic member of the Automatic Control Group of the Laboratory of Automatic Control and Industrial Computing LA2I, Department of Electrical Engineering, Mohammadia School of Engineering, Morocco. His research works concern essentially automatic control of non-linear systems for the management of renewables ressources. Email : h_mazoudi@yahoo.fr.

**Noureddine Elalami** received his Engineering Diploma in Automatic Control and Industrial Computing from Mohammadia School of Engineering, Rabat, Morocco, in 1980. He had a bachelor’s degree and then a master in Applied Mathematics from the Faculty of Sciences, Rabat, Morocco in 1981. In 1986, he obtained his state Doctorate in Automatic Control from the University of Perpignan, France. Suprvisor: Professor Marcel Amouroux, President of the jury: Last Professor Jaques-Louis LIONS. From 1980 until 1986, he was an assistant Professor at Mohammadia School of Engineering, and since 1986 he is a Professor of Applied Mathematics, Automatic Control, Nonlinear Systems and Signal Theory at the same School. He has many researches published in international reviews concerning the control of Bilinear and non-linear systems and their applications in engineering fields (biological systems, electrical systems, chemical systems, mechanical systems). Since January 2012, he is the Director of the Automatic Control and Industrial Computing Laboratory LA2I. Email : prelalami@gmail.com.