Using hybrid genetic and Nelder-Mead algorithm for decoupling of MIMO systems with application on two coupled distillation columns process

Atef A. Lasheen, Ahmed M. El-Garhy, Elsayed M. Saad, and Saad M. Eid

Abstract— Multiple Input Multiple Output (MIMO) systems are characterized by significant interactions (i.e.: coupling) between their inputs and their different outputs. The control of MIMO systems is usually implemented using sets of Single Input Single Output (SISO) loop controllers, which requires proper input output pairing and development of decoupling compensation unit. In this paper, a generalized decoupling technique is proposed. The proposed technique uses relative gain array (RGA) to select proper pairing and hybrid genetic and Nelder-Mead algorithm (HGNMA) to estimate the optimal elements' values of the steady state decoupling compensator unit that minimize internal couplings of MIMO system. HGNMA utilizes the concept of minimizing the summation of the integral square outputs (ISOs) of non-proper paired outputs with respect to specific input. One HGNMA is assigned to each input with its own fitness function. Each HGNMA services to minimize its fitness function. The proposed technique is applied on 4 input/4 output two coupled distillation columns process, it proves remarkable success in minimizing the interaction between every input and all outputs except that output has been proper paired with.

Keywords — MIMO, RGA, Decoupling, Genetic Algorithm Nelder-Mead Algorithm.

I. Introduction

THE problem of loop interactions and decoupling control of MIMO systems has been extensively studied [1]-[8], where multivariable processes are controlled.

All of them dealing with the possible design procedures which can be summarized in; determine the RGA of the process, the appropriate selection of input-output pairs and design the appropriate decoupling compensation unit. To design the decoupling compensation unit it is necessary to estimate the elements values of the steady state decoupling matrix using detailed analytical techniques with very high mathematical burdens in high order MIMO systems. Different optimization techniques can be used to estimate the elements values of the steady state decoupling matrix. Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) were used in [9].

In this paper, a hybrid method is used, the GA in conjunction with the Nelder-Mead simplex algorithm are combined together to have the better local and global optimization searching abilities simultaneously.

HGNMA is used to estimate the optimal values of the elements of the steady state decoupling compensation matrix. The proposed HGNMA technique can be easily applied on any high order MIMO process. The paper is organized as follows: Section II addresses the decoupling concepts, Section III introduces the Hybrid Genetic Nelder-Mead Algorithm (HGNMA), Section IV explains the proposed decoupling scheme based on HGNMA, and Section V describes the thermally coupled distillation columns process as a case study. In Section VI the proposed scheme is simulated, applied on the case study, and the results are evaluated. Finally; the conclusion is given in Section VII.

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II. Decoupling

Processes with only one output being controlled by a single manipulated variable are classified as single-input single-output (SISO) systems. Many processes, however, do not conform to such a simple control configuration, where in the process industries for example, any unit operation capable of manufacturing or refining a product cannot do so with only a single control loop, and in most cases, the control system has more than one manipulated variable and more than one control input, and the interactions between these loops are such that the model cannot be further reduced. A system with Multiple Inputs and Multiple Outputs (MIMO), sometimes also called a multivariable system.

One of the most challenging aspects of the control of MIMO systems is the interaction between different inputs and outputs. This loop interaction is naturally, i.e. as a result of their physical and chemical make-up, or may also arise as a consequence of process design, and cause instability in the control system. This problem can be alleviated by a proper choice of input-output pairings to minimize the effect of each input on the outputs. This paper considers only the structures of input-output pairing of multivariable processes used in control systems design**.** The problem of decoupling control of multi-input/multi-output (MIMO) systems by state variable feedback has been extensively studied in literatures [1]-[8], where more detailed treatment of multivariable system models can be found.

The MIMO system can be described by the following state model:

$$Y_{\mathrm{m}}(\mathrm{s}) = G_{m*n}(\mathrm{s})\, u_{\mathrm{n}}(\mathrm{s}), \qquad (1)$$

Or in the matrix form as:

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_m \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) & ..... & G_{1n}(s) \\ G_{21}(s) & G_{22}(s) & ..... & G_{2n}(s) \\ \vdots & \vdots & \vdots & \vdots \\ G_{m1}(s) & G_{m2}(s) & ..... & G_{mn}(s) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \qquad (2)$$

Where:

Y(s): vector of outputs in S-domain of (M) outputs**,**

u(s): vector of inputs in S-domain of (N) inputs,

$G(s)$: system transfer function matrix of (M*N) dimension.

From (2) it can be concluded that each input has an effect on every output of the system (outputs are coupled).

To control the MIMO process the problem of interaction can be alleviated by minimizing the coupling effects which is known as "process decoupling" this includes two steps [8]:

1) Choose optimum, or 'best', pairings of inputs with outputs.
2) Development of decoupling compensators.

The goal of decoupling control is to eliminate complicated loop interactions so that a change in one process variable will not cause corresponding changes in other process variables. To do this a non-interacting or decoupling control scheme is used. In this scheme, a compensation unit called a decoupler is used before the process. This decoupler is the inverse of the gain array and allows for all measurements to be passed through it in order to give full decoupling of all of the loops. For simplicity and without loss of generality, the complete schematic of the 2 Input/ 2 Output decoupled control system based on Zalkind /Luyben assumptions is shown in Fig. 1.

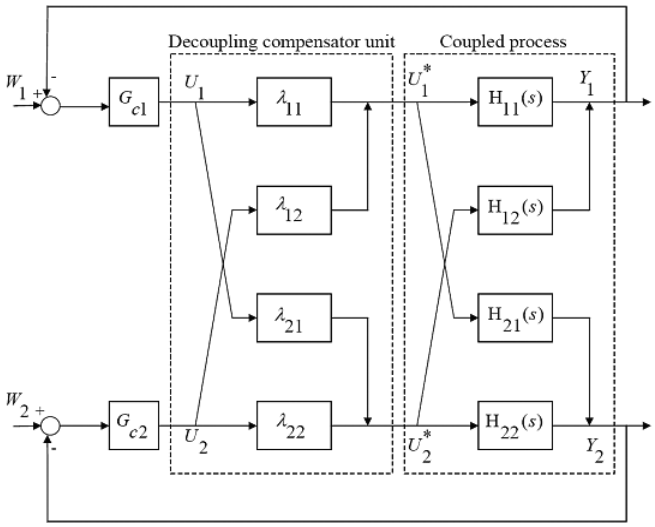

Fig. 1 2 Input/ 2 Output decoupled control system.

In decoupled control systems, each output is independently controlled by a single input. If a plant transfer matrix is diagonally dominant, it may be possible to design a good controller by considering each input-output pair as a separate loop. This approach is sometimes called decentralized control. An important issue in decentralized control design is the appropriate selection of input-output pairs. The main task in the development of decoupling compensators is to determine the values of the elements of the steady state decoupling compensation matrix [8], [9].

One way of choosing the pairing is via the relative gain array (RGA)**.** The relative gain is the ratio of the open-loop gain of some particular loop while other controllers are in manual to the same gain evaluated with the other controllers in automatic. RGA technique is not only a valuable tool for the selection of manipulative-controlled variable pairings, it has also been used to predict the behavior of controlled responses, the detailed steps of determining RGA matrix are given in [10]–[14]. APPENDIX I includes the detailed steps for getting RGA for 2 Input/2 Output process which can be easily applied for higher order processes.

Based on the assumptions of Zalkind [3] and Luyben [12], the steady state decoupling compensation matrix for N Input/N Output system has the following form:

$$\Lambda_{ss} = \begin{bmatrix} 1 & \lambda_{12} & \lambda_{13} & .... & \lambda_{1N-1} & \lambda_{1N} \\ \lambda_{21} & 1 & \lambda_{23} & .... & \lambda_{2N-1} & \lambda_{2N} \\ \lambda_{31} & \lambda_{32} & 1 & .... & \lambda_{3N-1} & \lambda_{3N} \\ & & & .... & & \\ & & & .... & & \\ \lambda_{N1} & \lambda_{N2} & & .... & \lambda_{NN-1} & 1 \end{bmatrix} \qquad (3)$$

The detailed procedures for deducing the compensation matrix for 2 Input/ 2 Output process is explained in APPENDIX II. These procedures can be extended to cover higher order MIMO processes. Although the decoupling method proposed by Zalkind and Luyben is straightforward

and produces λs independent of loop controllers, it suffers from overwhelming mathematical burdens for higher order MIMO processes [9]. In this paper, the decoupling method is modified using HGNM technique to cope with higher MIMO processes with number of inputs equals to the number of outputs.

III. HGNMA

An effective optimization technique is dependent on its searching ability for global optimum solution and its accuracy. Genetic algorithms can be very powerful to find a global optimum area but are not very fast to solve local optimization problems. However, it is sometimes very difficult to find the minimum of a function using a genetic algorithm because bad solutions can be very near to the global optimum so that when the genetic algorithm is unlucky it may have some problems to find and remain in good areas.

Local optimization techniques such as the Nelder-Mead Simplex have some common characteristics with genetic algorithm as they do not use the successive derivatives of the function and deals with a population of points instead of a single point. Furthermore, they are quite efficient to find a local optimum very quickly. In recent years, to enhance the global optimization searching ability of genetic algorithm, the genetic algorithm (GA) and the simplex method are both categorized into the primitive stage, that is, both of them are a direct search method without gradient information. Thus, it has a fast searching ability and has been widely applied to improve conditions for complicated processes [15], [16].

A. Genetic Algorithm

Genetic algorithms (GA) is directed random search techniques used to look for parameters that provide a good solution to a problem, it holds a population of solutions (often known as individuals or chromosomes). The separate parts of individuals are known as genes. Each individual is assigned a fitness value, which indicates the quality of the solution the chromosome represents. During the execution of a GA, population is continually replaced by new populations. The new populations are created by applying operators (crossover and mutation) to members of the Existing population.

Crossover is seen as the most important operator, it takes two individuals (the parents) and transfers genetic material between parents to produce new individuals (children). An individual's chance of being chosen as a parent is proportional to its fitness. This is done so that the principle of natural selection is mimicked; that is the fittest members of the population are allowed more opportunity to breed in the hope that they will pass their good genetic material to the next population. If this happens enough the population should gradually improve as fitter, and fitter individuals are created.

The process involved in GA optimization problems is based on that of natural evolution and broadly works as follows:

1) Randomly generate an initial population of potential solutions.
2) Evaluate the suitability or 'fitness' of each solution.
3) Select two solutions based on favor of fitness.
4) Crossover the solutions at a random point on the string to produce two new solutions.
5) Mutate the new solutions based on a mutation probability.
6) Go to step (2).

The above steps for optimization are shown in the following flow chart, Fig. 2.

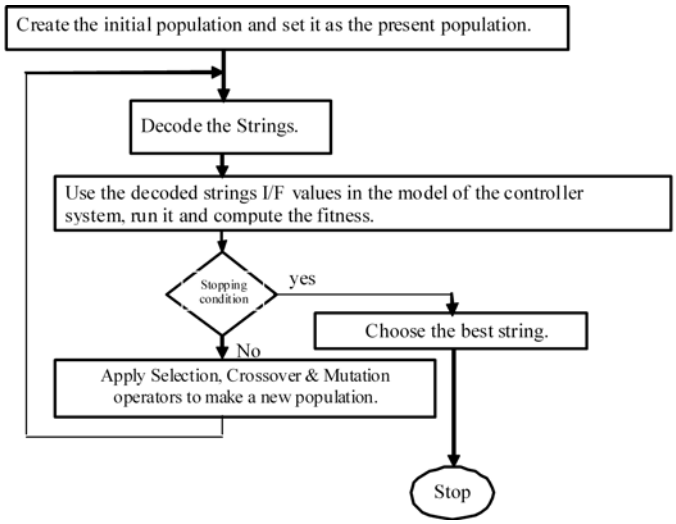

Fig. 2 Typical genetic algorithm flow chart.

B. The Nelder-Mead Algorithm

The Nelder–Mead simplex algorithm is a classical powerful local descent algorithm, making no use of the objective function derivatives. The algorithm uses a geometric construct, called simplex to achieve function optimization. A ''simplex'' is a geometrical figure consisting in, n-dimensions, of (n +1) points [16]. If any point of a simplex is taken as the origin, the other (n) points define vector directions that span the n-dimension vector space. If we randomly draw as initial starting point, then we generate the other (n) points through a sequence of elementary geometric transformations (reflection, contraction, expansion and multi-contraction), the initial simplex moves, expands or contracts. To select the appropriate transformation, the method only uses the values of the function to be optimized at the vertices of the simplex considered. After each transformation, the current worst vertex is replaced by a better one.

This algorithm has some common characteristic with the genetic algorithm, as they do not need the derivatives of the function and deals with a population of points instead of a single point. Furthermore, they are quite efficient to find a local optimum very quickly. Also, it is easy to be

programmed and fast technique. Due to its simplicity and robustness, the Nelder-Mead method is much more efficient than alternative traditional methods.

To start the algorithm we need to choose the first point to start. The algorithm is then supposed to make is own way downhill through the unimaginable complexity of an N-dimensional topology. At the beginning of iteration, a nondegenerate simplex S1 is given, along with its (n+1) vertices, each of which is a point in the search space.The first step is ordering and labeling these vertices as x_1, x_2,..., x_{n+1}, such that: $f(x_1) \leq f(x_2) \ldots \leq f(x_{n+1})$; where x_1 refers as the best point or vertex, x_{n+1} as the worst point, and x_n as the next worst point. Similarly, we refer to $f(x_1)$ as the best function value, and so on. The result of each iteration is either a single new vertex (point), the accepted point, which replaces x_{n+1} in the set of vertices for the next iteration, or a set of n new points (if a shrink is performed) that, together with x_1, from the simplex at the next iteration as indicated in the flow chart in Fig. 3.

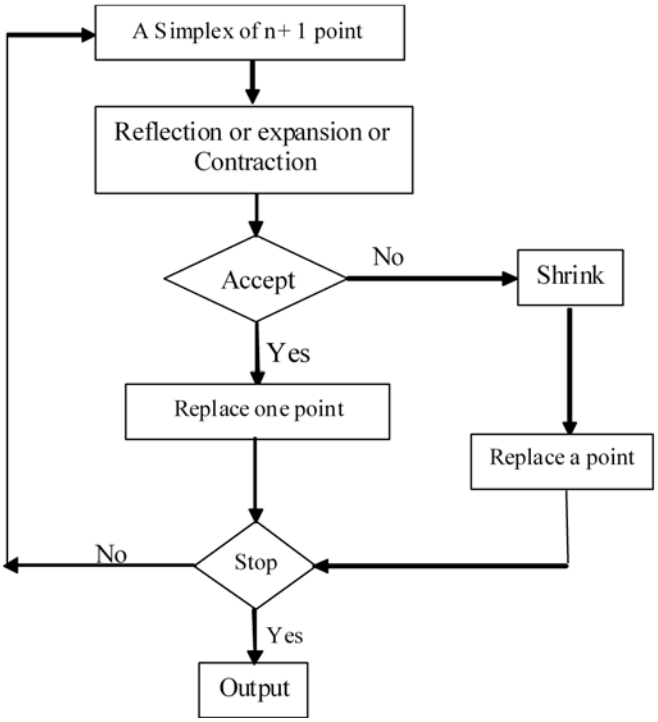

Fig. 3 Structure of Nelder-Mead simplex method.

There are many methods to utilize the idea of hybridizing local search techniques with the genetic algorithm. One idea is to use one method to find the individuals (solutions) for the new population and then apply the other method to improve this new population. The other idea of hybridizing process is to do some modifications in the genetic operations; selection, crossover, and mutation using local search methods.

In this paper we use the first idea, where the GA generates the solutions for the new population and then the Nelder-Mead technique is used to improve the best solution which exists in the new population. The Nelder-Mead technique generates a new space around the best point obtained from the GA, and search within this space about a better point. The main idea of this hybrid approach is to avoid creating random movements by using local information about promising search directions. This approach introduces two concepts: exploration – exploitation. In an exploration phase, the GA covers the whole search space, and detects a good area. The exploitation phase is then performed inside this good area by using the Nelder–Mead technique. Applying the Nelder–Mead search enhance the exploitation process, and accelerate the GA procedure.

Fig. 4 shows the management of the search space with the hybrid genetic Nelder–Mead algorithm (HGNMA).

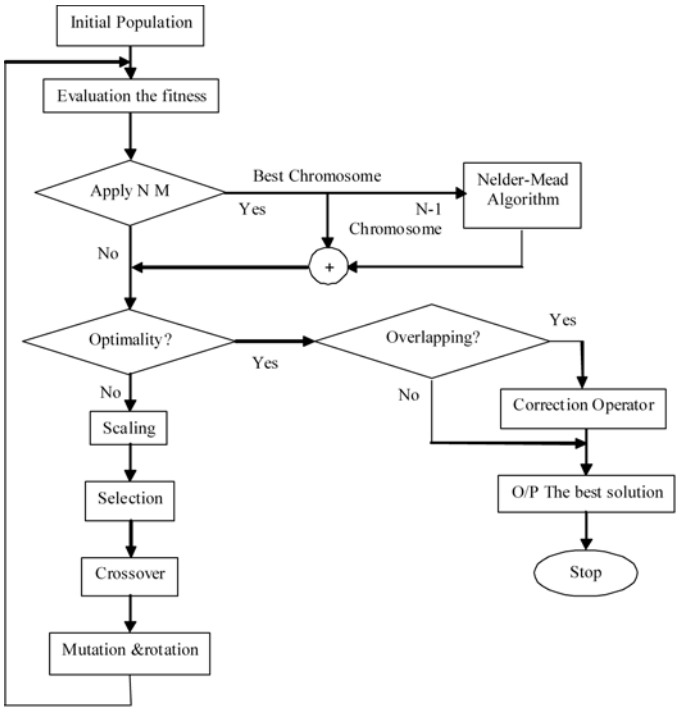

Fig. 4 Management of the search space with HGNMA.

IV. Proposed Decoupling Scheme Based on HGNMA Technique

The proposed HGNMA scheme utilizes the concept of minimizing the summation of the integral square outputs (ISOs) of non-proper paired outputs with respect to specific input. One HGNMA is assigned to each input with its own fitness function. Each HGNMA algorithm serves to minimize its own fitness function. The fitness function related to each specific input consists of the summation of integral square outputs (ISOs) due to that input except the output that has been proper paired with, this assures maintaining control on a specific output that has been proper paired with that input while minimizing the effect of that input on the remaining outputs. For *N*-Input/*N*-Output process, (N×N) ISOs are produced with (N − 1) number of λs in each ISO. Number of (N) fitness functions (one fitness function for each input Uj) is prepared to be used by (N) number of HGNMA algorithms to estimate a total

number of (N × (N −1)) decoupling compensation elements "λs" [9] .

With the following assigned subscripts:

i: subscript is assigned to the specific output,

j: subscript is assigned to specific input,

q: subscript is assigned to the output that has been proper paired with specific input j .

Then; the fitness function for specific input U_j can be written as:

$$\text{FITNESS}_j = \sum_{i=1}^{N} \text{ISO}_{ij} \quad j = 1, 2, 3, \ldots, N \qquad (4)$$

The detailed procedures for estimating ISOs are explained in APPENDIX III.

V. Case Study

Distillation units are the most widely used in separation techniques for fluid mixtures in chemical and petrochemical industries. Schematically, a distillation column is composed of a cascade of trays between which liquid and vapour phases flow in counter-current directions according to hydrodynamic diagrams depending on tray model. These interactions lead to a mass transfer so that the less volatile components are recoverable at the lower trays, whereas the lightest are recovered mainly in the upper trays of the column in addition to the condenser which is called distillate.

The main disadvantage of the distillation is its high–energy requirements. Several techniques are used to overcome this problem like integration of the distillation column with the overall process where significant energy savings can be reached, as the use of complex distillation arrangements such as Thermally Coupled Distillation Sequences (TCDS), heat integrated distillation systems, and the heat pumping techniques. The thermally coupled distillation configurations have received considerable attention because of their efficiency to reduce the energy required for the separation of ternary mixtures. The structure of the TCDS systems offers some control challenges arising from the transfer of vapor (or liquid) streams between the columns [17]-[19].

The model of a thermally coupled distillation column with side withdrawal and an additional rectifying column that we use for simulation purposes has been derived in [3], where further details about the control of coupled columns can be found.

The plant consists of two coupled distillation columns, main column A and rectifying column B, shown in Fig. 5, serving for the separation of a ternary mixture of the more volatile "methanol; MeOH", intermediate volatility "ethanol; EOH" and the less volatile "propanol; POH". The main column consists of 40 trays (including boiler and condenser stage). The side withdrawal is located at tray 11, and the feed enters the column at tray 22. The rectifying column consists of 10 trays and an additional condenser stage, where almost pure products can be withdrawn: methanol from the top of the main column, propanol from the bottom of the main column and ethanol from the top of the side column.

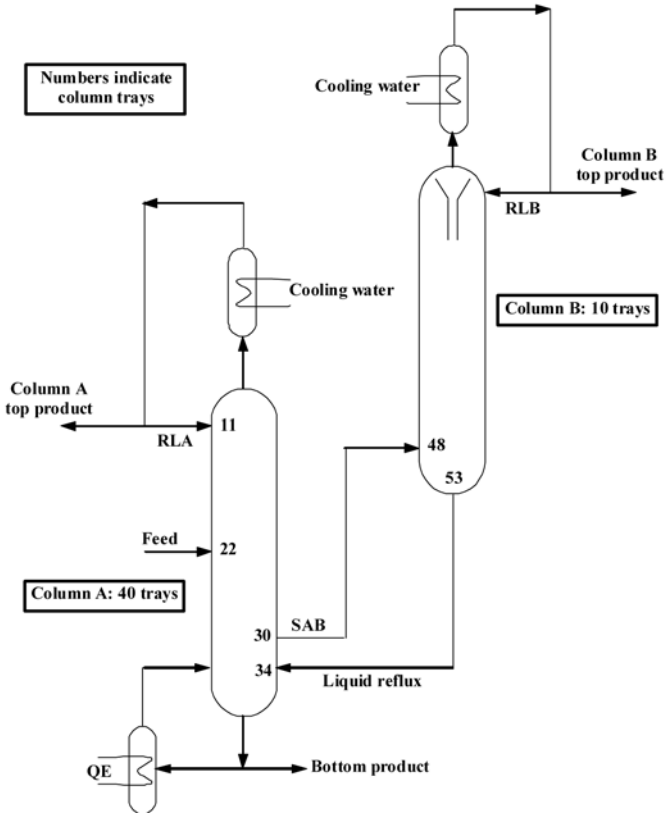

Fig. 5 Two thermally coupled column.

The model is derived under some typical assumptions:

1) Chemical and thermal equilibrium on each stage.
2) Constant liquid holdup on all stages.
3) Negligible vapor holdup.
4) Perfect mixing with ideal gas phase.
5) Constant pressure throughout the columns.
6) Total condenser behavior.
7) Saturated feed and reflux liquid flows.

Thus, for any sequence, the control of the lightest component of the ternary mixture was manipulated with the top reflux flowrate, the heaviest component with the reboiler heat duty and the control of the intermediate component, on the other hand, depended on the reflux flowrate of the side rectifier. However changes in reflux also affect bottom product composition and component fractions in the top product stream are also affected by changes in heat input.

As described in [2] there are 4 process inputs available for multivariable control as following:

1) Heat input to the reboiler (QE).
2) The vapor flow rate in the vapor transfer line (SAB).
3) The reflux ratio in the main column (RL1).
4) The reflux ratio in the second column (RL2).

The temperature is measured on each tray of both columns where it responds quickly to disturbances in opposite to concentration measurements which very often have dead times, and cause further control problems, for these reasons plates temperature are chosen as controlled variables. Thus

there are four temperature trays measurement taken as controlled variables (outputs); T11, T30, T34 and T48.

The transfer matrix of the two thermally coupled distillation columns scheme given by [3], proves very high interactions between each input and all the outputs which can cause system instability, and be written in the following form:

$$\begin{bmatrix} T_{11} \\ T_{30} \\ T_{34} \\ T_{48} \end{bmatrix} = \begin{bmatrix} \frac{2.6}{1.69s+1} & \frac{-6.098}{3.5s+1} & \cdots \\ \frac{7.32(1.05s+1)}{(10.4s+1)(0.14s+1)} & \frac{-1.45}{0.4s+1} & \cdots \\ \frac{4.6(0.53s+1)}{(2.78s+1)(0.09s+1)} & \frac{-2.37(0.23s+1)}{(2s+1)(0.3s+1)} & \cdots \\ \frac{2.11}{0.92s+1} & \frac{-2.11(0.06s+1)}{(2.38s+1)(0.05s+1)} & \cdots \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} \cdots & \frac{-4.99(0.2s+1)}{(4.5s+1)(0.06s+1)} & \frac{0.071}{3.5s+1} \\ \cdots & \frac{-1.57(0.23s+1)}{(1.34s+1)(0.2s+1)} & \frac{-0.14}{1.92s+1} \\ \cdots & \frac{-2.7}{1.75s+1} & \frac{-0.36(0.02s+1)}{(2.47s+1)(0.04s+1)} \\ \cdots & \frac{-1.75}{2.16s+1} & \frac{-0.3(1.89s+1)}{(4.35s+1)(0.16s+1)} \end{bmatrix} \begin{bmatrix} QE \\ SAB \\ RLA \\ RLB \end{bmatrix}$$

The specifications for top and bottom product purity can be met through keeping the tray temperatures within a specified range around their steady state values. Normally, the process is decoupled into a group of independent loops and suitable controller is assigned to each loop [20]–[22].

VI. Simulation and Results

The resulted RGA matrix is:

$$RGA = \begin{bmatrix} -0.042925 & 0.56288 & 0.40831 & 0.071739 \\ 1.4559 & 0.55583 & -0.9138 & -0.097971 \\ -0.46472 & -3.111 & 4.4832 & 0.09256 \\ 0.051706 & 2.9923 & -2.9777 & 0.93367 \end{bmatrix}$$

Based on the values of *RGA*, the proper pairing is determined as:

(T_{11} – SAB), (T_{30} – QE), (T_{34} – RLA), (T_{48} –RLB)

Based on the proper pairing, the following four fitness functions are deduced:

$$\begin{aligned} FITNESS_1 &= \mathrm{ISO}_{11} + \mathrm{ISO}_{31} + \mathrm{ISO}_{41} \\ &= 1.0\times10^{19}(5.3039 - 5.4790\lambda_{21} - 6.2332\lambda_{31} - 0.8320\lambda_{41} + \\ &\quad 0.4306\lambda_{21}\lambda_{41} + 3.2229\lambda_{21}\lambda_{31} + 0.4893\lambda_{31}\lambda_{41} + 1.4183{\lambda_{21}}^2 + \\ &\quad 1.8322{\lambda_{31}}^2 + 0.0327{\lambda_{41}}^2) \end{aligned}$$

$$\begin{aligned} FITNESS_2 &= \mathrm{ISO}_{22} + \mathrm{ISO}_{32} + \mathrm{ISO}_{42} \\ &= 1.0\times10^{19}(1.4185 - 5.4821\lambda_{12} + 3.2233\lambda_{32} + 0.4306\lambda_{42} - \\ &\quad 0.8323\lambda_{12}\lambda_{42} - 6.2367\lambda_{12}\lambda_{32} + 0.4893\lambda_{32}\lambda_{42} + 5.3123{\lambda_{12}}^2 + \\ &\quad 1.8325{\lambda_{32}}^2 + 0.0327{\lambda_{42}}^2) \end{aligned} \quad (6)$$

$$\begin{aligned} FITNESS_3 &= \mathrm{ISO}_{13} + \mathrm{ISO}_{23} + \mathrm{ISO}_{43} \\ &= 1.0\times10^{17}(1.0080 - 2.6799\lambda_{13} + 2.4094\lambda_{23} + 0.3346\lambda_{43} - \\ &\quad 3.1301\lambda_{13}\lambda_{23} - 0.4275\lambda_{13}\lambda_{43} + 0.4015\lambda_{23}\lambda_{43} + 2.2319{\lambda_{13}}^2 + \\ &\quad 1.4427{\lambda_{23}}^2 + 0.0284{\lambda_{43}}^2) \end{aligned}$$

$$\begin{aligned} FITNESS_4 &= \mathrm{ISO}_{14} + \mathrm{ISO}_{24} + \mathrm{ISO}_{34} \\ &= 1.0\times10^{19}(0.0324 - 0.8283\lambda_{14} + 0.4267\lambda_{24} + 0.4861\lambda_{34} - \\ &\quad 5.4545\lambda_{14}\lambda_{24} - 6.2137\lambda_{14}\lambda_{34} + 3.2005\lambda_{24}\lambda_{34} + 5.2984{\lambda_{14}}^2 + \\ &\quad 1.4047{\lambda_{24}}^2 + 1.8230{\lambda_{34}}^2) \end{aligned}$$

Four HGNMAs; namely: HGNMA1, HGNMA2, HGNMA3 and HGNMA4 are implemented based on the above fitness functions. The Number of individuals and the maximum number of generations of different HGNMAs are listed in Table I. The optimal final resulted values of steady state decoupling compensation elements are listed in Table II. Fig. 6 illustrates the evolution of fitness function. To compare the responses of the system before and after decoupling with the optimum decoupling matrix, all inputs are subjected to step inputs. The changes of step inputs are intended to be at different time instants as shown in Fig. 7 to be able to evaluate their effects on the different outputs. Fig 8 displays the response of the system before decoupling, while Fig 9 displays the response after decoupling. Fig. 9 proves that the resulted values of steady state decoupling compensation elements based on HGNMA achieve perfect decoupling. The change in any specific input affects only the output that has been proper paired with, while the effects on the remaining outputs are remarkably minimized.

Table I The number of individuals and the maximum number of generations for HGNMAs.

HGNMA	Number of individuals	Maximum no. of generations
HGNMA1	10	5
HGNMA2	20	5
HGNMA3	10	5
HGNMA4	20	10

Table II The resulted values of fitness functions, number of iterations and steady state decoupling compensation elements.

Fitness function	Final value of fitness function	Number of iterations	Values of steady state decoupling elements (λs)		
$FITNESS_1$	27.636815	532	$\lambda_{21} = -1.927265$	$\lambda_{31} = 2.926305$	$\lambda_{41} = 3.518318$
$FITNESS_2$	15.062809	607	$\lambda_{12} = 0.178861$	$\lambda_{32} = 0.886474$	$\lambda_{42} = -10.946445$
$FITNESS_3$	1.847233	434	$\lambda_{13} = 0.060843$	$\lambda_{23} = -0.790557$	$\lambda_{43} = 0.154844$
$FITNESS_4$	0.566512	1012	$\lambda_{14} = -0.007874$	$\lambda_{24} = 0.455676$	$\lambda_{34} = -0.546730$

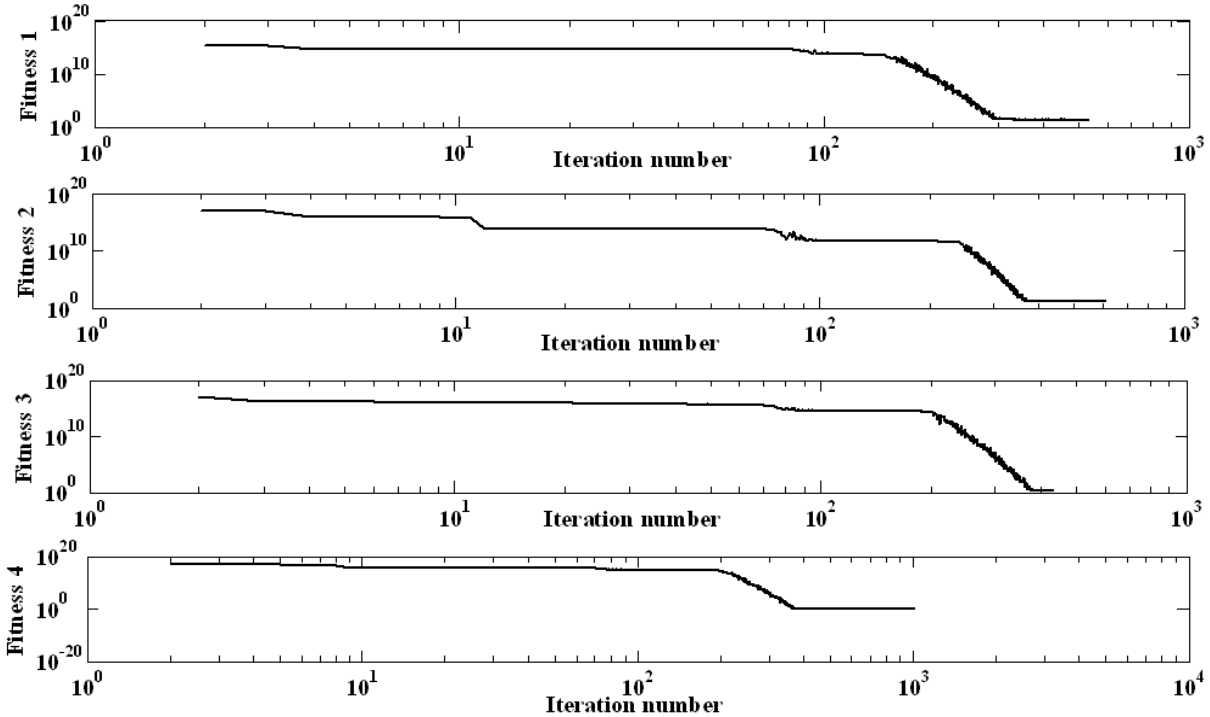

Fig. 6 The evolution of fitness functions in each iteration

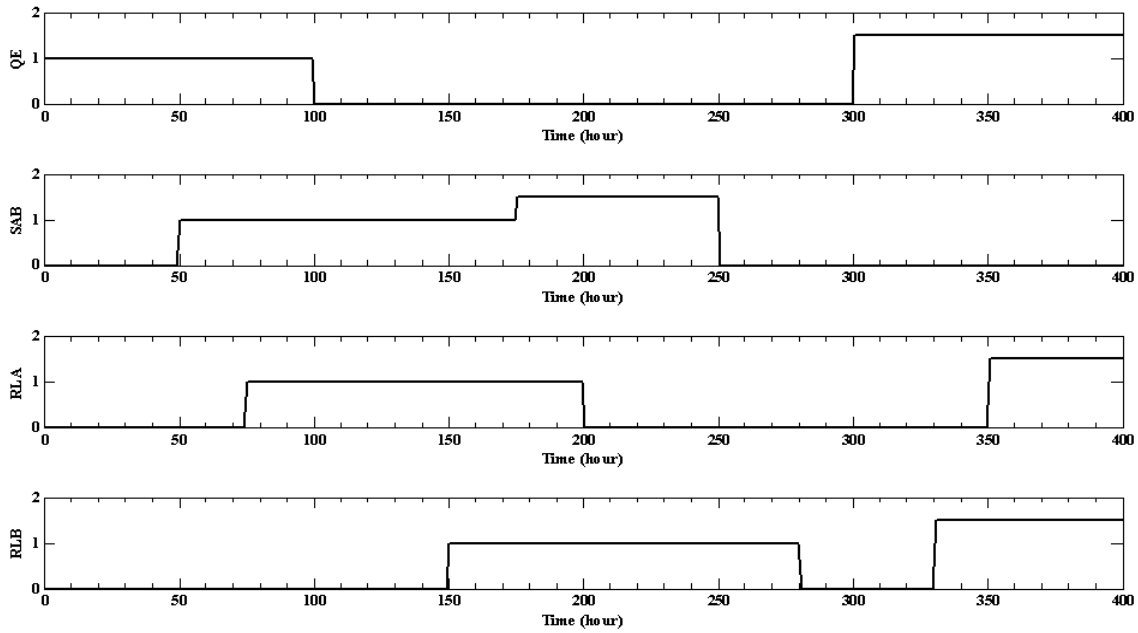

Fig. 7 Step changes in system inputs

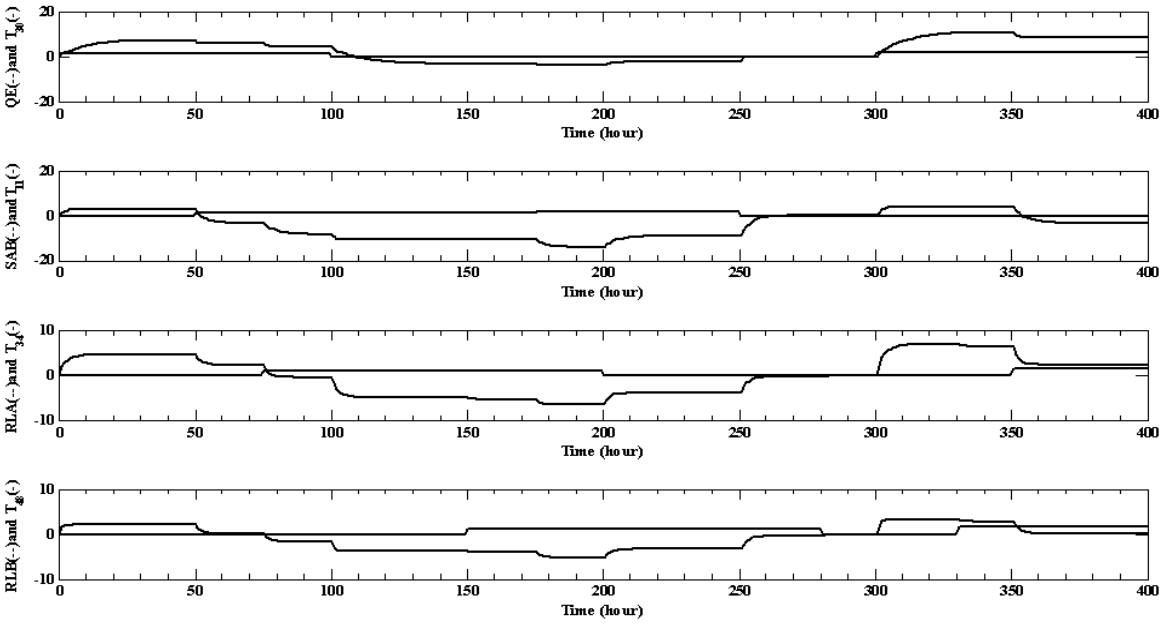

Fig. 8 System outputs due to step changes in system inputs before decoupling

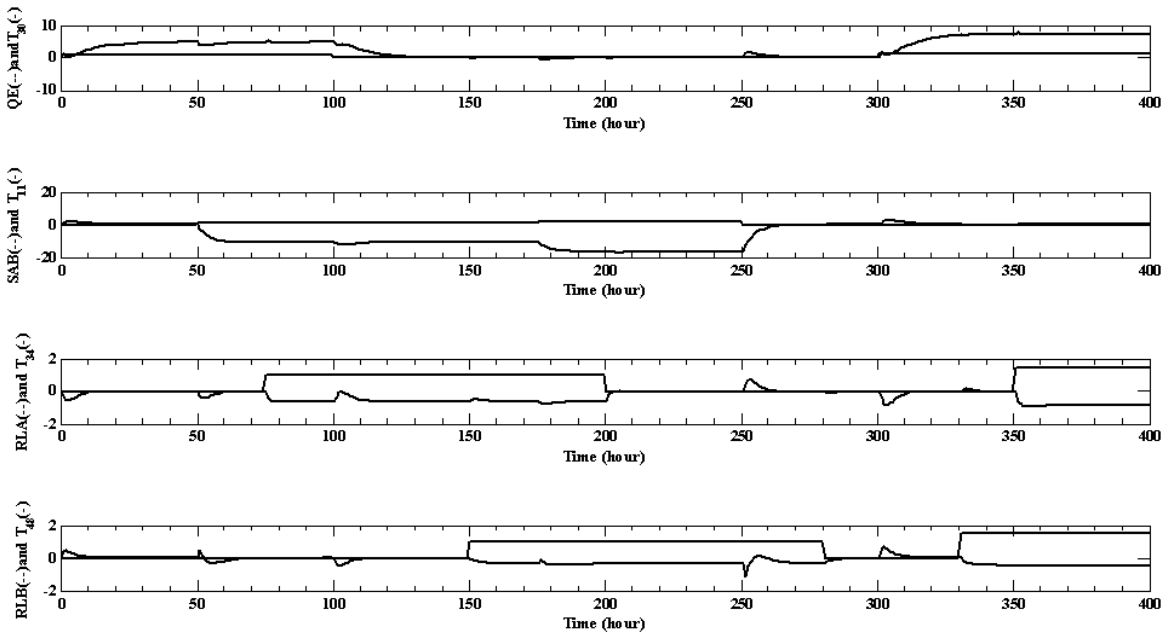

Fig. 9 System output due to step changes in system inputs after decoupling

VII. CONCLUSION

A proposed technique based on group of HGNMs is used to estimate the optimum values of steady state decoupling compensation elements that minimize the interactions between each input and its unpaired outputs. From simulations, it has been demonstrated that applying the hybrid algorithm yields to optimum values with less mathematical burdens and remarkable small number of iterations as compared to other random search techniques such as GA and PSO since the HGNM algorithm exploits the higher ability of the GA in global search and the effiency of NM in local search.

APPENDIX I: RGA FOR 2 INPUT/ 2 OUTPUT PROCESS

RGA considers the steady-state character of interactions. For simplicity and without loss of generality, determination of RGA for 2 Input/ 2 Output process can be explained as follows:

Consider 2 Input/ 2 Output system described by

$$Y(s)=\begin{bmatrix}Y_1(s)\\Y_2(s)\end{bmatrix}=\begin{bmatrix}H_{11}(s) & H_{12}(s)\\H_{21}(s) & H_{22}(s)\end{bmatrix}\begin{bmatrix}U_1(s)\\U_2(s)\end{bmatrix} \tag{7}$$

For a small step input with one variable, say u_1, we can hold either u_2 or y_2 constant and find the effect on y_1. Specifically, we find

$$\left(\frac{\Delta y_1}{\Delta u_1}\right)_{u_2} = \lim_{s\to 0} sY_1(s) = \lim_{s\to 0} H_{11}(s) \tag{8}$$

when $u_2 = 0$ and $U_1(s) = 1/s$. In addition, with the same input but the constraint $y_2 = 0$, we find

$$Y_2(s) = H_{21}(s).\frac{1}{s} + H_{22}(s).U_2(s) = 0 \tag{9}$$

So that

$$U_2(s) = -\frac{H_{21}(s)}{sH_{22}(s)} \tag{10}$$

and hence

$$\left(\frac{\Delta y_1}{\Delta u_1}\right)_{y_2} = \lim_{s\to 0} sY_1(s) = \lim_{s\to 0}\left\{H_{11}(s) - \frac{H_{12}(s)H_{21}(s)}{H_{22}(s)}\right\} \tag{11}$$

We then define the relative gain γ_{ij} between y_i and u_j as

$$\gamma_{ij} = \left(\frac{\Delta y_i}{\Delta u_j}\right)_{y_{\bar{i}}}^{-1}\left(\frac{\Delta y_i}{\Delta u_j}\right)_{u_{\bar{j}}} \tag{12}$$

The bar notation represents the 'other value' so when $i = 2$ then $\bar{i} = 1$ .Thus for $i = 1,2$ and $j = 1,2$

$$\begin{aligned}
\gamma_{11} &= \left(\frac{\Delta y_1}{\Delta u_1}\right)_{y_2}^{-1}\left(\frac{\Delta y_1}{\Delta u_1}\right)_{u_2} \\
\gamma_{12} &= \left(\frac{\Delta y_1}{\Delta u_2}\right)_{y_2}^{-1}\left(\frac{\Delta y_1}{\Delta u_2}\right)_{u_1} \\
\gamma_{21} &= \left(\frac{\Delta y_2}{\Delta u_1}\right)_{y_1}^{-1}\left(\frac{\Delta y_2}{\Delta u_1}\right)_{u_2} \\
\gamma_{22} &= \left(\frac{\Delta y_2}{\Delta u_2}\right)_{y_1}^{-1}\left(\frac{\Delta y_2}{\Delta u_2}\right)_{u_1}
\end{aligned} \tag{13}$$

The RGA matrix for 2 Inputs/ 2 Outputs system can be expressed in a matrix form $\mathrm{RGA}_{2\times 2}$ as follows

$$\mathrm{RGA}_{2\times 2} = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \tag{14}$$

In $\mathrm{RGA}_{2\times 2}$ the following relations are fulfilled

$$\begin{aligned}
\gamma_{11} + \gamma_{12} &= 1 \\
\gamma_{11} + \gamma_{21} &= 1 \\
\gamma_{21} + \gamma_{22} &= 1 \\
\gamma_{12} + \gamma_{22} &= 1
\end{aligned} \tag{15}$$

APPENDIX II: STEADY STATE DECOUPLING COMPENSATION MATRIX FOR 2 INPUT/ 2 OUTPUT PROCESS.

To explain the decoupling method proposed by Zalkind and Luyben and without loss of generality, return back to the 2 Input/ 2 Output decoupled control system shown in Fig. 1, G_{c1} and G_{c2} represent the forward path controllers for the decoupled control system. Let the forward control diagonal matrix be denoted G_c with output U, and the output of the decoupling compensators unit be denoted U^*.

The system is described by the following relationships:

$$Y = HU^* \tag{16}$$

$$U^* = \Lambda U \tag{17}$$

$$U = G_c[W - Y] \tag{18}$$

where; Λ : decoupling compensation matrix.

W: set values input.

Equations (16) – (18) yield to:

$$Y = H\Lambda G_c[W - Y] \tag{19}$$

The objective is artificially creating a situation where the forward path controllers 'think' that they are controlling two independent loops. Since G_c is a diagonal matrix, the objective will be achieved if there exist a matrix X such that

$$X = H\Lambda = \mathrm{diag}[x_1, x_2] \tag{20}$$

Equation (20) yields to:

$$\Lambda = H^{-1}X \tag{21}$$

where; $H^{-1} = \mathrm{adj}(H)/\det(H)$ (22)

$$\mathrm{adj}(H) = \begin{bmatrix} H_{22} & -H_{12} \\ -H_{21} & H_{11} \end{bmatrix} \tag{23}$$

$$\det(H) = H_{11}H_{22} - H_{12}H_{21} \tag{24}$$

Since X = diag [x_1, x_2], therefore

$$\Lambda = \begin{bmatrix} H_{22}x_1 & -H_{12}x_2 \\ -H_{21}x_1 & H_{11}x_2 \end{bmatrix} \tag{25}$$

The simplest form of decoupling compensation matrix at steady state Λ_{ss} has unity diagonal elements:

$$\lambda_{11} = \lambda_{22} = 1 \tag{26}$$

This leads to the following off-diagonal elements at steady state:

$$\lambda_{12} = \frac{-H_{12}(s=0)}{H_{11}(s=0)} \tag{27}$$

$$\lambda_{21} = \frac{-H_{21}(s=0)}{H_{22}(s=0)} \tag{28}$$

The decoupling compensation matrix at steady state can be written in the following form:

$$\Lambda_{ss} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \tag{29}$$

i.e.

$$\Lambda_{ss} = \begin{bmatrix} 1 & \dfrac{-\mathrm{H}_{12}(s=0)}{\mathrm{H}_{11}(s=0)} \\ \dfrac{-\mathrm{H}_{21}(s=0)}{\mathrm{H}_{22}(s=0)} & 1 \end{bmatrix} \tag{30}$$

APPENDIX III: DETAILED PROCEDURES FOR ESTIMATING ISOS

Based on the expression of steady state decoupling compensation matrix for N Input/ N Output process given in (3), we can write:

$$[H(s)]\Lambda_{ss} = \begin{bmatrix} \mathrm{F}_{11} & \mathrm{F}_{12} & \mathrm{F}_{13} & \cdots & \mathrm{F}_{1N-1} & \mathrm{F}_{1N} \\ \mathrm{F}_{21} & \mathrm{F}_{22} & \mathrm{F}_{23} & \cdots & \mathrm{F}_{2N-1} & \mathrm{F}_{2N} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ \mathrm{F}_{N1} & \mathrm{F}_{N2} & \mathrm{F}_{N3} & \cdots & \mathrm{F}_{NN-1} & \mathrm{F}_{NN} \end{bmatrix} \tag{31}$$

where;

$\mathrm{F}_{i1} = \mathrm{Fn}\,(\mathrm{s}, \lambda_{i1}), i = 1,2,\ldots,N$; with no appearance of λ_{11}.
$\mathrm{F}_{i2} = \mathrm{Fn}\,(\mathrm{s}, \lambda_{i2}), i = 1,2,\ldots,N$; with no appearance of λ_{22}.
$\mathrm{F}_{iN} = \mathrm{Fn}\,(\mathrm{s}, \lambda_{iN}), i = 1,2,\ldots,N$; with no appearance of λ_{NN}.

Assume $(N \times 1)$ inputs to the decoupled process such that

$$[U(s)]_{N\times 1} = \begin{bmatrix} U_1(s) \\ U_2(s) \\ U_3(s) \\ \vdots \\ U_N(s) \end{bmatrix} \tag{32}$$

then, we have $(N \times 1)$ outputs, such that

$$[Y(s)]_{N\times 1} = \begin{bmatrix} Y_1(s) \\ Y_2(s) \\ Y_3(s) \\ \vdots \\ Y_N(s) \end{bmatrix} = [\mathrm{H}(s)]\Lambda_{ss}[U(s)] \tag{33}$$

Now, the outputs $Y(s)$ are calculated as step responses for specific step input as follows:

for $U_1(s) = \dfrac{1}{s}$, and $U_2, U_3, \ldots, U_N$ are zeroes, then

$$Y_1 = \frac{\mathrm{F}_{11}}{s}, Y_2 = \frac{\mathrm{F}_{21}}{s}, \ldots, Y_N = \frac{\mathrm{F}_{N1}}{s} \tag{34}$$

for $U_2(s) = \dfrac{1}{s}$, and $U_1, U_3, \ldots, U_N$ are zeroes, then

$$Y_1 = \frac{\mathrm{F}_{12}}{s}, Y_2 = \frac{\mathrm{F}_{22}}{s}, \ldots, Y_N = \frac{\mathrm{F}_{N2}}{s} \tag{35}$$

$\vdots$

consecutively, for $U_N(s) = \dfrac{1}{s}$, and $U_1, U_2, \ldots, U_{N-1}$ are zeroes, then

$$Y_1 = \frac{\mathrm{F}_{1N}}{s}, Y_2 = \frac{\mathrm{F}_{2N}}{s}, \ldots, Y_N = \frac{\mathrm{F}_{NN}}{s} \tag{36}$$

Using Parseval theory [23], the integral square outputs (ISOs) for each input are calculated such that:

for input U_1:
$\mathrm{ISO}_{i1} = \mathrm{Fn}\,(\lambda_{i1}), i = 1,2,\ldots,N$ with no appearance of λ_{11}.
for input U_2:
$\mathrm{ISO}_{i2} = \mathrm{Fn}\,(\lambda_{i2}), i = 1,2,\ldots,N$ with no appearance of λ_{22}.
$\vdots$
consecutively,
for input U_N:
$\mathrm{ISO}_{iN} = \mathrm{Fn}\,(\lambda_{iN}), i = 1,2,\ldots,N$ with no appearance of λ_{NN}.

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