

Simulation of electromagnetic devices using advanced algorithms

Ion T. Cârstea, Daniela P. Cârstea, Alexandru Adrian I. Cârstea

Abstract— This work presents numerical algorithms for simulation of distributed-parameter systems with direct applications in electrical engineering. The algorithms are developed in the context of the finite element method. Many works in the professional literature present coupled models for the electromagnetic devices and this work is toward this direction with emphasis on the development of efficient algorithms in numerical computation of the coupled models.

Our work describes the solution of coupled electromagnetic and heat dissipation problems in two dimensions and cylindrical-coordinates system for devices with cylindrical symmetry.

The purpose of the work is to define both conventional algorithms and parallel algorithms for coupled problems in context of the finite element method. The mathematical models for electromagnetic field are based on potential formulations. Some numerical results are presented.

Keywords— Coupled fields, Finite-element method, Domain decomposition.

I. INTRODUCTION

The reality forces us to deal with complex coupled systems where two or more physical systems interact. Two or more fields coexist in the same geometry, in the same electromagnetic device. These fields interact. For example, induction heating is used for surface treatment of materials. In this practical application, the eddy currents generated by an electromagnetic inductor are used as the thermal heat sources through the Joule effect. More, any change in the physical or geometric parameters of an electromagnetic device will affect both magnetic and thermal fields. In our target examples the physical phenomena are electromagnetic and thermal. The physical properties of the materials are strongly dependent on the temperature, especially the following characteristics: electric conductivity, magnetic permeability, and specific heat and thermal conductivity.

In this work we limit our discussion to coupled electromagnetic and thermal fields. Mathematical models for the problems in which the electromagnetic field equations are

coupled to other partial differential equations, such as those describing thermal field, fluid flow or stress behaviour, are described by equations that are coupled [1]. The coupling between the fields is a natural phenomenon and only in a simplified approach the field analysis can be treated as independent problem.

In several cases, it is possible a decoupling and a cascade solution of the coupled equations. Another attractive and efficient approach of solving coupled differential equations is to consider the set as a single system. In this way a single linear algebraic system for the whole set of differential equations is obtained after discretization, and is solved to a single step. If one or more equations are non-linear, non-linear iterations of the whole system are required.

The equations of the electromagnetic fields and heat dissipation in electrical engineering are coupled because the most of the material properties are temperature dependent and the heat sources represent the effects of the electromagnetic field [1].

The thermal effects of the electromagnetic field are both desirable and undesirable phenomenon. Thus, in conducting parts of some electromagnetic devices (coils of the large-power transformers, current bars, cables conductors, conductors of the electric machines etc) the heating is an undesirable phenomenon. The heat is generated by ohmic losses of the driving currents and eddy currents induced in conducting materials. But in induction heating devices for welding the heating is a desirable phenomenon. The thermal effect of the electromagnetic field is the treatment base for many electric materials in industry [5].

With the terminology of the system theory, we identify two kinds of the heat sources (and commands in an inverse problem):

- **Distributed sources** (electrical currents)
- **Boundary sources** (Dirichlet's condition, Neumann's condition, convection and radiation)

In the heating of the electromagnetic devices, the **internal heat sources** are represented by [2]:

- **Ohmic losses from driving (source) currents**
- **Ohmic losses from eddy currents** induced in conducting materials of the time variable magnetic field
- **Dielectric losses** due to friction in the molecular polarisation process in solid dielectrics that form the insulation of the high-voltage apparatus

Manuscript received December 13, 2006; Revised received May 16, 2007
 Ion Cârstea is an Associate Professor with University of Craiova, Romania.
 E_mail: incrst@yahoo.com

Daniela P. Cârstea is a Professor with High Railways Industrial Group, Craiova, Romania. E_mail: danaerst@yahoo.com

Alexandru Adrian Cârstea is with the University of Craiova, Romania

- **Hysteresis loss** in magnetic problems. It is due to magnetic domain friction in ferromagnetic materials.

The **boundary sources** (commands) can be [2]:

- **Dirichlet command**, that is, an imposed temperature on the boundary of the spatial domain
- **Neumann command** that involves an imposed flux temperature on the boundary of the spatial domain
- **Convective command** (the temperature of the ambient medium or a cooling fluid, a parameter of the cooling fluid as the speed etc)
- **Radiation commands** (the temperature of the ambient medium or other parameters that are outside the spatial domain of the field problem and influences the temperature of a device by radiation phenomenon).

II. MATHEMATICAL MODELLING OF THE COUPLED FIELDS

For numerical simulation of the coupled systems we must have in mind some practical aspects:

- Mathematical models of electromagnetic field and thermal field
- Mathematical tools for field problems
- Mathematical methods for coupled problems

A complete mathematical model for coupled electromagnetic-thermal fields involves Maxwell's equations and the heat conduction equation. Combining these equations yields a coupled system of non-linear equations.

A complete physical description of electromagnetic field is given by Maxwell's equations in terms of five field vectors: the magnetic field \mathbf{H} , the magnetic flux density \mathbf{B} , the electric field \mathbf{E} , the electric field density \mathbf{D} , and the current density \mathbf{J} . In low-frequency formulations, the quantities satisfy Maxwell's equations [4]:

$$\nabla \times \mathbf{H} = \mathbf{J} \quad (1)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2)$$

$$\text{div } \mathbf{B} = 0 \quad (3)$$

$$\text{div } \mathbf{D} = \rho_c \quad (4)$$

with ρ_c the charge density, σ – the electric conductivity, and μ the magnetic permeability. For simplicity we give up to the bold notations for vectors.

The second set of relationships, called the constitutive relations, is for linear materials:

$$\mathbf{B} = \mu \mathbf{H}; \mathbf{D} = \varepsilon \mathbf{E}; \mathbf{J} = \sigma \mathbf{E}$$

The B-H relationship is often required to represent non-linear materials. The current density \mathbf{J} in Eq. (1) must represent both currents impressed from external sources and the internally generated eddy currents.

The formulation with vector and scalar potentials has the mathematical advantage that boundary conditions are more often easily formed in potentials than in the fields themselves. The magnetic vector potential is a vector \mathbf{A} such that the flux density \mathbf{B} is derivable from it by the operator *curl* or $(\nabla \times)$.

The mathematical models for the electromagnetic field

problems may be included in the following formulations:

- Integral equation formulations (Fredholm integral equations)
- Differential equation formulations (partial differential equations of elliptic or parabolic type)
- Hybrid formulations

The complexity of the mathematical model for electromagnetic field was one of the main reasons to find and develop new computational methods. All methods can be included in one of the following classes [4]:

- Manipulation of the equations so that some unknowns are eliminated
- Definition of some potential functions from where the field unknowns can be obtained by simple processing
- Finding of some assumptions that simplifies the computation for practical problems

The potential formulations seem attractive because of their computational advantages. One of these consists in the fact the boundary conditions are easily framed in the potentials than in the field themselves.

A. The eddy-current problems

The time-varying magnetic field within a conducting material causes circulating currents to flow within the material. These currents called eddy-currents can be unwanted or desirable phenomena. Thus, the eddy-currents in electrical machines give rise to unwanted power dissipation. On the other hand the induction heating is a wanted phenomenon in industry of the metal treatment.

Industrial equipment in which the eddy currents are essentially can be included in one of the following classes:

- **long structures**, in which the electric field and the current density possess only one component
- **complex structures** in which we use models 3D

In the *long structures*, the currents are generated by an electric field applied at the terminals of the conductor, or by a time-varying magnetic field linking the loop formed by the conductors. These structures belong to electric transmission network or the distribution networks (bus bars, large-power cables etc). In these problems the applied voltage of the bar or cable is known and we seek to compute the current density distribution within the conductor in order to determine some electromagnetic quantities of interest (the electrodynamic forces, mutual inductances, local heating etc).

The *complex structures* create difficulties in simulation and computation of their characteristics although these structures possess construction simplicity. One of these structures is the device for electric heating by electromagnetic induction. In this type the applications it is necessary to compute accurately the eddy currents. If the eddy-currents distribution is non-uniform, the resulting high-temperature gradients may crack the workpiece.

The problems are different in the two different types of applications but for any given application the presence of the saturable iron sheets introduces saturation phenomena and the problem becomes non-linear.

For each class we can apply general mathematical methods but it is more efficient to develop a particular algorithm for each kind of classes.

The **effects** of the eddy currents are:

- The time-varying magnetic **flux density** is **non-uniform** within the conductor. The alternating magnetic flux is concentrated toward the outside surface of the material (phenomenon known as the skin effect).
- **Power losses** are increased in the material

Eddy current computation appears in two types of problems:

- **Stationary** problems where the structures are fixed and source currents are time varying
- **Motion** problems where the field source is a coil in moving

Many practical engineering problems involve geometric shape and size invariant in one direction. Let z denote the Cartesian co-ordinate direction in which the structure is invariant in size and shape. This is the case of a **plane-parallel field** or **translational field** problem, where A has one component, namely A_z . This component is independent of the z co-ordinate and the Coulomb gauge is automatically imposed and V is independent of x and y . In such a case both the magnetic vector potential and the source current J_s reduce to a single component oriented entirely in the axial direction and vary only with the co-ordinates x and y [4].

Consequently, the component A_z (for simplicity we give up the subscript z) satisfies the diffusion equation in fixed bodies:

$$\nabla \cdot (\nu \nabla A) - \sigma \frac{\partial A}{\partial t} = -J_s \quad (5)$$

or, in cartesian co-ordinates:

$$\frac{\partial}{\partial x} \left(\nu \frac{\partial A}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu \frac{\partial A}{\partial y} \right) - \sigma \frac{\partial A}{\partial t} = -J_s \quad (6)$$

The boundary conditions are set-up for the single component A and can be Dirichlet and/or Neumann's condition. The interface conditions between two materials with different properties are:

$$A_1 = A_2; \quad \nu_1 \frac{\partial A_1}{\partial N} = \nu_2 \frac{\partial A_2}{\partial N}$$

where n is the normal at the common surface of the two regions with different material properties.

B. Modelling of time-dependent fields

The time dependent electromagnetic field problems are usually solved using differential models of diffusion type. Many practical problems of great interest in electromagnetics involve time-harmonic fields and this case will be considered in this work.

In general, computer software for time-varying problem can be classified into two classes [2]:

- **time-domain** programs
- **frequency-domain** programs

Time-domain programs generate a solution for a specified

time interval at different time moments. Frequency-domain programs solve a problem at one or more fixed frequencies.

The first class has some disadvantages. One of these consists in the large amount of data that must be stored to recover the field behaviour. Although the second class has an essential advantage (a compact and a cheap program in terms of the computer resources), the area of problems that can be solved is limited. It is applicable only to linear problems (all phenomena are sinusoidal).

The usual mathematical model for time dependent electromagnetic field problems is with Maxwell's equations in their normal differential form. For low frequency the displacement current term in Maxwell's equations can be neglected. At a surface of a conducting material the normal component of current density J_n can be assumed to be zero.

III. MATHEMATICAL MODELLING OF THE THERMAL FIELD

The thermal field is described by the heat conduction equation [4]:

$$\frac{\partial}{\partial t} [(c\gamma)(T) \cdot T] + \nabla \cdot [-k(T) \cdot \nabla T] = q \quad (7)$$

where: $T(x, t)$ is the temperature in the spatial point x at the time t ; point k is the tensor of thermal conductivity; γ is mass density; c is the specific heat that depends on T ; q is the density of the heat sources that depends on T . In the coupled problems we use the formula:

$$q = \rho(T) \cdot J^2 \quad (8)$$

with ρ the electrical resistivity of the material. Equation (7) is solved with boundary and initial conditions. The boundary conditions can be of different types: Dirichlet's condition for a prescribed temperature on the boundary; Neumann's condition; convection condition; radiation condition, and mixed condition [1]. These boundary conditions have the following form on different parts of the boundary surface S :

Dirichlet's condition:

$$T(x, y, z, t) \Big|_{S_1} = T_D(x, y, z, t)$$

Neumann's condition:

$$\left[k \frac{\partial T}{\partial n} + q_n \right] \Big|_{S_2} = 0$$

Convection:

$$\left[k \frac{\partial T}{\partial n} + h(T - T_\infty) \right] \Big|_{S_3} = 0$$

Radiation:

$$\left[k \frac{\partial T}{\partial n} + \epsilon \sigma_B (T^4 - T_\infty^4) \right] \Big|_{S_4} = 0$$

where the boundary surface S is:

$$S = S_1 \cup S_2 \cup S_3 \cup S_4$$

The significances of the quantities that appear in the boundary conditions are: T_D is a known function defined on the boundary S_1 ; h is the convection coefficient; T_∞ is the ambient temperature; q_n is the normal heat flux; σ_B is

Boltzman's constant in radiation and ϵ is the emissivity coefficient. The coefficients of the heat transfer as h and ϵ depend on the temperature and the surface quality. For these we use empirical formulas based on the experiments.

For many eddy-current problems the magnetic flux penetration into a conductor without internal sources of the magnetic field is confined mainly to surface layer. This is the skin effect. The skin depth δ depends on the material properties μ , ω and σ so that for the small depths all effects of the magnetic field are confined to a surface layer.

In steady-state low-frequency eddy current problems in magnetic materials, the mathematical model is the diffusion equation defined by Eq. (6).

The skin effect can be exploited in two directions:

To reduce the space domain in analysis with a fine mesh close to conductor surfaces

To reduce the material volume since a significant proportion of the conductor is virtually unused

The penetration depth is given by the formula [4]:

$$\delta = \sqrt{\frac{2}{\omega\sigma\mu}} \quad (9)$$

For example, in a semi-infinite slab of conductor with an externally applied uniform alternating field, parallel to the slab, the amplitude of flux decays exponentially. In other words for problems with the skin depth very small all the effect of the field is confined to a surface layer. In a numerical model based on finite element method (FEM) this effect can be exploited by the use of domain decomposition at the level of the problem. In this way we reduce the run-time of a program based on FEM.

Designer engineers use the formula (9) considering the permeability and the conductivity as numbers. In reality the two physical parameters change during heating. The changes in the value of δ affect the loss in the material and depend on the process (conduction or induction). For example, if the conductivity decreases by x , the depth increases by \sqrt{x} , that is the current penetrates deeper into the metal. If the magnetic material heats, its resistivity (the inverse of the conductivity) rises but its relative permeability remains substantially constant up to the Curie point. In this point it drops suddenly to unit.

Another simplifying assumption for the designer engineers is based on that all heat enters at the surface of the conductor. In reality, this is only true if the frequency of the magnetic field source is very high and the depth of heating is small compared with the geometrical dimensions of the conductor. This fact can be exploited in numerical simulation of these devices by reduction of the analysis domain.

For an accurate computation of the penetration depth of the magnetic field we must consider two practical conditions:

- The heat is distributed in the conducting part
- There is an important heat lost by radiation at the conductor surface

Radiation can be regarded as a simple surface loss subtracting from the surface power input. The Stefan-

Boltzmann's law gives the radiation loss. If the body is radiating to a surface at absolute temperature T_∞ Kelvin, the radiation loss is defined by:

$$P_r = \epsilon_r c_0 (T^4 - T_\infty^4)$$

where ϵ_r is the emissivity coefficient of the surface (dimensionless), and T is the absolute surface temperature in grades Kelvin (K). The constant c_0 is $5.67 \cdot 10^{-8} \text{ W/m}^2\text{K}^4$. For low temperatures, the radiation loss is negligible but in the induction-heating device it must be considered.

Consequently, it is convenient to use coupled models and accurate methods for computation of the heat penetration in the conductors, especially in the induction heating devices.

A. Transient problems

Many engineering applications are described by parabolic partial derivatives equations. When applying the FEM to time dependent problems, the time variable is usually treated in one of two ways:

- Time is considered as an *extra dimension* and shape functions in space and time are used
- The *nodal variables* are considered as *functions of time* and the shape functions in space are used.

A common approach for transient problems is to solve time dependent differential equations by finite differences approximation of time derivative terms, combined with some weighted residual method in space.

A widely used finite difference scheme for the first-order equations is the so-called θ . Certain values of θ correspond to known methods for time stepping:

- $\theta = 0$ the forward difference method;
- $\theta = 1/2$ Crank-Nicholson's method;
- $\theta = 2/3$ central difference method;
- $\theta = 1$ the backward difference method.

B. θ -rule combined with Galerkin's method

We illustrate the method by applying the θ -rule in time and Galerkin's method in space to the following heat conduction equation [4]:

$$\frac{\partial T}{\partial t} = \nabla(k\nabla T) + q \quad x \in \Omega, \quad t > 0 \quad (10)$$

$$T(x,0) = f(x) \quad x \in \Omega \quad (11)$$

$$-k \frac{\partial T}{\partial n} = g(x,t) \quad x \in \Gamma, \quad t > 0 \quad (12)$$

We shall present the numerical models obtained by two strategies.

Applying the θ -rule to the heat equation (10) results in the following spatial problem:

$$T^{(0)} = f(x)$$

$$\begin{aligned} \frac{T^{(m)} - T^{(m-1)}}{\delta t} &= \theta [\nabla(k\nabla T^{(m)}) + q^{(m)}] \\ &+ (1-\theta) [\nabla(k\nabla T^{(m-1)}) + q^{(m-1)}] \quad x \in \Omega \\ -k \frac{\partial T^{(m)}}{\partial n} &= g(x, t_m) \quad x \in \Gamma, \quad t > 0 \end{aligned} \quad (13)$$

where the superscript m denotes the iteration number, that is $T^{(m)} = T(x, t_m)$.

Discretizing Eq. (13) by the method of weighted residuals, with $T^{(n)}$ approximated by [7]

$$T^{(m)} = \sum_{i=1}^r T_i^{(m)} N_i(x)$$

gives an algebraic equations system:

$$([M] + [K])\{T\}^{(m)} = \{b\}^{(m-1)}$$

where the matrices [M] and [K] have the entries:

$$M_{ij} = \int_{\Omega} N_i N_j d\Omega$$

$$K_{ij} = \theta \cdot \delta t \cdot \int_{\Omega} k \nabla N_i \cdot \nabla N_j d\Omega$$

Instead of first discretizing in time by a finite difference method, first we can apply the discretizing in space by the weighted residual method, with T approximated by:

$$T(x, t) = \sum_{i=1}^r T_i(t) N_i(x)$$

By this procedure a first order differential equations system is obtained in the form:

$$\frac{d\{T\}}{dt} + [A]\{T\} = \{f\}$$

The θ -method for the time integration leads to [4]:

$$\begin{aligned} ([I] + \theta \cdot \delta t \cdot [A])\{T\}^{(m)} &= \\ ([I] - \delta t(1-\theta)[A])\{T\}^{(m-1)} &+ \{g\}^{(m)} \\ \{g\}^{(m)} &= \delta t \cdot (\theta\{f\}^{(m)} + (1-\theta)\{f\}^{(m-1)}) \end{aligned}$$

For $\theta=0$ the forward Euler's scheme is obtained and we can get $T^{(m)}$ explicitly; otherwise, a linear system must be solved at each time step. When $\theta > 1/2$ there is no stability restriction on time step δt , which can be convenient for the simulation algorithm. The choice of $\theta=1/2$ leads to an optimal combination of stability and accuracy.

IV. COUPLED MODELS FOR MAGNETIC AND THERMAL FIELDS

With a correct formulation of the mathematical models and a good selection of the mathematical tools for a specified field problem, we must select the method for the numerical solution of the field problem. Ones of these methods for field problems are moment's method, finite element method (FEM), boundary element method (BEM), hybrid method BEM-FEM, finite volume method (FVM), and edges element method (EEM).

In our works we considered the FEM [6]. This method can be viewed as a particular case of the general method of

moments, or a case of the Rayleigh-Ritz method.

When applying the FEM to time dependent problems, the time variable is usually treated in one of two ways:

- Time is considered as an *extra dimension* and shape functions in space and time are used
- The *nodal variables* are considered as *functions of time* and the shape functions in space are used.

For magnetic field we consider the A-formulation, that is we define the magnetic vector potential A by $B = \text{curl } A$. More, the domain is the same for temperature and the electromagnetic field although in practice the interest is for different field domains.

In order to solve the transient coupled set of equations a numerical model can be developed using the finite element method [7]. The finite element discretization in space is used, leading to a system of first-order differential equations:

$$[S_A] \left\{ \frac{\partial A}{\partial t} \right\} + [K_A] \{A\} + \{f_J\} = 0 \quad (14)$$

$$[S_T] \left\{ \frac{\partial T}{\partial t} \right\} + [K_T] \{T\} + [K_{AT}] \{A\} = 0 \quad (15)$$

where the matrices have the entries defined in accordance the FEM. The subscripts A and T refer to the magnetic and thermal field respectively. The vector $\{f_j\}$ is generated by the heat source.

$$q = \rho \|J\|^2 = \rho \nabla \times H \cdot \nabla \times H$$

The two equations are coupled and non-linear. Finally, the two models can be considered as a coupled system defined in matrix form [1]:

$$\begin{bmatrix} [S_A] & [0] \\ [0] & [S_T] \end{bmatrix} \begin{Bmatrix} \left\{ \frac{\partial A}{\partial t} \right\} \\ \left\{ \frac{\partial T}{\partial t} \right\} \end{Bmatrix} + \begin{bmatrix} [K_A] & [0] \\ [K_{AT}] & [K_T] \end{bmatrix} \begin{Bmatrix} \{A\} \\ \{T\} \end{Bmatrix} + \begin{Bmatrix} \{f_J\} \\ \{0\} \end{Bmatrix} = 0$$

In a discrete form the unknowns are the nodal values of the temperature T and the magnetic vector potential A. The non-linear equations for T and A are straightforwardly obtained by a Galerkin's finite element method. For the case of 2D steady-state problems we do the following approximations at the element level [1]:

$$T(x, y) = \sum_{j=1}^r N_j(x, y) T_j$$

$$A(x, y) = \sum_{j=1}^r N_j(x, y) A_j$$

where the interpolation functions N_j are basis functions in the mesh over Ω , and r is the number of nodes of an element.

The usual procedure for the FEM applications leads to a system of $2p$ equations where p is the total number of the unknowns in each field problem. Finally, the coupled problem is described by a system of algebraic systems in the form [2]:

$$f_A(A_1, \dots, A_p, T_1, \dots, T_p) = 0 \quad (16)$$

$$f_T(A_1, \dots, A_p, T_1, \dots, T_p) = 0 \quad (17)$$

where the subscript denotes the original problem (A – for

the magnetic field in the magnetic vector potential formulation; T – for the thermal field).

V. ITERATIVE ALGORITHMS FOR COUPLED FIELDS

The finite element method has three distinct logical stages: pre-processing, processing (solution) and post-processing. Each stage has an inherent parallelism that can be exploited for parallel computing. New algorithms for the parallel computers were developed and presented in the professional literature. We shall limit discussion to one of them: domain decomposition [8]. This algorithm uses the subdomain-to-subdomain iteration. Although the procedure is well known, we must modify it for coupled problems.

A. Conventional algorithms

The numerical model for coupled problem defined by Eq. (16) and Eq. (17), can be solved by two different basic strategies [1]:

- Solving the equations for T_i and A_i simultaneously
- Solving the equations for the two fields in sequence with an outer iteration, technique known as operator-splitting technique (for example Newton-Raphson procedure)

In the area of the first strategy, Gauss-Seidel and Jacobi methods are well known. We present these methods in brief.

The Gauss-Seidel algorithm for coupled fields has the following pseudo-code [1]:

For $m:=1, 2, \dots$ until **convergence** DO
Solve

$$f_A(A_1^{(m)}, \dots, A_p^{(m)}; T_1^{(m-1)}, \dots, T_p^{(m-1)}) = 0 \quad \text{with}$$

respect to $A_1^{(m)}, \dots, A_p^{(m)}$

Solve

$$f_T(A_1^{(m)}, \dots, A_p^{(m)}; T_1^{(m)}, \dots, T_p^{(m)}) = 0$$

with respect to $T_1^{(m)}, \dots, T_p^{(m)}$

In other words, the system is solved firstly with respect to A, using the values of T from the previous iteration. Afterwards, the equation derived from the thermal field model is solved using the computed values of A from the current iteration. The equations $f_A=0$ or/and $f_T=0$ are non-linear and must be solved by an iterative procedure (for example Newton-Raphson's method).

The algorithm Jacobi-type is similar to Gauss-Seidel method, except that at the iteration m when we must solve the model for T, the values for A are from the previous iteration, that is $A^{(m-1)}$. The algorithm has the following pseudo-code:

For $m:=1, 2, \dots$ until **convergence** DO

Solve

$$f_A(A_1^{(m)}, \dots, A_p^{(m)}; T_1^{(m-1)}, \dots, T_p^{(m-1)}) = 0 \quad \text{with respect to}$$

$$A_1^{(m)}, \dots, A_p^{(m)}$$

Solve

$$f_T(A_1^{(m-1)}, \dots, A_p^{(m-1)}; T_1^{(m)}, \dots, T_p^{(m)}) = 0 \quad \text{with respect}$$

to $T_1^{(m)}, \dots, T_p^{(m)}$

This algorithm has an inherent parallelism so that can be implemented in a parallel program. Practically, we decomposed the coupled problem in two subproblems: one for the magnetic field, another for the thermal field. At a time step of the algorithm, the numerical models for the two fields can be solved independently.

B. Advanced algorithms

The domain decomposition method is the best among three possible decomposition strategies for the parallel solution of PDEs, namely, operator decomposition, function-space decomposition and domain decomposition. This is one of the motivations to present the principles of the domain decomposition methods in this section [3].

The domain decomposition could be determined from mathematical properties of the problem (real boundaries or interfaces between subdomains), or from the geometry of the problem (pseudo-boundaries). For elliptic partial differential equations, there exists a mathematical approach based on the ideas given earlier in 1890 by Schwarz [8]. In Schwarz procedure there is an inherent parallelism with a data communication time for the passage of pseudo-boundary data between the subproblems.

There is no general rule for the domain or/and operator decomposition. It is defined in a somewhat random fashion. The problems and solutions that appear in the decomposition techniques depend on the following aspects [3]:

- If it is used *domain decomposition* or the *operator decomposition*
- If the partition has *disjoint* or *overlapping* sub-domains
- *The type of boundary conditions* that are set up on the pseudo-boundaries of the sub-domains
- If the decomposition is *static* or *dynamic*

A general criterion for the decomposition does not exist so that the experience of the engineer can be a useful reference for many algorithms and software products.

C. Decomposition techniques

The desire of the scientific community for faster processing on larger amounts of data has driven the computing field to a number of new approaches in this area. The main trend in the last decades has been toward advanced computers that can execute operations simultaneously, called parallel computers. For these new architectures, new algorithms must be developed and the domain decomposition techniques are powerful iterative methods that are promising for parallel computation. Ideal numerical models are those that can be divided into independent tasks, each of which can be executed independently on a processor. Obviously, it is impossible to define totally independent tasks because the tasks are so inter-coupled that it is not known how to break them apart. However, algorithmic skeletons were developed in this direction that enables the problem to be decomposed among different processors. The mathematical relationship between

the computed sub-domain solutions and the global solution is difficult to be defined in a general approach.

In the area of the coupled fields we define two levels of decomposition that is we define a hierarchy of the decompositions [1]:

- One at the level of the problem
- The other at the level of the field

In other words, we decompose the coupled problem in two sub-problems: a magnetic problem and a thermal problem, each of them with disjoint or overlapping spatial domains. This is the first level of decomposition. At the next level, we decompose each field domain in two or more subdomains. The decomposition is guided both by the different physical properties of the materials, and the difference of the mathematical models. At this level of decomposition the Steklov-Poincaré's operator can be associated with field problem [8]. This operator reduces the solution of the coupled subdomains to the solution of an equation involving only the interface values. One efficient and practical solution of elliptical partial differential equations is the dual Schur complement method [3].

VI. SOFTWARE PRODUCTS

A finite element (FE) program may be developed in a modular form (see the block diagram from the Fig. 1). FEM involves three stages:

- Pre-processing
- Solution (or processing)
- Post-processing

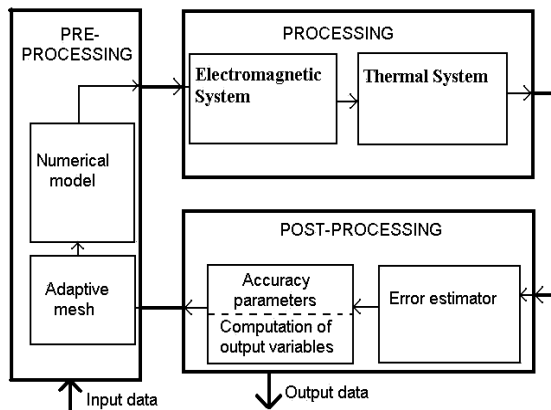


Fig. 1– Block diagram for software CAD

Each stage involves more steps that are not shown in the block-diagram. The details of the finite element programs are presented in a large professional literature so that it is not the purpose to present them in this work.

The influence of the temperature on the material properties can be used in development of efficient programs in terms of the computing resources: memory and the execution time. Some relevant aspects in the design of the CAD software for coupled magneto-thermal problems are:

The thermal source in the heat equation can be defined by the time-mean of the ohmic power loss. The motivation is simple: the time constant of the magnetic phenomenon is small compared to the diffusion time of the heat transfer.

A cascade solution may be more efficient than a fully coupled model. In some applications there is a strict coupling between magnetic and thermal equation at each time instant, but in many situations we can do separate analyses of the magnetic field and the thermal field.

It can be used a predefined temperature profile of a material for updating the magnetic field at specified temperatures. For example, at Curie temperature the material properties change dramatically [1]. After this critical point the magnetic field equation must be updated. The material characteristics are shown in the Fig. 2

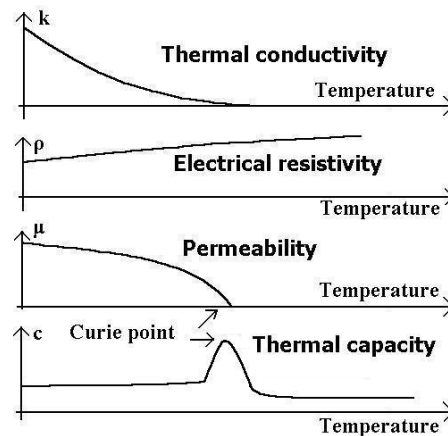


Fig.2 - Characteristics vs. temperature

The analysis domain can be divided in more subdomains with different solvers for each subdomain. In other words we can divide the analysis domain in accordance with the mathematical model of the problem.

The numerical model can be obtained by θ -rule combined with the Galerkin's method.

We must have a measure of confidence in the numerical solution. An approach for this requirement is a control of the solution accuracy. In adaptive mesh generation, the mesh is refined iteratively on the basis of error estimates. Advantages of this approach are:

- The solution is accompanied by an error estimate that is a measure of the confidence
- The solution is cheap because the nodes are added only where the accuracy is necessary
- The uninitiated can use complex programs without any fore-knowledge of the refinement strategies

The disadvantages are:

- The matrix size is increased
- The software complexity is increased
- The CPU time increases with the estimating errors

There are basically the following methods of refinement [3]:

- h - refinement (subdividing elements)
- p - refinement (increasing the polynomial order p)
- r - refinement (nodal positions are moved)
- hp-refinement (a combination of h- and p-refinements)

VII. STRESS ANALYSIS

Stress analysis problem is the utmost one that imports the temperature field from the heat transfer problem and the magnetic forces from the time-harmonic magnetic problem. The conducting medium is subjected to both temperature change and Lorenz force. Due to this magnetic and thermal loading the device components become deformed. The electrodynamic force is a vector normal to the magnetic induction **B** and the electrical current **I** according to the formula $\mathbf{F} = \mathbf{I} \times \mathbf{B}$.

In a stress analysis problem the displacement, strain and stress are of great importance. The physical quantities for stress analysis are:

- Displacement vector δ
- Strain vector ϵ and its principal values
- Stress vector σ and its principal values
- Some relevant criteria (Tresca criterion, Drucker-Prager criterion, Mohr-Coulomb criterion, Von Mises stress)

For axisymmetric problems, the displacement field is assumed to be defined by the two components of the displacement vector in direction O_r and O_z . Only three components of strain and stress tensors are independent in both plane stress and plane strain cases and four components for the axisymmetric problems due to the radial deformation.

The equilibrium equations for axisymmetric problems are:

$$\frac{1}{r} \frac{\partial(r\sigma_r)}{\partial r} + \frac{\partial\tau_{rz}}{\partial z} = -f_r$$

$$\frac{1}{r} \frac{\partial(r\tau_{rz})}{\partial r} + \frac{\partial\sigma_z}{\partial z} = -f_z$$

where σ_r , σ_z , τ_{rz} are the stress components, and f_r , f_z are components of the volume force vector [10].

Temperature strain is determined by the coefficients of thermal expansion and temperature difference between strained and strainless states. Components of the thermal strain for axisymmetric problem and orthotropic material are defined by the following equation [10]:

$$\epsilon_0 = \begin{Bmatrix} \alpha_z \\ \alpha_r \\ \alpha_\theta \\ 0 \end{Bmatrix} \cdot \Delta T$$

where α_z , α_r , α_θ are the coefficients of thermal expansion along the corresponding axes for orthotropic material, and ΔT is the temperature difference between strained and strainless states.

For linear elasticity, the stresses are related to the strains by

the constitutive law (Hooke's law):

$$\{\sigma\} = [D](\{\epsilon\} - \{\epsilon_0\}) \quad (18)$$

where [D] is a matrix of elastic constants (Young's modulus, Poisson's ratio, shear modulus), and $\{\epsilon_0\}$ is the column vector for the initial thermal strain.

VIII. SOME INDUSTRIAL APPLICATIONS

In any electromagnetic device there are power losses that are transformed in heating so that the modelling of device involves coupled mathematical models. In electrical engineering the coupled electromagnetic and thermal fields represent both desirable phenomena and undesirable phenomena. Two examples illustrate this assertion: induction heating and the high-voltage (HV) electrical transformers.

Induction heating describes the thermal conductivity problem in which the heat is generated by eddy currents induced in conducting materials, by a varying magnetic field. Induction heating is an efficient procedure for bulk-heating metals to a set temperature [5]. The heating is generated by the eddy-currents induced from a separate source of alternating current.

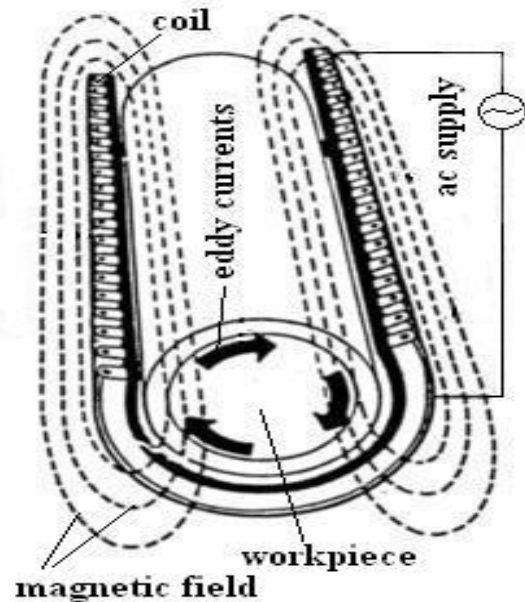


Fig.3 - Device for induction heating

Figure 3 shows a long cylindrical workpiece excited by a close-coupled axial coil [6]. The device has a cylindrical symmetry so that the problem can be reduced to a 2D-problem in the plane O_rz . An axial section is presented in Fig. 4 with: 1- the workpiece, 2 - the air and 3 - the coil. The coil is assimilated with a massive conductor. In this case we cannot ignore the eddy currents in the coil. We consider a low-frequency current in the coil so that the penetration depth is large. In this case we can decompose the whole domain of the field problem into *overlapped subdomains* for the two coupled-fields. The domain for the magnetic field can be reduced to a quarter of the device bounded by a boundary at a finite distance from the device. For the thermal field we

consider the workpiece as the analysis domain. The penetration depth of the magnetic field in the workpiece imposes the overlapping domains for the two fields [6]. The numerical model is considered in a cylindrical co-ordinates with the vertical axis Or and the horizontal axis Oz.

The mathematical model for the electromagnetic field using A-formulation is a 2D-scalar model in (r-z) plane:

$$\frac{\sigma}{r} \frac{\partial(rA)}{\partial t} - \nabla \cdot \left[\frac{\nu}{r} \nabla(rA) \right] = J_s \quad (19)$$

For the harmonic-time case, mathematical model for electromagnetic field is:

$$\frac{\partial}{\partial r} \left[\frac{\nu}{r} \frac{\partial(rA)}{\partial r} \right] + \frac{\partial}{\partial z} \left[\frac{\nu}{r} \frac{\partial(rA)}{\partial z} \right] - j\sigma\omega \frac{\nu}{r} (rA) = -J_s \quad (20)$$

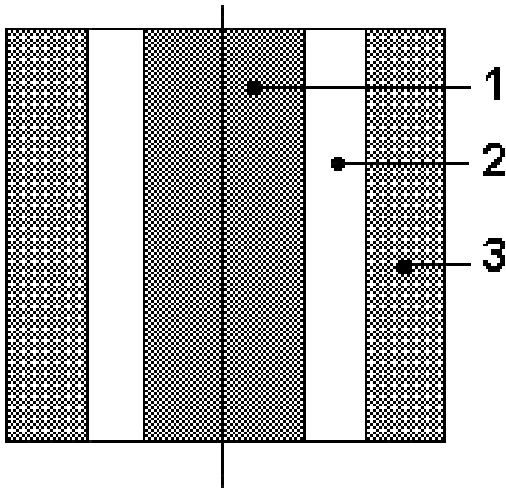


Fig. 4 – Axial section of the device

As the second example we consider a three-phase transformer in oil where the coils and limbs have rotational symmetry about the axis of the limb, that is, a transformer with cylindrical windings. The windings are wound on former cylinders and a mounted concentrically to the stepped leg of the core. An axisymmetric model is represented in Fig. 1 (an axial section by a surface perpendicular to symmetry plane). Because of the geometrical symmetry, only a half of the window is used for analysis (see Fig. 2). The tank wall is made of mild steel and symmetry axis denoted by 1 is axis Oz in a cylindrical co-ordinate system Orz. The two transformer windings, LV (low voltage) and HV (high voltage), can be balanced (as in our example) or unbalanced. The low voltage windings are usually placed next to the core. The high voltage windings are either mounted separately or wound directly over the low voltage windings (as in our example). There is a gap between the HV and LV windings, which is used for axial cooling of the windings.

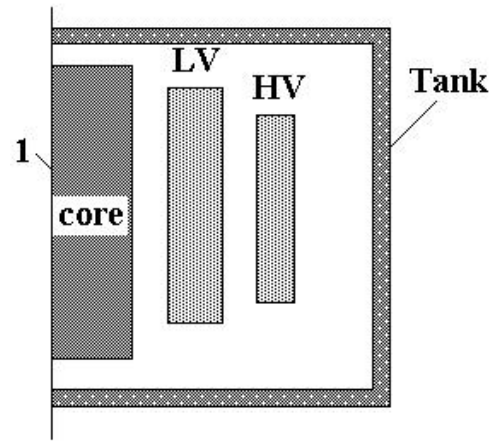


Fig. 4 - The analysis domain for the transformer

The transformer under consideration is a 400 MVA, three-phase, wound core, oil immersed, power transformer, shown in the Fig. 4. In Fig. 5 the meshed domain for triangular finite element model is shown with the horizontal axis as rotation axis [10]. The considered transformer has the voltage ratio 400 kV/24 kV.

We consider the particular case of isotropic elastic material so that in relation (18) D is a symmetric matrix whose entries are functions of only two independent parameters. These parameters are either Young's modulus E and Poisson ratio ν , or the bulk B and shear G moduli. The following relations hold among these parameters:

$$B = \frac{E}{3(1-2\nu)}; \quad G = \frac{E}{2(1+\nu)}$$

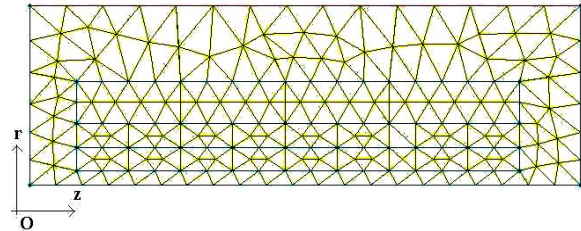


Fig. 5 - The meshed domain

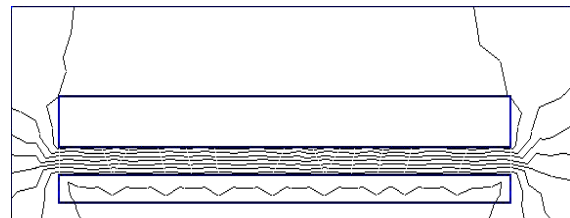


Fig. 6 - Magnetic field lines

The stress problem is the determination of the displacement vector δ of the strain field ϵ and stress tensor produced in the device by the magnetic field and temperature field. In Fig. 6 the field lines for the magnetic problem are shown. We analysed the case of high permeability for tank and yoke so

that a Neumann's boundary condition was considered. The vectors of Lorenz force are plotted in Fig. 7.

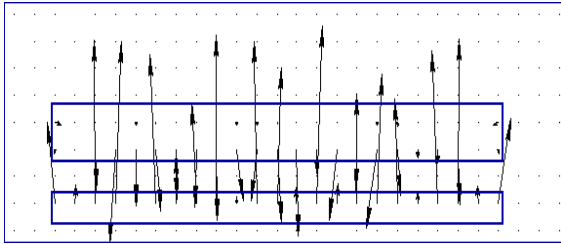


Fig. 7 - Vectors of Lorenz's forces

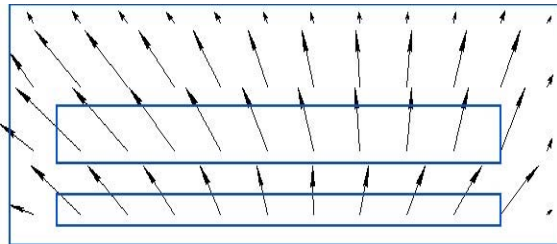


Fig. 8 - Displacement vectors

The absolute value of displacement is computed with the relation:

$$\delta = \sqrt{\delta_z^2 + \delta_r^2}$$

where δ_z and δ_r are the components along coordinate axes.

In Fig. 8 the displacement vectors are shown [10]. The force at the end of the winding builds up principally in the axial direction. During a short-circuit, the windings turns, turn insulation and spacers are subjected to alternate compression and relaxation.

IX. CONCLUSION

The problem of coupled fields in electrical engineering is a complex problem in terms of computing resources. In practice the coupled fields are treated independently in some simplified assumptions. The accuracy of the numerical computation is poor. With the new computer architectures, a multidisciplinary research is possible. Some iterative procedures were presented with emphasis on the coupled problems.

In coupled problems a hierarchy of decomposition can be defined with a substantial reduction of the computation complexity. The finite element method was used for the numerical result. The program Quickfield was used in our target examples [10].

In our future research we shall extend our results to coupled magnetic, thermal and stress analysis for important devices from the energy distribution systems as the electrical cables

and reactors. More, we have in our projects some important objectives as optimal design of the electromagnetic devices using coupled models and gradient techniques. Also, we published some works in the area of CAD for optimal control of the heat transfer in large-power cables [4].

We shall extend the results of this research to electrical transformers with non-linear magnetisation curves. In some previous works we presented the computation of the electrodynamic forces in the large power transformers using uncoupled models [4]. This approach was a design basis for many designers motivated by the computation complexity and a limited computing power of the conventional computers. With the new computers we can use coupled models for transformers and other devices of high voltage and large power.

In this work we limited our presentation to conventional algorithms. But FEM has an inherent parallelism in any stage: pre-processing, processing and post-processing. These features will be exploited in our next software. We developed our own software for mesh generation using multiblock method with good results for parallel computing [9].

REFERENCES

- [1]. Cârstea, I., Cârstea, D., Cârstea, A.A., " Simulation of electromagnetic devices using coupled models". In Volume: *SYSTEM SCIENCE and SIMULATION in ENGINEERING*. Published by WSEAS Press, 2007. ISSN: 1790-5117; ISBN: 978-960-6766-14-5. Pg. 71-79.
- [2]. Cârstea, I., "Advanced Algorithms for Coupled Problems in Electrical Engineering". In: *Mathematical Methods and Computational Techniques in Research and Education*. Published by WSEAS Press, 2007. ISSN: 1790-5117; ISBN: 978-960-6766-08-4. Pg. 31-38
- [3]. Cârstea, I., "Domain decomposition for coupled fields in electrical engineering". In: *FINITE ELEMENTS. Theory and Advanced Applications*. Published by WSEAS Press, 2007. ISBN: 975-960-8457-84-3. Pg. 54-61.
- [4]. Cârstea, D., Cârstea, I., "CAD of the electromagnetic devices. The finite element method." Editor: Sitech, 2000. Romania.
- [5]. Marchand, Ch., Foggia, A., "2D finite element program for magnetic induction heating". In: *IEEE Transactions on Magnetics*, vol. 19, no.6, November 1983.
- [6]. Cârstea, D., Cârstea, I., "A domain decomposition approach for coupled fields in induction heating devices". In: *Proceedings of the 7-th International Conference on Applied Electromagnetics (PES 2005)*, pg. 95-96. May 23-25, 2005. Nis, Serbia-Montenegro.
- [7]. Segerlind.L.J., *Applied Element Analysis*, John Wiley and Sons, 1984, USA.
- [8]. Quarteroni, A., Valli, A., "Domain Decomposition Methods for Partial Differential Equations". Oxford Science Publication. 1999
- [9]. Cârstea, D., "Multiblock Method for Database Generation in Finite Element Programs". In Volume: *Mathematical Methods and Computational Techniques in Research and Education*. Published by WSEAS Press, 2007. ISSN: 1790-5117; ISBN: 978-960-6766-08-4.
- [10]. *** *QuickField program, version 5.4*. Page web: www.tera-analysis.com. Company: Tera Analysis.