Domain decomposition for numerical simulation of induction heating devices

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Abstract**—** The work presents a domain decomposition technique for coupled fields. The mathematical model for magnetic field is based on time-harmonic Maxwell equations in vector magnetic potential formulation for axisymmetric fields. The model for the heat transfer is the heat conduction equation. A numerical model based on the finite element method is developed. The dynamic interfaces of the subdomains in the induction heating simulation can be exploited in the sense of reduction of the computational effort. The analysis domain is divided into two overlapping subdomains for the two coupled-fields considering physical significance of the pseudo-boundary of the two subdomains.

Keywords**—** Coupled fields, Finite-element method, Domain decomposition.

I. Introduction

To analyse a large complex system it is necessary for economic reasons in terms of computer resources, to divide the system into a number of small components or subsystems. Each component is considered as though it was separate problem and it is solved independently. The subsystems are finally coupled together in such way that interface conditions are satisfied at common boundaries.

The decomposition technique could be determined from mathematical properties of the problem, or from the geometry of the problem. There is no general rule for the domain or/and operator decomposition. It is defined in a somewhat random fashion but in the case in discussion there are physical considerations for domain decomposition.

Although a tremendous variety of numerical methods have been proposed for simulation of these systems, the most recently invented parallel computational strategies are largely based on the finite element (FE) and multigrid methods. The programs for the simulation of the distributed-parameter systems have an inherent parallelism when FE method is used.

We shall limit our discussion to the solution of partial differential equations (PDE) in 2D space using a distributed unstructured mesh of triangles with linear approximations within an element (piecewise linear elements), although the ideas presented here do extend to higher order elements. Although we present a single target example, the issues of the parallel implementation of FE programs are not strictly dependent upon this particular case.

In our presentation we consider an important industrial application of the coupled fields: the induction heating of the bulk metals [1]. Induction heating describes the thermal conductivity problem in which the heat sources are the eddy currents induced in conducting materials by a varying magnetic field. This is a convenient method for bulk-heating metals to an imposed temperature. We assume that the sources of the magnetic field have sinusoidal time dependence. More, we consider the quasistatic case when we neglect the effects due to displacement currents.

The mathematical model for electromagnetic and thermal fields are partial derivative equations. Finite element method [FEM] leads to coupled systems of algebraic equations. The method is very flexible with respect to geometric shape of the domain and essential boundary conditions [2]

Although the final numerical models are algebraic equations that are coupled, the numerical models for coupled problem can be obtained by one of the following approaches:

- Starting with the space discretization
- Starting with discretization in time

In the first strategy, the partial derivative equations are transformed, by finite element discretization in space, to a system of ordinary differential equations. (ODE). In this way we can use the large number of time-discretization methods developed in the professional literature of ODE world.

In the second approach, we start with one of the finite difference schemes for time discretization. One of the most known methods is the so-called θ-rule [1]. For different values of θ we obtain different schemes as the forward Euler's scheme, the backward Euler's scheme and Crank-Nicholson's scheme. Each of them has some limitations because of the stability and error in the time approximation.

II. Coupled electromagnetic-thermal fields

The electromagnetic and thermal fields coexist in the same geometry, in the same electromagnetic device. These fields interact. In our target example shown in Fig. 1, the eddy currents generated by an electromagnetic inductor are used as the thermal heat sources through the Joule effect.

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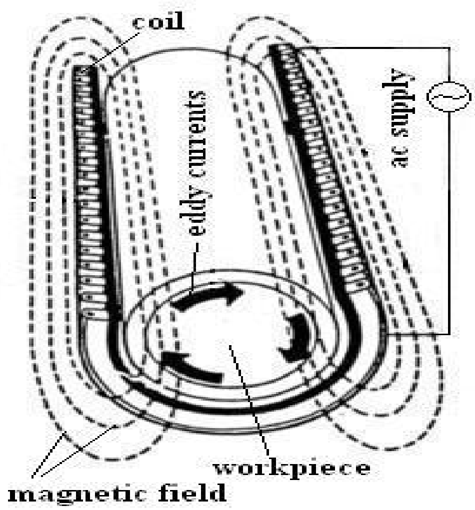

Fig. 1 – Induction heating device

A complete mathematical model for coupled electromagnetic-thermal fields involves Maxwell's equations and the heat conduction equation [1]. Combining these equations yields a coupled system of non-linear equations. The mathematical models of the two fields are coupled because the most of the material properties are strongly dependent on temperature. Especially the following characteristics depend on the temperature: electric conductivity, magnetic permeability, and specific heat and thermal conductivity.

Another important coupling term is the heat source. In our application the heat in device is generated by ohmic losses from the eddy currents.

As a general conclusion, any change in the physical or geometric parameters of an electromagnetic device will affect both magnetic and thermal fields.

In our work we use 2D-models that can be applicable to parallel-plane and axisymmetric fields. We assume that the sources of the magnetic field have sinusoidal time dependence. In this case in A-formulation, with A the magnetic vector potential, the 2D eddy current problem is described by the following equation [2]:

$$\nabla \times (\nu \nabla \times A) + j\omega\sigma A = J_S + \sigma V \qquad (1)$$

where: ω is the angular velocity of the steady-state alternating-current problem, J_s is the amplitude of the external current density, V is a scalar potential and J=-jωσA is the density of the eddy currents. In Eq. (1), A is the complex notation of the vector potential Oz, or azimuthal component. The subscript of A is not used for the simplicity.

In our target example we can do the following assumptions:

- The device geometry has a rotational symmetry
- Materials have isotropic physical properties
- Source current density has only one azimuthal component in the θ-direction
- The edges of materials (workpiece, coil etc.) are parallel to the axes of the coordinates system

In other words, we have a *rotational* or *axisymmetric field* so that the magnetic potential A has an azimuthal component but not others. In Fig. 2 the axial section of the device is shown. The mathematical model reduces to the equation [1]:

$$\frac{\partial}{\partial r}\left(\frac{\nu r}{r}\frac{\partial (rA)}{\partial r}\right) + \frac{\partial}{\partial z}\left(\nu z\frac{\partial A}{\partial z}\right) - \sigma\frac{\partial A}{\partial t} = -J_S \qquad (2)$$

The boundary conditions are Dirichlet and Neumann's conditions.

Mathematical model for the heat transfer is the heat (diffusion) equation, which has the form [8]:

$$(c\gamma)\frac{\partial T}{\partial t} = \nabla\cdot(k\nabla T) + q \quad x \in \Omega, t > 0 \qquad (3)$$

with initial and boundary conditions. In Eq. (3) the significances of the variables are: T (x, t) is the temperature in point of coordinates x at the time t; k is the thermal conductivity; γ is the density; c is thermal capacity and q is the density of the heat source. Both c and q depend on the temperature. The analysis domain is denoted by Ω.

The heat is generated by ohmic losses from the eddy currents and can be expressed in terms of the time derivative of A [8]:

$$q = \sigma\left(\frac{\partial A}{\partial t}\right)^2 \qquad (4)$$

In the case of 2D harmonic problems, the coupling term q is written as:

$$q = \sigma\omega^2 \frac{A\cdot A^*}{2} \qquad (5)$$

where A* is the complex conjugate of A.

Eq. (4) is solved with boundary conditions that can be: Dirichlet, Neumann, convection and radiation. In induction heating the radiation cannot be neglected. The Stefan-Boltzman's law gives P the radiation loss:

$$P = 5.67\times 10^{-8}\varepsilon(T^4 - T_a^4)$$

where ε is the emissivity coefficient of the surface (dimensionless), T is the absolute surface temperature (K) and T_a is the ambient temperature.

Many analysis techniques for coupled problems in induction heating were based on the simplifying assumption that all heat entered at the workpiece surface. In reality the penetration depth of the magnetic field depends on the frequency of the supply. The above assumption is true if the frequency is very high so that the depth of heating is small compared with geometrical dimensions of the workpiece (radius in our example, or thickness for a plate).

In analysis of the device shown in Fig. 1, we must consider the heat distribution in the workpiece and heat lost by radiation from the workpiece surface.

III. Numerical Models for Coupled Fields

The coupled problem in the induction heating devices is partly parabolic and partly elliptic so that this special case can be called a parabolic-elliptic problem. A discrete formulation can be obtained by the finite element method (FEM). By

Galerkin's procedure, we obtain a numerical model. There is a large amount of good books about FEM so that it is not case to present FEM. Essentially, the analysis domain is divided in linear triangular elements and the unknowns A and T are determined in the mesh nodes.

The two field problems can be solved in different space domains that are overlapping. The situation is generated by physical considerations. For example, the thermal field is of interest in the workpiece and the magnetic field is of interest at the workpiece surface. More, at high frequency of the coil current, the penetration depth of the magnetic field is small so that only a small layer of the workpiece can be considered in the analysis domain of the magnetic field. The penetration depth of the magnetic field is dependent on the temperature and time so that we have an analysis domain with a dynamic boundary.

In general, the time dependent problems after a spatial discretization can lead to a lumped-parameter model [1]:

$$f_A(A_1,..., A_p, T_1,..., T_p) = 0 \qquad (5)$$

$$f_T(A_1,..., A_p, T_1,..., T_p) = 0 \qquad (6)$$

where the subscript denotes the original problem (A – for the magnetic field in the magnetic vector potential formulation; T – for the thermal field).

It is not the purpose of this work to present the large number of numerical methods for the systems (5) and (6). For coupled problems we must find new algorithms (especially for parallel computers that are available commercially), or to modify the conventional algorithms. As example, we present an algorithm Jacobi-type with the following pseudo-code [4]:

For m: =1, 2, … until convergence *DO*
Solve

$$f_A(A_1^{(m)},...,A_p^{(m)};T_1^{(m-1)},...,T_p^{(m-1)}) = 0$$

with respect to $A_1^{(m)}, \dots A_p^{(m)}$
Solve

$$f_T(A_1^{(m-1)},..., A_p^{(m-1)}; T_1^{(m)},..., T_p^{(m)}) = 0$$

with respect to $T_1^{(m)}, \dots T_p^{(m)}$

It is obviously that that at the iteration number m when we must solve the model for T, the values for A are from the previous iteration, which is $A^{(m-1)}$. This particular aspect can be exploited in a parallel implementation of the algorithm. At a certain time moment the two equations systems (5) and (6) can be solved simultaneously so that they can be solved on different processors of a parallel computer.

A. *Case of low frequency*

We consider the low frequency case when the penetration depth of the magnetic field in workpiece is large [5]. As target example we consider a long cylindrical workpiece excited by a close-coupled axial coil (see Fig. 1). The problem is an axisymmetric heating device (see the Fig. 2) with: 1- the workpiece, 2 – the air, and 3 – the coil.

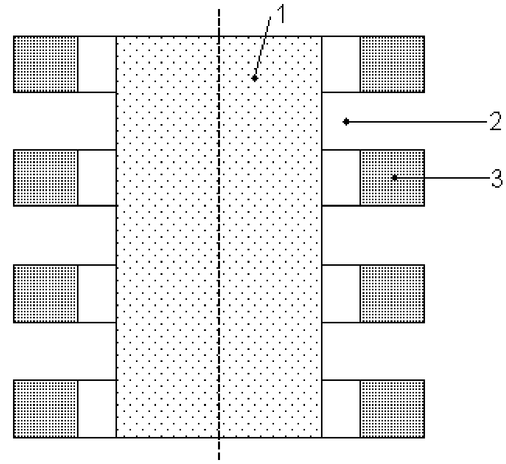

Fig.2 – Axial section of the device

The coil can be assimilated with a massive conductor. In this case we cannot ignore the eddy currents in the coil. We consider a low-frequency current in the coil so that the penetration depth is large. In this case we can decompose the whole domain of the field problem into *overlapped subdomains* for the two coupled-fields.

The domain for the magnetic field is a quarter of the device bounded by a boundary at a finite distance from the device. For the thermal field we consider the workpiece as the analysis domain [1]. The penetration depth of the magnetic field in the workpiece imposes the overlapping domains for the two fields. The numerical model is considered in a cylindrical co-ordinates with the vertical axis Or and the horizontal axis Oz.

The radiation plays an important role in induction heating at high temperature. Convection losses are small in through-heating, as the workpiece is contained in a shell that does not permit air movement. In the case the workpiece is in the open air, the convection losses are very important.

In Fig. 3 the temperatures versus time in two points are shown [10]. The curve 1 represents the temperature in the point (200,0) on the external workpiece surface. The curve 2 represents the temperature in the point (0,0) from the centre of the workpiece. The initial temperature was 40 ^{0}C. The workpiece is a steel cylinder and the coil material is copper. The current intensity is 30000 [A] and the time duration is 600 [s]. We considered natural convection at the external surface of the workpiece. The convection coefficient is 50.

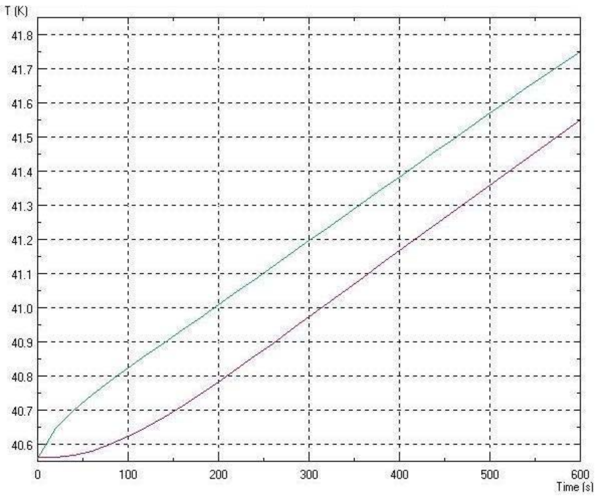

Fig. 3 – Temperatures vs. time in workpiece

B. *Case of high frequency*

For many eddy-current problems the magnetic flux penetration into a conductor without internal sources of the magnetic field is confined mainly to surface layer. This is the skin effect. The skin depth *d* depends on the material properties μ, ω and σ so that for the small depths all of the effects of the magnetic field are confined to a surface layer. The skin effect can be exploited in two directions [2]:

- To reduce the space domain in analysis with a fine mesh close to conductor surfaces
- To reduce the material volume since a significant proportion of the conductor is virtually unused

The penetration depth is given by the formula:

$$d = \sqrt{\frac{2}{\omega\sigma\mu}} \qquad (7)$$

The value of *d* changes during heating and becomes small at high frequency. In the last case the spatial domain for the magnetic field can be reduced. More, a part of the domain boundary varies in time so that at each time step a new mesh must be generated. A solution to avoid the regeneration of the mesh is to estimate the maximum value of δ from the bounds of the working frequency and to use a static boundary. In any case a substantial reduction of the spatial domain of the magnetic problem is reduced in comparison with the low working frequencies. The thermal field can be limited to the workpiece where the distributed heat sources are in a small layer.

C. *Efficient models*

The influence of the temperature on the material properties can be used in development of efficient programs in terms of the computing resources: memory and the execution time. Some relevant aspects in the design of the CAD software for coupled magneto-thermal problems are:

- The thermal source in the heat equation can be defined by the time-mean of the ohmic power loss. The motivation is simple: the time constant of the magnetic phenomenon is small compared to the diffusion time of the heat transfer.
- A cascade solution may be more efficient than a fully coupled model. In some applications there is a strict coupling between magnetic and thermal equation at each time instant, but in many situations we can do separate analyses of the magnetic field and the thermal field.
- It can be used a predefined temperature profile of a material for updating the magnetic field at specified temperatures. For example, at Curie temperature the material properties change dramatically. After this critical point the magnetic field equation must be updated.
- The analysis domain can be divided in more subdomains with different solvers for the coupled problems([6], [9]).

In Fig. 4 the variations of some principal properties versus temperature are shown (by *r* we denoted the material resistivity) [8]. For example, the resistivity of the metals varies with the temperature by a law that we can approximate by a linear form:

$$r = r_0[1 + \alpha(T - T_0)]$$

where r_0 is the resistivity at a specified temperature (frequently the ambient temperature equal to 20 ^{0}C).

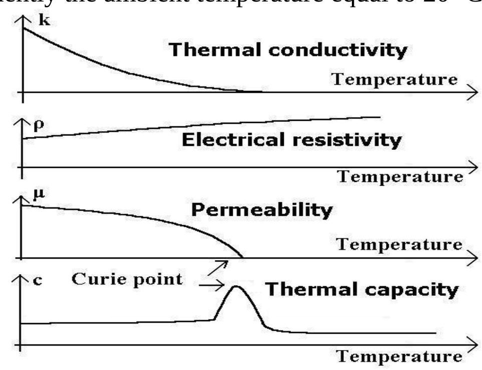

Fig. 4 – Material characteristics versus temperature

From the curves presented in Fig.4, it is obviously that the magnetic field quantities must not update at every time moment. In this way a cascade solution of the coupled problem is better than other approaches.

IV. Software products for coupled problems

We limit the discussion to the software based on FEM. Any software product of type CAD has three distinct stages

- Pre-processing
- Processing (solution)
- Post-processing

In Fig. 5 the conventional algorithmic skeleton for the coupled problems is shown. Our interest is for the solution of the numerical model (processing stage) although in some previous works we presented our own software product for automatic mesh generation [2]. The multiblock method is adequate for parallel implementation in a high-performance software.

Each stage of finite element method has an inherent parallelism so that a parallel program is the best approach for numerical algorithms based on FEM.

There are many software products in the area of CAD based on FEM but there is room for improvement of these products. An error estimator is a difficult task and the accuracy control of the numerical results is an open problem. There are some general strategies as h-method, p-method or hybrid method presented in professional literature. It is obviously that an adaptive mesh involves an increased computational effort because of the regeneration of the whole database for the program.

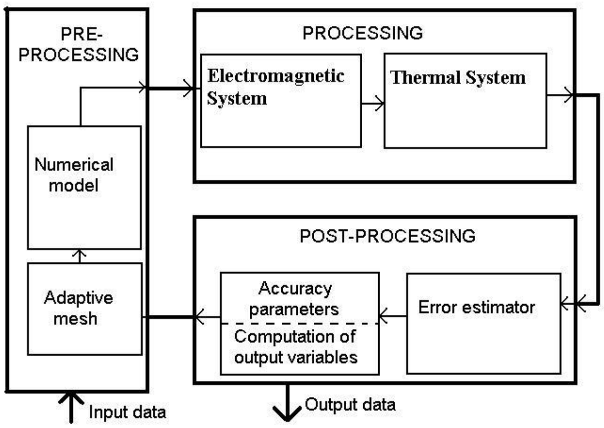

Fig. 5 – Block diagram for software CAD

Some relevant aspects in the design of the CAD software for coupled magneto-thermal problems are:

- The thermal source term in the heat equation can be defined by the time-mean of the ohmic power loss.
- A cascade solution may be more efficient than a fully coupled model.
- It can be used a predefined temperature profile of a material for updating the magnetic field at specified temperatures.
- The analysis domain can be divided in more subdomains with different solvers for each subdomain

V. Decomposition techniques

An important measure of the algorithm and program performance is the number of operations and the execution time. Consequently we must invent new algorithms for the new architectures as parallel computers. Domain decomposition is one of the most efficient approaches for the parallel computers. In this approach the entire analysis domain is divided into several parts (subdomains), which can be disjoint or overlapped [1]. The subdomains are connected to each other by interface boundaries.

The partition of the analysis domain is an open problem. There is no general rule for the domain decomposition and many experts in this area do decomposition in a somewhat random fashion. The problems and their solutions differ by the boundary conditions on the subdomain interfaces, the choice of the partition (disjoint or overlapped subdomains), if the decomposition is static or dynamic and the decomposition granularity. The solutions of these problems can influence with different weighted factors the whole performance of the algorithm, and finally, the program performance.

In the FEM context, the requirements of partitioning should be:

- Minimisation of the number of neighbours for each subdomain
- Minimisation of the number of nodal values at interfaces between subdomains

Balancing the size of each subdomain that will have a direct consequence the balancing the computational load between different processors

In the context of the FEM, the subdomains of a partition can be viewed as super-elements that are managed independently. In other words, each subdomain may have more elements. In assembly stage of finite element (FE) program, element matrices are generated for each element and assembled finally. At the level of the subdomain, the element matrices are assembled into a sub-matrice. All these sub-matrices are to be assembled into the final matrices of the numerical model for the whole domain.

Ideal numerical models are those that can be divided into independent tasks, each of which can be executed independently on a processor. Obviously, it is impossible to define totally independent tasks because the tasks are so inter-coupled that it is not known how to break them apart.

In coupled problems from engineering, the partition of the whole analysis domain can be guided by physical considerations. More, some aspects of the partition, as the dynamics of the pseudo-boundaries (the boundaries between subdomains), can be imposed.

In magneto-thermal problems we must follow a special strategy imposed by physical phenomena from device. Thus, the domain decomposition must me applied with some particularities:

- We can use a decomposition hierarchy
- The interfaces are imposed by physical properties of the system components
- The pseudo-boundaries have a dynamics that depends on the temperature, that is we have dynamic interfaces

We consider a decomposition of tree type with two levels:

- One at the level of the problem
- The other at the level of the field

In other words we decompose the coupled problem in two sub-problems: an electromagnetic problem and a thermal problem, each of them with disjoint or overlapping spatial domains. This is the first level of decomposition and decomposition is guided by the different nature of the two fields that interact. The analysis domain for the electromagnetic problem depends on the frequency of the supplying source. At high frequencies only a layer of the thick equal to penetration depth of the magnetic field is considered. The thermal field is computed in the workpiece, that is a part of the whole domain.

At the next decomposition level, we decompose each field domain in two or more subdomains. The decomposition is guided both by the different physical properties of the materials, and the difference of the mathematical models. At this level of decomposition the Steklov-Poincaré's operator can be associated with field problem [9]. This operator reduces the solution of the coupled subdomains to the solution of an equation involving only the interface values. One efficient and practical solution of elliptical partial differential equations is the dual Schur complement method.

The second aspect mentioned above has an engineering motivation. The pseudo-boundaries are imposed by the material interfaces where some physical restrictions must be fulfilled. Thus, if the device is built from different materials, a pseudo-boundary must be the interface of the materials.

The third aspect of the decomposition is imposed by the effect of the temperature on the magnetic field penetration in conductor. Concerning the target example, with a steel workpiece, at each time moment the temperature is modified and value of depth is changed. In other words we have dynamic interfaces at the first decomposition level. More, the two analysis domains are overlapped and the overlapping area varies in time. In other words for dynamic decomposition, the analysis domain Ω at the time t^* is expressed as a union of n subdomains [7]:

$$\Omega(t^*) = \bigcup_{i}^{n(t^*)} \Omega_i(t^*)$$

On each subdomain a related equation with initial and boundary conditions is defined. The difficulty of the dynamic decomposition consists in the fact that new domain decomposition must be performed at the next time step.

A. Domain decomposition at low frequency

An iteration-by-subdomain algorithm obtains the solution of the field problem. Let us consider a partition with 2 subdomains, that is $\Omega=\Omega_1 \cup \Omega_2$. Denoting by u_i the restriction of the unknown u (that can be A or T) to Ω_i, i=1,2, the original problem is equivalent to two similar problems with the original problem provided that the following transmission conditions are imposed on the interface [7]:

$$F(u_1) = F(u_2)$$

$$G\left(\frac{\partial u_1}{\partial n}\right) = G\left(\frac{\partial u_2}{\partial n}\right)$$

where F and G are known functions.

In magnetic field problems we can use alternating Dirichlet-Neumann pseudo-boundaries, that is, the functions F and G have the form F(u)=*u and* G(u)= $\partial u/\partial n$. Here n is the unit normal on the interface directed from Ω_1 to Ω_2 .

Let us consider the numerical solution of the general partial differential equation:

$$Lu = f$$

where L is a differential operator and u (x, t) is a function that belongs to a specified functions space, and satisfies given boundary and initial conditions. We denote by C the boundary of the analysis domain and C_{12} the interface boundary. For simplicity we consider a Dirichlet's condition on the boundary equal to a known value g. The iteration-by-subdomain algorithm assumes the following steps [2]:

For k=0,1,... until convergence do

$Lu_1^{k+1} = f$ *in* Ω_1

$u_1^{k+1}=g$ *on* $C\text{-}C_{12}$

$F(u_1^{k+1})=F(u_2^{k})$ *on* C_{12}

$Lu_2^{k+1} = f$ *in* Ω_2

$u_2^{k+1}=g$ *on* $C\text{-}C_{12}$

$G(u_2^{k+1})=G(u_1^{k+1})$ *on* C_{12}

end do

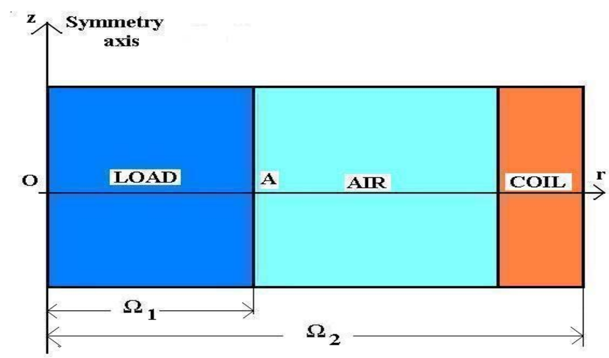

Fig. 6 – Domain decomposition at low frequency

In Fig. 6 the domain decomposition at low frequency is shown for the first level of decomposition. The domain for the electromagnetic field is Ω_2. At the second level of decomposition we can use a decomposition guided by different criterion. One of them is the nonlinearity of the mathematical model. Thus, the analysis domain of the magnetic field can be divided into subdomains:

- A linear subdomain (coil, air)
- A non-linear subdomain (workpiece)

The two subproblems are separately solved but they are coupled by continuity (transmission) conditions on the common boundary (interface).

B. Domain decomposition at high frequency

For many eddy-current problems the magnetic flux penetration into a conductor without internal sources of the magnetic field is confined mainly to surface layer whose thick is the penetration depth ***d*** (skin effect).

In Fig. 7 the scheme for domain decomposition is shown. It is obviously that the overlapping partition is dynamic (depends on the temperature). The skin depth *d* depends on

- The material properties μ and σ
- The frequency of the ac supply

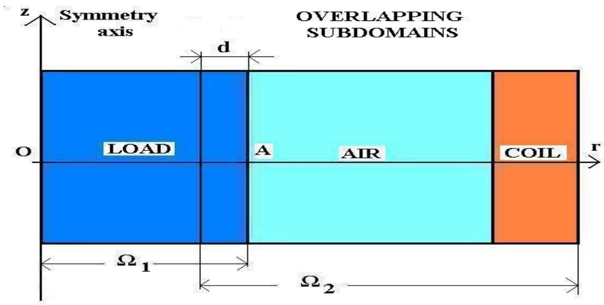

Fig. 7 – Domain decomposition at high frequency

The skin effect can be exploited in two directions:

- To reduce the space domain in analysis with a fine mesh close to conductor surfaces
- To reduce the material volume since a significant proportion of the conductor is virtually unused

VI. Numerical results

We present the induction heating of tubes in a furnace with a coil with 8 windings connected in series [10]. The height of the workpiece is 5600 [mm]. In our numerical simulation the eddy currents in the coil were considered. The analysis domain with the mesh is shown in Fig. 8. On the outer boundary a Dirichlet's condition was imposed. The geometry and the physical properties are defined as follows: electric conductivity of the workpiece (steel) - 1.10^7 [S/m], specific heat - 200 [J/Kg.K], density - 7800 [Kg/m^3], relative magnetic permeability is 500, and thermal conductivity – 100 [W/K.m].

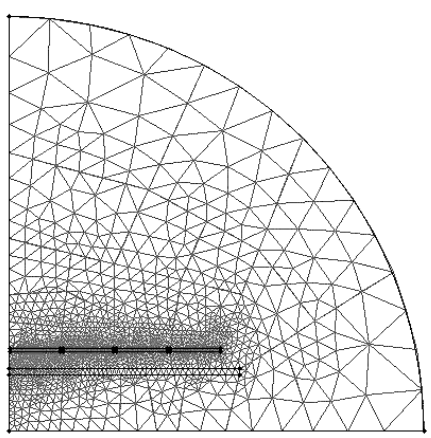

Fig. 8 - Meshed domain of a large furnace

The coil material is copper. The amplitude of the source current is 30000 [A] at a frequency of 10 000 [Hz]. The mesh has 2300 nodes and was generated using the software Quickfield [10]. In Fig. 9 the final axial temperature on the external workpiece surface is plotted, starting from the centreline (axis Or in our case).

The non-uniformity of the axial distribution of the surface temperature is a consequence of the non-uniformity of coil heating at high-density currents.

The temperatures differ on the internal and outer surfaces of the workpiece because the external surface is near the coil. In practice, a refractory material is put in the gap between the coil and workpiece. More, in large power coils the windings conductors have internal channels for cooling with water.

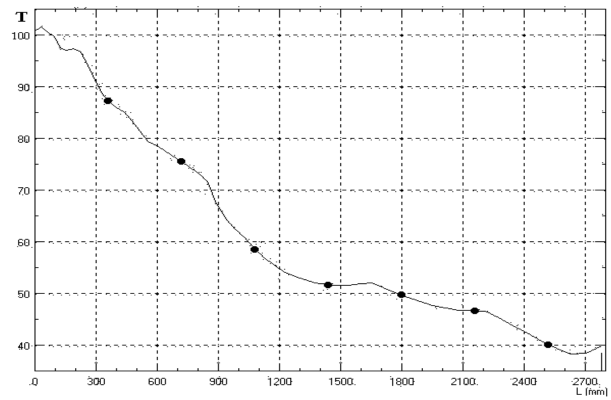

Fig. 9 – Final temperature of the workpiece

VII. Conclusions

The problem of coupled fields in electromagnetic devices is a complex problem so that an analytical solution is impossible. The numerical simulation is the single approach for the analysis of the device. In this work we tried to present some computational aspects in coupled magneto-thermal fields in the context of the finite element method. Although we limited the presentation from the programmer's viewpoint of a conventional computer, the results can be extended to parallel computing.

Our target example is of great importance for many industrial applications of the eddy-currents.

Domain decomposition offers an efficient approach for large-scale problems or complex geometrical configurations. This method in the context of the finite element programs leads to a substantial reduction of the computing resources as the time of the processor.

In coupled problems a hierarchy of decomposition was defined with a substantial reduction of the computation complexity. Each level of decomposition has its own advantages generated by the large number of conventional algorithms for each field problem.

Although we limited the presentation to the domain decomposition considering physical properties of the field problem, the partitioning of the domain can be performed according to the mathematical models of the field problem (operator decomposition). There are different models in different subregions of the electromagnetic device. For example, in the electromagnetic system the mathematical model can be an elliptic or parabolic equation, that is, we have an elliptic-parabolic problem.

For time-dependent partial differential equations the decomposition of the spatial domain is an evolutionary process. The dynamic interfaces increase the algorithm and program complexity. An approximation of the dynamic decomposition by a static decomposition is possible for a known range of the source frequencies.

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