Algorithms for numerical analysis of high-voltage insulation

Ion T. Cârstea, Daniela P. Cârstea, Alexandru Adrian I. Cârstea

Abstract— Insulation (dielectric) heating is caused by losses due to friction of the molecular polarisation process in dielectric materials. A polluted insulation has a finite resistance so that the leakage current in the insulation heats the insulation. The problem of heating is a coupled thermal-electric problem. The paper presents an algorithm based on a 2D model for coupled fields in the insulation of a high-voltage cable in some operating conditions (loaded and unloaded). The heat transfer in insulation is described by the heat conduction equation where the heat sources are both internal sources generated by the leakage current in a resistive insulation, and the boundary heat sources of the convective and Dirichlet/Neumann type.

Keywords— Coupled fields, Finite-element method, Insulation.

I. Introduction

The goal of this work is to determine the electrical field in high-voltage direct-current (HVDC) cables using numerical solutions and its thermal effects. An analytical solution is possible only in particular cases. The numerical solution is a way to obtain accurate results and to study in laboratory conditions many operating regimes. The numerical methods are widespread because they are well suited for any geometry and accurate. By numerical simulation we reduce the number of development tests, which is a reduction of the design stage is made by a minimum effort.

The computation of field distribution in direct-current (DC) cables is more complex than in alternating current (AC) cables [1]. The field distribution in DC cables depends on the insulation permittivity and conductivity, the geometry of the cable and the applied voltage U. A DC cable can operate in different regimes so that we limit our discussion at the case of application of the step voltage. The distribution of the field is at first a pure capacitive distribution. After the voltage reached its final value, the electric field has a resistive distribution. Between these two stages the field is time-dependent and is called intermediate field. In all three stages, a load current may be present that heats the conductor of the cable. An accurate model involves a coupled-problem because the electric and thermal fields interact. The heat source is defined by ohmic losses generated by the electrical current in conductor and insulation of the cable. The problem is described by a coupled thermal-electric set of equations. The coupling between the two fields is the thermal effect of the electrical current or a material property as the electrical conductivity. A computation algorithm is presented for coupled problems in two dimensions.

A numerical algorithm based on the finite element method (FEM) is presented describing the solution of two-dimensional systems. In our example we consider only steady-state regime for the electric field although many transient regimes appear in the behaviour of the electromagnetic devices. The assumption is acceptable because the time constants for the electric phenomenon are less than the time constants for the thermal field.

The problem of insulation heating involves two approaches that we present in brief [6].

The *capacitive* case. In this case, all the involved insulation media can be assumed to be perfect insulation without free charges. The mathematical model is given by Laplace's equation, written with the potential *V*. The conductivity of the medium is zero. In other words the insulation is assumed to be perfectly insulating so that neither its permittivity nor the voltage frequency mattered. A constant permittivity or a non-linear permittivity material with a ε(x) profile, or ε(E) can be numerically calculated. The finite element method is a convenient method to solve Laplace's equation. The link between each insulation layer is given by the field continuity relationship at each interface.

The *resistive* case. In this case the resistive contribution is not negligible and the potential distribution obeys the Laplace'[s equation. This case is important in an accurate computation of the coupled fields.

Insulation heating is caused by losses due to friction of the molecular polarisation process in dielectric materials. A polluted insulation has a finite resistance so that the leakage current in the insulation heats the insulation.

For our discussion, we consider a high-voltage cable that is employed from relatively low distribution voltages up to 500 kV. The cable consists in a metallic conductor and one or more insulation layers (see Fig. 1). The insulation layers may

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have different physical properties and some layers play the role of the control of electric field distribution, especially in insulation breakdown. In HV industry, we meet some particular configurations where the cable consists in a metallic conductor, a semi-conducting layer extruded over the conductor, a polymeric insulation, an additional semi-conducting layer, and a ground or neutral conductor.

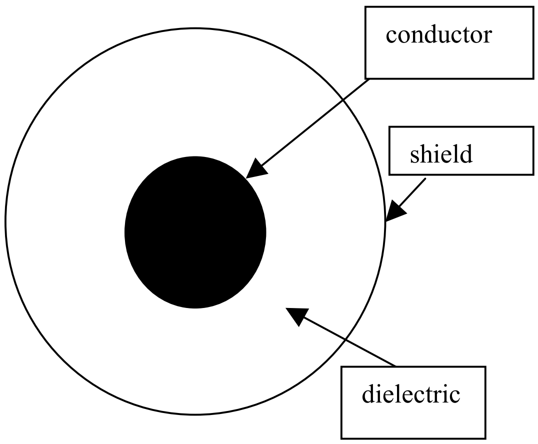

Fig. 1 – Cross-section of the cable

We consider a parallel-plane model. In this case the analysis domain is a cross section in the cable [7]. The problem of heating is a coupled thermal-electric problem. At the application of a voltage U, the field changes from a purely capacitive distribution to a purely resistive field. The field between the initial and final has a time variation. Generally speaking, there is no perfect insulation so that a leakage current exists. This current generates a local heating of the dielectric (insulation) although the most important heating source is the load current (the current in the conductor).

II. Mathematical models

The electric field distribution can be obtained by approximation of the Maxwell equations. These approximations take different forms in accordance with material properties of the equipment. In modelling of these physical systems we must consider both perfect insulations and imperfect (or polluted) insulations.

In our target example the analysis domain is plotted in the Fig. 4. The mesh with triangular elements is presented. We limited the analysis domain to a quarter because of the device symmetry.

A. The capacitive model

In the linear and isotropic insulations where there is no current, from Maxwell equations, Laplace's equation for the scalar potential function V holds [8]:

$$\frac{\partial}{\partial x}\left(\varepsilon_x \frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varepsilon_y \frac{\partial V}{\partial y}\right) = 0 \qquad (1)$$

In the region where the current density is not zero, the dynamics of the electric potential is described by the diffusion-like equation:

$$\nabla J = \nabla\left(\sigma E + \frac{\partial D}{\partial t}\right) = 0 \qquad (2)$$

Boundary conditions for imperfect insulations are:

$$\left(\sigma_1 + \varepsilon_1 \frac{\partial}{\partial t}\right)E_{1n} = \left(\sigma_2 + \varepsilon_2 \frac{\partial}{\partial t}\right)E_{2n} \qquad (3)$$

B. The resistive model

The static field distribution can be modelled by the following equations [6]:

$$\nabla \times \overline{E} = 0; \quad \overline{E} = \rho \overline{J} \qquad (4)$$

with: ρ - the material resistivity, E - the electric strength and J – the current density.

A 2D-field model was developed for a resistive distribution of the electric field. An electric vector potential P is introduced by the relation [2]:

$$\overline{J} = \nabla \times \overline{P}$$

Laplace's equation describes the field distribution (for anisotropic materials):

$$\frac{\partial}{\partial x}\left(\rho_x \frac{\partial P}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho_y \frac{\partial P}{\partial y}\right) = 0 \qquad (5)$$

Mathematical model for the thermal field is the conduction equation [2]:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right) + q = \gamma c \frac{\partial T}{\partial t} \qquad (6)$$

with: T (x, y, t) - temperature in the point with co-ordinates (x, y) at the time t; kx , ky – thermal conductivities; γ-specific mass; c – specific heating; q – heating source.

The heating source is defined by:

$$q = \rho J^2 = \rho(J_x^2 + J_y^2) = \rho\left[\left(\frac{\partial P}{\partial x}\right)^2 + \left(-\frac{\partial P}{\partial y}\right)^2\right] \qquad (7)$$

It is obviously that there is a natural coupling between electrical and thermal fields. Thus, the resistivity in equation (5) is a function of T, and the heating source q in Eq. (6) depends on J. Numerical models for the two field problems can be obtained by the finite element method. In insulation applications we consider the dependency of the temperature by the form:

$$\sigma = \sigma_0 \exp(\alpha T)\exp(\gamma E)$$

where: σ_0 stands for the conductivity at a temperature of 0^0 C and field strength of 0 kV/mm; *α* denotes the temperature dependency coefficient and *γ* denotes the field dependency coefficient.

III. Analytical solutions

For homogeneous insulation we can use analytical formulas both for electrical field computation and thermal field. The limits of the analytical computation are well known so that we don't present them here.

We consider a HV cable at 150 kV with two insulation layers at the application of a step voltage of the value U, without load. The steady state of the field in the first layer can be computed Insulation heating is caused by losses due to friction of the molecular polarisation process in dielectric materials. A polluted insulation has a finite resistance so that the leakage current in the insulation heats the dielectric. with the following relation []:

$$E_1 = U \frac{\sigma_2}{r(\sigma_2 \ln \frac{r_2}{r_1} + \sigma_1 \ln \frac{r_3}{r_2})} \tag{8}$$

A similar expression is for the second layer. In Fig. 3 the field distribution is plotted for different values of the first-layer conductivity, that is: σ2>σ10>σ11>σ12.

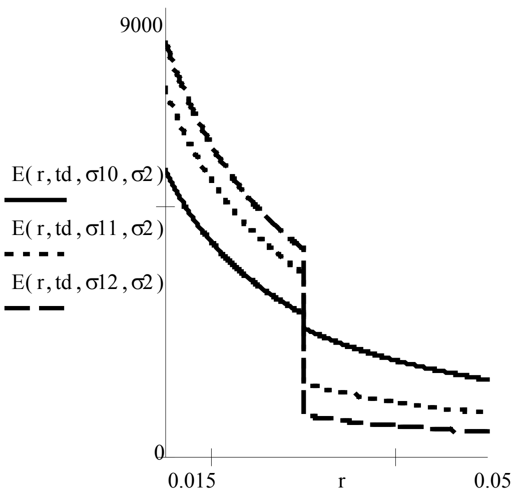

Fig. 2 – Resistive field distribution

The physical properties of this example (relative permittivity, electric resistivity, and thermal conductivity) are:

- The first layer: ε_{r1}=5.5; ρ_1=1.10^{12} [Ω. m]; k_1=0.08 [W/m.K]
- The second layer: ε_{r2}=3.5; ρ_2=1.10^{14}[Ω. m]; k_2=0.18 [W/m.K]

The electrical conductivity depends on the temperature so that we must compute the temperature distribution. For this we compute the leakage current density as:

$$J_1=\sigma_1.E_1; \quad J_2=\sigma_2.E_2$$

The field distribution is a hyperbolic function if there is no temperature drop in insulation. With no temperature drop in insulation, the maximum value of the electric field strength is near the conductor (the curve E1 in Fig. 3).

For large loads that lead to high temperatures in conductor, the field near the lead sheath may become higher than the highest field strength near the conductor. An analytical relation is proposed in the references [3]:

$$E(r,\Delta t) = U.\frac{k(\Delta t)}{r2\left[1-\left(\frac{r1}{r2}\right)^{k(\Delta t)}\right]}.\left(\frac{r}{r2}\right)^{k(\Delta t)-1} \tag{9}$$

where: r1 is the inner insulation radius, r2 is the outer insulation radius, Δt is the temperature drop of insulation, and k (t) is defined as:

$$k(\Delta t) = \frac{\alpha.\Delta t}{\ln \frac{r2}{r1}}$$

In Fig.3, the electric field is plotted for different temperature drops in insulation. E1(r) represents the electric field in the cable without temperature drop in insulation. The temperature drop in insulation modifies the electrical field distribution. In other words, a load can influence the capacitive field distribution [3].

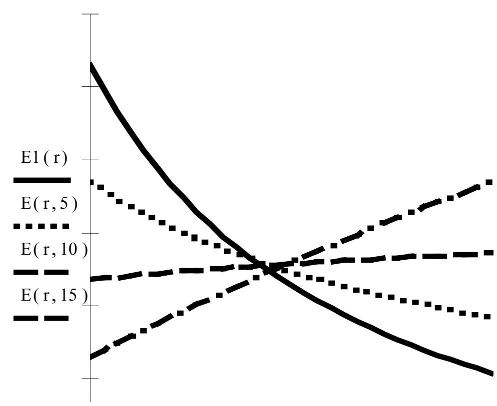

Fig. 3 – Electric strength for different temperature drops

In our examples the computations were performed on a 150 kV cable with r1=20 mm, r2=50 mm and the temperature coefficient α=0.1 ^{0}C.

IV. NUMERICAL MODELS

The differential model for non-homogeneous insulation cannot be solved analytically. A numerical model can be obtained by Galerkin's procedure.

In general the time dependent problems after a spatial discretization can lead to a lumped-parameter model. For example, the heat equation, after spatial discretization, lead to a system of ordinary differential equations by the form ([4] – [5]):

$$[S]\left\{\frac{\partial T}{\partial t}\right\} + [R]\left\{T\right\} + \{b\} = 0 \tag{10}$$

where [R] and [S] are matrices and b is the vector of the free terms.

The algorithm in pseudo-code has the following structure:

1. *Choose the initial value of the temperature*
2. *Repeat*

{Computations for electrical field}
- *Compute the resistivity ρ*
- *Solve the numerical model for electric potential P*

{Computations for thermal field}
- *Compute the heating source q*
- *Solve the numerical model for the temperature*

3. *Until the convergence_test is TRUE*

The convergence test is the final time of the physical process.

We present the numerical model for the heat equation. A spatial discretization leads to the ordinary differential equation (10). The time discretization of the temperature can be obtained by a formula of finite difference [9]:

$$\frac{\partial T}{\partial t} = \frac{T^{(k)} - T^{(k-1)}}{\delta t}$$

With this approximation, the heat equation (6) becomes:

$$(\gamma c)T^{(k)} = \nabla(k\nabla T^{(n-1)}) + \frac{(c\gamma)T^{(k-1)}}{\delta t} + q^{(k-1)}$$

A refinement of the numerical algorithm in pseudo-code can have the following form [1]:

- *Put the iteration counter k on 0 and the initial time t_0.*
- *k=k+1*
- *Compute the resistivity value.*
- *Solve the numerical model for the electric potential P*
- *Compute the heat source q*
- *Update the numerical model for the temperature*
- *Solve the numerical model for the thermal field. The result is the temperature at the moment t_k.*
- *Increase the time with the step δt in order to obtain the following step t_k.*
- *If the time t_k is less than an imposed limit, then jump to the step 2, else stop.*

V. NUMERICAL RESULTS

Our example is a high-voltage direct-current (HVDC) cable with two insulation layers [10]. The leakage current in insulation is caused by the finite resistivity of the insulation.

The geometrical properties of the cable are: internal radius of the first layer is 15 [mm]; internal radius of the second layer is 16 [mm]; external radius of the second layer is 20 [mm].

The physical electrical properties are: voltage of the cable is U=150 [kV]; resistivity of the first layer is $1.\ 10^{9}$ [Ω. m]; resistivity of the second layer is 2.10^{10} [Ω. m]

The thermal properties are: thermal conductivity of the first layer is 0.271[W/K.m]; specific heat c=1800 [J/Kg.K]; specific mass γ=1300 [Kg/m^3]; thermal conductivity of the second layer is 0.17[W/K.m]; specific heat c=1600 [J/Kg.K]; specific mass γ=1200 [Kg/m^3].

At the application of a high voltage the field has a capacitive distribution initially [5]. This distribution is for a short time so that it is not interest for the temperature distribution. Finally the field has a resistive distribution. Between these limits there is an intermediate field that can be computed by an iterative procedure.

The analysis domain is the insulation space. The symmetry of the problem can reduce the analysis domain to a quarter (Fig.4).

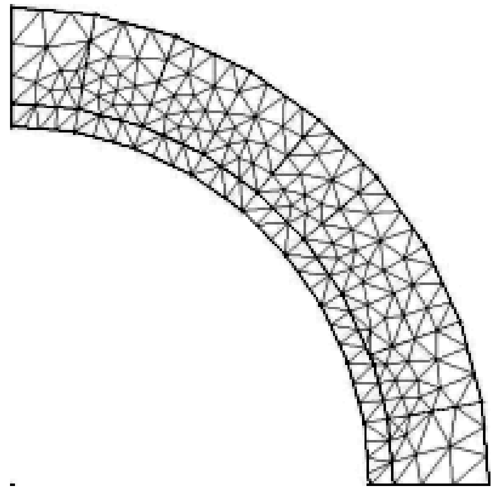

Fig. 4 – Meshed analysis domain

The heat source is the thermal effect of the current in the insulation and the load current of the cable. It is obviously that the ohmic losses in the cable conductor are the most important heat source.

A. Constant heat flux

In our first case we consider that there is a constant heat flux on the interface conductor-insulation. The source of this flux is the Joule-Lenz's effect of the load in the cable. The mathematical model for the heat transfer is the conduction equation (6). The boundary conditions are Neumann's condition at the interface conductor-insulation, and convective condition at the boundary insulation-environment.

The Neumann's condition can be computed by the conductor losses in the case the cable was loaded before switching of the step voltage, that is the current in the cable has been raised long before and the temperature distribution in the cable is stable. In this case the value of the heat flux is computed with the relation [1]:

$$p = \frac{P_{cond}}{2\pi r_0}$$

with P_{cond} - the ohmic losses per cable meter in the inner conductor as Joule-Lenz's effect, and r_0 - the radius of the cable conductor.

Thus, Neumann's condition is:

$$\left.\frac{\partial T}{\partial n}\right|_{C_1} = -p$$

with C_1 – the interface of the cable conductor and insulation and n the unit normal at the interface.

At the interface insulation-environment we consider a convective condition by the form:

$$\left.\frac{\partial T}{\partial n}\right|_{C_2} = h(T - T_\infty)$$

with h – the convective coefficient, T_∞ - the ambient temperature and C_2 – the boundary of the cable and the external medium.

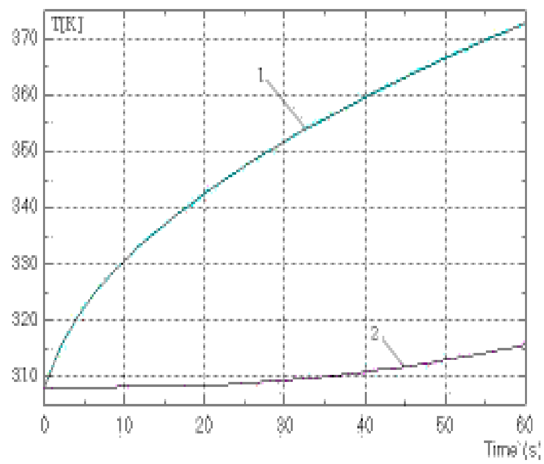

Fig. 5 – Temperature versus time

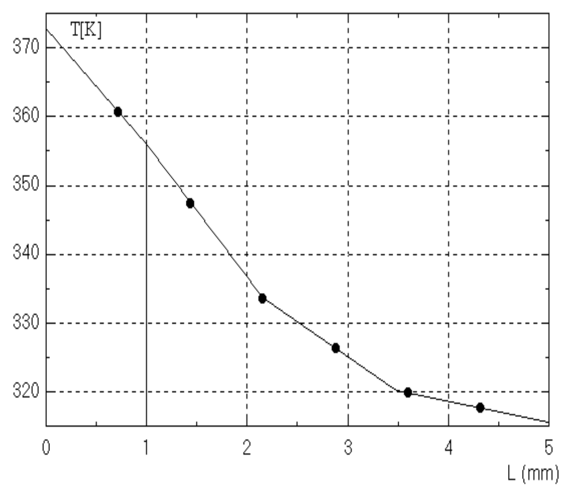

Fig. 6 – Final temperature versus radius

In Fig. 5 the temperatures vs. time at external surface of the conductor the curve 1 (green), and environment surface – the curve 2 (red) are plotted [4]. The time interval was 60 [s]. The convection coefficient h was 12 for an environment temperature 308 [K] (35 ^{0}C).

In the Fig. 5 the final distribution of the temperature in the radial direction is plotted [4]. The width of the insulation is 5 [mm].

B. *Constant temperature*

Another practical assumption in electrical engineering is a Dirichlet boundary condition at the interface conductor-insulation. For our target example we considered a constant temperature of the conductor surface and a convective condition at the boundary insulation - environment. In numerical simulation the conductor temperature was considered as 100 ^{0}C (373.16 K).

In Fig. 7 the temperature versus time is plotted in two interest points [10]. The first curve denoted 1 (green) is the temperature at the external surface of the first layer. The curve 2 (red) represents the temperature at the external surface of the cable.

From the engineering viewpoint the assumption of Neumann's condition seems more realistic. The heat flux at the conductor surface can be estimated more accurate than the conductor temperature.

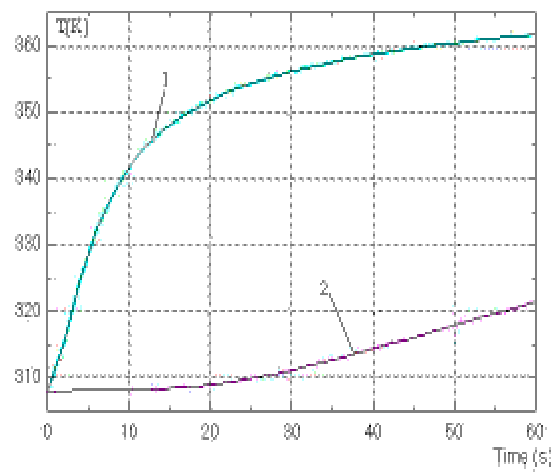

Fig. 7 – Temperature versus time for Dirichlet condition

An accurate model can be obtained by including the conductor in the analysis domain [2]. This approach increases the computational effort.

In these two examples we considered a steady-state regime of the electric field. This is a practical situation but there are cases where the voltage has step variations so that a transient regime appears as a natural situation.

VI. Conclusion

In this paper we presented an algorithm for coupled electric and thermal fields in the insulation of the large power cables. A parallel-plane model was considered both for electrical field and thermal field. The numerical models were obtained by the finite element method in a 2D space.

As target example we considered a cable with two-insulation layer. The resistivity of the insulation was considered as finite value. In this case the ohmic losses of the leakage current in insulation generate supplementary losses. The principal heat source remains the losses in the cable conductor.

As first example we considered a constant heat flux on the interface conductor-insulation. In practice, the heat flux is dependent on the temperature of the cable conductor. By the numerical simulation we can consider all practical cases in the operating regimes of the cable. In the second example we considered a constant temperature of the conductor.

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