A Model of Vehicle Fuel Consumption at Conditions of the EUDC

Michael Ben Chaim, Efraim Shmerling

***Abstract*—**A mathematical model for evaluating vehicle fuel consumption on a 100 km interval at standard operating conditions for the EUDC (Extra-Urban Driving Cycle) is presented. The extra-urban cycle fuel consumption has two significant components: fuel consumption at average speed and during accelerations, and therefore, in the model, it is determined separately for two different operating modes: average speed and accelerations. In each of these modes fuel consumption is calculated based on the efficiency of the engine. Unlike previously developed models, which determine fuel consumption based on specific fuel consumption, ours determines fuel consumption based on the efficiency of the engine, which makes the model more adequate since it incorporates engine mode changes. The efficiency of the engine is expressed as a function of the speed mode of the engine and of the degree of power utilization of the engine.

***Keywords*—**vehicle, fuel economy, fuel consumption, extra-urban driving cycle

I. Introduction

REDUCING the transportation sector energy consumption is an important part of reducing overall energy consumption. It requires development of new, more fuel efficient vehicle models and more efficient operating of existing vehicles. This makes the development of fuel consumption estimation methods very important. The most simple and conveniently implemented method is based on utilization of mathematical models.

Evaluating fuel efficiency is an important procedure during ground vehicle design and operation. Based on this evaluation, usually performed via mathematical modeling and simulation, main constructive parameters of the vehicle may be determined at the design stage and steps to reduce fuel consumption may be taken. Since one of the main goals of vehicle design is minimizing fuel consumption for expected operating conditions, development of analytical models that allow accurate prediction of vehicle consumption appears to be highly desirable.

A well-known approach to estimating fuel consumption is inverse simulation [1] – [4], where the driving cycle-to-tank chain is represented by power transferring functional blocks with predetermined efficiency, as shown in Fig. 1.

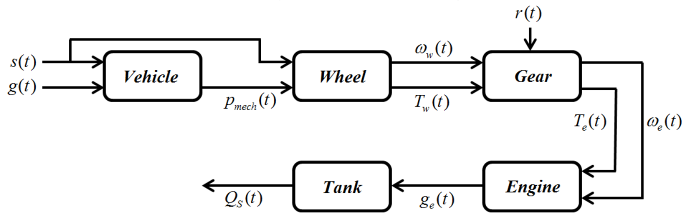

Fig. 1. Inverse simulation flow

$s(t)$ – driving speed versus time (driving profile),
$g(t)$ – road grade versus time (driving profile),
p_{mech}(t) – mechanical power on the wheel,
$\omega_w(t)$ – wheel angular speed,
$Tw(t)$ – mechanical torque on the wheel,
$r(t)$ –gear ratio,
$\omega_e(t)$ – engine speed,
$T_e(t)$ – engine torque,
$g_e(t)$ – specific fuel consumption.

Here, engine specific fuel consumption (Fig. 2) is usually represented by an appropriate two-dimensional lookup table, rather than obtained analytically. Alternatively, a similar two-dimensional map is often used for expressing engine efficiency rather than specific fuel consumption. Hence, simulation software must be used in order to determine the vehicle mileage. It would be more convenient if the fuel consumption could be determined from analytic expressions only, eliminating the need for simulation.

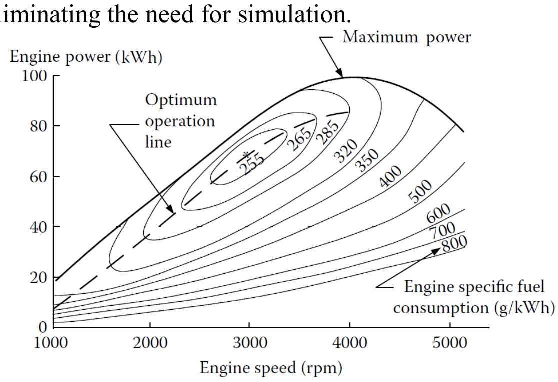

Fig. 2. Typical internal combustion engine specific fuel consumption map [3]

In [5] a dynamic model of the THS powertrain is developed and then applied for model-based control development. Two control algorithms are introduced: one based on the stochastic

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dynamic programming method, and the other based on the equivalent consumption minimization strategy. The performance of these two algorithms is assessed by comparing against the dynamic programming results, which are non-causal but provide theoretical benchmarks for fuel consumption. [6] focuses on technology analysis and simulation to mitigate the transportation impacts on energy and environment, with the major goal of estimating the technology contribution towards the 125 g/km CO_2 target in Europe. The authors analyze cheap- and low-complexity measures, while keeping the same power/weight ratio, for several vehicle categories. The measures are: regenerative braking; fuel cut while coasting; engine stop/start; and engine downsizing and turbocharging. Simulation of these mechanisms for several road vehicle categories and driving cycles, allow us to conclude that with the last three mechanisms fuel consumption and CO_2 emissions can be reduced by 15–49%, compared to the original vehicle. HC, CO and NO_x emissions can be reduced by similar percentages. Regenerative braking is valuable only if the additional weight is compensated by diminishing the body weight. The simulations confirm that the use of "slightly" modified conventional vehicles can reduce fuel consumption and carbon dioxide emissions, without the complexity and high cost of full-hybrid powertrains. Attempts for creating mathematical models for estimating fuel efficiency have been widely made in the literature. For example, it was proposed in [6] , [7] to evaluate fuel consumption Q_S measured in liters per 100 km, on the basis of hourly fuel consumption and engine power via the following relation,

$$Q_s = \frac{g_e \cdot (P_{rl} + P_w + P_a)}{10 \cdot V_a \cdot \eta_T \cdot \rho_f}, \qquad (1)$$

where

g_e is the specific fuel consumption, g·kWh^{-1},

P_{rl} is the power required to overcome the rolling resistance of the road, kW,

P_w is the power required to overcome the resistance of the air, kW,

P_a is the power required to overcome the resistance of the inertial acceleration, kW,

η_T is the efficiency of the transmission,

ρ_f is the fuel density, kg·l^{-1},

V_a is the average speed of the vehicle, km·h^{-1}.

Equation (1) assumes that the vehicle constantly operates in acceleration mode and the engine power is determined according to this assumption. Specific fuel consumption is assumed to be constant and optimal. In [8] - [10] it is proposed to calculate fuel consumption based on specific hourly fuel consumption and energy expenditure, while the authors of [2], [3] have suggested determining energy expenditure based on a computer simulation software ADVISOR; good results were obtained for the qualitative analysis of fuel economy. Energy expenditure determination based on statistical modeling was proposed in [1] - [4].

Particularly noteworthy is the work of Guzzella et.al. [4], where fuel consumption of vehicles for the European Driving Cycle MVEG-95 was determined on the basis of energy expenditure during the movement of the vehicle. The authors proposed the following relation,

$$E_{MVEG-95} \approx A_f c_D \cdot 1.9 \cdot 10^4 + m_v f_r \cdot 8.4 \cdot 10^2 + m_v \cdot 10,$$
$$[kJ/100km], \qquad (2)$$

where

m_v is car mass, kg;

f_r is the rolling resistance coefficient;

c_D is the coefficient of aerodynamic resistance of the car;

A_f is the characteristic area of the car, m^2.

Similarly to (1), eq. (2) assumes that the vehicle constantly operates in acceleration mode and that the efficiency is constant and optimal. The first term in the right-hand side of (2) is the energy required for overcoming the resistance of the air, the second term is the energy required for overcoming the resistance of the road, and the third term is the energy required for overcoming the inertial acceleration.

The authors of [11] explore the influence of driving patterns on fuel consumption using a portable emissions measurement system on ten passenger cars. It is shown that vehicle fuel consumption per unit distance is optimum at speeds between 50 and 70 km/h, fuel consumption increasing significantly with acceleration. A VSP-based model was developed to calculate vehicle fuel consumption in this study, and produced good results compared to the measured data.

In [12] vehicle driving states were taken as a Markov process to reduce the influences of uncertainty and small variations in driving speed on the driving cycle model. Collected data were classified into model events of idling, acceleration, deceleration, and constant velocity using maximum likelihood estimation. The model events with similar average speeds were categorized into six states, and their transition probabilities were calculated. Pseudo-random numbers satisfying distribution of the state transition probabilities were generated to extend the length of driving cycle. The application of the driving cycle model to the roads in Hefei (a city in China) shows that the average error in obtained typical driving cycles was only 7.81%.

Basically, the bottleneck of all the proposed approaches is the need to include an engine consumption map, which was overcome by assuming constant specific fuel consumption for all operating modes, which is obviously inaccurate. The current work is interesting from a methodological point of view, since an attempt is being made to analytically calculate energy expenditure and fuel consumption, taking into account the instantaneous specific fuel consumption.

Currently, the major set of regulations governing vehicle operating modes for estimating fuel consumption of vehicles are the rules of the UN Economic Commission for Europe [13]. The above mentioned models and formulas for calculating fuel consumption, which do not take into consideration the changes in the mode of motion, are unfit for evaluating fuel consumption in accordance with the accepted regulations. Development of a mathematical model which can be used for this purpose is the main contribution of this article.

The rest of the manuscript is organized as follows. In section 2, the proposed model for calculating fuel consumption in accordance with UN ECE regulations is described. In this model fuel consumption is determined separately for two different vehicle operating modes: constant speed and accelerations.

In section 3, verification of the adequacy and accuracy of the obtained formula is presented. To assess it, calculation of fuel consumption using the derived formula was carried out and the results were compared to experimental data provided by the manufacturers. In order to carry out calculations via the proposed formula, parameters common for all automobiles are first identified. The vehicle-specific parameters used are the type of engine, automobile mass, maximum power and shaft speed at maximum power. Based on the comparison of calculations carried out using the proposed model to data from the manufacturers, it is concluded that the proposed mathematical model is suitable for practical use.

Unlike previously developed models, ours determines fuel consumption based on the efficiency of the engine, which makes the model more adequate since it incorporates engine mode changes. The efficiency of the engine is expressed as a function of the speed mode of the engine and of the degree of power utilization of the engine.

II. Equation for Estimating Fuel Consumption

To determine fuel consumption we accept the assumption that the car consumes fuel only to cover 100 km at a constant speed of the cycle and to increase the kinetic energy during accelerations.

The formula for calculating fuel consumption is based on the UN ECE regulations for EUDC (Table I, Fig. 3) [13].

As in [3], we divide the energy expenditure into two parts, the first one being the energy required for overcoming the resistance of the air and the energy required for overcoming the resistance of the road, and the second – the energy required for overcoming the resistance of the inertia of the weight of the vehicle during accelerations (Fig. 4 and 5).

In these calculations it was assumed that the number of accelerations is lower than the number given in the UN ECE for the Urban Cycle, and the average speed is higher (Table I, Fig. 3) [13]:

Table I. Parameters for EUDC

Characteristics	Unit	EUDC
Distance	km	6.955
Duration	s	400
Average Speed	km/h	62.6
Maximum Speed	km/h	120

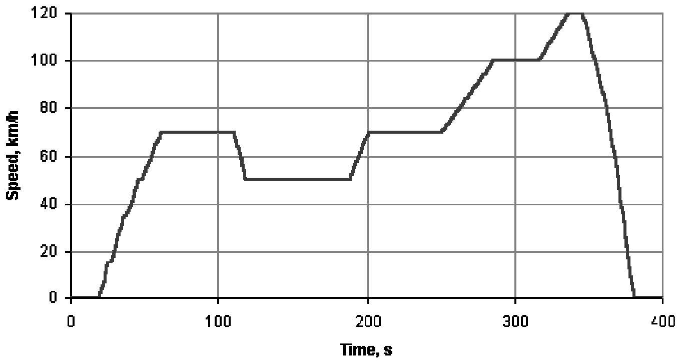

Figure 3. Scheme of EUDC

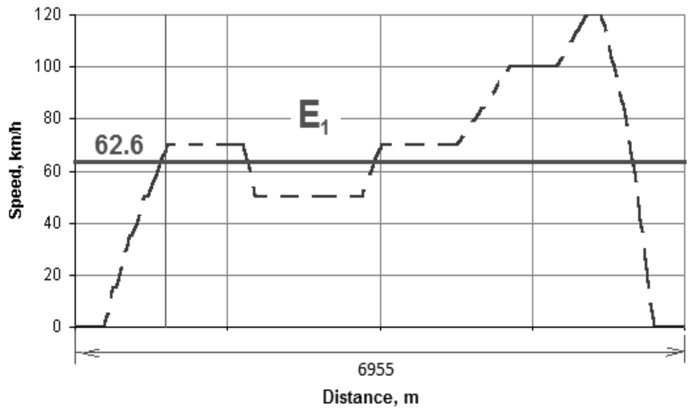

Fig. 4. Scheme for calculating energy E_1

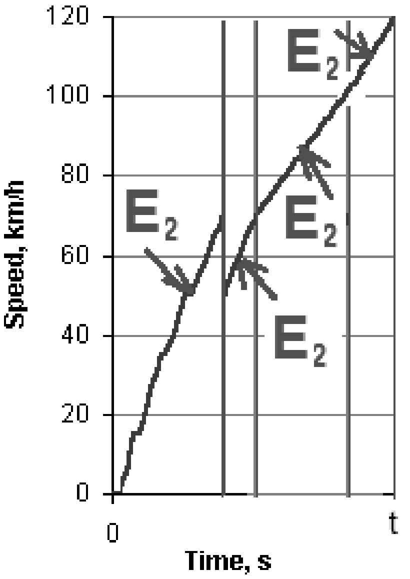

Fig. 5. Scheme for calculating energy E_2

As stated above, according to the accepted assumption, the automobile engine operates in two main modes, the first of which is movement at average speed, and the second – series of accelerations. The equation for estimating fuel consumption must take this into account. In the formula the energy expenditure is expressed as a sum:

$$E_s = E_1 + E_2,$$

where

E_1 is the energy required to overcome the forces of resistance at average speed on the 100 km interval,

E_2 is the kinetic energy required for episodic accelerations on the 100 km interval, J.

Fuel consumption per 100 kilometers has the form:

$$Q_{S(e)} = \frac{E_S}{H_L},$$

where

H_L is the calorific value of one liter of fuel.

The energy required to overcome the forces of resistance at average speed on the 100 km interval:

$$E_1 = \frac{1}{\eta_T \eta_{P,n}} \left(m_a \cdot g \cdot c_r + \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V_a^2 \right) \cdot S,$$

where

η_T is the efficiency of the transmission,

V_a is the average speed of the vehicle, m/sec,

m_a is car mass, kg,

c_r is the rolling resistance coefficient,

C_D is the coefficient of aerodynamic resistance of the car,

A_f is the characteristic area of the car, m^2.

The values of parameters c_r and A_f are determined by empirical equations [14]:

$$c_r = 0.0136 + 0.40 \cdot 10^{-7} V_a^2,$$

$$A_f = 1.6 + 0.00056(m_a - 765).$$

S is the car mileage, which equals 100000 m, i.e. 100 km,

$\eta_{P,n}$ is the efficiency of the engine, which depends on the degree of power utilization and the engine speed mode in the following way:

$$\eta_{P,n} = \eta_e \mu_P \mu_n,$$

where

η_e is the engine's peak efficiency,

μ_P is the coefficient through which the influence of the degree of power utilization (the part-load) on the peak efficiency of the engine is expressed,

μ_n is the coefficient through which the influence of engine speed mode on the peak efficiency of the engine is expressed.

In order to obtain functions μ_P and μ_n, the dependences $\mu_P = f(P/P)$ and $\mu_n = f(n/n_N)$ were analyzed for a number of modern gasoline and diesel engines, information about which is available in the literature. As a result of the data analysis [14] – [18], the following table (Table II) was obtained.

Table II. μ_P and μ_n coefficient values

P_i/P_e, n_i/n_e, %	μ_P, Gasoline	μ_P, Diesel	μ_n
0.20	0.47	0.64	0.87
0.30	0.59	0.72	0.92
0.40	0.71	0.79	0.96
0.50	0.82	0.89	0.98
0.60	0.90	0.92	0.99
0.70	0.97	0.97	1.00
0.80	1.00	1.00	0.99
0.90	0.97	0.95	0.98
1.00	0.90	0.80	0.96

The table (Table II) data was approximated and the following results were obtained:

a formula for calculating μ_P for diesel engines:

$$\mu_P = 0.5968 - 0.1666 \frac{P_i}{P_e} + 2.4968 \left(\frac{P_i}{P_e} \right)^2 - 2.1128 \left(\frac{P_i}{P_e} \right)^3,$$

a formula for calculating μ_P for gasoline engines:

$$\mu_P = 0.234 + 1.0592\frac{P_i}{P_e} + 0.8149\left(\frac{P_i}{P_e}\right)^2 - 1.2121\left(\frac{P_i}{P_e}\right)^3,$$

and a formula for calculating μ_n for diesel and gasoline engines:

$$\mu_n = 0.7107 + 0.9963\left(\frac{n_i}{n_P}\right) - 1.0582\left(\frac{n_i}{n_P}\right)^2 + 0.3124\left(\frac{n_i}{n_P}\right)^3$$

where

P_i is the engine power required for the given mode (P) of motion,

P_e is the engine power by the performance characteristics of the engines, corresponding to vehicle speed V_{a_i},

n_P is the engine speed at maximum power of engine, $\min^{-1}$,

n_i is the engine speed at average speed of vehicle, V_{a_i}, $\min^{-1}$.

According to the definition

$$\frac{P_i}{P_e} = \frac{\left(m_a g c_r + 0.5 c_D A_f V_a^2 + m_a a \gamma_m\right)V_a}{P_e}.$$

Here the numerator is the engine power required for the given mode of motion, and the denominator is the engine power by the performance characteristics of the engine for the corresponding vehicle speed. It is a function of engine speed and maximum engine power and is determined by the empirical formula [7, 8]:

$$P_e = 10^3 P_{\max}\left[a\left(\frac{n}{n_P}\right) + b\left(\frac{n}{n_P}\right)^2 - c\left(\frac{n}{n_P}\right)^3\right]$$

where

$P_{\max}$ is the engine's maximum power, kW,

a, *b*, *c* are the polynomial coefficients, different for different types of engines (see Table III).

Table III. Polynomial coefficients [7]:

Gasoline engine	Diesel engine	Coefficients
1.0	1.0	a
1.0	0.5	b
1.0	0.5	c

n_P is the engine speed at maximum power of engine, $\min^{-1}$,

n_i is the engine speed at average speed of vehicle, V_a, $\min^{-1}$.

The formula for determining it has the following form

$$n = \frac{9.55 V_a \xi_{ax} \xi_n}{r_d},$$

where

r_d is the rolling radius of the tire, *m*,

ξ_{ax} is the finale drive gear ratio,

ξ_n is gear ratio in the gearbox,

ρ is air density, $N \cdot s^2 / m^4$,

g is the acceleration of gravity, m / s^2.

Graphically:

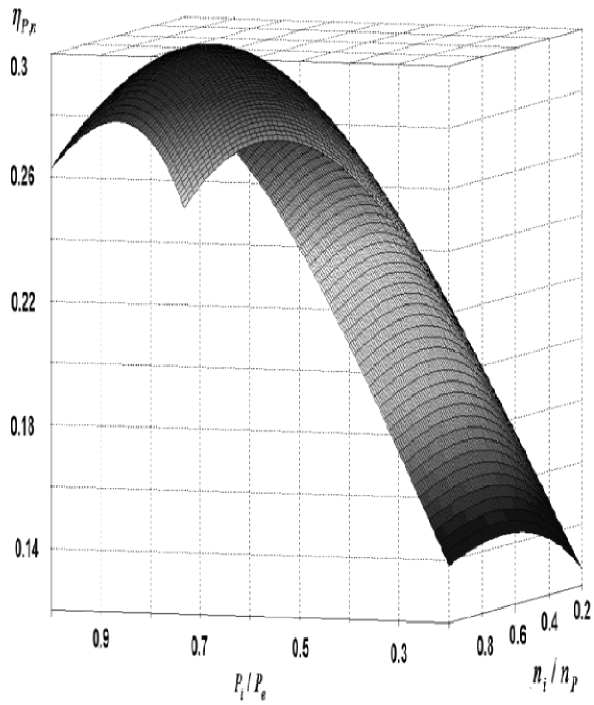

Fig. 6. Variable efficiency of gasoline engines

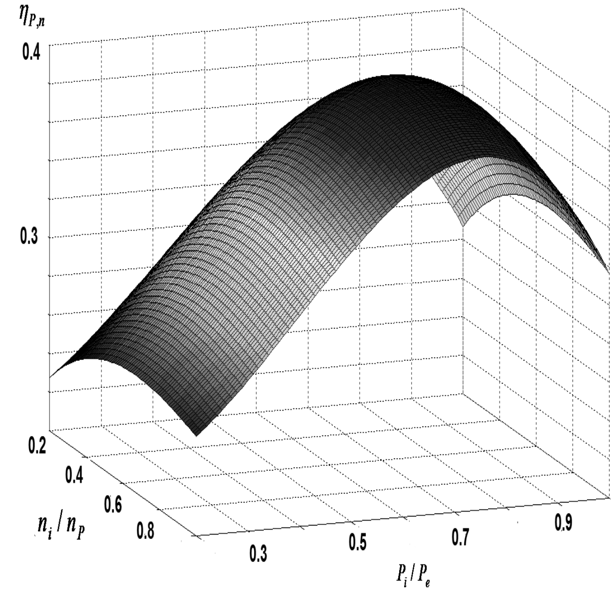

Fig. 7. Variable efficiency of diesel engines

Kinetic energy required for episodic accelerations on the 100 km interval:

$$E_2 = \frac{qm_a}{\eta_T \eta_e} \sum_{i=1}^{k} \frac{a_i \gamma_{mi} S_i}{\mu_{P_i} \mu_{n_i}},$$

where

γ_{m_i} is the mass factor of the vehicle,

a_i is the acceleration of the vehicle, m/s^2,

S_i is the acceleration distance of the vehicle, *m*,

k is the number of acceleration intervals,

q is the number of accelerations in each acceleration interval,

μ_{P_i} is the coefficient through which the influence of the degree of power utilization (the part-load) on the peak efficiency of the engine is expressed in each acceleration interval,

μ_{n_i} is the coefficient through which the influence of engine speed mode on the peak efficiency of the engine is expressed in each acceleration interval.

The parameters for calculating fuel consumption according to the formula were established graphically by Fig. 1. For the first part of the energy expenditure of the vehicle E_1, average speed and distance are defined as $V_a = 62.6\ km/h$ and $S = 100\ km$, respectively. For the second part of the energy expenditure of the vehicle E_2, parameters were defined for four ranges of acceleration (i.e., k = 4), according to the EUDC cycle. The calculation results are summarized in Table IV.

Table IV. Vehicle motion parameters during accelerations according to the EUDC cycle

	Acceleration interval, km/h			
Vehicle motion parameters during acceleration	0-70	55-70	70-100	100-120
Acceleration time, t, s	40	15	35	20
Acceleration, m/s^2, a_i	0.5	0.3	0.25	0.28
Acceleration distance, m, S_i	740	220	650	325
Average speed during acceleration, km/h, $V_{a\,i}$	35	63	85	110
Number of accelerations in interval, q	14.37	14.37	14.37	14.37

The total energy required for driving 100 kilometers is:

$$E_S = \frac{1}{\eta_T \eta_{P,n}} \left(m_a \cdot g \cdot f_{rl} + \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V_a^2 \right) \cdot S + \frac{qm_a}{\eta_T \eta_e} \sum_{i=1}^{k} \frac{a_i \gamma_{mi} S_i}{\mu_{P_i} \mu_{n_i}} ,$$

When we substitute E_S, the equation for fuel consumption defined by energy expenditure takes the following form:

$$Q_{S(e)} = \frac{1}{\eta_T \eta_{P,n} H_L} \left(m_a \cdot g \cdot f_{rl} + \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V_a^2 \right) \cdot S + \frac{qm_a}{\eta_T \eta_e H_L} \sum_{i=1}^{k} \frac{a_i \gamma_{mi} S_i}{\mu_{P_i} \mu_{n_i}} .$$

III. Concluding Remark

To validate the obtained formula, estimates of fuel consumption obtained by calculations via the formula were compared to experimental data available from manufacturers [19]. The vehicle-specific parameters that we used were the type of engine, automobile mass, maximum power and engine speed at maximum power.

Table V. The values of the parameters m_a, η_e, η_T used in our calculation and the corresponding fuel consumption (EUDC)

Type of Engine	m_a	η_e	η_T
Diesel	1500	0.40	0.95
Gasoline	1500	0.30	0.95
Type of Engine	γ_{m_i}	A_f, m^2	c_r
Diesel	1.06-1.12	1.8	0.015
Gasoline	1.06-1.12	1.8	0.015

Table VI. The values of the parameters P_e, n_p, ξ_{ax} used in our calculation and the corresponding fuel consumption (EUDC)

Type of Engine	P_e,kW	N_p,min^{-1}	ξ_{ax}
Diesel	100	4500	3.5
Gasoline	100	6000	3.5
Type of Engine	ξ_{n_i}	$c_d, Ns^2/m^4$	$Q_s, l/100km$
Diesel	1.0-3.5	0.30	5.3
Gasoline	1.0-3.5	0.30	7.5

Table VII. Fuel consumption rates of different vehicles based on experimental data vs. the results obtained using the formula

Vehicle	Fuel consumption based on experimental data	Fuel consumption by formula
Volkswagen Polo Sedan	4.8	4.9
Toyota Yaris	4.5	4.8
Toyota Sienna AWD	10.7	11.2
Toyota Camry AWD3.5	6.8	7.3
Hyundai Genesis Coupé 2.0 T	7.1	7.5

Table VIII. Technical specifications of different vehicles

Vehicle	Technical Specifications		
	Mass, kg	Max. Power, kW	rpm
Volkswagen Polo Sedan	1106	62.6	5000
Toyota Yaris	1005	73.1	6000
Toyota Sienna AWD	2080	197.6	6000
Toyota Camry AWD3.5	1570	196.9	6000
Hyundai Genesis Coupé 2.0 T	1570	157.3	6000

We compared the results of our calculations to data on modern automobiles manufactured in 2011 by leading automotive firms. The discrepancy between the results of our calculations and the experimental data was between 2-7%, which indicates that the level of accuracy of estimates obtained via the formula is sufficiently good.

Based on the above comparison we concluded that the obtained formula is sufficiently accurate and fit for evaluating fuel consumption at extra-urban operating conditions.

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