Geometry optimization of piezoceramic laser shutter

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Abstract — Usually numerical modeling and simulation of multicomponent piezoelectric actuators lead to the large number of recurred calculations with different geometrical parameters of the actuator. The exchanges in the modal shape sequence are a general case problem concerning to all mechanical structures. This problem is also important for optimization, since calculations are tied both to eigenfrequencies and eigenforms. If the eigenfrequency is chosen incorrectly, the piezoactuator will not function, so it is very important to numerically determine eigenforms.
In this paper are overlooked piezoelectric actuators concept and urgency, proposed model of a laser shutter and piezoelectric laser gate, analysis of geometrical parameters optimization is done. What is more, influence of geometrical parameters and advantage of the domination coefficients are presented, calculations, results and conclusions are given.

Keywords — Domination coefficients, eigenfrequencies, geometry optimization, laser shutter, piezoelectric actuators.

I. INTRODUCTION

Piezoelectric actuators are widely used in high precision mechanical systems such as positioning devices, manipulating systems, control equipment and etc. Piezoelectric actuators have advanced features such as high resolution, short response time, compact size, and good controllability [1, 2, 3].

The performance of these devices strongly depends on the features of the actuator, which is the main part of the piezomechanical system. The synthesis of needful oscillation fields of the actuator can be obtained optimizing the geometrical parameters, the vector of poliarization and the topology of excitation zones of the actuator. The piezoelectric effect and the hysteresis effect play an important role in the dynamical behavior of these actuators. So it is very important to know what modal shape will be excited when modelling piezoelectric actuators [4].

Vibration amplitudes of the bulk piezoceramic element usually are at range of nanometers, so very often piezoceramics is combined with amplifier mechanism to enlarge stroke of the actuator. However some problems appear applying amplifier mechanisms, i.e. difference between wave propagation speed of piezoceramics and amplifier, accuracy issues of gluing and mounting and etc.

In general many design principles of piezoelectric actuators are proposed. Summarizing the following types of piezoelectric actuators can be specified: traveling wave, standing wave, hybrid transducer, and multi-mode vibrations actuators.

Piezoelectric actuator of multimode vibrations type is presented and analyzed in this paper [5], [6], [7].

Summarizing the analysis of the theoretical research it can be stated that due to special qualities of piezoceramics the field of application for piezoceramic actuators is very wide. At the time much attention is being paid to the piezoceramics produced in industry, so new possibilities for creating fast and precise vibration devices of small dimensions appear. Although the range of theoretical research is very wide, the change in eigenform sequence observed while solving optimization problems remains insufficiently researched being at the same time an important issue.

II. PROBLEM DEFINITION

Exchanges in the modal shape sequence could be determined analyzing various constructions of piezoelectric actuators. For example, let's consider longitudinal – flexural oscillations of the beam actuator. Using this type of actuator, changes of the modal shape sequence could be found analytically [8].

Using the technical oscillation theory of the beam the longitudinal oscillations are found by solving the second order differential equation [9], [10]:

$$m(x)\frac{\partial^2 \xi}{\partial^2 t} - \frac{\partial}{\partial x}\left(Es_i \frac{\partial \xi}{\partial x} \right) = 0 \tag{1}$$

Longitudinal oscillations of the beam can be expressed as follows [2], [4]:

$$\omega_k = \frac{k}{2l}\sqrt{\frac{E}{\rho}} \tag{2}$$

E - the Jung modulus; k - the mode number of the longitudinal oscillations; l - the length of the beam; ρ mass density.

Flexural oscillations of the beam are found by solving the second order differential equation [9], [10]:

$$\frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2 \xi_{len}}{\partial x^2} \right) + \rho S \frac{\partial^2 \xi_{len}}{\partial^2 t} = 0 \tag{3}$$

Flexural oscillations of the beam are described by the expression [9], [10]:

$$\omega_n = \frac{\pi h\,(n+0.5)^2}{4l^2}\sqrt{\frac{E}{3\rho}} \tag{4}$$

h - the height of the beam; n – the mode number of the flexural oscillations;

If certain values of k and n are defined, then h/l ratio of the beam could be calculated. From the equations (2) and (4) following equation could be obtained:

$$\frac{h}{l}=\frac{2\sqrt{3}\,k}{\pi(n+0.5)^2} \tag{5}$$

As an example:

$$k=1;\ n=2\ \Rightarrow\ \frac{h}{l}=0.1765 \tag{6}$$

But h/l ratio could be changed, for example, increasing or reducing the height of the beam. In this case k value remains the same, but n value changes. This means that the sequence of modal shapes changes when the geometrical parameters of the beam vary. For example, when the length and height ratio is 0.6<l/h<2.4 we have an ordinary modal shape sequence and in other case second and third modal shape of two dimensional actuator changes (Fig. 1).

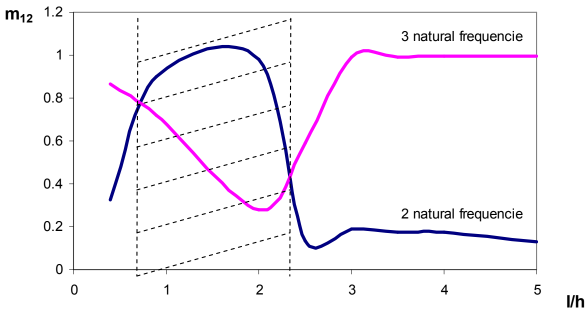


Fig. 1 Dependence of the dominating coefficient m_{12} on the length and height ratio l/h of two-dimensional piezoelectric actuator

Usually, for numerical analysis of piezoactuators software such as ANSYS is used. By the algorithm of eigenvalue problem eigenfrequencies for systems are sorted in the ascending order; thereby the sequences of eigenforms change. This rule for sorting frequencies is disadvantageous when numerical analysis of multidimensional piezoactuators needs to be automated. This problem is also important for optimization, since calculations are tied both to eigenfrequencies and eigenforms. If the eigenfrequency is chosen incorrectly, the piezoactuator will not function, so it is very important to numerically determine eigenforms and place them inside the eigenform matrix of the construction model [11], [19].

To solve this problem the following algorithm is proposed: find the sum of the amplitude squares of piezoactuator oscillations in all directions of the degrees of freedom for a point, i.e., the full system energy in all directions [9], [11], [12]:

$$S_k^n=\sum_{i=1}^{r}\left(A_{ik}^n\right)^2 . \tag{7}$$

where n – the eigenfrequency for a system, k – the number of degrees of freedom in a node, A_{ik}^n – the value of the eigenform vector for the ith element.

Then the ratio is calculated:

$$m_j^n=\frac{S_j^n}{\sum_{i=1}^{k}S_i^n} \tag{8}$$

where m_j^n – the oscillation domination coefficient corresponds to the nth eigenform. The index j of domination coefficients indicate, in which direction the energy under investigation is the largest. j can assume such values: 1 corresponds to the x coordinate, 2 – y, and 3 – z, etc. Having calculated domination coefficients in all directions of degrees of freedom and having compared them to each other, we can determine the dominant oscillation type. The domination coefficients calculated according to formula (8) are normalized, so their limits vary from 0 to 1. It is very convenient for analyzing the influence of various parameters on domination coefficients.

To clearly determine the eigenform and its place in the eigenform matrix of the construction model, it is not enough to calculate only the oscillation domination coefficients. Domination coefficients only help to differentiate eigenforms by dominating oscillations, for example, radial, tangential, axial, etc.

Because of this an additional criterion is introduced into the process of determining eigenform, individual for each eigenform, i.e., the number of nodal points or nodal lines for the form. That depends on the dimensionality of the eigenform. During calculations the number of nodal points of beam-like and two-dimensional piezoactuators is determined by the number of sign changes in oscillation amplitude for the full length of the piezoactuator in the directions of coordinate axes (Fig. 3).

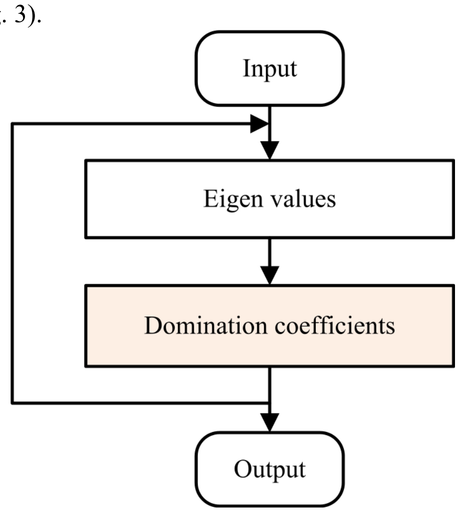


Fig. 2 The scheme for determining integrating parameters

Summarizing the algorithm for determining eigenforms of piezoactuator oscillations, we can note that it is composed of two integral stages: calculating domination coefficients (Fig. 2) and determining the number of nodal points or lines of the eigenform (Fig. 3).

This algorithm is not tightly bound to multidimensional piezoactuators, so it can be successfully applied in analysing oscillations of any constructions. When solving dynamics problems of piezoactuators for high precision microrobots where repeated calculations with higher eigenfrequencies are involved, it is proposed to modify the general algorithm introducing the stage of determining eigenforms with the help of domination coefficients [9].

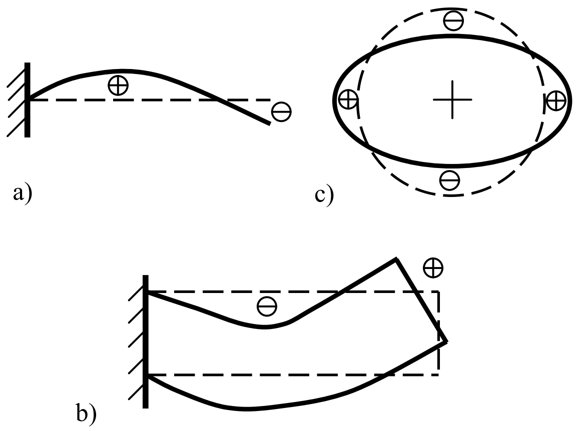


Fig. 3 Modal shape identification of the piezo actuators: a) beam, b) plate, c) cylinder.

Based on the analysed algorithm, calculation of cylinder shape piezo actuator was carried out.

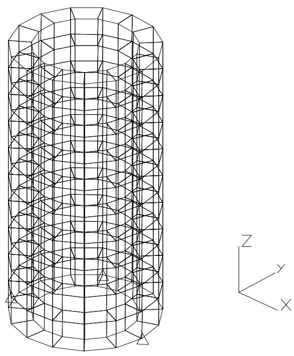


Fig. 4 Finite elements model of cylinder piezo actuator.

Also dominating coefficients and their dependence from the geometrical parameters of the cylinder were calculated. Finite elements model of the analyzed cylinder actuator is presented in Fig. 4.

Analysis of modal shapes sequence of the cylinder piezoelectric actuator must be done depending on cylinder wall thickness and internal radius ratio (Fig. 5) because two different sets of cylinder modal shapes are defined.

When the ratio of the wall thickness and radius is 0.25 < h/Rvid < 0.5, transition from thin layered cylinder modal shapes to thick layered cylinder modal shapes happen.

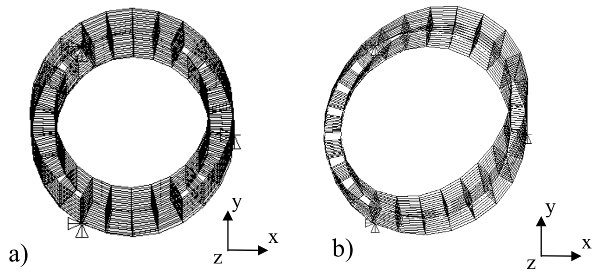


Fig. 5 First modal shape of cylinder piezoelectric actuator: a) 0.5 < h/Rvid (2,66 kHz), b) h/Rvid < 0.25 (2,87 kHz).

This process independs from cylinder boundary conditions. When cylinder geometrical parameters meet aforementioned ratio the sequence of modal shapes is unstable and it changes when any of parameters changes (Fig. 6). Dominating coefficient indexes 1, 3 corresponds radial and axial directions respectively.

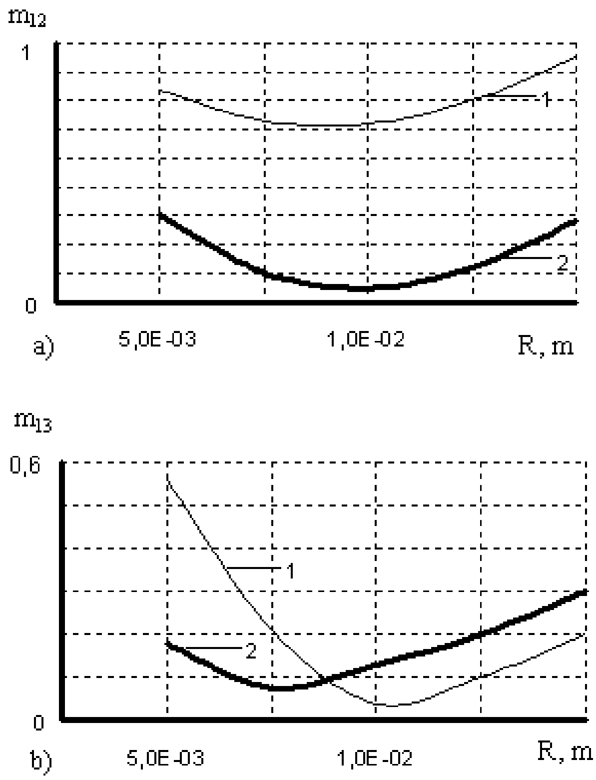


Fig. 6 Dependence of the dominating coefficient m13 of cylinder piezoelectric actuator for the third natural frequency on: a) length; b) internal radius. Wall thickness of the cylinder actuator is 0.0025m.

During numerical analysis it was determined that increasing of the length of the cylinder, increase the amplitudes of dominating oscillations, but hasn't influence to modal shape sequence changes. Internal radius of the cylinder actuator defines the ratio of the wall thickness and radius, so its influence to the modal shape sequence reveal when ratio reach already mentioned values.

III. THE LASER GATE DESIGN

The model of a piezoelectric laser gate is composed of two glued piezoceramic plates of rectangular shape, marked with number 1 (see Fig. 7) – made of Pz-26 [13] piezoceramic material; the polarization vector is directed along the width of the plate. At their end the mass from steel is glued, marked with number 2. The detailed properties of these materials are provided in Table 1.

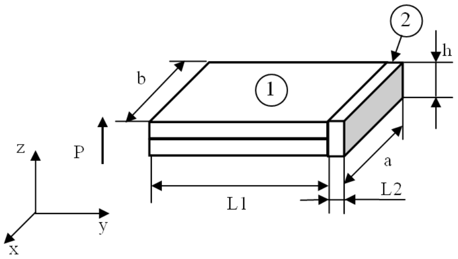


Fig. 7. The scheme of a laser gate: P – the vector of polarisation, b and a – width of the fixed and free ends of the plate, respectively, h– height, L1 and L2 – lengths of the piezoceramic plate (1) and mass (2), respectively

TABLE I
THE PROPERTIES OF THE MATERIAL USED FOR MODELLING

Material property	Piezoceramics Pz-26 (1)	Titanium (2)
Jung modulus N/m2	8.5 x 1010	204x109
Puason coefficient	0.39	0.3
Density kg/m3	7600	7800
Dielectric permittivity, x10-9F/m	ε11= 15.937; ε33= 15.937	

Piezoelectric matrix C/m2	e13=-3.09;
	e33= 16.0;
	e52= 11.64	

Elasticity matrix, x1010 N/m2	c11= 14.68; c12= 8.108;
	c13= 8.105; c33= 13.17;
	c44=3.29; c66=3.14	

The electrode throughout the whole area of the glued piezoceramic plates is grounded. The upper and lower surfaces of the plate are overed in electrodes that are connected to the 100 V voltage, see Fig. 8. Such electrode placement is used for exciting the first flexional form in the piezoactuator.

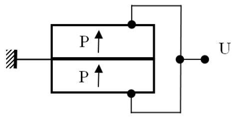


Fig. 8. The scheme of electrode placement

Geometric parameters of a laser gate have to be chosen in such a way that the eigenfrequency of the first flexional form be as high as possible, since this way its rapidity is guaranteed. Also, during analysis the oscillation amplitude A has to remain unchanged or change unsignificantly. The resulting construction would satisfy system‘s technical characteristics and be rational from a technological standpoint.

IV. THE LASER SHUTTER DESIGN

The model of a laser shutter is composed of rectangular shape piezoceramic plate, marked with number 1 (see Fig. 9) – made of CTS-23 [13] piezoceramic material; the polarization vector is directed along the length of the plate and two partial cylinders at both sides, made from titanium, marked with number 2. The detailed properties of these materials are provided in Table 2.

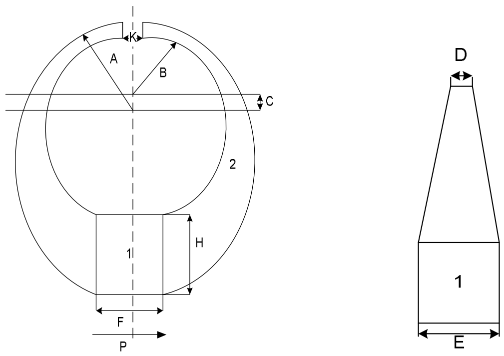


Fig. 9. The scheme of a laser shutter: P – the vector of polarisation,

A – outer radius, B – inner radius, C – difference between A and B radius, D – partial cylinder (2) upper beam width, E – partial cylinder (2) lower beam width, F – rectangular (1) length, H – rectangular (1) height.

The electrodes are connected to the both sides of grounded piezoceramic plate. The electrodes are connected to the 500 V voltage, see Fig. 10. Such electrode placement is used for exciting the first flexional form in the piezoactuator.

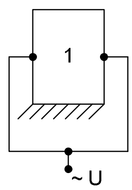


Fig. 10. The scheme of electrode placement

Geometric parameters of a laser shutter have to be chosen in such a way that the eigenfrequency of the first flexional form be as high as possible, since this way its rapidity is guaranteed. Also, during analysis the oscillation amplitude A has to remain unchanged or change unsignificantly.

TABLE II
THE PROPERTIES OF THE MATERIAL USED FOR MODELLING

Material property	Piezoceramics CTS-23 (1)	Titanium (2)
Jung modulus N/m^2	8.2764 x 10^{10}	118x10^9
Puason coefficient	0.33	0.32
Density kg/m^3	7600	4500
Dielectric permittivity, x10^{3}F/m	ε_{11}= 1.2; ε_{22}= 1.2; ε_{33}= 1.1	
Piezoelectric matrix x10^{-3}C/m^2	e_{13}= -13.6; e_{23}= -13.6; e_{33}= 27.1; e_{42}= 37.0; e_{51}= 37.0	

The resulting construction would satisfy system‘s technical characteristics and be rational from a technological standpoint.

V. FEM MODELING OF THE ACTUATOR

Finite element method (FEM) was used to perform numerical modeling of the actuator. It was used to carry out modal frequency and harmonic response analysis and to calculate trajectories of contact point (Fig. 6) movements. Basic dynamic equation of the piezoelectric actuator are derived from the principle of minimum potential energy by means of variational functionals and can be written as follows [14], [15], [16]

$$[M]\{\ddot{u}\}+[K]\{u\}+[T]\{\phi\}=\{0\}. \tag{9}$$

$$[T]^T\{u\}-[S]\{\phi\}=\{0\}. \tag{10}$$

where [M], [K], [T], [S], [C] are matrices of mass, stiffness, electro elasticity, capacity, damping respectively; $\{u\}$, $\{\phi\}$, $\{F\}$, $\{Q\}$ are vectors of nodes displacements, potentials, structural mechanical forces and charge.
Here:

$$[K]=\int_V [B]^T[c^E][B]\,dV \tag{11}$$

$$[T]=\int_V [B]^T[e][B_E]\,dV \tag{12}$$

$$[S]=\int_V [B_E]^T[\varepsilon^S][B_E]\,dV \tag{13}$$

$$[M]=\rho\int_V [N]^T[N]\,dV \tag{14}$$

$$[C]=\alpha[M]+\beta[K] \tag{15}$$

Driving force of the piezoelectric actuator is obtained from piezoceramical element. Finite element discretization of this element usually consists of a few layers of finite elements. Therefore nodes coupled with electrode layers have known potential values in advance and nodal potential of the remaining elements are calculated during the analysis. Dynamic equation of piezoelectric actuator in this case can be expressed as follows [16], [17], [18]

$$\begin{gathered}[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}+[T_1]\{\phi_1\}+[T_2]\{\phi_2\}=\{0\}\\ [T_1]^T\{u\}-[S_{11}]\{\phi_1\}-[S_{12}]\{\phi_2\}=\{Q_1\}\\ [T_2]^T\{u\}-[S_{12}]^T\{\phi_1\}-[S_{22}]\{\phi_2\}=\{0\}.\end{gathered} \tag{16}$$

here

$$[T]=[T_1 \quad T_2]. \tag{17}$$

$$[S]=\begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22}\end{bmatrix}. \tag{18}$$

where $\{\varphi_1\}\{\varphi_2\}$ are accordingly vectors of nodal potentials known in advance and calculated during numerical simulation.

Eigenfrequencies and modal shapes of the actuator are derived from the modal solution of the piezoelectric system [16[, [17]

$$\det([K^*]-\omega^2[M])=\{0\}. \tag{19}$$

where [K*] is modified stiffness matrix and it depends on nodal potential values of the piezoelements.

Harmonic response analysis of piezoelectric actuator is carried out applying sinusoidal varying voltage on electrodes of the piezoelements. Structural mechanical loads are not used. Equivalent mechanical forces are obtained because of inverse piezoe-fect and can be calculated as follows [16], [17]

$$\{F\}=-[T]\{\varphi_1\} \tag{20}$$

here

$$\{\phi_1\}=\{U\}\sin(\omega_k t) \tag{21}$$

where {U} is vector of voltage amplitudes, applied on the nodes coupled with electrodes. The vector of mechanical forces can be calculated as follows

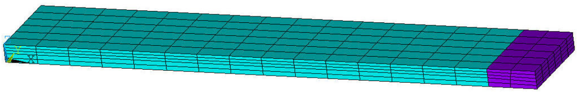
$$\{F_{eq}\}=([T_2][S_{22}]^{-1}[T_2]^T-[T_1])\{U\}\sin\omega_k t \tag{22}$$

Results of structural displacements of the piezoelectric actuator obtained from harmonic response analysis are used for determining the trajectory of contact point movement.

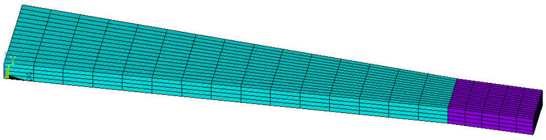
VI. CALCULATIONS AND REZULTS

The purpose of this experiment was to determine the dominating coefficients ability to be used in piezo laser shutter and piezo laser gate design.

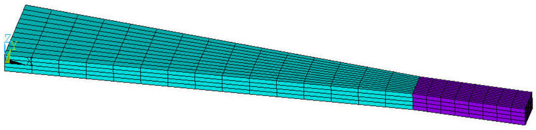
Geometric parameters of a laser gate have to be chosen in such a way that the eigenfrequency of the first bending form be as high as possible, ensuring short response time. Also, during analysis the oscillation amplitude A has to remain unchanged or change unsignificantly. The resulting construction would satisfy system‘s technical characteristics and be rational from a technological standpoint.



a) a=b



b) a=b/2



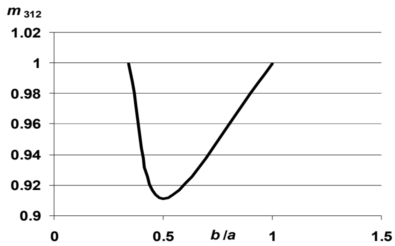
c) a=b/3

Fig. 11. FEM models of piezoelectric laser gate

Based on the analysed algorithm, calculation of piezoelectric laser gate was carried out. Finite elements model and first bending forms of the analyzed piezoelectric laser gate is presented in Fig. 11 and Fig. 13.

The largest domination coefficients are in the z direction and that means that flexional oscillations dominate. Having compared the influence of geometric parameters on domination coefficients and oscillation amplitude (Fig. 12 a and b), we can claim that with the help of domination coefficients it is possible to determine the point where the vibration amplitude is the largest.

a)



b)

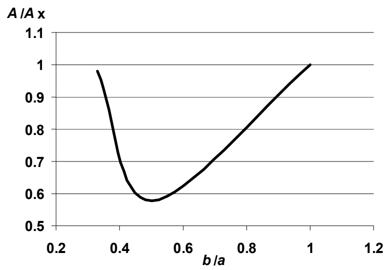
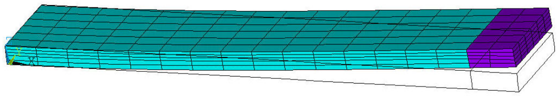
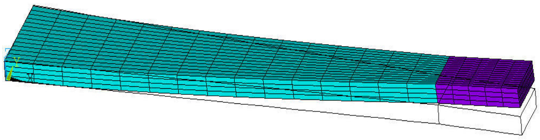


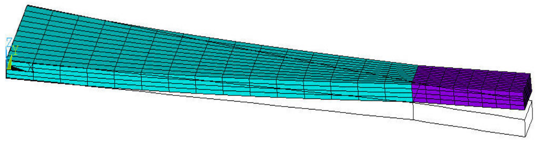
Fig.12. The influence of geometric parameters on: domination coefficients (a) and oscillation amplitude (b)



a) a=b



b) a=b/2

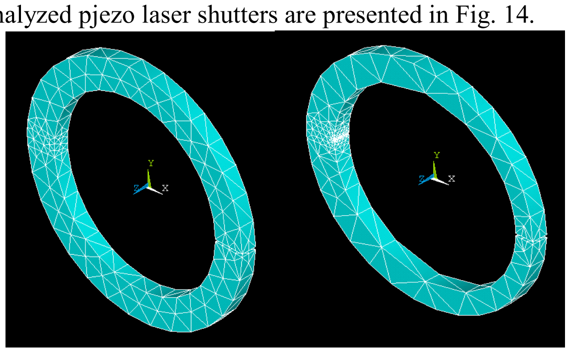


c) a=b/3

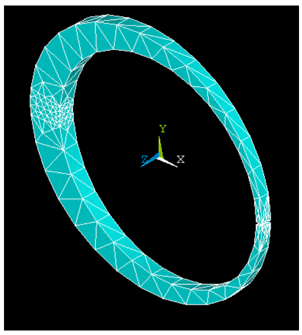
Fig. 13. First bending form of piezoelectric bimorph transducers in case when: a) a=b, b) a=b/2, c) a=b/3.

Geometric parameters of a laser shutter have to be chosen in such a way that the eigenfrequency of the first flexional form be as high as possible, since this way its rapidity is guaranteed. Also, during analysis the oscillation amplitude *A* has to remain unchanged or change unsignificantly.

During analysis the dimensions of partial cylinders ends of the plate have been changed. Finite elements models of the analyzed pjezo laser shutters are presented in Fig. 14.



a) B/C=30/0, D=5 b) B/C=32/2, D=4



c) B/C=34/4, D=3

Fig. 14 The Pjezo laser shutter FEM models

For each case considered an eigenvalue problem have been solved and harmonic analysis performed; the amplitudes for the contact point, and the system eigenfrequencies have been calculated for each construction considered.
Domination coefficients have been also calculated using formula (8). A more detailed analysis is provided below.

TABLE II
THE DOMINATION COEFFICIENTS

B/C	Sτ	Sφ	Sz
30/0	0,698541	0,301458	0,000001
32/2	0,725792	0,274203	0,000006
34/4	0,573052	0,426939	0,000008

Fig. 15 present dominating coefficient dependence from the internal radius B and the external radius A. Based on it, geometrical parameters of the piezo laser shutter could be found with the needful oscillation type and frequencie (Fig. 16).

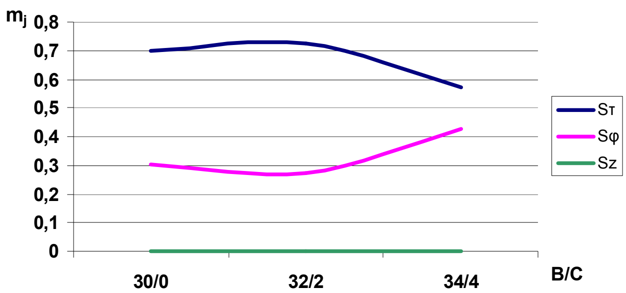


Fig.15 The influence of geometric parameters on domination coefficients.

The largest domination coefficients are in the *y* direction, and that means that flexional oscillations dominate (Fig. 15). Having compared the influence of geometric parameters on domination coefficients and eigenfrequencies, we can claim that with the help of domination coefficients we can determine the point where laser shutter ends is closed.

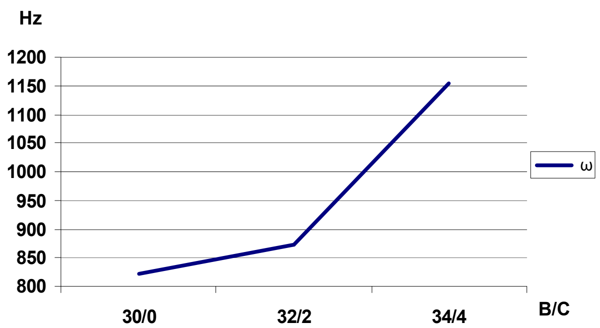
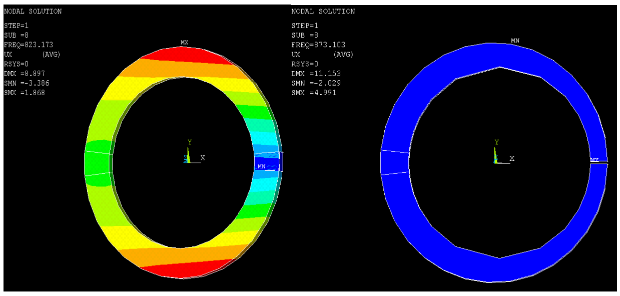
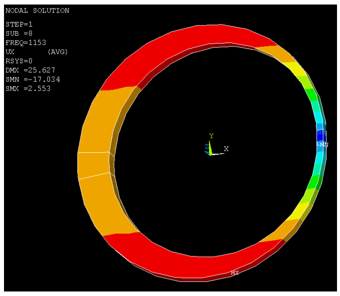


Fig.16. The influence of geometric parameters on eigenfrequencies.

Having compared the influence of geometric parameters on domination coefficients and eigenfrequencies, we can claim that with the help of domination coefficients we can determine the point where laser shutter ends is closed (Fig. 17).



a) B/C=30/0, D=5 b) B/C=32/2, D=4



c) B/C=34/4, D=3

Fig. 17. The Pjezo Laser shutter first flexional form, when the dimensions of partial cylinders ends of the plate have been changed.

VII. Conclusions

There isn't standard program developed for the piezoelectric system design. Identification of modal shapes sequence is necessary step in order to automate numerical experiments of the multicomponent piezo actuators.

An algorithm of modal shape identification has been proposed that could be applied to all mechanical structures. This algorithm can be used as an additional stage in FEM software.

Acknowledgment

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