Experimental impact force location and identification using inverse problems: application for a circular plate

Z. Boukria[a], P. Perrotin[a], A. Bennani[b]

a-LOCIE, CNRS FRE3220-Université de Savoie

Polytech' Savoie, 73376 Le Bourget du Lac Cedex, France

b-Université de Lyon, France

Université Lyon1, LBMC, 17 rue de France – 69627 Villeurbanne cedex

Abstract—Estimating load by direct measurements for structures that are in use is in practice very difficult or impossible, either because the impact location is inaccessible or because the projectile is deformable and therefore cannot be instrumented. The problem of identifying impact force on mechanical structures is the inverse of the direct problem: the use of measured responses on a given structure to identify the causes, that is, the implicated impact forces. The approach needs to create experimentally or numerically, the transfer functions between the impact and the measurement points on the structure so as to measure the responses, and to find the load by deconvolution of the signal. It is known that this type of problem is poorly conditioned. To obtain a stable solution with a physical sense, it must be stabilized using conventional regulation methods, such as the Tikhonov method. The problem of characterizing the impact becomes more complex when the impact location is unknown; so it is necessary to create the transfer functions between several impacts and measuring points, and minimize the objective function, which can locate the impact and then identify the force impact history.

This study develops an experimental method of identifying the impact force on two simple structures: a circular plate, using the transfer function obtained experimentally between the strain response and the force history applied to a point on the structure, and the Tikhonov method for the inverse problem. To locate the impact force, we used an experimental method based on the minimization of an objective function created from the transfer functions between several impact locations, forming a mesh structure with several measuring points.

***Keywords*—**Inverse problem, experimental location, identification, transfer function, plate, Tikhonov regularization.

I. Introduction

Impact force location and identification applied to a real structure is an inverse problem and is very important for design and engineering applications. The technique for impact force identification using measured responses on a structure has been proposed by many researchers.

Doyle [1,2,3,4] presented a method in a series of studies to determine the impact force on structures such as plates subjected to transverse impact. Strain gauges were used in these experiments to measure strain in selected locations. The relationship between the measured strain and the applied force was calculated from the classical theory of plates [3,4] and then techniques in the time or frequency domain were then used to reconstruct the force history.

Chang and Sun [5] proposed a method of reconstructing the impact force using the Green functions generated with experimental signal deconvolution. The advantage of this method is that the material, shape, and boundary conditions are considered by the Green functions. This method was recently improved with the explicit Green function in the process of deconvolution [6].

For impact location, the technique used (as in the field of acoustics and seismology) is based on the arrival time difference method [7]. This technique requires much more precision in determining the absolute or relative time of arrival of the first wave.

Yen and Wu [8, 9] developed a method of locating and identifying the impact force from the strain recorded at many points on a rectangular plate. A reciprocal relationship between all stress pairs was developed to locate the source, without precise prior knowledge of the force and the force history was then determined.

Monitoring structures is increasingly necessary to monitor damage. To track a structure, one must know the intensity of effort and the impact location. This study develops an experimental method of identifying the impact force on a simple structure such as a plate, using a transfer function obtained experimentally between the strain response and the force history applied to one of the structure's points, and using the Tikhonov method for the inverse problem.

To locate the impact force, an experimental method was used, based on the minimization of an objective function created from the transfer functions between the several impact locations, forming a mesh structure and several measuring points.

II. Impact Force Identification

A. Approach

To identify the impact force on a structure, the transfer function-based approach is used. Transfer functions can be determined analytically [10], [11], experimentally [12], or numerically. An experimental determination with a vibration test or with impacts has the advantage of being applicable to all types of structures, even those with any kind of complex boundary condition.

The response of the structure to be measured can be acceleration, displacement, or strain.

For a linear system, response E_j at a point j is related to impact force F_i applied to point i by the convolution equation:

$$\left(E_j\right)=\left[H_{ij}\right]\left(F_i\right) \tag{1}$$

$$\left[H_{ij}\right]=\begin{pmatrix} H_{ij}(\Delta t) & 0 & & & 0 \\ H_{ij}(\Delta 2t) & H_{ij}(\Delta t) & \ddots & & \\ H_{ij}(\Delta 3t) & H_{ij}(\Delta 2t) & \ddots & \ddots & \\ \vdots & \vdots & \ddots & \ddots & 0 \\ H_{ij}(n\Delta t) & H_{ij}((n-1)\Delta t) & \dots & \dots & H_{ij}(\Delta t) \end{pmatrix} \tag{2}$$

Where H_{ij} is the transfer function of the structure between points i and j; it contains the system's dynamic characteristics and depends on time, measurements, and impact locations.

The matrix H is very poorly conditioned or rank-deficient and so finding a solution is not guaranteed for every response, possibly leading to an unstable solution. One must therefore regularize the problem for to achieve a physically acceptable solution.

B. Experimental set-up

To validate the approach, an aluminum circular plate was impacted with a hammer at different points. The measurements of impact force and strain at various points were recorded continuously with a data acquisition system at a 25-kHz frequency rate. Strain was measured using strain gauges. The Dytran Model 5850B impact hammer used was made of plastic, with a 5-kHz bandwidth.

The data acquisition system (Fig. 1) is an autonomous STELA bag developed by SAPHIR and is composed of an NI cDAQ 9172 support with accelerometer and strain gauge modules.

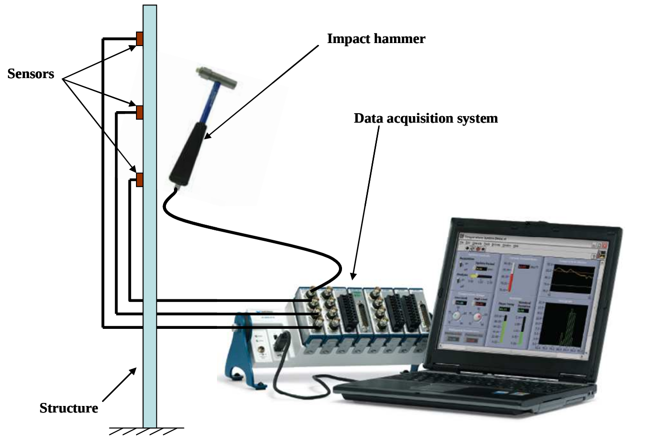

Fig.1 Data acquisition system

The plate is made of aluminum Al5054, measures 450 x 450 x 4.4 mm2, is embedded in a circular plate measuring 410 mm2 and is instrumented with strain gauges (radial and orthogonal): 120 Ω with a gauge factor of 2.10 (Fig. 2).

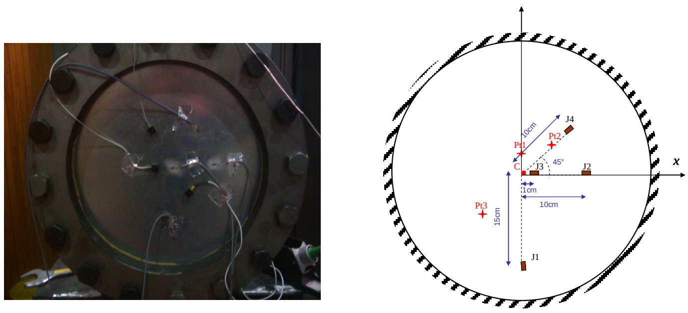

Fig. 2 Experimental set-up for impact force identification on a plate

The coordinates of the impact points and strain gauge positions are shown in Table 1.

Point	X (cm)	Y (cm)	R (cm)
C	0	0	0
Pt1	0	3	3
Pt2	5.5	5	7.43
Pt3	−7	−7.5	10.26
J1	0	15	15
J2	10	0	10
J3	1	0	1
J4	7.07	7.07	10

Table 1. The coordinates of the impact points and strain positions

The impact force and the deformation at various points are recorded continuously at an acquisition frequency of 25 kHz.

C. Results and discussion

1. Frequency response function (FRF)

Experimentally it is possible to obtain a transfer function for a pair of impact point-measurement points by measuring force and strestrain. By applying a Fourier transform to the results, the frequency response function (FRF) can be created, representing the ratio between the output signal (response measured at one point) and the input signal (applied force at one point).

$$FRF= FFT(E)/FFT(F) \quad (3)$$

The FRF matrix inversion has certain limits in the time domain, which normally requires appropriate windowing to reduce leakage errors.

The FRF allows us to obtain the first vibration modes for a structure, corresponding to the peaks of the FRF. These peaks also correspond to the peaks of the spectrum distortion. It is possible to create the transfer functions between an impact point and a measuring point for the circular plate. Figure 3 shows the FRF obtained using an impact in the plate center and deformation recorded on strain gauge J2.

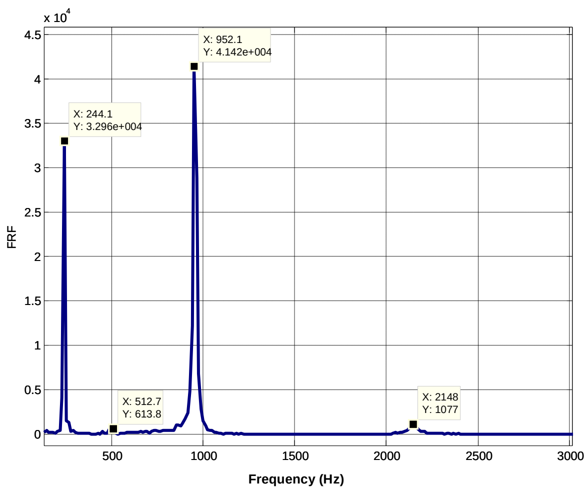

Fig.3 FRF for impact at the plate center and strain on J2

Table 2 summarizes the first three modes of vibration obtained for the plate.

Modes	Frequency (Hz)
1	244.1
2	952.1
3	2148

Table 2. The first three vibration modes

It is possible to verify the accuracy of the transfer functions obtained by calculating the strain from the transfer function and the impact force used, which represents the direct problem.

For an impact applied to point Pt1 and deformation recorded on the strain gauge J2, the FRF obtained is shown in Figure 4.

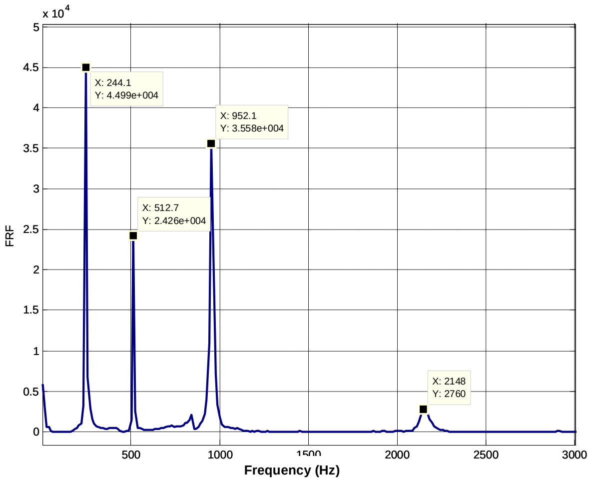

Fig.4 FRF for impact at Pt1 and strain on J2

For the same structure studied, a new vibration mode (512.7 Hz) appears, because a node in this mode coincides with the center of the plate. Consequently, when the impact is made at the plate center, this mode cannot be activated.

The direct problem is to calculate the strain using the FRF and the impact force applied to the reference impact point. The comparison of the deformation obtained and that measured shows a small difference in reconstruction, probably because of the difficulty in applying the impact at the exact reference impact point (Fig. 5).

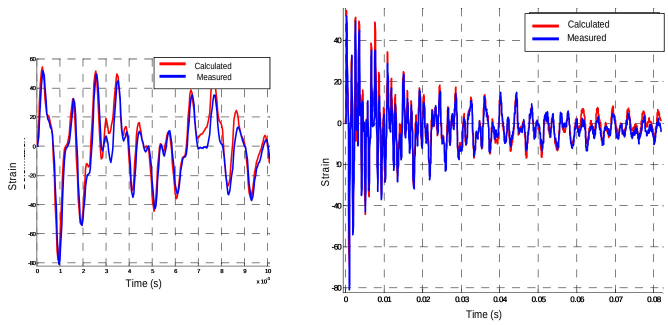

Fig.5 Comparison between measured and calculated strain – direct problem

The resolution of the system by simply reversing the direction of least squares leads to an unstable, oscillating or divergent

(Fig. 6) impact test at the center of the plate and strain recorded on strain gauge J1.

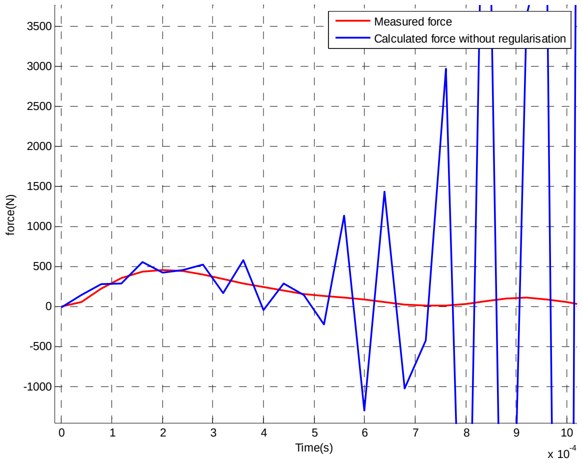

Fig.6 Calculated force without regularization

The problem is analyzed using the singular value decomposition (SVD) of different matrices Hij. It should be recalled that the SVD of a matrix H is a decomposition of the form:

$$H = U\Sigma V^T = \sum_{i=1}^{n} u_i \sigma_i v_i^T \qquad (4)$$

where:

- $U=(u_1,u_2,...,u_n)$: square orthogonal matrix formed by n orthonormal vectors ($U^TU=I_n$), which are the eigenvectors of the matrix HH^T;
- $V=(v_1,v_2,...,v_n)$: square orthogonal matrix formed by n orthonormal vectors ($V^TV=I_n$), which are the eigenvectors of the matrix H^TH;
- $\Sigma = diag(\sigma_1, \sigma_2,..., \sigma_n)$: diagonal matrix whose diagonal terms are the singular values of H, which are the square roots of eigenvalues H^*H, and ranked in descending order

$$\sigma_1 \geq \sigma_1 \geq ... \geq \sigma_n .$$

The singular values σ_i and vectors u_i and v_i satisfy certain properties, the most important being:

$$Hv_i = \sigma_i u_i , \qquad \|Hv_i\|_2 = \sigma_i , \qquad i=1,..,n$$

This relationship leads to a simple expression for the solution of the system $H*F = E$:

$$f = \sum_{i=1}^{n} \frac{u_i^T e}{\sigma_i} v_i \qquad (5)$$

In practice, in most cases and especially for the matrix obtained by discretizing the Fredholm equation of the first kind, one can see that:

- The singular values decrease to zero.
- An increase in the size of matrix H causes an increase in the number of small singular values.
- Vectors v_i and u_i are increasingly oscillating as index i increases, i.e. as σ_i decreases.

Given that the coefficients $|u_i^T e|$ corresponding to the small singular values σ_i does not decrease faster than them, the solution will be dominated by highly oscillating terms: find the problems associated with a high disturbance frequency.

A graph of singular FRF values (impact at plate center and strain on J1) shows a very significant and sudden difference between the smallest and largest singular values, which explains the instability and divergence of the solution (Fig. 7).

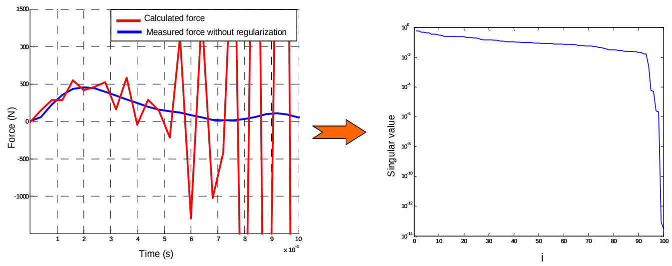

Fig.7 Singular FRF value – impact at the center and strain on J1

The solution obtained by inversion is not always disrupted, as can be seen in the second example which represents the solution in the case of an impact force applied at Pt1 and deformation registered on gauge J3 (Fig.8). The singular values decrease gradually and continuously without a sudden jump and provide a correct and stable solution.

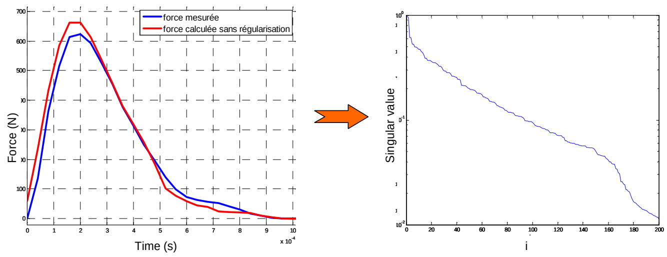

Fig.8 Singular FRF value – impact at Pt1 and strain on J3

In practice, to obtain a physically acceptable solution, the problem must be regularized.

2. Regularization

The aim of any regularization method is to reduce the contribution of these small singular values. The Tikhonov approach [13] is used to resolve poorly posed problems.

In its standard form, the key idea is to accept a non-zero residual standard and to impose a low norm $\|f\|$ to the solution. In fact, it replaces the original poorly posed problem with a close problem that is clearly posed and better conditioned, so that the solution depends consistently on the data and is robust.

Using the singular value decomposition of H, the regularized solution is written as:

$$f_\beta = \sum_{i=1}^{n} \frac{\sigma_i^2}{\sigma_i^2+\beta^2}\frac{u_i^T e}{\sigma_i} v_i = \sum_{i=1}^{n} fact_i \frac{u_i^T e}{\sigma_i} v_i \qquad (6)$$

The filter factor $fact_i = \frac{\sigma_i^2}{\sigma_i^2+\beta^2}$ between 0 and 1 controls the attenuation of each component of the SVD: β fixed between 0 and 1, then:

$$\begin{cases} \sigma_i \gg \beta \Rightarrow fact_i = 1+\theta(\beta^2/\sigma_i^2) \simeq 1 \\ \sigma_i \ll \beta \Rightarrow fact_i = \frac{\sigma_i^2}{\beta^2} + \theta(\sigma_i^4/\beta^4) \simeq \frac{\sigma_i^2}{\beta^2} \end{cases} \qquad (7)$$

For singular values close to β, $fact_i$ is between the two extremes expressed above. Thus, the first elements of the SVD corresponding to singular values greater than β will contribute fully to the solution f_β, whereas the last components corresponding to less than β are extremely weak and contribute little to the solution.

The regularization of the inverse problem will depend on the choice of the parameter β. This is a compromise between stability and the likelihood of achieving a solution.

Here the L-curve method is applied to determine the regularization parameter graphically. It was first applied by Lawson and Hanson [14] and more recently by Hansen [15] for regularization in the Tikhonov sense; it is based on the principle of seeking the optimum of a function composed of two terms, a residue called RN (residual norm) and the norm of the solution, designated by SN (semi-norm).

$$RN = \left\|\{E\}-[H]\{f\}\right\|^2 \qquad (8)$$

$$SN = \left\|\{f\}\right\|^2 \qquad (9)$$

For this method, the optimal regularization parameter corresponds to the point of maximum curvature for an (SN,RN) plot.

3. Resolution method

To apply the process of rebuilding an effort to impact on a structure, an impact is applied to a point j of the structure, and the response is recorded on a sensor at a point i. This impact will serve as a reference impact and allow the calculation of the transfer function for a pair: impact point (j) and measurement point (i). The transfer matrix is then formed and decomposed into singular values.

To rebuild a new impact force applied to the point j using the measurement recorded on point i, the Tikhonov method is used after calculation of the optimal regularization parameter with the L-curve method.

The steps of the resolution method are shown in Figure 9.

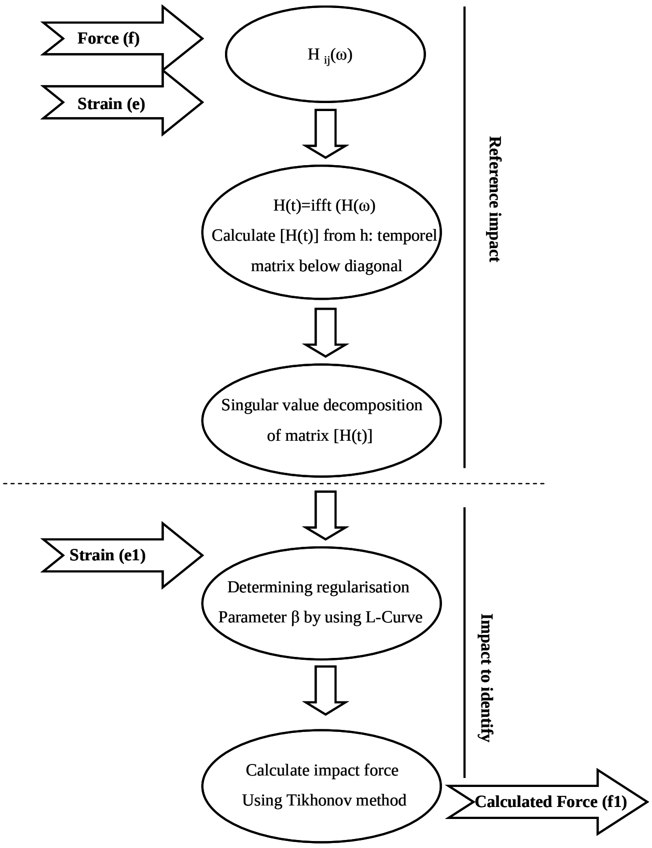

Fig.9 Identification resolution scheme

For example, for an impact in the plate center and strain measured at J1, the optimal regularization parameter obtained is shown in Figure 10.

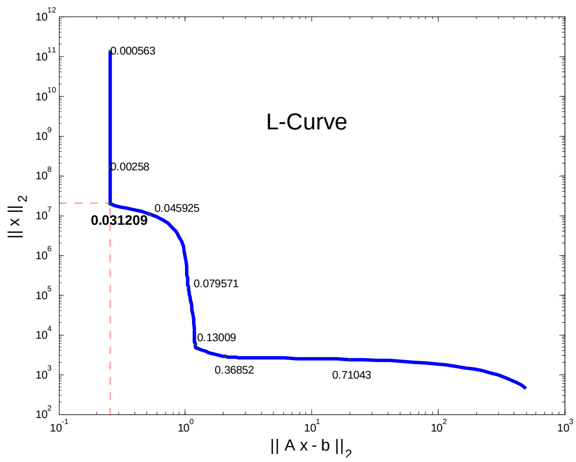

Fig.10 L-curve

For a new impact on the plate center, the resolution procedure is applied and it reconstructs the impact force. The result obtained is compared to the experimental force in Figure 11.

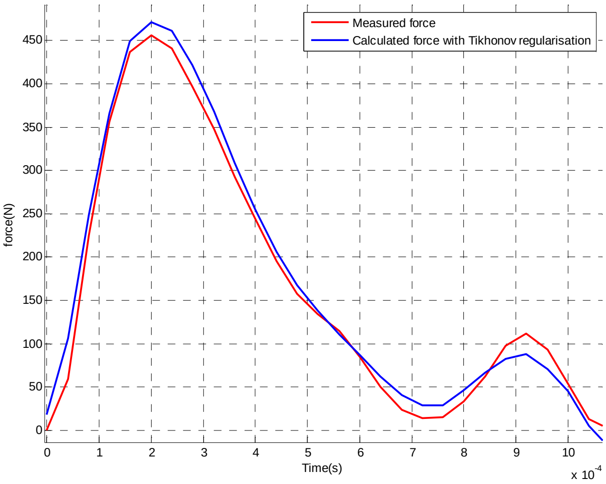

Fig.11 Calculated force with Tikhonov regularization (impact at the center of the plate and strain recorded on strain gauge J1)

In this figure, the contribution of regularization can be observed: it smoothes disturbances and provides a correct, stable solution.

In some particular cases, the solution is satisfactory without regularization. This is mainly due to the position of the gauge and its distance from the impact point.

III. Impact Force Identification and Location

A. Method

The problem of impact characterization on a structure becomes more complex when the impact location is unknown. To solve this problem, the Hu method [16] can be used: it is based on the minimization of an objective function created from the transfer functions between the several impact locations, forming a mesh structure, and several measuring points (at least three sensors [17]).

When applying the technique to identify the impacts for several sensors:

$$F = \min_{\{\tilde{f}\}} \sum_{i=1}^{m} \left\| \{\tilde{\varepsilon}_i\} - [H_i]\{\tilde{f}\} \right\|^2 + \beta \left\| \tilde{f} \right\|^2 \qquad (10)$$

$\{\tilde{\varepsilon}_i\}$ strain measured by sensor i at point (x_{0i},y_{0i},z_{0i})

$[H_i]$ transfer function between $\{\tilde{\varepsilon}_i\}$ and $\{\tilde{f}\}$

β regularization parameter.

To determine the impact location, it is assumed that impact force $\{\tilde{f}_e\}$ was applied at a point on the mesh structure; it is calculated from formula (10) and measured strain $\{\tilde{\varepsilon}_i\}$. An error vector could therefore be constructed between the estimated and measured strains:

$$E = \sum_{i=1}^{m} \frac{\left\| [H_i]\{\tilde{f}_e\} - \{\tilde{\varepsilon}_i\} \right\|^2}{\left\| \{\tilde{\varepsilon}_i\} \right\|^2} \qquad (11)$$

The impact location was obtained by minimizing the objective function E. In the process of minimization, $\{\tilde{f}_e\}$ was recalculated for each impact point assumed using equation (11). Once the impact location had been obtained, the impact force history can be calculated using equation (11).

The experimental approach consists of applying impacts throughout the grid structure and recording strain using gauges installed on the structure. This allows the creation of the structure’s transfer functions that are saved as a historical vibration. To locate and identify the impact force of an unknown impact, the recorded strain from each gauge was used and the approach followed the scheme in Fig.12, thus automating the method:

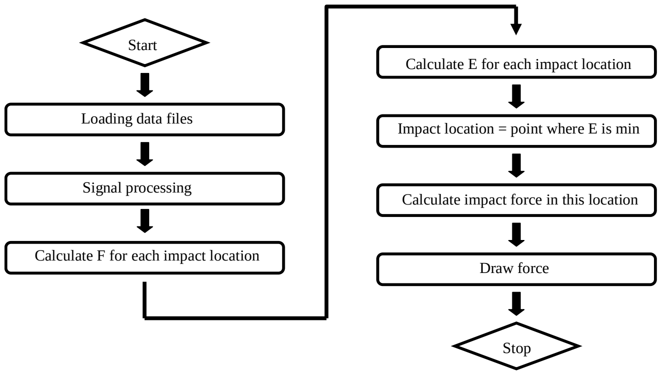

Fig.12 Characterization resolution scheme

B. Experimental set-up

First, a grid was drawn on the plate and four strain gauges were installed to record the strain response because, for a two-dimensional problem (plate), a minimum of three sensors is required (Fig.13).

Impacts were applied to all points of the grid and transfer functions were thus created between the impact location and the measurements.

Fig.13 Experimental set-up for impact location

C. Results

Once the impact force has been calculated and assuming that it is applied to each grid point of the structure, the error function can be calculated. It can be observed that the value of the error function is minimal at point Pt35; it therefore represents the point at which the impact to characterize is applied. The red points on Figure 14 represent the maximum error vector, and those in blue represent the minimum values. In this application, the maximum value of E is limited to 20 for a better view and interpretation of the curve.

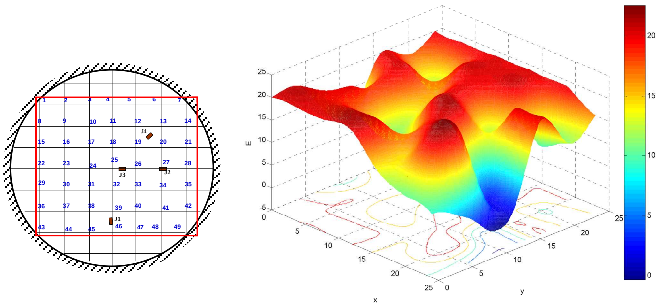

Fig.14. Error function

Once the point of impact has been found, the approach of identifying the impact force at that point is applied. For an unknown impact, e.g., on point 35 on the grid and after applying the procedure for characterizing and using the recorded strain, a printed screen of the results is obtained (Figure 15). The impact point is indicated by the number of points on the grid that appear on the screen and the calculated force is compared graphically to the measured force.

The approach is applied to multiple impact points that coincide with the intersections of the grid on the structure studied. For these cases, the exact impact location was obtained and the identified force was very close to the measured force.

If the point of impact is outside these points, the impact location is obtained on the grid intersection point that is the closest to the exact location. The reconstructed impact force in this case is not always close to that measured, and its quality strongly depends on the distance between the exact point of impact and the grid point on which it is located.

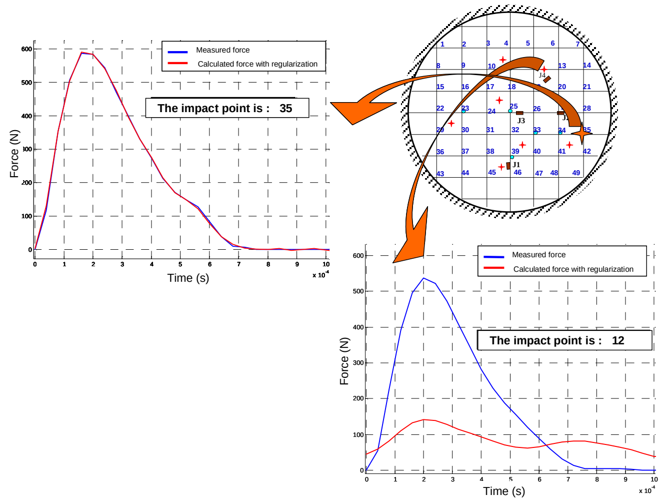

Fig.15. Example of results – impact location and force identification

The impact locations obtained are summarized in Figure 16.

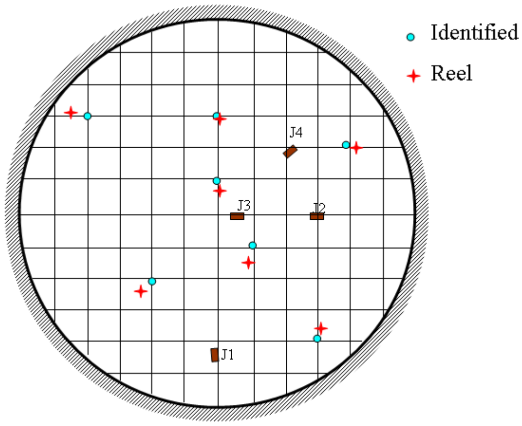

Fig.16 Results of impact locations on the plate

D. Impact location outside grid point

To locate an impact that is not on a grid point, the mesh of the grid must be refined. It is also possible to use a numerical model that controls the mesh. In an experimental approach that has the advantage of finding results with the structure's existing boundary conditions, it is difficult to increase the number of grid points for a large structure. Shape functions can be used to find transfer functions at any point (Fig. 17).

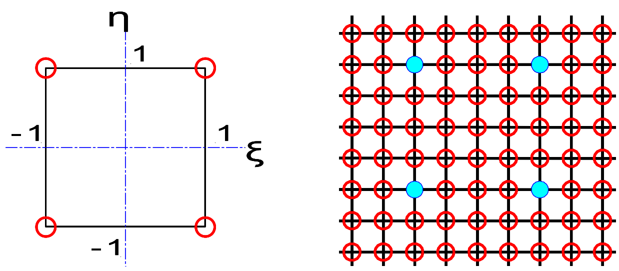

Fig.17 Using shape function to calculate transfer function outside a grid point

After obtaining the transfer function at four nodes for one element (a rectangular region), the transfer function at any location can be calculated within this element using shape functions [18]. A four-noded element should possess bi-linear shape functions.

$$\left[G(\xi_i,\eta_i)\right]=\sum_{i=1}^{4}N_i(\xi_i,\eta_i)\left[G_i\right] \tag{12}$$

$$\begin{aligned}N_1&=\frac{1}{4}(1-\xi)(1-\eta), N_2=\frac{1}{4}(1+\xi)(1-\eta)\\ N_3&=\frac{1}{4}(1-\xi)(1+\eta), N_4=\frac{1}{4}(1+\xi)(1+\eta)\end{aligned} \tag{13}$$

Once the transfer function has been calculated for all grid points, the impact location and impact force are obtained by applying the characterization procedure. For the previous example, the exact point of impact is obtained and the calculated force is close to the measured force (Fig.18).

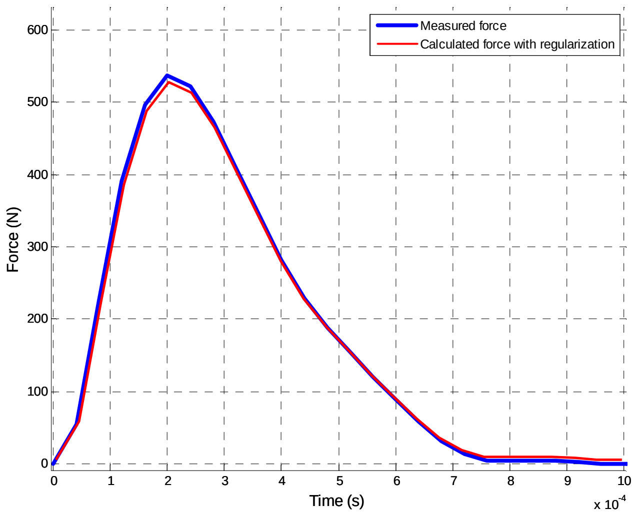

Fig.18 Impact location outside grid point

IV. Conclusion

This study shows that it is possible to experimentally identify and locate the impact force applied to simple structures such as a plate with linear material, using measured responses and transfer functions. An experimental determination by means of a vibration test or impact has the advantage of being applicable to all types of structures, even complex ones, with arbitrary boundary conditions.

Using the Tikhonov regularization method and the L-curve method to determine the optimal regularization parameter, it is possible to obtain good results. Using experimental transfer functions created between different impact points forming a grid on the structure, as well as strains recorded on different sensors, the impact is located on the grid and the force is calculated at this location.

For an impact outside an impact point, the use of shape function to calculate the transfer function at any point allows one to refine the grid mesh and obtain the exact impact location.

In a subsequent experiment, we will test the robustness of the method by examining the optimal solution in terms of the number and position of the sensors. In addition, we will apply this experimental approach to describe the impact on a reinforced concrete slab to establish a system for detecting hazards such as falling blocks on a rock-shed structure.

References

[1] J.F. Doyle, “An experimental method for determining the dynamic contact law”, Experimental Mechanics, vol. 24, no. 4, pp. 265-270, 1984.
[2] J.F. Doyle, “Further developments in determining the dynamic contact law”, Experimental Mechanics, vol. 24, no. 4, pp. 265-270, 1984.
[3] J.F. Doyle, “Determining the contact force during the transverse impact of plates” Experimental Mechanics, vol. 27 (1987), no. 1, pp. 68-72, 1987.
[4] J.F. Doyle, “Experimentally determining the contact force during the transverse impact of an orthopic plate”, Journal of Sound and Vibration, vol. 118, no. 3, pp. 441-448, 1987.
[5] C. Chang and C.T. Sun, “Determining transverse impact force on a composite laminate by signal deconvolution”, Experimental Mechanics, vol. 29, pp. 414-419, 1989
[6] E. WU and C.S. Yen, “Two methods for determining impact force history on elastic plates”, Experimental Mechanics, vol. 35, pp. 11-18, 1995.
[7] Y.H. Pao, “Theory of acoustic emission, ASME Elastic Wave and Non-Destructive Testing of Materials”, AMD, vol. 29, pp. 107-128, 1978.
[8] C.S. Yen and E. Wu, “On the inverse problem of rectangular plates subjected to elastic impact, part I: Method development and numerical verification”, Journal of Applied Mechanics, vol. 62, no. 3, pp. 692-698, 1995.
[9] C.S. Yen and E. Wu, “On the inverse problem of rectangular plates subjected to elastic impact, part II: Experimental verification end further applications”, Journal of Applied Mechanics, vol. 62, no. 3, pp. 699-705, 1995.
[10] M. Géradin and D. Rixen, *Théorie des vibrations: application à la dynamique des structures*, MASSON, Paris, 1993.
[11] E. Jacquelin, A. Bennani and M. Massenzio, “Analysis of a reconstruction problem”, Structural Engineering and Mechanics, vol. 21, no. 3, pp. 237-254, 2005.
[12] E. Jacquelin, A. Bennani and P. Hamelin, “Force reconstruction: analysis and regularization of a deconvolution problem”, Journal of Sound and Vibration, vol. 265, pp. 81-107, 2003.
[13] A.N. Tikhonov, V.Y. Arsenin, *Solutions of ill-posed problems*, Wiley, 1977.
[14] C.L. Lawson, R.J Hanson, *Solving least squares problems*, SIAM, Philadelphia, P A, 1995.
[15] P.C. Hansen, “The L-curve and its use in the numerical treatment of inverse problems”, Tech. Report, IMM-REP 99-15, Dept. of Math. Model. Tech. Univ. of Denmark, 1999.
[16] N. Hu and H. Fukunaga, “A new method for health monitoring of composite structures through identification of impact force”, Journal of Advanced Science, vol. 17, no.1&2, pp. 82-89, 2005.
[17] A. Salehian, “Identification the location of a sudden damage in composite laminates using wavelet approach”, thesis report, Worcester Polytechnic Institute, June 2003.
[18] H. Fukunaga, T. Umino and N. Hu. “Impact Force Identification of CFRP Stiffened Panel under Multiple Loading”. Proc. Fifth Int. Workshop on Structural Health Monitoring, 2007, pp. 177-184.