Stresses in an Anisotropic Elastic Plate due to Strip-Loading

Dinesh Kumar Madan, Shamta Chugh and Kuldip Singh

***Abstract*—** The closed-form expressions for the stresses at any point of monoclinic elastic plate interfacing differently with the base due to strip-loading are obtained. The interface between the elastic plate and the base is assumed to be either 'perfectly bonded' or 'smooth-rigid' or 'rough-rigid'. As particular cases the stresses in orthotropic elastic layered half-space, isotropic elastic layered half-space and due to shear line-load in monoclinic elastic half space have been obtained. Numerically, in the monoclinic elastic half-space, the variation of shear stresses with the horizontal distance has been studied.

***Keywords*—** Monoclinic material, Strip-loading, Shear line-load.

I. Introduction

As mentioned by Crampin [1], monoclinic symmetry is the symmetry of two sets of non orthogonal parallel cracks, where the plane of symmetry is perpendicular to the lines of intersection of the two sets of crack faces. Monoclinic symmetry of the systems of cracks may be found near the surface of the Earth where lithostatic pressures have not closed cracks perpendicular to the maximum compressional stress.

The solution of the problem of the deformation of a horizontally layered elastic material under the action of the surface loads has been finding wide applications in engineering, geophysics and soil mechanics. When the source surface is very long in one direction in comparison with the others, the use of two-dimensional approximation is justified and consequently calculations are simplified to a great extent and one gets a closed form analytical solution. A very long strip-source and a very long line-source are examples of such two-dimensional sources. Love [2] obtained expressions for the displacements due to a line source in an isotropic elastic medium. Maruyama [3] obtained the displacement and stress fields corresponding to long strike-slip faults in a homogenous isotropic half-space. Okada [4,5] provided compact analytic expressions for the surface deformation and internal deformation due to inclined shear and tensile faults in a homogenous isotropic half-space. Garg et al. [6] obtained an analytical solution for the deformation of an orthotropic layered half-space caused by along strike-slip fault. Ting [7] derived the Green's functions for a line force and a screw dislocation for the anti-plane deformation of a monoclinic elastic medium consisting of a single half-space or two half-spaces in 'perfect' contact. The calculation of anti-plane deformation due to a line source in a monoclinic medium is much more difficult than the corresponding calculation for a source in an orthotropic medium because of the presence of the mixed derivatives in the equation of equilibrium.

In the present paper, we have obtained the closed-form expressions for the stresses in an horizontal monoclinic elastic plate of an infinite lateral extent lying over a base due to strip-loading. In geophysics, the elastic plate represents the crust of the earth. The interface between the plate and the base may be either 'perfectly bonded', 'smooth-rigid' or 'rough-rigid'. The deformation of the monoclinic elastic plate corresponding to each type of the interface has been obtained. The deformation of a monoclinic elastic uniform half-space due to strip-loading can be obtained from our results as particular case. As particular cases: the stresses in orthotropic elastic layered half-space, isotropic elastic layered half-space and due to shear line-load in monoclinic elastic half space have been obtained. Numerically, we have studied the variation of stresses.

II. Fundamental Equations

The constitutive matrix equation of a monoclinic material has the following form [8]

$$\begin{bmatrix}\tau_1\\ \tau_2\\ \tau_3\\ \tau_4\\ \tau_5\\ \tau_6\end{bmatrix}=\begin{bmatrix}c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16}\\ c_{12} & c_{22} & c_{23} & 0 & 0 & c_{26}\\ c_{13} & c_{23} & c_{33} & 0 & 0 & c_{36}\\ 0 & 0 & 0 & c_{44} & c_{45} & 0\\ 0 & 0 & 0 & c_{45} & c_{55} & 0\\ c_{16} & c_{26} & c_{36} & 0 & 0 & c_{66}\end{bmatrix}\begin{bmatrix}e_1\\ e_2\\ e_3\\ e_4\\ e_5\\ e_6\end{bmatrix}. \tag{1}$$

In equation (1), we used Voigt's convention by which the tensional indices are replaced by matrix indices in the expression of the stress and shear components τ_i and e_i $(i=1,2,3,4,5,6)$. The elements $c_{ij}, i, j=1,2,3,4,5,6$ of the stiffness matrix from (1) represent the elasticity's of the monoclinic material.

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The field's equations of a monoclinic material in anti-plane strain equilibrium state are:

-displacement equations:

$$u_1 = u_2 = 0,\ u_3 = u_3(x_1, x_2); \tag{2}$$

-strain equations:

$$e_{11} = e_{22} = e_{33} = e_{12} = 0,\ \ e_{31} = \frac{1}{2} u_{3,1},\ e_{23} = \frac{1}{2} u_{3,2}; \tag{3}$$

-stress equations:

$$\tau_{11} = \tau_{22} = \tau_{33} = \tau_{12} = 0,\ \ \tau_{31} = c_{45}\, u_{3,2} + c_{55}\, u_{3,1},\ \tau_{23} = c_{44}\, u_{3,2} + c_{45}\, u_{3,1}. \tag{4}$$

Consequently, Cauchy's first two equations are identically satisfied and the third equation becomes

$$\tau_{13,1} + \tau_{23,2} = 0\ . \tag{5}$$

Using equations (4) and (5), the equilibrium equation satisfied by u_3 can be written in the following form:

$$u_{3,11} + \frac{c_{45}}{c_{55}} u_{3,12} + \frac{c_{44}}{c_{55}} u_{3,22} = 0\ . \tag{6}$$

III.FORMULATION AND SOLUTION OF THE PROBLEM

We consider a horizontal monoclinic elastic plate of thickness H lying over a base. The origin of Cartesian co-ordinates system $(x_1\, x_2\, x_3)$ is taken at the upper boundary of the plate and x_1-axis is drawn into the medium. The monoclinic elastic plate occupies the region $0 < x_1 \le H$ and the region $x_1 > H$ is the base over which the plate is lying (Fig. 1).

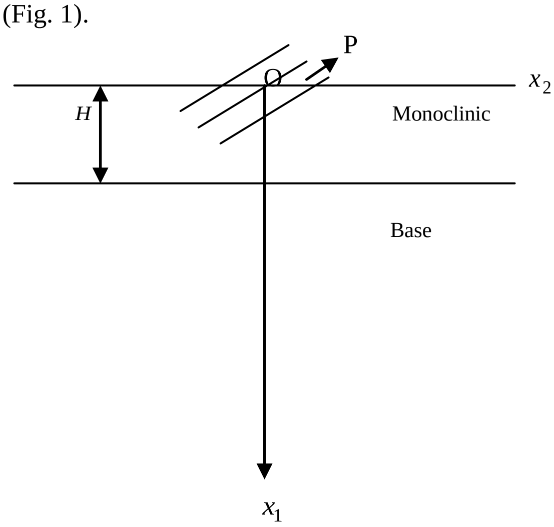

Fig. 1. Section of the model by the plane $x_3 = 0$.

Let a shear-load P per unit area is acting over the strip $|x_2| \le h$ of the surface $x_1 = 0$ in the positive x_3 – direction. The boundary condition at the surface $x_1 = 0$ is

$$\tau_{31} = \begin{cases} -P, & |x_2| \le h \\ 0, & |x_2| > h \end{cases} \tag{7}$$

The interface $x_1 = H$ between the plate and the base may be either 'smooth-rigid' or 'rough-rigid' or 'perfectly bonded'.

A. Interface Conditions

When the interface $x_1 = H$ is of the smooth-rigid type, the condition is [9]

$$\tau_{31}(x_1 = H) = 0. \tag{8}$$

When the interface is in rough-rigid contact at $x_1 = H$, the condition is [9]

$$u_3(x_1 = H) = 0. \tag{9}$$

When the plate and the base are in perfectly bonded at $x_1 = H$, the continuity of the displacement and shear stress τ_{31} implies [10]

$$u_3(x_1 = H-) = u_3(x_1 = H+).$$
$$\tau_{31}(x_1 = H-) = \tau_{31}(x_1 = H+). \tag{10}$$

We shall find the deformation field at any point of the monoclinic elastic plate corresponding to each type of contact between the plate and the base due to strip-loading.

The Fourier transform of $X(x_1, x_2)$ is defined as

$$\overline{X}(x_1, k) = \int_{-\infty}^{\infty} X(x_1, x_2) e^{i k x_2} d x_2\ , \tag{11}$$

so that

$$X(x_1, x_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \overline{X}(x_1, k) e^{-i k x_2} d k\ . \tag{12}$$

Taking the Fourier transform of (6), we get

$$\frac{d^2 \bar{u}_3}{d x_1^2} - 2\left(\frac{c_{45}}{c_{55}} ik\right) \frac{d \bar{u}_3}{d x_1} - \frac{c_{44}}{c_{55}} k^2 \bar{u}_3 = 0\ . \tag{13}$$

The solution of the ordinary differential equation (13) is

$$\bar{u}_3 = \left(C_1 e^{m_1 |k| x_1} + C_2 e^{-m_1 |k| x_1}\right) e^{i m_2 k\, x_1}\ . \tag{14}$$

where $m_1 = \sqrt{m_3 - m_2^2}$, $m_2 = \frac{c_{45}}{c_{55}}$, $m_3 = \frac{c_{44}}{c_{55}}$ and C_1 and C_2 may be functions of *k*.

By using inverse Fourier transform, we have

$$u_3 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(C_1 e^{m_1 |k| x_1} + C_2 e^{-m_1 |k| x_1}\right) e^{-i(x_2 - m_2 x_1) k} d k\ . \tag{15}$$

Using equation (15) and equation (4), the shear stresses are

$$\tau_{31} = \frac{T_1}{2\pi} \int_{-\infty}^{\infty} \left(C_1 e^{m_1 |k| x_1} - C_2 e^{-m_1 |k| x_1}\right) e^{-i(x_2 - m_2 x_1) k} |k|\, d k, \tag{16}$$

$$\tau_{32} = \frac{T_1}{2\pi} \left[m_2 \int_{-\infty}^{\infty} \left(C_1 e^{m_1 |k| x_1} - C_2 e^{-m_1 |k| x_1}\right) e^{-i(x_2 - m_2 x_1) k} |k| d k \right. \tag{17}$$
$$\left. - i m_1 \int_{-\infty}^{\infty} \left(C_1 e^{m_1 |k| x_1} + C_2 e^{-m_1 |k| x_1}\right) e^{-i(x_2 - m_2 x_1) k} k\, d k \right].$$

where $T_1 = m_1 c_{55}$. Using the boundary condition (7), we have

$$\bar{\tau}_{31} = \frac{-2P}{\pi} \sin kh. \tag{18}$$

Therefore,

$$\tau_{31} = \frac{-P}{\pi} \int_{-\infty}^{\infty} \left(\frac{\sin kh}{k} \right) e^{-ikx_2} dk \; . \tag{19}$$

From (16) and (18), we obtain

$$C_1 - C_2 = -\frac{2P}{T_1} \left(\frac{\sin kh}{k|k|} \right). \tag{20}$$

Smooth-Rigid Interface

When the contact between the plate and the base at $x_1 = H$ is smooth-rigid, the interface condition (8) and equation (16) yield

$$C_1 e^{m_1|k|H} - C_2 e^{-m_1|k|H} = 0. \tag{21}$$

From (20)-(21), we have

$$C_1 = \frac{2P \sin kh}{T_1 \, k|k|} \left(\frac{e^{-2m_1|k|H}}{1 - e^{-2m_1|k|H}} \right),$$

$$C_2 = \frac{2P \sin kh}{T_1 \, k|k|} \left(\frac{1}{1 - e^{-2m_1|k|H}} \right). \tag{22}$$

The displacement of monoclinic elastic plate for a smooth-rigid interface can be obtained from (15)-(17) and (22). The integral expression for the displacement is obtained as:

$$u_3 = \frac{P}{\pi T_1} \int_{-\infty}^{\infty} \frac{\sin kh}{k|k|} \left[e^{-m_1|k|x_1} + \sum_{n=1}^{\infty} e^{-m_1|k|(2nH + x_1)} + \sum_{n=1}^{\infty} e^{-m_1|k|(2nH - x_1)} \right] e^{-i(x_2 - m_2 x_1)k} dk \, . \tag{23}$$

and the analytical expressions for the shear stresses τ_{31} and τ_{32} in an elastic plate are obtained as:

$$\tau_{31} = -\frac{P}{\pi} \left[\tan^{-1} \frac{2h m_1 x_1}{(x_2 - m_2 x_1)^2 + m_1^2 x_1^2 - h^2} + \sum_{n=1}^{\infty} \left\{ \tan^{-1} \frac{2h m_1 (2nH + x_1)}{(x_2 - m_2 x_1)^2 + m_1^2 (2nH + x_1)^2 - h^2} - \tan^{-1} \frac{2h m_1 (2nH - x_1)}{(x_2 - m_2 x_1)^2 + m_1^2 (2nH - x_1)^2 - h^2} \right\} \right], \tag{24}$$

$$\tau_{32} = -\frac{P}{\pi} \Bigg[m_2 \Bigg\{ \tan^{-1} \frac{2h m_1 x_1}{(x_2 - m_2 x_1)^2 + m_1^2 x_1^2 - h^2} + \sum_{n=1}^{\infty} \left(\tan^{-1} \frac{2h m_1 (2nH + x_1)}{(x_2 - m_2 x_1)^2 + m_1^2 (2nH + x_1)^2 - h^2} - \tan^{-1} \frac{2h m_1 (2nH - x_1)}{(x_2 - m_2 x)^2 + m_1^2 (2nH - x_1)^2 - h^2} \right) \Bigg\} + \frac{m_1}{2} \Bigg\{ \log \frac{[(x_2 - m_2 x_1) + h]^2 + [m_1 x_1]^2}{[(x_2 - m_2 x_1) - h]^2 + [m_1 x_1]^2} + \sum_{n=1}^{\infty} \left(\log \frac{[(x_2 - m_2 x_1) + h]^2 + [m_1 (2nH + x_1)]^2}{[(x_2 - m_2 x_1) - h]^2 + [m_1 (2nH + x_1)]^2} + \log \frac{[(x_2 - m_2 x_1) + h]^2 + [m_1 (2nH - x_1)]^2}{[(x_2 - m_2 x_1) - h]^2 + [m_1 (2nH - x_1)]^2} \right) \Bigg\} \Bigg]. \tag{25}$$

Rough-Rigid Interface

When the contact is rough-rigid then after using the interface condition (9) in equation (15), we obtain

$$C_1 e^{m_1|k|H} + C_2 e^{-m_1|k|H} = 0. \tag{26}$$

From (20) and (26), the values of C_1 and C_2 are found to be

$$C_1 = -\frac{2P \sin kh}{T_1 \, k|k|} \left(\frac{e^{-2m_1|k|H}}{1 + e^{-2m_1|k|H}} \right)$$

$$C_2 = \frac{2P \sin kh}{T_1 \, k|k|} \left(\frac{1}{1 + e^{-2m_1|k|H}} \right) . \tag{27}$$

Using (27) in equation (15), the displacement is

$$u_3 = \frac{P}{\pi T_1} \int_{-\infty}^{\infty} \frac{\sin kh}{k|k|} \left[e^{-m_1|k|x_1} + \sum_{n=1}^{\infty} (-1)^n e^{-m_1|k|(2nH + x_1)} - \sum_{n=1}^{\infty} (-1)^n e^{-m_1|k|(2nH - x_1)} \right] e^{-i(x_2 - m_2 x_1)k} dk.. \tag{28}$$

The analytical expression for the stresses are obtained as:

$$\tau_{31} = -\frac{P}{\pi} \left[\tan^{-1} \frac{2h m_1 x_1}{(x_2 - m_2 x_1)^2 + m_1^2 x_1^2 - h^2} + \sum_{n=1}^{\infty} (-1)^n \left\{ \tan^{-1} \frac{2h m_1 (2nH + x_1)}{(x_2 - m_2 x_1)^2 + m_1^2 (2nH + x_1)^2 - h^2} - \tan^{-1} \frac{2h m_1 (2nH - x_1)}{(x_2 - m_2 x_1)^2 + m_1^2 (2nH - x_1)^2 - h^2} \right\} \right], \tag{29}$$

$$\begin{aligned}
\tau_{32} = -\frac{P}{\pi}\Bigg[& m_2\Bigg\{ tan^{-1}\frac{2hm_1x_1}{(x_2-m_2x_1)^2+m_1^2x_1^2-h^2} \\
& +(-1)^n\sum_{n=1}^{\infty}\Bigg(tan^{-1}\frac{2hm_1(2nH+x_1)}{(x_2-m_2x_1)^2+m_1^2(2nH+x_1)^2-h^2} \\
& - tan^{-1}\frac{2hm_1(2nH-x_1)}{(x_2-m_2x_1)^2+m_1^2(2nH-x_1)^2-h^2}\Bigg)\Bigg\} \\
& -\frac{m_1}{2}\Bigg\{ log\frac{[(x_2-m_2x_1)+h]^2+[m_1x_1]^2}{[(x_2-m_2x_1)-h]^2+[m_1x_1]^2}+ \\
& +\sum_{n=1}^{\infty}(-1)^n\Bigg(log\frac{[(x_2-m_2x_1)+h]^2+[m_1(2nH+x_1)]^2}{[(x_2-m_2x_1)-h]^2+[m_1(2nH+x_1)]^2} \\
& + log\frac{[(x_2-m_2x_1)+h]^2+[m_1(2nH-x_1)]^2}{[(x_2-m_2x_1)-h]^2+[m_1(2nH-x_1)]^2}\Bigg)\Bigg\}\Bigg].
\end{aligned} \tag{30}$$

Perfect Contact

The displacement in the monoclinic elastic half-space $x_1 > H$ is

$$u'_3 = \frac{1}{2\pi}\int_{-\infty}^{\infty} C'_2\, e^{-m'_1|k|x_1}\, e^{-i(x_2-m'_2x_1)k}\,dk, \tag{31}$$

where

$m'_1=\sqrt{m'_2-m'^2_3}$, $m'_2=\dfrac{c'_{45}}{c'_{55}}$, $m'_3=\dfrac{c'_{44}}{c'_{55}}$ and the coefficient C'_2 is to be determined from the boundary conditions. Then

$$\tau'_{31} = -\frac{T'_1}{2\pi}\int_{-\infty}^{\infty} C'_2\, e^{-m'_1|k|x_1} e^{-i(x_2-m'_2x_1)k}|k|\,dk. \tag{32}$$

where $T'_1 = m'_1 c'_{55}$.

Equations (10), (15), (16), (31) and (32) yield the relations

$$\left(C_1e^{m_1|k|H}+C_2e^{-m_1|k|H}\right)e^{ikm_2H} = C'_2e^{-m'_1|k|H}e^{ikm'_2H}. \tag{33}$$

$$T_1\left(C_1e^{m_1|k|H}-C_2e^{-m_1|k|H}\right)e^{ikm_2H} = -T'_1C'_2e^{-m'_1|k|H}e^{ikm'_2H}. \tag{34}$$

Solving (20), (33) and (34), we get

$$\begin{aligned}
C_1 &= \frac{2P\,sin\,kh}{T_1\,k|k|}\left(\frac{Ve^{-2m_1|k|H}}{1-Ve^{-2m_1|k|H}}\right), \\
C_2 &= \frac{2P\,sin\,kh}{T_1\,k|k|}\left(\frac{1}{1-Ve^{-2m_1|k|H}}\right), \\
C'_2 &= \frac{4P\,sin\,kh}{T_1\,k|k|}\left(\frac{e^{-A|k|H+iB\,kH}}{1-Ve^{-2m_1|k|H}}\right)\left(\frac{T}{T+1}\right),
\end{aligned} \tag{35}$$

where

$T=T_1/T'_1,\ V=(T-1)/(T+1),\ A=m_1-m'_1$ and $B=m_2-m'_2$. (36)

Using (31) in equations (15), (16) and (17), we obtain the deformation field as follow:

$$\begin{aligned}
u_3 = \frac{P}{\pi T_1}\int_{-\infty}^{\infty}\frac{sin\,kh}{k|k|}\Bigg[& e^{-m_1|k|x_1}+\sum_{n=1}^{\infty}V^ne^{-m_1|k|(2nH+x_1)} \\
& +\sum_{n=1}^{\infty}V^ne^{-m_1|k|(2nH-x_1)}\Bigg]e^{-i(x_2-m_2x_1)k}dk.,
\end{aligned} \tag{37}$$

$$\begin{aligned}
\tau_{31} = -\frac{P}{\pi}\Bigg[& tan^{-1}\frac{2hm_1x_1}{(x_2-m_2x_1)^2+m_1^2x_1^2-h^2} \\
& +\sum_{n=1}^{\infty}V^n\Bigg\{ tan^{-1}\frac{2hm_1(2nH+x_1)}{(x_2-m_2x_1)^2+m_1^2(2nH+x_1)^2-h^2} \\
& - tan^{-1}\frac{2hm_1(2nH-x_1)}{(x_2-m_2x_1)^2+m_1^2(2nH-x_1)^2-h^2}\Bigg\}\Bigg],
\end{aligned} \tag{38}$$

$$\begin{aligned}
\tau_{32} = -\frac{P}{\pi}\Bigg[& m_2\Bigg\{ tan^{-1}\frac{2hm_1x_1}{(x_2-m_2x_1)^2+m_1^2x_1^2-h^2} \\
& +\sum_{n=1}^{\infty}V^n\Bigg(tan^{-1}\frac{2hm_1(2nH+x_1)}{(x_2-m_2x_1)^2+m_1^2(2nH+x_1)^2-h^2} \\
& - tan^{-1}\frac{2hm_1(2nH-x_1)}{(x_2-m_2x_1)^2+m_1^2(2nH-x_1)^2-h^2}\Bigg)\Bigg\} \\
& -\frac{m_1}{2}\Bigg\{ log\frac{[(x_2-m_2x_1)+h]^2+[m_1x_1]^2}{[(x_2-m_2x_1)-h]^2+[m_1x_1]^2} \\
& +\sum_{n=1}^{\infty}V^n\Bigg(log\frac{[(x_2-m_2x_1)+h]^2+[m_1(2nH+x_1)]^2}{[(x_2-m_2x_1)-h]^2+[m_1(2nH+x_1)]^2} \\
& + log\frac{[(x_2-m_2x_1)+h]^2+[m_1(2nH-x_1)]^2}{[(x_2-m_2x_1)-h]^2+[m_1(2nH-x_1)]^2}\Bigg)\Bigg\}\Bigg].
\end{aligned} \tag{39}$$

for $0 \le x_1 \le H$ and for $x_1 > H$

$$\begin{aligned}
u'_3 = \frac{2P}{\pi T_1}\left(\frac{T}{T+1}\right)\int_{-\infty}^{\infty}\frac{sin\,kh}{k|k|}\Bigg[& e^{-m_1|k|x_1-A|k|H+iBkH} \\
& +\sum_{n=1}^{\infty}V^ne^{-m_1|k|(2nH+x_1)-A|k|H+iBkH}\Bigg]e^{-i(x_2-m'_2x_1)k}dk,
\end{aligned} \tag{40}$$

$$\begin{aligned}
\tau'_{31} = -\frac{2P}{\pi(1+T)}\Bigg[& tan^{-1}\frac{2h(m'_1x_1+AH)}{(x_2-m'_2x_1-iBH)^2+(m'_1x_1+AH)^2-h^2} \\
& +\sum_{n=1}^{\infty}V^n\,tan^{-1}\frac{2h(2n\,m_1H+m'_1x_1+AH)}{(x_2-m'_2x_1-iBH)^2+(2nm_1H+m'_1x_1+AH)^2-h^2}\Bigg],
\end{aligned} \tag{41}$$

$$\begin{aligned}
\tau'_{32} = -\frac{2P}{\pi(1+T)}\Bigg[& m'_2\Bigg\{ tan^{-1}\frac{2h\,(m'_1x_1+AH)}{(x_2-m'_2x_1-iBH)^2+(m'_1x_1+AH)^2-h^2} \\
& +\sum_{n=1}^{\infty}V^n\,tan^{-1}\frac{2h\,(2nHm_1+m'_1x_1+AH)}{(x_2-m'_2x_1-iBH)^2+(2nHm_1+m'_1x_1+AH)^2-h^2}\Bigg\} \\
& -\frac{m'_1}{2}\Bigg\{ log\left[\frac{(x_2-m'_2x_1-iBH+h)^2+(m'_1x_1+AH)^2}{(x_2-m'_2x_1-iBH-h)^2+(m'_1x_1+AH)^2}\right]. \\
& +\sum_{n=1}^{\infty}V^n\,log\left[\frac{(x_2-m'_2x_1-iBH+h)^2+(2nHm_1+m'_1x_1+AH)^2}{(x_2-m'_2x_1-iBH-h)^2+(2nHm_1+m'_1x_1+AH)^2}\right]\Bigg\}\Bigg]
\end{aligned} \tag{42}$$

We obtain that the deformation fields for smooth-rigid and rough-rigid interfaces can also be obtained from the deformation field for the perfectly bonded case respectively, on substituting $V = 1$ and $V = -1$.

IV. PARTICULAR CASES

A. Orthotropic Elastic Layered Half-Space

Taking $c_{45} = 0 \left(i.e.\ m_1 = m_3 = \sqrt{\dfrac{c_{44}}{c_{55}}}, m_2 = 0 \right)$ in the equations (37)-(42), we obtain the deformation for an orthotropic elastic layered half-space.

B. Isotropic Elastic Layered Half-Space

Taking $c_{45} = 0$ and $c_{44} = c_{55} = \mu$ $(i.e.\ m_1 = m_3 = 1,\ m_2 = 0)$ in the equations (37)-(42), we obtain the deformation field for an isotropic elastic layered half-space .

C. Shear Line-Load in Monoclinic Elastic Half Space

Taking $P = \dfrac{P_0}{2h}$ (shear Line-load) and proceeding to limit $h \to 0$, we obtain the deformation field caused by shear line-load P_0, per unit length, acting at the boundary $x_2 = 0$ of the semi-infinite monoclinic elastic medium in the positive $x_3 -$ direction

$$u_3 = \frac{P_0}{\pi T_1} \int_0^\infty k^{-1} \cos(x_2 - m_2 x_1) e^{-m_1 k x_1} dk = -\frac{P_0}{2\pi T_1} \log\left[(x_2 - m_2 x_1)^2 + m_1^2 x_1^2 \right], \tag{43}$$

$$\tau_{31} = -\frac{P_0 m_1 x_1}{\pi \left[(x_2 - m_2 x_1)^2 + m_1^2 x_1^2 \right]}, \tag{44}$$

$$\tau_{32} = -\frac{P_0 m_1 x_2}{\pi \left[(x_2 - m_2 x_1)^2 + m_1^2 x_1^2 \right]}. \tag{45}$$

V. NUMERICAL RESULTS

In this section, we wish to show the variation of following stresses with the horizontal distance in a monoclinic elastic half-space due to strip-loading on taking $m_1 = m_1'$ and $m_2 = m_2'$ i.e. $T = 1$ and $V = 0$ in the equations (40)-(42)

$$u_3 = \frac{P}{\pi T_1} \int_{-\infty}^{\infty} \frac{\sin kh}{k|k|} e^{-m_1 |k| x_1} e^{-i(x_2 - m_2 x_1)k} dk., \tag{46}$$

$$\tau_{31} = -\frac{P}{\pi} \left[\tan^{-1} \frac{2 h m_1 x_1}{(x_2 - m_2 x_1)^2 + m_1^2 x_1^2 - h^2} \right], \tag{47}$$

$$\tau_{32} = -\frac{P}{\pi} \left[m_2 \tan^{-1} \frac{2 h m_1 x_1}{(x_2 - m_2 x_1)^2 + m_1^2 x_1^2 - h^2} - \frac{m_1}{2} \log \frac{\left\{ (x_2 - m_2 x_1 + h)^2 + m_1^2 x_1^2 \right\}}{\left\{ (x_2 - m_2 x_1 - h)^2 + m_1^2 x_1^2 \right\}} \right] \tag{48}$$

Further, we use the elastic constants (in Gpa) for Dolomite given by Rasolofosaon and Zinszner [11]

$c_{11} = 65.53$ $c_{12} = 65.53$ $c_{13} = 12.19$
$c_{16} = 2.94$ $c_{22} = 50.77$ $c_{23} = 11..61$
$c_{33} = 60.11$ $c_{26} = -0.19$ $c_{36} = 0.84$
$c_{44} = 23.51$ $c_{45} = 1.49$ $c_{55} = 24.57$
$c_{66} = 20.21$.

Figures 2(a)-(c) show the variations of shear stress τ_{31} with the horizontal distance x_2 for different values of $h = 0.25$, 0.50, 0.75 and 1 by taking x_1 = 0.25, 0.75 and 1.5.

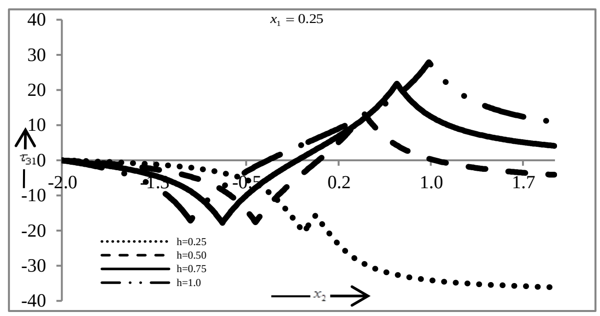

Fig. 2(a)

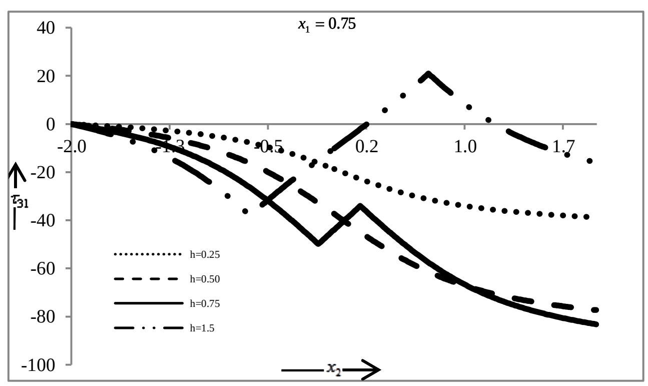

Fig. 2(b)

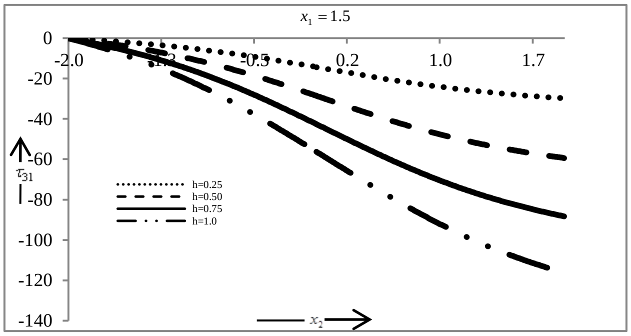

Fig. 2(c)

Fig.2. Variation of stress τ_{31} with the horizontal distance x_2 for different values of $h = 0.25, 0.50, 0.75$ and 1 for (a) x_1 = 0.25, (b) x_1 =0.75 and (c) x_1 =1.5.

Figures 3(a)-(c) show the variations of shear stress τ_{32} with the horizontal distance x_2 for the same values. It has been found from all the figures that the distance between stresses increases in magnitude.

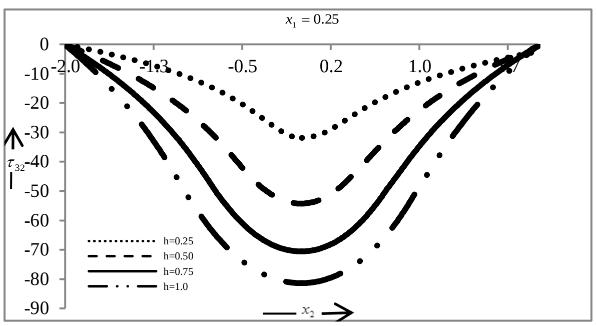

Fig. 3(a)

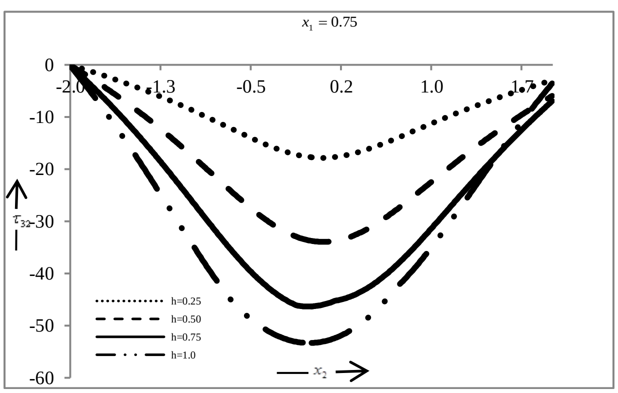

Fig. 3(b)

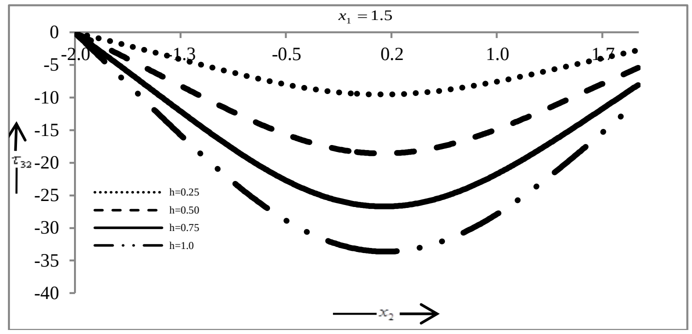

Fig. 3(c)

Fig.3. Variation of stress τ_{32} with the horizontal distance x_2 for different values of $h = 0.25, 0.50, 0.75$ and 1 for (a) x_1 = 0.25, (b) x_1 =0.75 and (c) x_1=1.5.

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