Probabilistic Reliability Analysis of High-Performance Reinforced Concrete Beam using Matlab software

David Pustka

Abstract**—**Significant improvement of computer technology in recent decades brought possibility to exploit fully probabilistic methods for reliability evaluation of structural systems and their members. One of these methods is SBRA (Simulation-Based Reliability Method) using direct Monte Carlo simulation technique. Application of this method allows utilize more precise calculation models, consistent description of random variables using truncated histograms and as well direct quantification of reliability. The following paragraphs of this paper describes application of this method in design and reliability assessment of high-performance reinforced concrete beam.

Keywords**—**Computer simulation, concrete, reinforcement, reliability.

I. Introduction

TODAY'S codes for design of civil engineering structures are predominantly based on partial factor design method. This method was developed more than a half of century ago in time of primitive (from today's viewpoint) calculation tools. Limited calculating possibilities of this method led to introduction of computational models, which we can intercept today as excessively simplified. Reliability of structures is influenced by a lot of quantities, which are less or more random. Partial factor design method describes these random quantities using characteristic values and corresponding partial factors. This simplification then leads to impossibility to quantify resulting reliability of evaluated structure.

Rapid development of computer technology (especially in recent decades) brought possibility to exploit computer simulation for fully probabilistic reliability analyses of structures - see e.g. [1]-[18]. One of probabilistic methods, which has been developed for more than two decades, is SBRA (Simulation-Based Reliability Assessment) method - see e.g. [4], [5], [7]. This method uses computer simulation Monte Carlo for modeling of various processes influenced by omnipresent randomness. Versatility of this method allows its use for solution of wide spectrum not only technical problems. Usefully this method can be used as well for reliability analysis of civil engineering structures. Advantage of this method is possibility to directly calculate probability of failure (in comparison with partial factor design method, where this indicator cannot be calculated). The SBRA method allows description of random quantities using histograms, which can be acquired e.g. by repeating of various experiments.

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Potential of the SBRA method within design and assessment of civil engineering structures is outlined in the following chapters. On example of simply supported beam made of high-performance reinforced concrete is described possible approach to fully probabilistic reliability analysis.

II. Solved Example - Input Data

The goal of considered example is design and reliability assessment of simply supported beam from the viewpoint of possible failure due to pure bending. The beam is considered to be of high-performance concrete (see e.g. [19]-[27]). Near the bottom surface is provided longitudinal tension bar reinforcement. In the first step it is calculated necessary number of reinforcing bars. Subsequently it is performed reliability assessment of this structural member.

Cross-section dimensions are considered to be the same along the whole length of the beam. The beam has only one bay between pin supports at the ends. The beam is loaded by a dead load, long-lasting and short-lasting live loads. All these loads are uniformly spread along the whole length of the beam. Effective length of the beam is considered as constant by $l_{\mathrm{eff}} = 6.55$ m. Modulus of elasticity of reinforcing steel is considered as well as constant by value $E_{\mathrm{s}} = 200$ GPa [21]. The following quantities are considered as random variables. Their nominal values are: cross-section width $b_{\mathrm{nom}} = 350$ mm, cross-section height $h_{\mathrm{nom}} = 700$ mm, diameter of longitudinal tension reinforcing bars located near the bottom surface of the beam $\phi_{\mathrm{s1,nom}} = 18$ mm, concrete cover of longitudinal reinforcing bars $c_{\mathrm{1,nom}} = 31$ mm, dead load $g_{\mathrm{1,nom}} = 20$ kNm^{-1}, long-lasting live load $q_{\mathrm{1,nom}} = 41$ kNm^{-1}, short-lasting live load $q_{\mathrm{2,nom}} = 39$ kNm^{-1}. Random nature of considered random quantities are represented by histograms shown in Fig. 1-10. Random quantities are calculated as multiplication of nominal value and random realization obtained from corresponding histogram using Monte Carlo method.

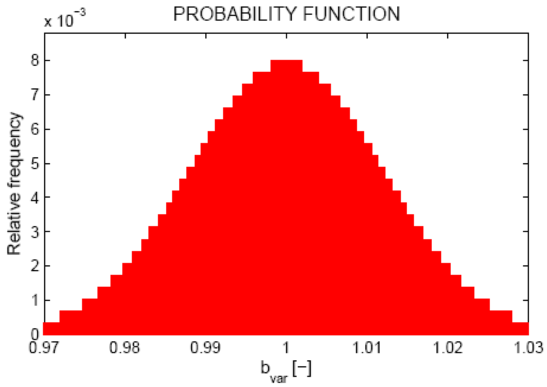

Fig. 1 Histogram representing cross-section width b_{var} [-]

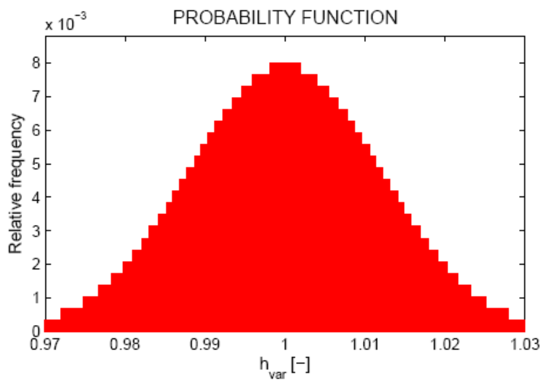

Fig. 2 Histogram representing cross-section height h_{var} [-]

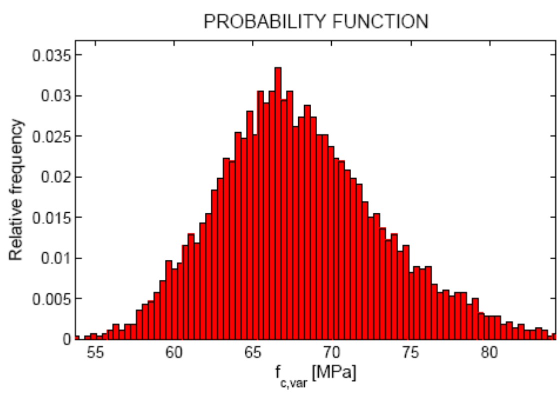

Fig. 3 Histogram of strength of concrete $f_{\mathrm{c,var}}$ [MPa]

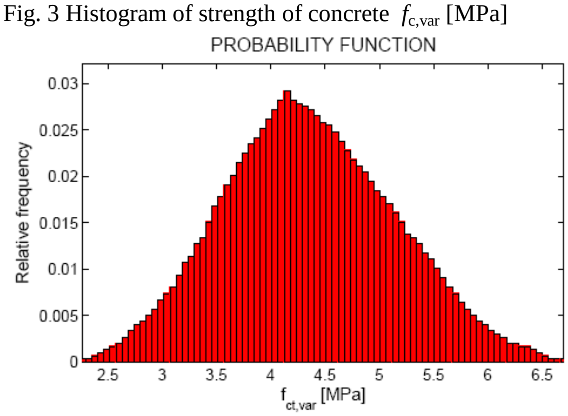

Fig. 4 Histogram of strength of concrete in tension $f_{\mathrm{ct,var}}$ [MPa]

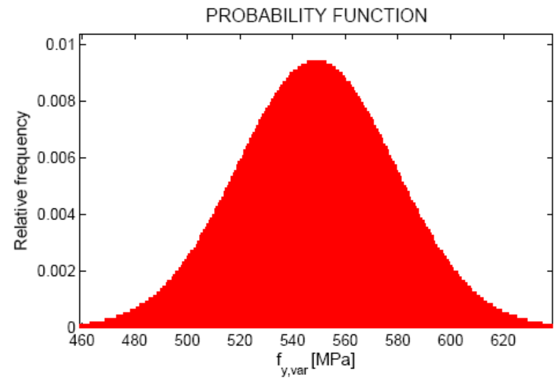

Fig. 5 Histogram of steel yield of reinforcing bars $f_{\mathrm{y,var}}$ [MPa]

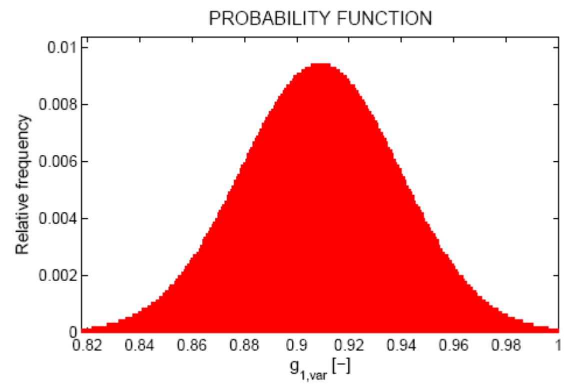

Fig. 6 Histogram representing dead load $g_{1,\mathrm{var}}$ [-]

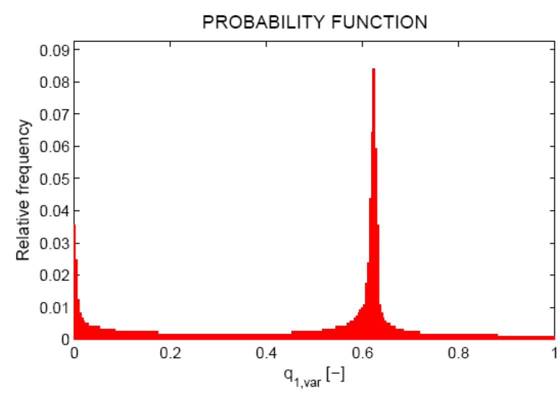

Fig. 7 Histogram of long-lasting live load $q_{1,\mathrm{var}}$ [-]

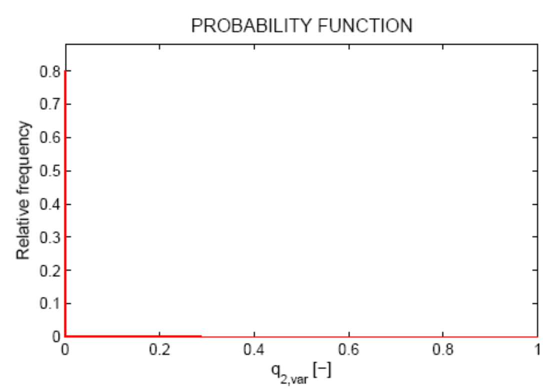

Fig. 8 Histogram of short-lasting live load $q_{2,\mathrm{var}}$ [-]

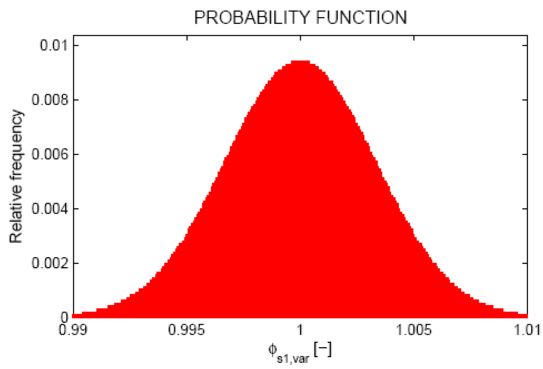

Fig. 9 Histogram of diameter of tension reinforcing bars $\phi_{s1,var}$ [-]

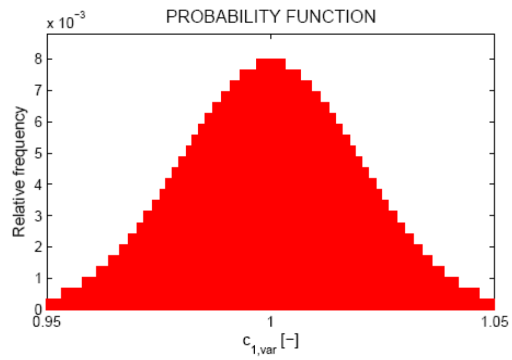

Fig. 10 Histogram of concrete cover $c_{1,var}$ [-]

Calculated statistics for input random variable quantities are shown in Fig. 11-20.

Variable:	b [m]
Minimum value:	0.33950003855584127
Maximum value:	0.36049996288896596
Bins of histogram:	1000
Mean:	0.35000316605633969
Variance:	1.6061712129381699e−05
Standard deviation:	0.0040077065922272427
Median:	908.5
Variance coeff.:	0.011450486683832242
Skewness:	0.0029919715213502416
Concentration coeff.:	2.6196134569180449
Kurtosis:	−0.38038654308195508
Quantile(7e−05):	0.33950003855584127
Quantile(0.5):	0.35001051119503818
Quantile(0.99993):	0.36049996288895669
Probability(0):	0

Fig. 11 Statistics for cross-section width b_{var} [-]

Variable:	h [m]
Minimum value:	0.67900027594294476
Maximum value:	0.72099990951092274
Bins of histogram:	1000
Mean:	0.70000800576901745
Variance:	6.4281024622600593e−05
Standard deviation:	0.0080175448001617421
Median:	896
Variance coeff.:	0.011453504437215396
Skewness:	−0.00045483718810774881
Concentration coeff.:	2.6204764925700821
Kurtosis:	−0.37952350742991792
Quantile(7e−05):	0.67900027594294476
Quantile(0.5):	0.7000211135645662
Quantile(0.99993):	0.72099990951094439
Probability(0):	0

Fig. 12 Statistics for cross-section height h_{var} [-]

Variable:	f_c [Pa]
Minimum value:	53766626.77704341
Maximum value:	84204895.132851854
Bins of histogram:	1000
Mean:	67996843.919184014
Variance:	26773505334173.34
Standard deviation:	5174312.0638567349
Median:	765
Variance coeff.:	0.076096356325104392
Skewness:	0.36418057285507932
Concentration coeff.:	2.9639004210320117
Kurtosis:	−0.036099578967988322
Quantile(7e−05):	53797095.514136314
Quantile(0.5):	67538495.943036243
Quantile(0.99993):	84174426.395758167
Probability(0):	0

Fig. 13 Statistics for strength of concrete $f_{c,var}$ [MPa]

Variable:	f_{ct} [Pa]
Minimum value:	2270203.0798734226
Maximum value:	6695706.5501103764
Bins of histogram:	1000
Mean:	4354220.0332858935
Variance:	614360067038.45544
Standard deviation:	783811.24452157191
Median:	898
Variance coeff.:	0.18001185942136969
Skewness:	0.1512487357303102
Concentration coeff.:	2.6793703846370236
Kurtosis:	−0.32062961536297641
Quantile(7e−05):	2274633.0132770631
Quantile(0.5):	4316832.3123553554
Quantile(0.99993):	6691276.6167069236
Probability(0):	0

Fig. 14 Statistics for strength of concrete in tension $f_{ct,var}$ [MPa]

Variable:	f_y [Pa]
Minimum value:	459288302.5801689
Maximum value:	638632877.22401273
Bins of histogram:	1000
Mean:	548960158.32616603
Variance:	869399298832219.37
Standard deviation:	29485577.81072332
Median:	772
Variance coeff.:	0.053711689935072464
Skewness:	−0.0031283463696372125
Concentration coeff.:	2.8333006588266336
Kurtosis:	−0.16669934117336638
Quantile(7e−05):	459467826.67891151
Quantile(0.5):	549050351.95147097
Quantile(0.99993):	638453353.12528789
Probability(0):	0

Fig. 15 Statistics for steel yield of reinforcing bars $f_{y,var}$ [Pa]

Variable:	g_1 [Nm^{-1}]
Minimum value:	16360.073244173251
Maximum value:	19999.988895635855
Bins of histogram:	1000
Mean:	18179.072380247857
Variance:	357914.62523841363
Standard deviation:	598.25966372338155
Median:	770.5
Variance coeff.:	0.032909251429869973
Skewness:	0.00012332905619552494
Concentration coeff.:	2.8270045827684802
Kurtosis:	−0.17299541723151979
Quantile(7e−05):	16363.716803383924
Quantile(0.5):	18178.209290298495
Quantile(0.99993):	19996.345336423728
Probability(0):	0

Fig. 16 Statistics for dead load $g_{1,var}$ [-]

Variable:	q_1 [Nm^{-1}]
Minimum value:	0.0006370806532599516
Maximum value:	41080.137097732004
Bins of histogram:	1000
Mean:	19582.926319506503
Variance:	112100638.14223386
Standard deviation:	10587.758881946353
Median:	444
Variance coeff.:	0.54066275434023936
Skewness:	−0.64952001415258476
Concentration coeff.:	2.2689703873849738
Kurtosis:	−0.73102961261502619
Quantile(7e−05):	0.0006370806532599516
Quantile(0.5):	24919.482814412648
Quantile(0.99993):	41080.13709773156
Probability(0):	0

Fig. 17 Statistics for long-lasting live load $q_{1,\mathrm{var}}$ [-]

Variable:	q_2 [Nm^{-1}]
Minimum value:	9.5896057458907442e−05
Maximum value:	39076.193906802815
Bins of histogram:	1000
Mean:	2154.0398412522509
Variance:	40427914.889439248
Standard deviation:	6358.2949671621282
Median:	89
Variance coeff.:	2.9518000760216832
Skewness:	3.7666684459060726
Concentration coeff.:	17.426617374993555
Kurtosis:	14.426617374993555
Quantile(7e−05):	9.5896057458907442e−05
Quantile(0.5):	39.115405016084239
Quantile(0.99993):	39037.078597683198
Probability(0):	0

Fig. 18 Statistics for short-lasting live load $q_{2,\mathrm{var}}$ [-]

Variable:	ϕ_{s1} [m]
Minimum value:	0.017820016807083557
Maximum value:	0.018179996591486031
Bins of histogram:	1000
Mean:	0.017999974288492353
Variance:	3.4980200037796842e−09
Standard deviation:	5.9144061441362687e−05
Median:	761
Variance coeff.:	0.0032857858846595325
Skewness:	−0.0013540664758810773
Concentration coeff.:	2.8288042501672725
Kurtosis:	−0.17119574983272745
Quantile(7e−05):	0.017820377147208084
Quantile(0.5):	0.018000186869347106
Quantile(0.99993):	0.018179636251361601
Probability(0):	0

Fig. 19 Statistics for diameter of tension reinforcing bars $\phi_{s1,\mathrm{var}}$ [m]

Variable:	c_1 [m]
Minimum value:	0.029450025002921602
Maximum value:	0.03254999329258431
Bins of histogram:	1000
Mean:	0.031000215647842153
Variance:	3.4960071464632431e−07
Standard deviation:	0.00059127042429528323
Median:	910.5
Variance coeff.:	0.019073106813579217
Skewness:	0.0010506467233191402
Concentration coeff.:	2.6239325454270177
Kurtosis:	−0.3760674545729823
Quantile(7e−05):	0.029450025002921602
Quantile(0.5):	0.030998457612071623
Quantile(0.99993):	0.032549993292584116
Probability(0):	0

Fig. 20 Statistics for concrete cover $c_{1,\mathrm{var}}$ [m]

III. Solved Example - Calculation

A. *Design of Reinforcement*

Calculation is performed for critical cross-section considered in the mid-span of the beam. Total uniformly distributed load acting on the beam is calculated as a sum of individual load components:

$$f_1 = g_1 + q_1 + q_2 \tag{1}$$

Bending effect in the mid-span cross-section of the beam is calculated using equation:

$$M_E = \frac{1}{8} f_1 \cdot l_{eff}^2 \tag{2}$$

Vertical distance of the center of gravity of longitudinal tension reinforcement to the tensioned edge of the concrete cross-section:

$$d_1 = c_1 + 0.5 \cdot \phi_{s1} \tag{3}$$

Vertical distance of the center of gravity of longitudinal tension reinforcement to the compressed edge of the concrete cross-section:

$$d = h - d_1 \tag{4}$$

Total required cross-section area of tension reinforcement to resist bending moment due to load:

$$A_{s1,req} = \frac{b \cdot d \cdot \eta \cdot f_{cd}}{f_{yd}} \cdot \left(1 - \sqrt{1 - \frac{2 \cdot M_{Ed}}{b \cdot d^2 \cdot \eta \cdot f_{cd}}}\right) \tag{5}$$

Cross-section area of single reinforcing bar of tension reinforcement:

$$A_{s1,\sin gle} = \pi \cdot \frac{\cdot \phi_{s1}^2}{4} \tag{6}$$

Required number of reinforcing bars of tension reinforcement:

$$n_{s1,req} = \frac{A_{s1,req}}{A_{s1,\sin gle}} \tag{7}$$

B. *Reliability Assessment*

Clear distance between longitudinal reinforcing bars:

$$a_{s1} = \frac{b - 2 \cdot c_1 - n_{s1} \cdot \phi_{s1}}{n_{s1} - 1} \tag{8}$$

Minimum clear distance between longitudinal reinforcing bars (requirement in [21]):

$$a_{s1,\min} = \max(1.2 \cdot \phi_{s1}; d_{g,\max} + 5 \cdot mm; 20 \cdot mm) \tag{9}$$

Reliability criterion - minimum clear distance between longitudinal reinforcing bars:

$$RF_{as1,\min} = a_{s1} - a_{s1,\min} \tag{10}$$

Axial distance of longitudinal reinforcing bars:

$$s_1 = a_{s1} + \phi_{s1} \tag{11}$$

Average width of tensioned part of the concrete cross-section:

$$b_t = b \tag{12}$$

Minimum total cross-section area of longitudinal tension reinforcement (requirement based on [21]):

$$A_{s1,\min} = \max(\frac{0.285459 \cdot f_{ct} \cdot b_t \cdot d}{f_y}; 0.0013 \cdot b_t \cdot d) \tag{13}$$

Total cross-section area of longitudinal tension reinforcement:

$$A_{s1} = n_{s1} \cdot \frac{\pi \cdot \phi_{s1}^2}{4} \tag{14}$$

Reliability criterion - minimum cross-section area of longitudinal tension reinforcement:

$$RF_{As1,\min} = A_{s1} - A_{s1,\min} \tag{15}$$

Maximum cross-section area of longitudinal reinforcement (requirement in [21]):

$$A_{s,\max} = 0.04 \cdot b \cdot h \tag{16}$$

Reliability criterion - maximum cross-section area of longitudinal reinforcement:

$$RF_{As1,\max} = A_{s1,\max} - A_{s1} \tag{17}$$

Factor for concrete strength in compression (based on [21]):
- if $f_c \leq 50$ MPa, then:

$$\eta = 1 \tag{18a}$$

- otherwise:

$$\eta = 1 - (f_c - \frac{58}{200} \cdot MPa) \tag{18b}$$

Factor for effective height of compressed part of the concrete cross-section (based on [21]):
- if $f_c \leq 50$ MPa, then:

$$\lambda = 0.8 \tag{19a}$$

- otherwise

$$\lambda = 0.8 - (f_c - \frac{58}{400} \cdot MPa) \tag{19b}$$

Distance of neutral axis to the compressed edge of the concrete cross-section:

$$x = \frac{A_{s1} \cdot f_y}{b \cdot \lambda \cdot \eta \cdot f_c} \tag{20}$$

Relative strain of steel reinforcement corresponding to steel yield:

$$\varepsilon_y = \frac{f_y}{E_s} \tag{21}$$

Limit relative strain of compressed concrete (based on [21]):

$$\begin{aligned}\varepsilon_{cu3} = {} & \frac{1}{1000} \cdot neg(sign(\frac{f_c}{MPa} - 58)) \cdot 3.5 + \\ & + zero(sign(\frac{f_c}{MPa} - 58)) \cdot 3.5 + \\ & + pos(sign(\frac{f_c}{MPa} - 58)) \cdot \\ & \cdot \left(2.6 + 35 \cdot \left(\frac{(90 - \frac{f_c}{MPa}) + 8}{100} \right)^4 \right)\end{aligned} \tag{22}$$

Limit ratio $x_{bal,1}/d$:

$$\xi_{bal,1} = \frac{\varepsilon_{cu,3}}{\varepsilon_{cu,3} + \varepsilon_y} \tag{23}$$

Maximum distance of neutral axis to the compressed edge of the concrete cross-section:

$$x_{bal,1} = \xi_{bal,1} \cdot d \tag{24}$$

Reliability criterion - location of neutral axis:

$$RF_x = x_{bal,1} - x \tag{25}$$

Lever arm of internal forces (vertical distance of the center of gravity of longitudinal tension reinforcement to the center of gravity of compressed part of the concrete cross-section):

$$z_1 = d - 0.5 \cdot x \tag{26}$$

Force in longitudinal tension reinforcement corresponding to the steel yield:

$$F_{s,1} = A_{s1} \cdot f_y \tag{27}$$

Bending resistance of the cross-section:

$$M_R = F_{s1} \cdot z_1 \tag{28}$$

Reliability criterion - assessment of safety in bending:

$$RF_M = M_R - M_E \tag{29}$$

Equations (9), (13), (16), (18) and (19) were considered according to requirements defined in EN 1992-1-1 (see [21]). Equations (13), (18) and (19) were against EN 1992-1-1 [21] modified with regard to possible application within probabilistic calculation. In equation (22) there were used functions, which were considered this way:

sign (x) = -1 for $x < 0$, = 1 for $x > 0$, = 0 for $x = 0$;
pos (x) = 1 for $x > 0$, = 0 for $x \leq 0$;
neg (x) = 1 for $x < 0$, = 0 for $x \geq 0$;
zero (x) = 1 pro $x = 0$, = 0 pro $x \neq 0$;

IV. Solved Example - Outputs

Resulting histograms of calculated random quantities obtained from computer simulation Monte Carlo are shown in Fig. 21-35. Computer simulation was performed for simulation steps equal to 10^6. Calculation was performed using probabilistic modules [28] running under commercial software Matlab (see e.g. [29]-[32]).

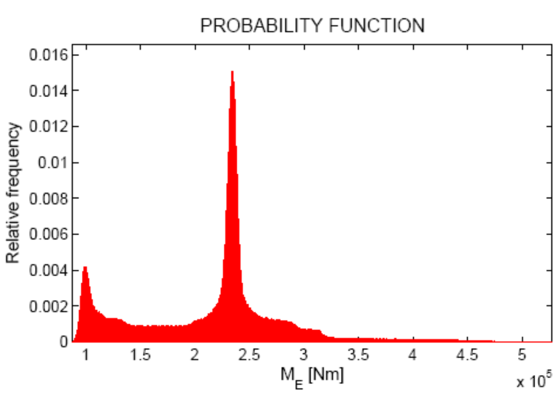

Fig. 21 Histogram of bending moment M_E [Nm]

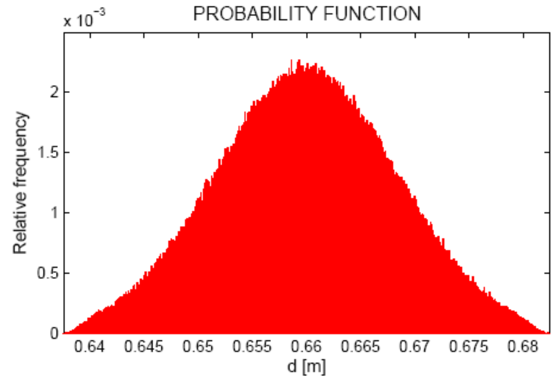

Fig. 22 Histogram of effective height d [m]

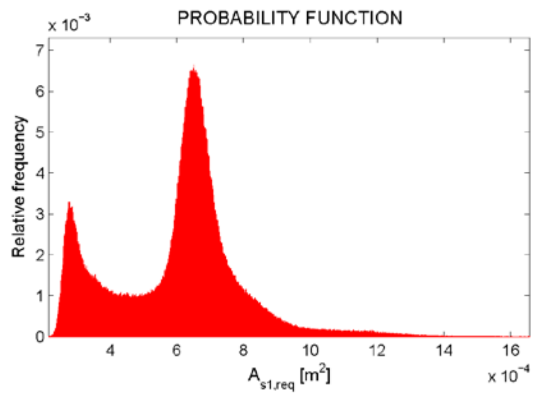

Fig. 23 Histogram of total required cross-section area $A_{s1,req}$ [m^2]

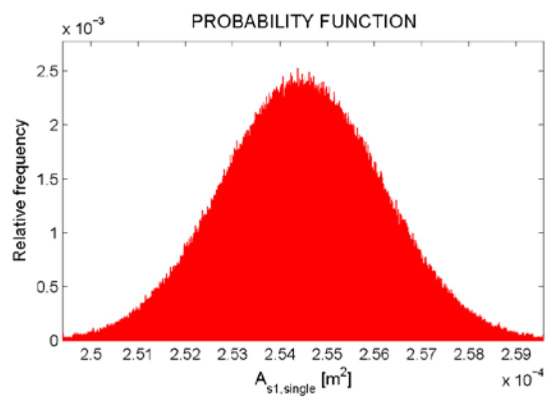

Fig. 24 Histogram of cross-section area of one bar $A_{s1,single}$ [m^2]

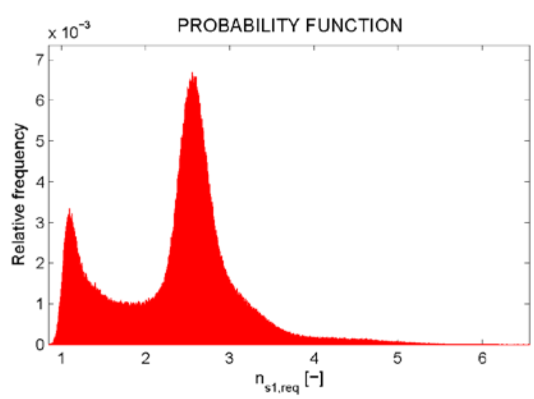

Fig. 25 Histogram of required reinforcing bars $n_{s1,req}$ [-]

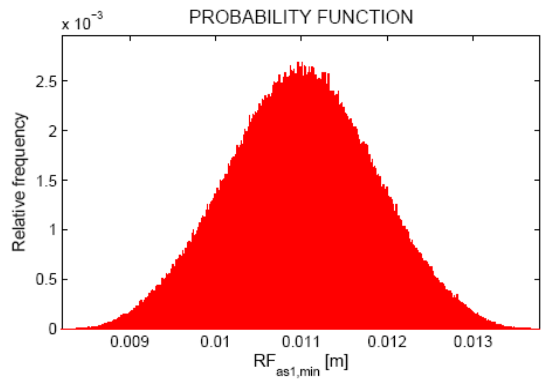

Fig. 26 Histogram $RF_{\mathrm{as1,min}}$ [m]

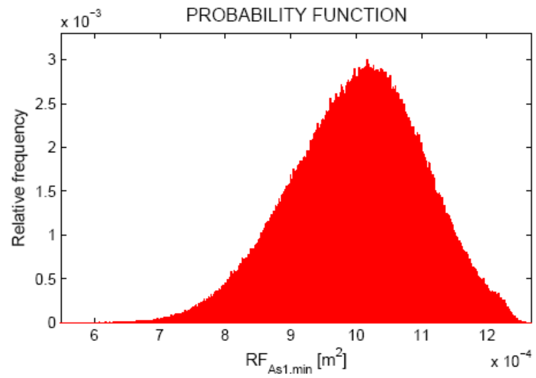

Fig. 27 Histogram of reliability function $RF_{\mathrm{As1,min}}$ [m^2]

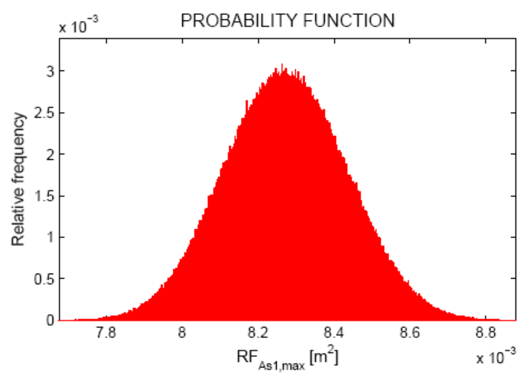

Fig. 28 Histogram of reliability function $RF_{\mathrm{As1,max}}$ [m^2]

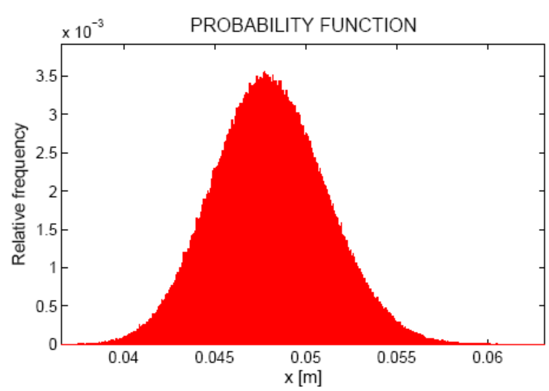

Fig. 29 Histogram of depth of neutral axis x [m]

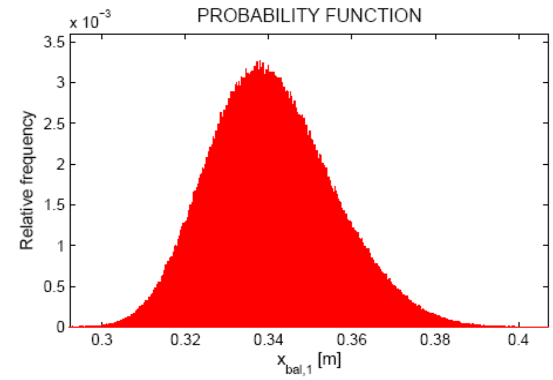

Fig. 30 Histogram of limiting depth of neutral axis $x_{\mathrm{bal,1}}$ [m]

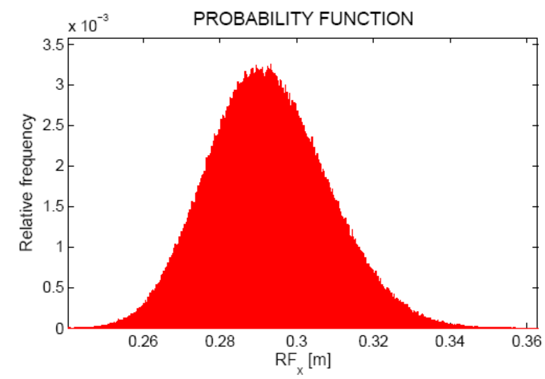

Fig. 31 Histogram of reliability function RF_{x} [m]

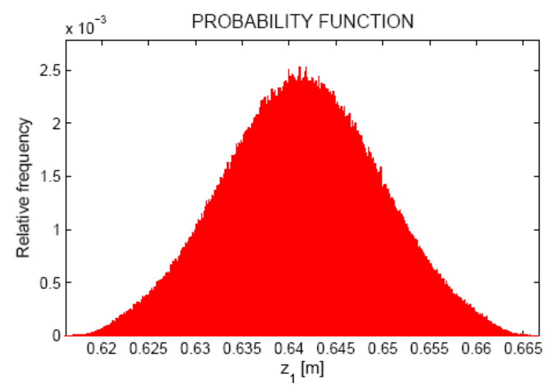

Fig. 32 Histogram of lever arm of internal forces z_1 [m]

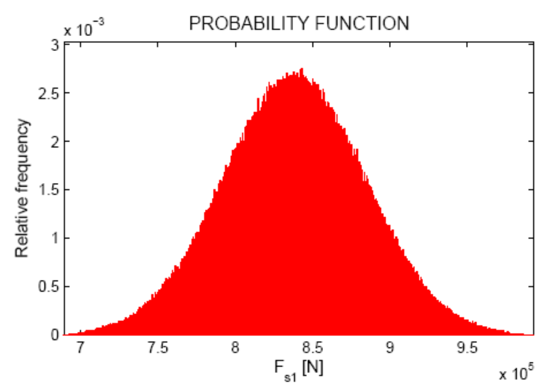

Fig. 33 Histogram of force in tension reinforcment F_{s1} [N]

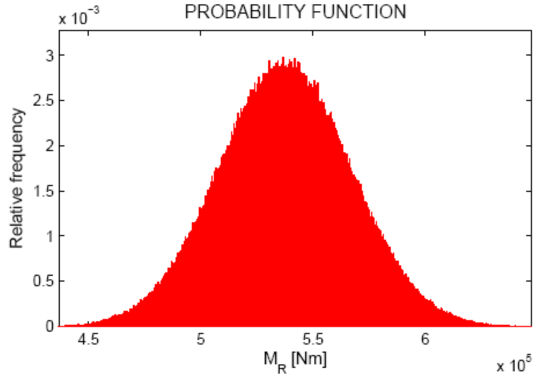

Fig. 34 Histogram of bending resistance M_R [Nm]

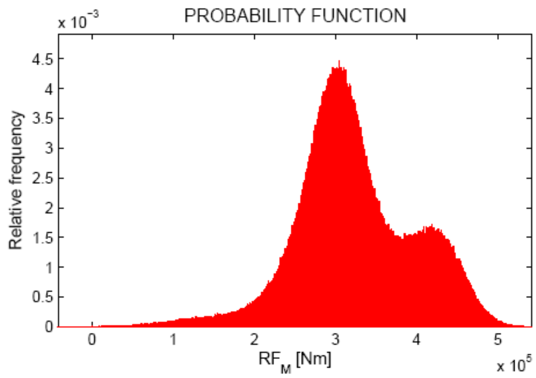

Fig. 35 Histogram of reliability function RF_M [Nm]

Statistical parameters obtained from computer simulation are shown in Fig. 36-50.

Variable:	M_E [Nm]
Minimum value:	87870.961841447759
Maximum value:	526734.16104393301
Bins of histogram:	1000
Mean:	214062.50719650765
Variance:	4397935394.4844837
Standard deviation:	66316.931431456353
Median:	605.5
Variance coeff.:	0.30980171306027893
Skewness:	0.10484462304174134
Concentration coeff.:	3.6199607336262889
Kurtosis:	0.6199607336262889
Quantile(7e−05):	89188.869346560343
Quantile(0.5):	231522.87989871937
Quantile(0.99993):	508722.75847406616
Probability(0):	0

Fig. 36 Statistics for load effect M_E [Nm]

Variable:	d [m]
Minimum value:	0.63747958190927079
Maximum value:	0.68244407997936807
Bins of histogram:	1000
Mean:	0.66000784474904417
Variance:	6.464324182937785e−05
Standard deviation:	0.0080401021031687066
Median:	867
Variance coeff.:	0.012181828090582479
Skewness:	−0.00046795634287367005
Concentration coeff.:	2.6246515024018078
Kurtosis:	−0.37534849759819222
Quantile(7e−05):	0.63824474353809146
Quantile(0.5):	0.65998433569811421
Quantile(0.99993):	0.68176893736571464
Probability(0):	0

Fig. 37 Statistics for effective height d [m]

Variable:	$A_{s1,req}$ [m^2]
Minimum value:	0.0002161932783232462
Maximum value:	0.0016573389229828425
Bins of histogram:	1000
Mean:	0.0006002874039972158
Variance:	3.6486516161369391e−008
Standard deviation:	0.00019101443966718692
Median:	0.00063598645409495983
Variance coeff.:	0.31820497714137086
Skewness:	0.18177175568148549
Concentration coeff.:	3.6937931694959301
Kurtosis:	0.69379316949593006
Quantile(7e−005):	0.00022629139595349356
Quantile(0.5):	0.00063598645409495983
Quantile(0.99993):	0.0015116375114606791
Probability(0):	0

Fig. 38 Statistics for $A_{s1,req}$ [m^2]

Variable:	$A_{s1,single}$ [m^2]
Minimum value:	0.00024940573627884601
Maximum value:	0.00025958351403747624
Bins of histogram:	1000
Mean:	0.00025447116312888988
Variance:	2.8008944523242963e−012
Standard deviation:	1.6735873004789133e−006
Median:	0.0002544691552438531
Variance coeff.:	0.0065767267296657885
Skewness:	0.01039758300853229
Concentration coeff.:	2.828959546559926
Kurtosis:	−0.17104045344007401
Quantile(7e−005):	0.00024941592424457037
Quantile(0.5):	0.0002544691552438531
Quantile(0.99993):	0.00025956313810603326
Probability(0):	0

Fig. 39 Statistics for $A_{s1,single}$ [m^2]

Variable:	$n_{s1,req}$ [−]
Minimum value:	0.85132979162183231
Maximum value:	6.5650410287278911
Bins of histogram:	1000
Mean:	2.3590693520479635
Variance:	0.56380442791148377
Standard deviation:	0.75086911503369469
Median:	2.4985258239407173
Variance coeff.:	0.31829039463458231
Skewness:	0.18255911254105825
Concentration coeff.:	3.6941216465358235
Kurtosis:	0.69412164653582353
Quantile(7e−005):	0.89136580629624895
Quantile(0.5):	2.4985258239407173
Quantile(0.99993):	5.9473425166082379
Probability(0):	0

Fig. 40 Statistics for $n_{s1,req}$ [-]

Variable:	$RF_{as1,min}$ [m]
Minimum value:	0.0081784600725566814
Maximum value:	0.013767675778218164
Bins of histogram:	1000
Mean:	0.011000576674362206
Variance:	7.0395165459104415e−07
Standard deviation:	0.00083901826832974511
Median:	720
Variance coeff.:	0.076270389559226437
Skewness:	0.0021730668720820849
Concentration coeff.:	2.6805326317273739
Kurtosis:	−0.31946736827262612
Quantile(7e−05):	0.0084414161668170413
Quantile(0.5):	0.010998244572710328
Quantile(0.99993):	0.013549478168087437
Probability(0):	0

Fig. 41 Statistics for reliability function $RF_{\text{as1,min}}$ [m]

Variable:	$RF_{As1.min}$ [m^2]
Minimum value:	0.0005579250341352656
Maximum value:	0.0012592595771260451
Bins of histogram:	1000
Mean:	0.0010021420720991337
Variance:	9.9089686583849681e−09
Standard deviation:	9.9543802712097386e−05
Median:	631.5
Variance coeff.:	0.09931028487396278
Skewness:	−0.23737083114813756
Concentration coeff.:	2.7944766225735655
Kurtosis:	−0.20552337742643445
Quantile(7e−05):	0.00063023480183100786
Quantile(0.5):	0.0010072284450602662
Quantile(0.99993):	0.0012508351381711649
Probability(0):	0

Fig. 42 Statistics for reliability function $RF_{\text{As1,min}}$ [m^2]

Variable:	$RF_{As1,max}$ [m^2]
Minimum value:	0.0077065392511281566
Maximum value:	0.0088731494043825382
Bins of histogram:	1000
Mean:	0.0082733774269202042
Variance:	2.5347778155917906e−08
Standard deviation:	0.00015920985571225767
Median:	561
Variance coeff.:	0.01924363503521731
Skewness:	0.025713703168048253
Concentration coeff.:	2.8125060979183742
Kurtosis:	−0.18749390208162575
Quantile(7e−05):	0.0077462437007884619
Quantile(0.5):	0.0082729115477534424
Quantile(0.99993):	0.0088170960636862663
Probability(0):	0

Fig. 43 Statistics for reliability function $RF_{\text{As1,max}}$ [m^2]

Variable:	x [m]
Minimum value:	0.03682596157264479
Maximum value:	0.062661919622752166
Bins of histogram:	1000
Mean:	0.048025873231965145
Variance:	9.3913778834412778e−06
Standard deviation:	0.0030645355085952711
Median:	366
Variance coeff.:	0.063810094483728666
Skewness:	0.11830057062444833
Concentration coeff.:	2.9449868845073164
Kurtosis:	−0.055013115492683617
Quantile(7e−05):	0.038015605286663846
Quantile(0.5):	0.047972405936605948
Quantile(0.99993):	0.059920566716536471
Probability(0):	0

Fig. 44 Statistics for depth of neutral axis x [m]

Variable:	$x_{bal,1}$ [m]
Minimum value:	0.29111892585862648
Maximum value:	0.40686253695893454
Bins of histogram:	1000
Mean:	0.3406869097113287
Variance:	0.0002151973825611246
Standard deviation:	0.014669607444002196
Median:	448.5
Variance coeff.:	0.043058911351868692
Skewness:	0.29695345773849119
Concentration coeff.:	2.947622217796682
Kurtosis:	−0.052377782203318013
Quantile(7e−05):	0.29691189938716928
Quantile(0.5):	0.33977990349838599
Quantile(0.99993):	0.39759377931324313
Probability(0):	0

Fig. 45 Statistics for limiting depth of netural axis $x_{\text{bal,1}}$ [m]

Variable:	RF_x [m]
Minimum value:	0.23927750995886846
Maximum value:	0.36315721233911924
Bins of histogram:	1000
Mean:	0.29266113308186409
Variance:	0.00023723940309692488
Standard deviation:	0.015402577806877812
Median:	432.5
Variance coeff.:	0.052629393061802143
Skewness:	0.22243230786358989
Concentration coeff.:	2.9553302599639668
Kurtosis:	−0.044669740036033319
Quantile(7e−05):	0.24485767673275374
Quantile(0.5):	0.2919790850455628
Quantile(0.99993):	0.35311291214612828
Probability(0):	0

Fig. 46 Statistics for reliability function RF_{x} [m]

Variable:	z_1 [m]
Minimum value:	0.61420460258718812
Maximum value:	0.66714499175791098
Bins of histogram:	1000
Mean:	0.64138789838540178
Variance:	6.6541986227663377e−05
Standard deviation:	0.0081573271497263969
Median:	763
Variance coeff.:	0.012718243001249711
Skewness:	−0.0015948770798741491
Concentration coeff.:	2.6464808286588188
Kurtosis:	−0.35351917134118116
Quantile(7e−05):	0.61780815260081035
Quantile(0.5):	0.64139020783701461
Quantile(0.99993):	0.66470729616045254
Probability(0):	0

Fig. 47 Statistics for lever arm z_1 [m]

Variable:	F_{s1} [N]
Minimum value:	689086.12253470381
Maximum value:	993972.56548714917
Bins of histogram:	1000
Mean:	838166.68062713416
Variance:	2057407337.1563694
Standard deviation:	45358.652285494216
Median:	669
Variance coeff.:	0.054116506100619396
Skewness:	0.0015109671266675718
Concentration coeff.:	2.8384829694497107
Kurtosis:	−0.16151703055028932
Quantile(7e−05):	697326.29666855314
Quantile(0.5):	838324.83184775279
Quantile(0.99993):	981764.90010364854
Probability(0):	0

Fig. 48 Statistics for strength in tension reinforcement F_{s1} [N]

Variable:	M_R [Nm]
Minimum value:	430480.26439370879
Maximum value:	646104.60898965818
Bins of histogram:	1000
Mean:	537544.23683562595
Variance:	843915052.69300485
Standard deviation:	29050.216052432464
Median:	547
Variance coeff.:	0.054042465832101635
Skewness:	0.0096214762384027736
Concentration coeff.:	2.8546892174534646
Kurtosis:	−0.1453107825465354
Quantile(7e−05):	444294.03621967428
Quantile(0.5):	537536.99604494497
Quantile(0.99993):	634880.91938107181
Probability(0):	0

Fig. 49 Statistics for bending reistatnce M_{R} [Nm]

Variable:	RF_M [Nm]
Minimum value:	−39014.494029507157
Maximum value:	546415.93430805486
Bins of histogram:	1000
Mean:	323481.62101358664
Variance:	5244637455.3968201
Standard deviation:	72419.869203118695
Median:	330.5
Variance coeff.:	0.22387630238837269
Skewness:	−0.079465258112383919
Concentration coeff.:	3.432394481857056
Kurtosis:	0.43239448185705598
Quantile(7e−05):	3178.6899948216083
Quantile(0.5):	314939.43861902755
Quantile(0.99993):	525319.3422958802
Probability(0):	5.5000000000000002e−05

Fig. 50 Statistics for reliability function RF_M [Nm]

V. Solved Example - Comments of Results

Design of necessary reinforcing bars was based on statistics shown in Fig. 40. From this Fig. it is evident that for probability 0.99993 the $n_{s1,req}$ equals approx. 5.947. Thus for further reliability assessment n_{s1} was considered equal to six reinforcing bars.

From Fig. 26-28 it is clear that minimum numbers of these functions are positive numbers. Due to this we can say, that these criterions are 100% fulfilled. From the last row of Fig. 46 it is evident, that for argument equal zero (zero quantile) probability (of failure) equals as well to zero. Reliability assessment of the critical cross-section of the beam in pure bending can be based on Fig. 50. From the last row it is clear that probability of failure (for quantile = 0) is equal $5.5 \cdot 10^{-5}$. If this probability of failure is less than design probability, we can consider critical cross-section as reliable (from the viewpoint of assumed limit state). Design probability should be available in corresponding design code or given by relevant authority.

VI. Conclusion

Almost all what surround us exhibit non-negligible marks of randomness. It is as well the case of civil engineering structures. Significant advancement in computer technology bring us possibility of utilization of new progressive computing methods for design and assessment of these structures. One of these progressive methods is SBRA method, which potential was briefly outlined in previous chapters. Advantage of this method is possibility of description of random quantities using general histograms, which can be acquired e.g. from various experiments. Perpetual grow of power of computer technology opens new possibilities for Monte Carlo simulations in still more and more demanding problems. Application of Monte Carlo simulation in reliability analysis allows direct calculation of probability of failure. Knowledge of this significant indicator can be used further e.g. in management of risk. Reinforced and prestressed concrete structures represent composite structures, which real behaviour is quite complicated. Fully probabilistic methods open way for more precise design and assessment of these structures. More information about development of the SBRA method is available on the web page [33] devoted to this method.

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