Frequency response of a viscoelastic plate under compressible viscous fluid loading

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Dedicated to the 100 anniversary of Academician Yury N. Rabotnov

Abstract**—** Forced vibrations of the system consisting of a viscoelastic plate-layer and a half-plane filled with a compressible viscous fluid are studied. The plane-strain state is considered in the case where the lineally-located time-harmonic forces act on the free face plane of the plate, and it is assumed that the mechanical relations for the plate-layer material are described through the Rabotnov fractional exponential operators. The motion of the plate is written by utilizing 2D exact equations of the theory of visco-elasto-dynamics, but the motion of the compressible viscous fluid is described by the linearized Navier-Stokes equations. It is assumed that the velocities and forces of the constituents are continuous on the contact plane between the plate and fluid. The dimensionless parameters which characterize the creep time and the long-term values of the elastic constants of the plate material are introduced. Moreover, the dimensionless parameters which characterize the compressibility and viscosity of the fluid are introduced as well. The corresponding boundary-value and contact problems which are obtained after employing the dynamical correspondence principle to the equations and relationships related to the plate are solved by applying the Fourier transformation with respect to the coordinate directed along the interface line. The inverse of this transformation is determined numerically. Numerical results on the interface stresses and velocities, and the influence of the foregoing dimensionless rheological parameters on these results are presented and discussed.

Keywords**—** Rabotnov fractional operators, viscous fluid, viscoelastic material, creep time.

I. Introduction

Investigations of problems related to the dynamics of plate-fluid interaction have great significance in the theoretical and application sense in aerospace, nuclear, naval, chemical and biological engineering. The first attempt in this field was made in [1], wherein vibrations of a circular elastic "baffled" plate in contact with still water were considered. It was assumed that this plate is clamped all around and placed in a matching circular aperture within an infinite rigid plane wall. The investigations were made by the use of the so-called "non-dimensional added virtual mass incremental" (NAVMI) method, according to which it is assumed that the modes of vibrations of the plate in contact with still water are the same as those in a vacuum, and the natural frequency is determined by the use of the Rayleigh quotient. In this case it is supposed that the squares of the natural frequencies of the plate are equal to the ratio between the maximum potential energy of the plate and the sum of the kinetic energies of both the plate and the fluid. Later this method was employed in many related investigations such as in papers [2]-[4] and in many others listed in these papers. Up to now it has also been used in investigations carried out without employing the NAVMI method. For instance, in a paper [5] the vibration and stability of the rectangular plate immersed in axial liquid flow was studied without employing the NAVMI method and the Galerkin method was applied to determine the expression of the flow perturbation potential. Then the Rayleigh-Ritz method was used to discretize the system.

Investigations carried out in [6] and other papers listed therein were also made without employing the NAVMI method. Note that in this paper the forced bending vibration of an infinite plate in contact with compressible (acoustic) inviscid fluid, where this fluid occupies a half-plane (half-space), was considered. This paper gives asymptotic analyses of the sound and vibration in the metal plate and compressible inviscid fluid system.

The other aspect of investigations related to the plate-fluid interaction regards wave propagation problems. Investigations carried out in [7] and other papers listed therein can be taken as examples. It should be noted that before the appearance of [7], the problems of time harmonic linear wave propagation in elastic structure-fluid systems were investigated within the framework of the theory of compressible inviscid fluid. A list of these studies and a review could be found in [7]. At the same time, the role of fluid viscosity in wave propagation in

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the plate-fluid system was first investigated in [7]. However, in this paper and all the papers indicated above, the equations of motion of the plate were written within the scope of the approximate plate theories by the use of various types of hypotheses such as the Kirchhoff hypotheses for plates. Consequently, the use of the approximate plate theories in these investigations decreases significantly the analyzed range of wave modes and their corresponding dispersion curves. It is evident that in many cases (for instance, in the cases where the wave length is less significant than the thickness of the plate) more accurate results in the qualitative and quantitative sense can be obtained by employing the exact equations for describing the plate motion. Moreover, in the foregoing investigations (except [4]) the initial strains (or stresses) in plates, which can be one of their characteristic particularities, are not taken into account. These two characteristics, namely, the use of the exact equations of plate motion and the existence of initial stresses in the plate, are taken into consideration in [8] and other papers, a review of which is given in [9]. Note that in these papers, in studying wave propagation in pre-stressed plate + compressible viscous fluid systems, the motion of the plate was written within the scope of the so-called three-dimensional linearized theory of elastic waves in initially stressed bodies. However, the motion of the viscous fluid was written within the scope of the linearized Navier-Stokes equations. Detailed consideration of related results was made in [10].

However, up to recent days, within this framework, there has not been any investigation related to the forced vibration of the pre-strained plate + compressible viscous fluid system. The first attempt in this field was made in [11], wherein the two-dimensional (plane-strain state) problem on the forced vibration of the pre-strained metal plate + compressible viscous fluid system, was studied. The motion of the plate is described by utilizing the three-dimensional linearized equations of the wave propagation in pre-stressed bodies and the motion of the fluid by utilizing linearized Navier-Stokes equations. Numerical results on the velocity distributions on the plate-fluid interface and the influence of the problem parameters and the frequency of the external force on these distributions are presented and discussed.

Nowadays polymer composite materials are intensively used in various branches of the modern industry related to the building of boats, ships, offshore structures, etc., when the fluid-structure interaction should be taken into account. However, all the foregoing investigations, as well as the studies carried out in [11], were focused on the interaction between a metal elastic plate and fluid, and therefore, in general, cannot be employed for understanding the behavior of the interaction between the polymer plate and fluid.

Consequently, investigation of problems related to the interaction between the plate type structure made of polymer materials and fluids may be interesting not only in the theoretical sense, but also in the practical sense in abovementioned branches of the modern industry.

In mainly, there are two moments which distinguish the interaction between a fluid and a plate made of polymer materials from the interaction between a fluid and a metal elastic plate. The first of them is the ratio of densities of the plate material and fluids, so as usual, the density of the polymers is not more than the density of fluids, but the density of the metals elastic material is greater significantly than that of the fluids. The second point relates to the time-dependent character of the mechanical properties of the polymer materials which are modeled through the well-known operators, the consideration of which can be found in the well-known monograph by Rabotnov [12]. However, up to know there is not any investigation related to the interaction between the plate made of linear viscoelastic material and fluids.

In the present paper, the attempt is made in this field and the problem related to the forced vibration of the system consisting of the plate made of a linear viscoelastic material and compressible viscous fluid is investigated. The plane-strain state in the plate is considered, and it is assumed that the viscoelasticity of the plate material is described through the Rabotnov fractional exponential operators [12]. In other words, in the present work, the analysis carried out in [11] is generalized for the case when the plate material is viscoelastic one.

Note that the corresponding problems related to pre-stressed elastic plate + elastic half-space systems were studied in [13]–[19], but the problem related to the system involving viscoelastic plate + viscoelastic half-space was studied in [20].

II. Matematical formulation of the problem

Consider a system consisting of a plate-layer made of viscoelastic material and a half-space filled by compressible viscous fluid. We associate the coordinate system $Ox_1x_2x_3$ (Fig. 1), and we will determine the position of the points of the constituents in this coordinate system. Assume that the plate thickness is h, and this plate occupies the region $\{|x_1| < \infty,\ -h < x_2 < 0,\ |x_3| < \infty\}$, but the fluid occupies the region $\{|x_1| < \infty,\ -\infty < x_2 < -h,\ |x_3| < \infty\}$. Moreover, assume that the lineal-located normal time-harmonic force acts on the plate's free face plane, and the distribution of this force with respect to the coordinate x_3 is homogeneous. Consequently, the plane-strain state occurs in the Ox_1x_2 plane.

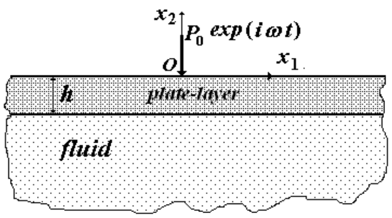

Fig. 1 The sketch of the plate + fluid system

Thus, within this we investigate the motion of the foregoing system. For this purpose, we write the equation of motion and other field equations for the plate.

Equation of motion:

$$\frac{\partial\sigma_{11}}{\partial x_1}+\frac{\partial\sigma_{21}}{\partial x_2}=\rho\frac{\partial^2 u_1}{\partial t^2},\quad \frac{\partial\sigma_{21}}{\partial x_1}+\frac{\partial\sigma_{22}}{\partial x_2}=\rho\frac{\partial^2 u_2}{\partial t^2}, \tag{1}$$

Constitutive relations:

$$\sigma_{11}=\lambda^*\varepsilon+2\mu^*\varepsilon_{11},\ \sigma_{22}=\lambda^*\varepsilon+2\mu^*\varepsilon_{22},$$
$$\sigma_{12}=2\mu^*\varepsilon_{12}, \tag{2}$$

where λ^* and μ^* are the following operators:

$$\begin{Bmatrix}\lambda^*\\ \mu^*\end{Bmatrix}\varphi(t)=\begin{Bmatrix}\lambda_0\\ \mu_0\end{Bmatrix}\varphi(t)+\int_0^t\begin{Bmatrix}\lambda_1\\ \mu_1\end{Bmatrix}(t-\tau)\varphi(\tau)d\tau\ . \tag{3}$$

In equation (3), λ_0 and μ_0 are the instantaneous values of Lame's constants as $t\to 0$, $\lambda_1(t)$ and $\mu_1(t)$ are the corresponding kernel functions for describing the hereditary properties of the material of the plate.

Strain-displacement relations:

$$\varepsilon_{11}=\frac{\partial u_1}{\partial x_1},\ \varepsilon_{22}=\frac{\partial u_2}{\partial x_1},$$
$$\varepsilon_{12}=\frac{1}{2}\left(\frac{\partial u_2}{\partial x_1}+\frac{\partial u_1}{\partial x_2}\right)\ \varepsilon=\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2} \tag{4}$$

Equations (1) – (4) are the complete system of equations of the theory viscoelasticity for isotropic bodies, and notation used in these equations is conventional.

According to [10], we consider the equations of motion of the Newtonian compressible viscous fluid: the density, viscosity constants and pressure of which are denoted by the upper index (1). Thus, we write the equation of motion and other field equations for the fluid.

Linearized Navier-Stokes equations for the fluid are:

$$\rho_0^{(1)}\frac{\partial v_1}{\partial t}-\mu^{(1)}\Delta v_1+\frac{\partial p^{(1)}}{\partial x_1}-(\lambda^{(1)}+\mu^{(1)})\frac{\partial e}{\partial x_1}=0,$$
$$\rho_0^{(1)}\frac{\partial v_2}{\partial t}-\mu^{(1)}\Delta v_2+\frac{\partial p^{(1)}}{\partial x_2}-(\lambda^{(1)}+\mu^{(1)})\frac{\partial e}{\partial x_2}=0, \tag{5}$$

Equation of continuity is:

$$\frac{\partial\rho^{(1)}}{\partial t}+\rho_0^{(1)}\left(\frac{\partial v_1}{\partial x_1}+\frac{\partial v_2}{\partial x_2}\right)=0, \tag{6}$$

Constitutive relations are

$$T_{11}=(-p^{(1)}+\lambda^{(1)}e)+2\mu^{(1)}e_{11},$$
$$T_{22}=(-p^{(1)}+\lambda^{(1)}e)+2\mu^{(1)}e_{22},\ T_{12}=2\mu^{(1)}e_{12}, \tag{7}$$

Deformation rate and velocity relations are:

$$e_{11}=\frac{\partial v_1}{\partial x_1},\ e_{22}=\frac{\partial v_2}{\partial x_2},\ e_{12}=\frac{1}{2}\left(\frac{\partial v_1}{\partial x_2}+\frac{\partial v_2}{\partial x_1}\right),$$
$$e=\frac{\partial v_1}{\partial x_1}+\frac{\partial v_2}{\partial x_2}, \tag{8}$$

State equation is

$$a_0^2=\frac{\partial p^{(1)}}{\partial\rho^{(1)}}. \tag{9}$$

In equations (5) and (6), $\rho_0^{(1)}$ is the fluid density before perturbation and

$$\Delta=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}. \tag{10}$$

The other notation in (5) – (9) is conventional.

According to [10], the solution of (5)-(10) is reduced to finding the two potentials φ and ψ, which are determined from the following equations:

$$\left[\left(1+\frac{\lambda^{(1)}+2\mu^{(1)}}{a_0^2\rho_0^{(1)}}\right)\Delta-\frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\right]\varphi=0,$$
$$\left(\nu^{(1)}\Delta-\frac{\partial}{\partial t}\right)\psi=0, \tag{11}$$

where $\nu^{(1)}$ is the kinematic viscosity, i.e., $\nu^{(1)}=\mu^{(1)}/\rho_0^{(1)}$.

The velocities v_1, v_2 and the pressure $p^{(1)}$ are expressed in terms of the potentials φ and ψ via the following expressions:

$$v_1=\frac{\partial\varphi}{\partial x_1}+\frac{\partial\psi}{\partial x_2},\ v_2=\frac{\partial\varphi}{\partial x_2}-\frac{\partial\psi}{\partial x_1},$$
$$p^{(1)}=\rho_0^{(1)}\left(\frac{\lambda^{(1)}+2\mu^{(1)}}{\rho_0^{(1)}}\Delta-\frac{\partial}{\partial t}\right)\varphi. \tag{12}$$

Supposing that $p^{(1)}=-(T_{11}+T_{22}+T_{33})/3$, we obtain

$$\lambda^{(1)}=-\frac{2}{3}\mu^{(1)} \tag{13}$$

It is assumed that

$$|v_i|\to 0,\ \left|\partial v_i/\partial x_j\right|\to 0,\ i,j=1,2 \text{ as } x_2\to-\infty \tag{14}$$

and there are no waves reflected from $x_2=-\infty$.

Moreover, it is assumed that the following boundary and contact conditions are satisfied:

$$\sigma_{21}\big|_{x_2=0}=0,\quad \sigma_{22}\big|_{x_2=0}=-P_0e^{i\omega t},$$
$$\left.\frac{\partial u_1}{\partial t}\right|_{x_2=-h}=v_1\big|_{x_2=-h},\ \left.\frac{\partial u_2}{\partial t}\right|_{x_2=-h}=v_2\big|_{x_2=-h},$$

$$\sigma_{21}\big|_{x_2=-h} = T_{21}\big|_{x_2=-h}, \sigma_{22}\big|_{x_2=-h} = T_{22}\big|_{x_2=-h}. \qquad (15)$$

This completes the formulation of the problem.

III. SOLUTION METHOD

First, we represent the displacements and the components of the strain tensor related to the plate, and the velocities and components of the strain rate tensor related to the fluid as

$$u_k(x_1,x_2,t) = \bar{u}_k(x_1,x_2)e^{i\omega t}, \ \varepsilon_{kn}(x_1,x_2,t) = \bar{\varepsilon}_{kn}(x_1,x_2)e^{i\omega t},$$
$$\varepsilon(x_1,x_2,t) = \bar{\varepsilon}(x_1,x_2)e^{i\omega t}, k;n=1,2$$
$$v_k(x_1,x_2,t) = \bar{v}_k(x_1,x_2)e^{i\omega t}, \ e_{kn}(x_1,x_2,t) = \bar{e}_{kn}(x_1,x_2)e^{i\omega t},$$
$$e(x_1,x_2,t) = \bar{e}(x_1,x_2)e^{i\omega t}, \qquad (16)$$

Below we will omit the over bar on the amplitudes of the sought values. Moreover, we use the relation

$$\int_0^t f_1(t-\tau) f_2(\tau) d\tau \approx \int_{-\infty}^t f_1(t-\tau) f_2(\tau) d\tau \qquad (17)$$

in (2) and (3). Thus, considering (16) and (17) in (2) and (3), we write

$$\sigma_{kn} = \lambda_0 \varepsilon(x_1,x_2)\delta_k^n e^{i\omega t} + 2\mu_0 \varepsilon_{kn}(x_1,x_2)e^{i\omega t}$$
$$+\varepsilon(x_1,x_2)\delta_k^n \int_{-\infty}^t \lambda_1(t-\tau)e^{i\omega\tau}d\tau$$
$$+2\varepsilon_{kn}(x_1,x_2)\int_{-\infty}^t \mu_1(t-\tau)e^{i\omega\tau}d\tau, \qquad (18)$$

where δ_k^n is a Kronecker symbol.

Using the transformation $t-\tau = s$, we can made the following manipulations of the integrals which enter into (18):

$$\int_{-\infty}^t \lambda_1(t-\tau)e^{i\omega\tau}d\tau = -\int_{\infty}^0 \lambda_1(s)e^{i\omega t}e^{i\omega s}ds$$
$$= e^{i\omega t}\int_0^\infty \lambda_1(s)e^{i\omega s}ds = e^{i\omega t}\left[\lambda_{1c}(\omega) - i\lambda_{1s}(\omega)\right], \qquad (19)$$

where

$$\lambda_{1c}(\omega) = \int_0^\infty \lambda_1(s)\cos(\omega s)ds, \ \ \lambda_{1s}(\omega) = \int_0^\infty \lambda_1(s)\sin(\omega s)ds. \ (20)$$

In a similar manner, we obtain

$$\int_{-\infty}^t \mu_1(t-\tau)e^{i\omega\tau}d\tau = -\int_{\infty}^0 \mu_1(s)e^{i\omega t}e^{i\omega s}ds$$
$$= e^{i\omega t}(\mu_{1c}(\omega) - i\mu_{1s}(\omega)) \qquad (21)$$

where

$$\mu_{1c}(\omega) = \int_0^\infty \mu_1(s)\cos(\omega s)ds, \ \ \mu_{1s}(\omega) = \int_0^\infty \mu_1(s)\sin(\omega s)ds. \ (22)$$

Thus, we obtain the complex elastic constants, the real and imaginary parts of which are determined through expressions (20) and (22). Considering (19) – (22), we can write the following expressions for the stresses in the plate:

$$\sigma_{kn} = \Big(\Lambda(\omega)\varepsilon(x_1,x_2)\delta_k^n + 2M(\omega)\varepsilon_{kn}(x_1,x_2)\Big)e^{i\omega t}, \qquad (23)$$

where

$$\Lambda(\omega) = \lambda_0 + \lambda_{1c}(\omega) - i\lambda_{1s}(\omega),$$
$$M(\omega) = \mu_0 + \mu_{1c}(\omega) - i\mu_{1s}(\omega). \qquad (24)$$

Thus, we obtain the relations (23) and (24) instead of (2) and (3). This means that the complete system of field equations (1), (4), (23) and (24) for the viscoelastic plate can also be obtained from the field equations written for the purely elastic system by replacing the elastic constants λ_0 and μ_0 with the complex constants $\Lambda(\omega)$ and $M(\omega)$, respectively. In other words, the foregoing mathematical calculation confirms the dynamic correspondence principle [21] for the problem under consideration, and the solution method used here coincides with this principle.

Note that the real parts of the complex constants, i.e., $\mathrm{Re}\,\Lambda(\omega)$ and $\mathrm{Re}\,M(\omega)$, are called the storage moduli, while the imaginary parts, i.e., $\mathrm{Im}\,\Lambda(\omega)$ and $\mathrm{Im}\,M(\omega)$, are called the loss moduli. The ratios $\mathrm{Im}\,\Lambda(\omega)/\mathrm{Re}\,\Lambda(\omega)$ and $\mathrm{Im}\,M(\omega)/\mathrm{Re}\,M(\omega)$ determine the phase shifting between the strains and stresses. A more detailed explanation of the mechanical meaning of these ratios can be given under the selection of concrete viscoelastic operators, what will be made in the next section.

Now we turn to the consideration of the determination of the amplitudes of the sought values. For this purpose we substitute (16) and (23) into the corresponding equations and relations, and replace the derivatives $\partial(\bullet)/\partial t$ and $\partial^2(\bullet)/\partial t^2$ with $i\omega(\bar{\bullet})$ and $-\omega^2(\bar{\bullet})$, respectively. We obtain the corresponding equations, boundary and contact conditions for the mentioned amplitudes. We employ the exponential Fourier transformation to these equations with respect to the x_1 coordinate

$$f_F(s,x_2) = \int_{-\infty}^{+\infty} f(x_1,x_2)e^{-isx_1}dx_1. \qquad (25)$$

Considering the problem symmetry with respect to $x_1 = 0$, the originals of the sought values could be represented as follows

$$u_1(x_1,x_2) = \frac{1}{\pi}\int_0^\infty u_{1F}(s,x_2)\sin(sx_1)ds,$$

$$u_2(x_1,x_2) = \frac{1}{\pi}\int_0^\infty u_{2F}(s,x_2)\cos(sx_1)ds,$$

$$\sigma_{11}(x_1,x_2) = \frac{1}{\pi}\int_0^\infty \sigma_{11F}(s,x_2)\cos(sx_1)ds,$$

$$\sigma_{22}(x_1,x_2) = \frac{1}{\pi}\int_0^\infty \sigma_{22F}(s,x_2)\cos(sx_1)ds,$$

$$\sigma_{12}(x_1,x_2)=\frac{1}{\pi}\int_0^\infty \sigma_{12F}(s,x_2)\sin(sx_1)ds ,$$

$$\varphi(x_1,x_2)=\frac{1}{\pi}\int_0^\infty \varphi_F(s,x_2)\cos(sx_1)ds ,$$

$$\psi(x_1,x_2)=\frac{1}{\pi}\int_0^\infty \psi_F(s,x_2)\cos(sx_1)ds ,$$

$$v_1(x_1,x_2)=\frac{1}{\pi}\int_0^\infty v_{1F}(s,x_2)\sin(sx_1)ds ,$$

$$v_2(x_1,x_2)=\frac{1}{\pi}\int_0^\infty v_{2F}(s,x_2)\cos(sx_1)ds ,$$

$$T_{11}(x_1,x_2)=\frac{1}{\pi}\int_0^\infty T_{11F}(s,x_2)\cos(sx_1)ds ,$$

$$T_{22}(x_1,x_2)=\frac{1}{\pi}\int_0^\infty T_{22F}(s,x_2)\cos(sx_1)ds ,$$

$$T_{12}(x_1,x_2)=\frac{1}{\pi}\int_0^\infty T_{12F}(s,x_2)\sin(sx_1)ds . \quad (26)$$

First, we consider the solution of the equations related to the Fourier transformation of the quantities for the plate-layer, i.e., the solution of the equations which are obtained from (26), (23), 24), (16), (4), and (1). Thus, substituting (26) in (23), (24), (16), (4) and (1), and doing some mathematical manipulations, we obtain the following equations with respect to the u_{1F} and u_{2F}:

$$Au_{1F}-B\frac{du_{2F}}{dx_2}+C\frac{d^2u_{1F}}{dx_2^2}=0 ,$$

$$Du_{2F}+B\frac{du_{1F}}{dx_2}+G\frac{d^2u_{2F}}{dx_2^2}=0 , \quad (27)$$

where

$$A=X^2-s^2\left[\Lambda(\omega)+2M(\omega)\right],\quad B=s\left[\Lambda(\omega)+M(\omega)\right],$$
$$C=M(\omega),\ D=X^2-s^2M(\omega),\ G=\Lambda(\omega)+2M(\omega),$$
$$X^2=\omega^2h^2/c_2^2\ ,\ c_2=\sqrt{\mu_0/\rho}\ . \quad (28)$$

Introducing the notation

$$A_0=\frac{AG+B^2+CD}{CG}\ ,\ B_0=\frac{BD}{CG},\ k_1=\sqrt{-\frac{A_0}{2}+\sqrt{\frac{A_0^2}{4}-B_0}}\ ,$$
$$k_2=\sqrt{-\frac{A_0}{2}-\sqrt{\frac{A_0^2}{4}-B_0}}\ , \quad (29)$$

we can write the solution of (27) as follows

$$u_{2F}=Z_1e^{k_1x_2}+Z_2e^{-k_1x_2}+Z_3e^{k_2x_2}+Z_4e^{-k_2x_2}\ ,$$

$$u_{1F}=Z_1a_1e^{k_1x_2}+Z_2a_2e^{-k_1x_2}+Z_3a_3e^{k_2x_2}+Z_4a_4e^{-k_2x_2}\ ,$$
(30)

where

$$a_1=\frac{-D-Gk_1^2}{Bk_1^2}\ ,\ a_2=-a_1 ,$$
$$a_3=\frac{-D-Gk_2^2}{Bk_2^2}\ ,\ a_4=-a_3 . \quad (31)$$

Using (30), (4) and (23), we also write expressions for the Fourier transformations σ_{12F} and σ_{22F} of the corresponding stresses which enter the boundary and contact condition (15)

$$\sigma_{12F}=Z_1M(\omega)\left(k_1a_1-s\right)e^{k_1x_2}+Z_2M(\omega)\left(-k_1a_2-s\right)e^{-k_1x_2}$$
$$+Z_2M(\omega)\left(k_2a_3-s\right)e^{k_2x_2}+Z_4M(\omega)\left(-k_2a_2-s\right)e^{-k_4x_2} ,$$
$$\sigma_{22F}=Z_1\left(s\Lambda(\omega)a_1+k_1(\Lambda(\omega)+2M(\omega))\right)e^{k_1x_2}$$
$$+Z_2\left(s\Lambda(\omega)a_2-k_1(\Lambda(\omega)+2M(\omega))\right)e^{-k_1x_2}$$
$$+Z_3\left(s\Lambda(\omega)a_3+k_2(\Lambda(\omega)+2M(\omega))\right)e^{k_2x_2}$$
$$+Z_4\left(s\Lambda(\omega)a_4-k_2(\Lambda(\omega)+2M(\omega))\right)e^{-k_2x_2} . \quad (32)$$

This completes the consideration of the Fourier transformation of the values related to the plate-layer. Now we consider the determination of the Fourier transformations of the quantities related to the fluid flow. First, we consider the determination of φ_F and ψ_F from the Fourier transformation of the equations in (11), which taking the relations (13) and

$$\varphi_F=\omega h^2\tilde{\varphi}_F\ ,\ \psi_F=\omega h^2\tilde{\psi}_F \quad (33)$$

into account could be written as follows

$$\frac{d^2\tilde{\varphi}_F}{dx_2^2}+\left(\frac{\Omega_1^2}{1+i4\Omega_1^2/(3N_w^2)}-s^2\right)\tilde{\varphi}_F=0 ,$$
$$\frac{d^2\tilde{\psi}_F}{dx_2^2}-\left(s^2+iN_w^2\right)_F\tilde{\psi}_F=0 , \quad (34)$$

where

$$\Omega_1=\frac{\omega h}{a_0}\ ,\ N_w^2=\frac{\omega h^2}{\nu^{(1)}} . \quad (35)$$

The dimensionless number N_w in (35) can be taken as Womersley number and characterizes the influence of the fluid viscosity on the mechanical behavior of the system under consideration. For purely hydrodynamic problems, when the Womersley number is large (around 10 or greater), it shows that the flow is dominated by oscillatory inertial forces. When the Womersley number is low, viscous forces tend to dominate the flow. However, for hydro-elastodynamic problems the mentioned "large" and "low" limits for the Womersley number can change significantly.

The dimensionless frequency Ω_1 in (35) can be taken as the parameter which characterizes the compressibility of the fluid on the mechanical behavior of the system under consideration. Thus, taking the condition (14) into consideration, the solutions to the equations in (34) are found as follows

$$\tilde{\varphi}_F = Z_5 e^{\delta_1 x_2}, \tilde{\psi}_F = Z_6 e^{\gamma_1 x_2}, \quad (36)$$

where

$$\delta_1 = \sqrt{s^2 - \frac{\Omega_1^2}{1 + i4\Omega_1^2 / \left(3N_w^2\right)}}, \quad \gamma_1 = \sqrt{s^2 + iN_w^2}. \quad (37)$$

Using (36) and (37) we obtain the following expressions for the Fourier transformations of the velocities, pressure and stresses of the fluid from the Fourier transformations of (5) – (12):

$$v_{1F} = \omega h\left[-Z_5 s e^{\delta_1 x_2} + Z_6 e^{\gamma_1 x_2}\right],$$

$$v_{2F} = \omega h\left[Z_5 \delta_1 e^{\delta_1 x_2} - Z_6 s e^{\gamma_1 x_2}\right],$$

$$T_{22F} = \mu^{(1)}\omega\left[Z_5\left(\frac{4}{3}\delta_1^2 + \frac{2}{3}s^2 - R_0\right)e^{\delta_1 x_2} + Z_6\left(-s\gamma_1 - \frac{2}{3}s\gamma_1\right)e^{\gamma_1 x_2}\right],$$

$$T_{21F} = \mu^{(1)}\omega\left[Z_5 s\delta_1 e^{\delta_1 x_2} + Z_6\left(s^2 + \gamma_1^2\right)e^{\gamma_1 x_2}\right],$$

$$p_F^{(1)} = \mu^{(1)}\omega R_0 Z_5 e^{\delta_1 x_2}, \quad (38)$$

where

$$R_0 = -\frac{4}{3}\frac{\Omega_1^2}{1 + i4\Omega_1^2/(3N_w^2)} - iN_w^2. \quad (39)$$

Substituting (30), (32), and (38) into the boundary and contact conditions (15), we obtain a set of equations with respect to the unknowns Z_1, Z_2, …, Z_6, in terms of which the sought values are determined. The mentioned equations can be expressed as follows

$$\left(\sigma_{12F}/\mu_0\right)\Big|_{x_2=0} = Z_1\alpha_{11} + Z_2\alpha_{12} + Z_3\alpha_{13} + Z_4\alpha_{14} = 0,$$

$$\left(\sigma_{22F}/\mu_0\right)\Big|_{x_2=0} = Z_1\alpha_{21} + Z_2\alpha_{22} + Z_3\alpha_{23} + Z_4\alpha_{24} = -P_0/\mu_0,$$

$$\frac{\partial u_{1F}}{\partial t}\Big|_{x_2=-h} - v_{1F}\big|_{x_2=-h} = i\omega(Z_1\alpha_{31} + Z_2\alpha_{32} + Z_3\alpha_{33} + Z_4\alpha_{34}) - \omega h(Z_5\alpha_{35} + Z_6\alpha_{36}) = 0,$$

$$\frac{\partial u_{2F}}{\partial t}\Big|_{x_2=-h} - v_{2F}\big|_{x_2=-h} = i\omega(Z_1\alpha_{41} + Z_2\alpha_{42} + Z_3\alpha_{43} + Z_4\alpha_{44}) - \omega h(Z_5\alpha_{45} + Z_6\alpha_{46}) = 0,$$

$$\left(\sigma_{21}/\mu_0\right)\Big|_{x_2=-h} - \left(T_{21}/\mu_0\right)\Big|_{x_2=-h} = Z_1\alpha_{51} + Z_2\alpha_{52} + Z_3\alpha_{53} + Z_4\alpha_{54}) - M_\mu(Z_5\alpha_{55} + Z_6\alpha_{56}) = 0,$$

$$\left(\sigma_{22}/\mu_0\right)\Big|_{x_2=-h} - \left(T_{22}/\mu_0\right)\Big|_{x_2=-h} = Z_1\alpha_{61} + Z_2\alpha_{62} + Z_3\alpha_{63} + Z_4\alpha_{64}) - M_\mu(Z_5\alpha_{65} + Z_6\alpha_{66}) = 0, \quad (40)$$

where

$$M_\mu = \frac{\mu^{(1)}\omega}{\mu_0}. \quad (41)$$

The expressions for coefficients α_{nm} $(n;m = 1,2,...,6)$ can be easily determined from (30), (32), and (38), and therefore we do not present they here. Thus, unknowns Z_1, Z_2,…, Z_6 in (40) can be determined via the formulae

$$Z_k = \frac{\det\left\|\beta_{nm}^k\right\|}{\det\left\|\alpha_{nm}^k\right\|}. \quad (42)$$

Note that the matrix $\left(\beta_{nm}^k\right)$ is obtained from the matrix $\left(\alpha_{nm}\right)$ by replacing the $k-th$ column of the latter by the column $(0, -P_0/\mu_0, 0, 0, 0, 0)^T$.

Now we consider the calculation of the integrals in (26). For this purpose, firstly we consider the following reasoning. If we take the Fourier transformation parameter s as the wavenumber, then equation

$$\det\left\|\alpha_{nm}\right\| = 0, \quad n, m = 1, 2, ..., 6 \quad (43)$$

coincides with the dispersion equation of the waves propagated in the direction of the Ox_1 axis in the system under consideration. It should be noted that, according to the well-known physical-and-mechanical considerations, equation (43) must have complex roots only. This character of the roots is caused by the viscoelasticity of the plate material and by the viscosity of the fluid. In other words, the integrated functions in (26) have not any singular points, and therefore these integrals can be calculated by the use of the well-known usual numerical calculation algorithm.

IV. Numerical Results and Discussions

A. Selection of the viscoelastic operators and complex constants

Assume that the viscoelasticity of the plate material is described via the Rabotnov fractional exponential operator [12], i.e., we propose that

$$\mu^*\eta(t) = \mu_0\left[\eta(t) - \frac{3\beta_0}{2(1+\nu_0)}R_\alpha^*\left(-\frac{3\beta_0}{2(1+\nu_0)} - \beta_\infty\right)\eta(t)\right],$$

$$\lambda^*\eta(t) = \lambda_0\left[\eta(t) + \frac{(1-2\nu_0)\beta_0}{2(1+\nu_0)\nu_0}R_\alpha^*\left(-\frac{3\beta_0}{2(1+\nu_0)} - \beta_\infty\right)\eta(t)\right],$$

$$E^*\eta(t) = E_0\left[\eta(t) - \beta_0 R_\alpha^*\left(-\beta_0 - \beta_\infty\right)\eta(t)\right],$$

$$\nu^*\eta(t)=\nu_0\left[\eta(t)+\frac{1-2\nu_0}{2\nu_0}\beta_0 R_\alpha^*\left(-\beta_0-\beta_\infty\right)\eta(t)\right], \qquad (44)$$

where

$$R_\alpha^*(x)\eta(t)=\int_0^t R_\alpha(x,t-\tau)\eta(\tau)d\tau ,$$

$$R_\alpha(x,t)=t^{-\alpha}\sum_{n=0}^{\infty}\frac{x^n t^{n(1-\alpha)}}{\Gamma((1+n)(1-\alpha))} , \qquad 0\le\alpha<1 . \qquad (45)$$

In (45), $\Gamma(x)$ is a gamma-function.

Note that the Rabotnov fractional exponential operators (45) allow us to describe the initial parts of the experimentally and theoretically constructed creep and relaxation graphs with the required accuracy. These operators also allow us to determine with very high accuracy the asymptotic values of these graphs. Operators in (44) and (45) are employed successfully to describe various polymer materials and epoxy-based composites with continuous fibers or layers. The values of the rheological parameters in (44) were determined for these materials in [12]. Moreover, these operators have many simple rules for complicated mathematical transformations, for example, the Laplace transformation and Fourier transformations which also will be used below in the present investigation. Note that utilizing these operators during the investigation of the stability loss and buckling delamination problems related to the viscoelastic composite materials was also considered in [22] and [23].

It follows from (44) that

$$\left(\lambda^*+\frac{2}{3}\mu^*\right)\eta(t)=\left(\lambda_0+\frac{2}{3}\mu_0\right)\eta(t) . \qquad (46)$$

As $(\lambda_0+2\mu_0/3)$ is the modulus of volume expansion (denote it by K_0), we can conclude that the selection of the operators in (44) corresponds to the case where the volumetric expansion of the materials of the plate is purely elastic. In other words, the constitutive relation in (2) can be rewritten as

$$\sigma(t)=K_0\varepsilon(t) , \qquad \sigma_{11}-\sigma=2\mu^*(\varepsilon_{11}-\varepsilon/3) ,$$
$$\sigma_{22}-\sigma=2\mu^*(\varepsilon_{22}-\varepsilon/3) , \quad \sigma_{12}=2\mu^*\varepsilon_{12} , \qquad (47)$$

where $\sigma_{11}-\sigma$, $\sigma_{22}-\sigma$, and σ_{12} are components of the deviatoric stresses, $\varepsilon_{11}-\varepsilon/3$, $\varepsilon_{22}-\varepsilon/3$, and ε_{12} are components of the deviatoric strains, and $\sigma=\sigma_{11}+\sigma_{22}$.

Thus, it follows from (47) that in the case under consideration, the operator μ^* is sufficient to describe the viscoelasticity of the plate material. In (44) and (45), α , β_0, and β_∞ are the rheological parameters of the plate material.

For explanation of the mechanical meaning of these parameters, following Rabotnov [12], we consider some properties of the operator (45). For this purpose, we note that the operator R_α^* (45) can also be determined as

$$R_\alpha^*(x)=\frac{I_\alpha^*}{1-xI_\alpha^*} \quad \text{or}$$
$$1+xR_\alpha^*(x)=\frac{1}{1-xI_\alpha^*} , \qquad (48)$$

where

$$I_\alpha^*\eta(t)=\int_0^t I_\alpha(t-\tau)\eta(\tau)d\tau , \; I_\alpha(t)=\frac{t^{-\alpha}}{\Gamma(1+\alpha)} , \; 0\le\alpha<1 \qquad (49)$$

According to (48), the operator $R_\alpha^*(x)$ is called in [12] as the resolvent operator emitted with the operator $I_\alpha^*(x)$.

The Laplace transformation $\bar{f}(p)=\int_0^\infty f(t)e^{-pt}dt$ of the functions $R_\alpha(x,t)$ (45) and $R_{1\alpha}(x,t)=\int_0^\infty R_\alpha(x,t-\tau)d\tau$ is

$$\bar{R}_\alpha(x,p)=\frac{1}{p^{1-\alpha}-x} ,$$
$$\bar{R}_{1\alpha}(x,p)=\frac{1}{p(p^{1-\alpha}-x)} . \qquad (50)$$

Under small values of time *t*, the first term in the series (45) is the dominant term, and therefore for the cases, where $t\to 0$, it could be written that

$$R_\alpha(x,t)\approx I_\alpha(t) . \qquad (51)$$

Moreover, it follows from (50) that in the case when $t\to\infty$, it could be written that

$$R_{1\alpha}(x,t)=\int_0^\infty R_\alpha(x,t-\tau)d\tau\to-\frac{1}{x} . \qquad (52)$$

Thus, taking (44)-(52) into account, we can conclude that the dimensionless rheological parameter α characterizes the mechanical behavior of the viscoelastic material around the initial state of the deformation, i.e., in the vicinity of $t=0$. Moreover, we can conclude that the dimension of the rheological parameter β_∞ coincides with the dimension of the rheological parameter β_0 and is proportional to $T^{\alpha-1}$, where T is the time dimension. At the same time, according to (44) -(52), we obtain the following expressions:

$$\lambda_\infty=\lim_{t\to\infty}\lambda^*1=\lambda_0\left(1+\frac{1-2\nu_0}{2\nu_0(1+\nu_0)(3/(2(1+\nu_0))+d)}\right),$$
$$\mu_\infty=\lim_{t\to\infty}\mu^*1=\mu_0\left(1-\frac{3}{2(1+\nu_0)(3/(2(1+\nu_0))+d)}\right),$$
$$E_\infty=\lim_{t\to\infty}E^*1=E_0\left(1-\frac{1}{1+d}\right),$$

$$\nu_\infty = \lim_{t\to\infty} \nu^* 1 = \nu_0 \left(1 + \frac{1-2\nu_0}{2\nu_0(1+d)}\right), \quad (53)$$

which characterize the long-term values of the mechanical constants. In (53), the notation

$$d = \beta_\infty / \beta_0 \quad (54)$$

is used. It follows from (53) and (54) that the ratio of the rheological parameters β_∞ and β_0, i.e., the ratio $\beta_\infty/\beta_0 (= d)$, characterizes the long-term values of the elastic constants.

The foregoing discussion shows that the expression

$$t_R = (\beta_\infty)^{\frac{-1}{1+\alpha}} \text{ and } t_C = \left(\beta_\infty + \frac{3}{2(1+\nu_0)}\beta_0\right)^{\frac{-1}{1+\alpha}}$$

could be taken as the characteristic relaxation time (denoted as t_R) and characteristic creep time (denoted by t_C), respectively. According to (54), it follows that $t_R > t_C$ and

$$t_R = t_C \left(\frac{3}{2(1+\nu_0)d} + 1\right)^{\frac{1}{1+\alpha}}.$$

Now we turn to consideration of the expressions for μ_c and μ_s. Considering (22) and (44), these expressions can be written as

$$\mu_c = \mu_0 \left[1 - \frac{3}{2(1+\nu_0)}\left(d + \frac{3}{2(1+\nu_0)}\right)^{-1} R_{\alpha c}(-\beta_{01} - \beta_\infty, \omega)\right],$$

$$\mu_s = -\mu_0 \frac{3}{2(1+\nu_0)}\left(d + \frac{3}{2(1+\nu_0)}\right)^{-1} R_{\alpha s}(-\beta_{01} - \beta_\infty, \omega), \quad (55)$$

where

$$\beta_{01} = \frac{3\beta_0}{2(1+\nu_0)}. \quad (56)$$

We recall that the ratio μ_s/μ_c is the loss tangent, i.e., $\tan\theta = \mu_s/\mu_c$, where the angle θ can be interpreted as providing the phase angle by which the deviatoric strain lags behind the deviatoric stress in steady-state harmonic oscillation in the viscoelastic materials under consideration. Substituting $(i\omega)$ for d in the Laplace transformation (50) of the core function (45) of the fractional exponential operator (44) and, following [12], doing some mathematical manipulations, we obtain

$$R_{\alpha c}(-\beta_{01} - \beta_\infty, \omega) = \frac{\xi^2 + \xi \sin\frac{\pi\alpha}{2}}{\xi^2 + 2\xi\sin\frac{\pi\alpha}{2} + 1},$$

$$R_{\alpha s}(-\beta_{01} - \beta_\infty, \omega) = \frac{\xi\cos\frac{\pi\alpha}{2}}{\xi^2 + 2\xi\sin\frac{\pi\alpha}{2} + 1}, \quad (57)$$

and

$$\xi = (QX)^{\alpha-1}, \; Q = \frac{c_2}{(\beta_{01} + \beta_\infty)^{\frac{1}{1-\alpha}} h}, \quad (58)$$

where X and c_2 are determined by (28).

According to the foregoing discussion on the dimensions of the rheological parameters β_∞ and β_0, we can conclude that Q and ξ (58) entering in (57) are dimensionless parameters.

Consider the mechanical sense of the parameter Q. It is evident from (54) and (58) that

$$Q = t_C c_2 / h, \quad (59)$$

whence it follows that for fixed c_2/h the increase (decrease) in the values of Q results in the increase (decrease) in the values of the characteristic creep time t_C. Therefore, we call the parameter Q as the dimensionless characteristic creep time.

Let us call, conditionally, the ratio h/t_C (denoted by c_c) as the "creep velocity" of the plate material. Then, the parameter Q can also be estimated as the ratio of the shear wave velocity c_2 in the corresponding purely elastic material to the "creep velocity" c_c of the plate material. Consequently, for fixed values of c_2, the increase in the values of the dimensionless parameter $Q (= c_2/c_c)$ corresponds to the decrease in the "creep velocity" c_c of the plate material.

As noted above, the dimensionless rheological parameter d (54) entering in (53) and (55) characterizes the long-term values of the mechanical properties, i.e., the values of λ_∞, μ_∞, E_∞, and ν_∞ are determined by (53), in so doing $\lambda_\infty > \lambda_0$, $\mu_\infty < \mu_0$, $E_\infty < E_0$, and $\nu_\infty > \nu_0$. Nevertheless, the magnitudes of λ_∞ and ν_∞ decrease, but the magnitudes of μ_∞ and E_∞ increase with d. At the same time, according to (53), we can write that $d = (E_\infty/E_0)/(1 - E_\infty/E_0)$ from which it follows that $d \to \infty$ as $E_\infty/E_0 \to 1$, and $d \to 0$ as $E_\infty/E_0 \to 0$.

To be more precise, the long-term values of the mechanical properties approach their corresponding instantaneous values at $t = 0$ with the parameter d. In other words, the relations

$$\lambda_\infty \to \lambda_0, \quad \mu_\infty \to \mu_0, \quad E_\infty \to E_0, \quad \nu_\infty \to \nu_0, \quad \lambda_c \to \lambda_0,$$
$$\mu_c \to \mu_0, \quad \lambda_s \to 0, \quad \mu_s \to 0 \quad \text{as} \quad d \to \infty \quad (59)$$

take place. Consequently, in the cases where $d \gg 1$, the forced vibration of the considered system must be very close to those of the corresponding system consisting of the purely elastic plate and compressible viscous fluid. Taking these discussions into account, we can conclude that the increase (decrease) in the values of the dimensionless parameter d results in the increase (decrease) in the long-term values of the elastic constants, for instance, in the values of E_∞. Moreover, it follows from the expression of the dimensionless parameter $\xi(=QX)^{\alpha-1}$ that

$$\text{at } \xi\to\infty:\ R_{\alpha c}(-\beta_{01}-\beta_\infty,\omega)\to 1,\quad R_{\alpha s}(-\beta_{01}-\beta_\infty,\omega)\to 0 \quad \text{as} \quad (QX)\to 0, \tag{60}$$

$$\text{at } \xi\to 0:\ R_{\alpha c}(-\beta_{01}-\beta_\infty,\omega)\to 0,\quad R_{\alpha s}(-\beta_{01}-\beta_\infty,\omega)\to 0 \quad \text{as} \quad (QX)\to \infty. \tag{61}$$

The relation (60) means that in the cases where $(QX)\ll 1$, the behavior of the considered plate-layer is very close to that of a purely elastic plate-layer with long-term values of the elastic constants. As well, the relation (61) means that in the cases where $(QX)\gg 1$, the behavior of the plate-layer is very close to that of the corresponding purely elastic system with instantaneous values of the elastic constants at $t=0$.

So, the influence of the viscoelasticity of the plate material on the frequency response (i.e., the external force dependence of the amplitudes of the fluid velocities, stresses and frequency ω) can be characterized via the parameters Q (the dimensionless creep time) and d (through which the long-term values of the mechanical constants are determined). In this case, the increase in the values of the parameters Q and d will correspond to the decrease in the viscous part of the viscoelastic deformations of the plate. According to expression Q given in (58), the influence of the rheological parameter α on the viscous part of the deformations can be taken into account with the help of the parameter Q.

B. Numerical results and their analysis

According to the discussions made in the previous section, the numerical results are obtained by evaluating of the integrals

$$\{\sigma_{22},\sigma_{11},u_2,T_{22},T_{11},v_2\} = \frac{1}{\pi}\mathrm{Re}\left\{e^{i\omega t}\int_0^\infty [\sigma_{22F},\sigma_{11F},u_{2F},T_{22F},T_{11F},v_{2F}]\cos(s\,x_1)ds\right\},$$

$$\{\sigma_{12},u_1,T_{12},v_1\} = \frac{1}{\pi}\mathrm{Re}\left\{e^{i\omega t}\int_0^\infty [\sigma_{12F},u_{1F},T_{12F},v_{1F}]\sin(s\,x_1)ds\right\}.$$

(62)

Note that during calculations the improper integrals (62) are replaced by the corresponding definite integrals, i.e., it is assumed that

$$\int_0^\infty(\bullet)ds \approx \int_0^{S_1^*}(\bullet)ds. \tag{63}$$

The values of S_1^* in (63) are determined from the convergence criteria of the improper integrals. In the present calculation procedure, it is established that the difference between the numerical results obtained for $S_1^*=100$ and $S_1^*>100$ is not greater than 10^{-6}. Therefore, all numerical results, which will be discussed below, are obtained for the case where $S_1^*=100$. Under numerical evaluations, the integrated interval $\left[0,S_1^*\right]$ is divided into a certain number of shorter intervals, and for each of them the Gauss integrating algorithm is employed. In this integration procedure, the values of the integrated expressions, i.e., the values of σ_{22F} ,…, v_{1F} in the Gauss integration nodes, are determined according to (40).

Under numerical investigation the values of the mechanical constants and density of the plate material are taken as $\mu_0=1.86\times10^9$ Pa, $\lambda_0=3.96\times10^9$ Pa, and $\rho_0=1160$ kg/m^3, but the material of the fluid is selected as Glycerin with the viscosity coefficient $\mu^{(1)}=1,393$ kg/(m·s), density $\rho=1260$ kg/m^3, and the sound speed $a_0=1459.5$ m/s [10].

After selection of the material parameters, the dimensionless parameters Ω_1, N_w (35) and M_μ (41) can be determined through the following quantities: h (the thickness of the plate-layer), ω (the frequency of the time-harmonic external forces). In the present work, we will assume that $h=0.001$ m and $5\,\text{Hz}\le\omega\le1000\text{Hz}$, and will investigate only the results which relate to the influence of the rheological parameters d and Q of the plate material on the frequency response of T_{22}, v_2, and v_1 in the case where the rheological parameter α in (44) and (45) is taken as $\alpha=0.5$. In these investigations, the values of T_{22}, v_2, and v_1 are calculated on the interface plane between the fluid and viscoelastic plate, i.e., at $x_2=-h$ (Fig.1). First, we consider the case when $\omega t=2\pi n$ $(n=0,1,2,...)$, i.e., the case where $\cos(\omega t)=1$ and $\sin(\omega t)=0$ in (62).

Thus, we analyze the graphs given in Figs. 2-4 which show the influence of the parameter d (54) on the frequency response of $T_{22}h/P_0$, $v_2h\mu_0/(P_0c_2)$ (where $c_2=\sqrt{\mu_0/\rho}$) and $v_1h\mu_0/(P_0c_2)$, respectively, in the case where $Q=10$. Under construction of these graphs the values of T_{22} and v_2 are calculated at $x_1/h=0$, but the values of the velocity v_1 at $x_1/h=25$. According to the meaning of the parameter d and according to the known mechanical considerations, the frequency response graphs under consideration must approach to those related to the purely elastic plate case. This prediction and mechanical considerations are proven with the results

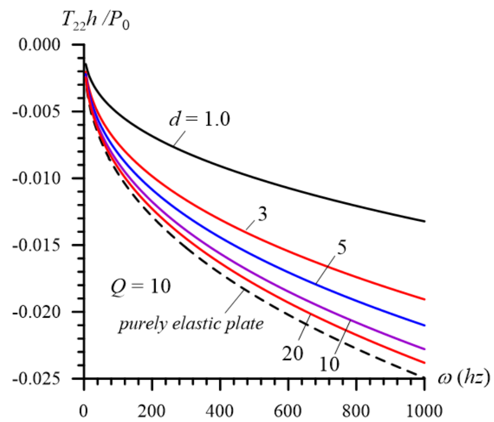

Fig. 2 The influence of the parameter d (54) on the frequency response of the stress T_{22}

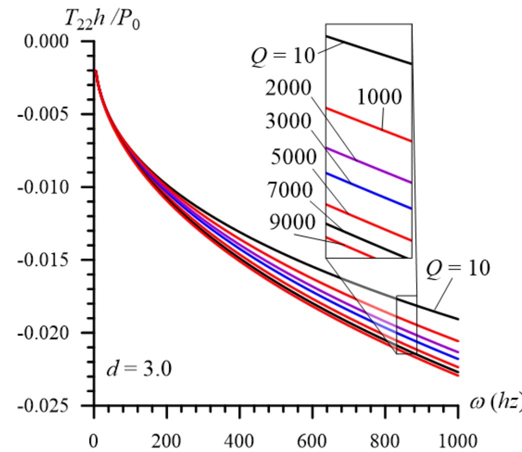

Fig. 5 The influence of the parameter Q (58) on the frequency response of the stress T_{22}

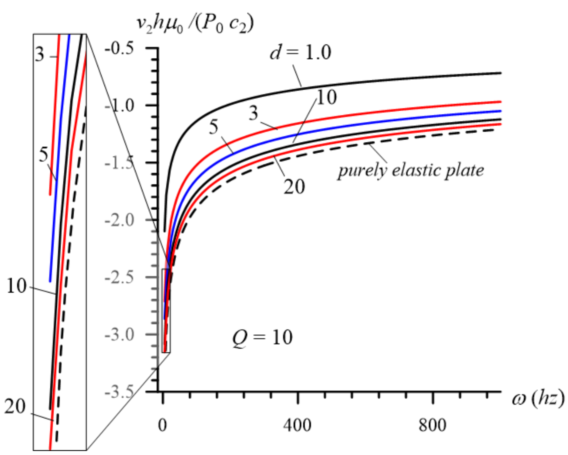

Fig. 3 The influence of the parameter d (54) on the frequency response of the velocity v_2

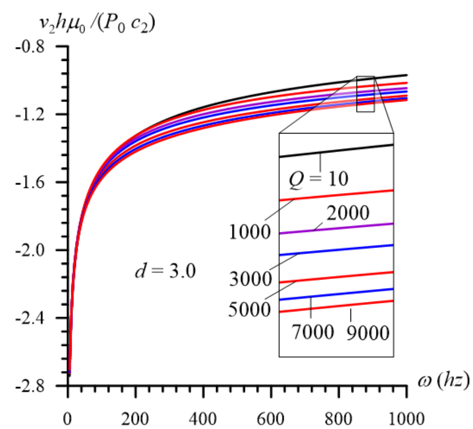

Fig. 6 The influence of the parameter Q (58) on the frequency response of the velocity v_2

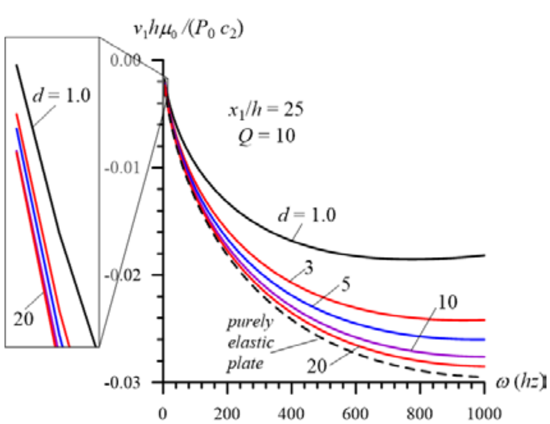

Fig. 4 The influence of the parameter d (54) on the frequency response of the velocity v_1

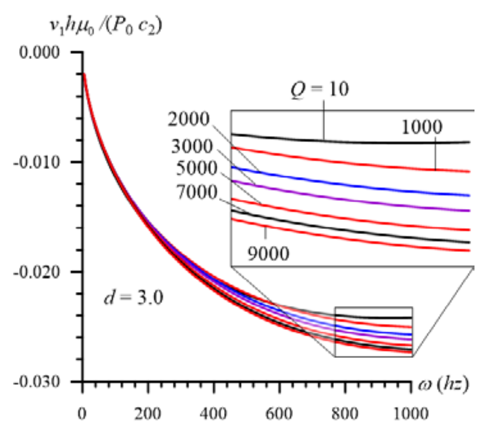

Fig. 7 The influence of the parameter Q (58) on the frequency response of the velocity v_1

given in Figs. 2-4. Moreover, it follows from these results that the absolute values of the studied quantities increase with the parameter d.

The agreement of the results presented in Figs. 2-4 with the mechanical considerations is also a validation of the trustiness of the algorithm and PC programs used in the present investigation.

Consider the results related to the influence of the parameter Q (58) on the frequency response of the studied quantities. These results are shown in Figs. 5-7 for $T_{22}h/P_0$, $v_2 h\mu_0/(P_0 c_2)$, and $v_1 h\mu_0/(P_0 c_2)$, respectively. To obtain these results, it is assumed that $d=3$ and, as above, the values of T_{22} and v_2 are calculated at $x_1/h=0$, but the values of the velocity v_1 at $x_1/h=25$.

It follows from these figures that the absolute values of the studied quantities increase with the parameter Q, i.e., with the dimensionless creep time (58) of the plate material. Moreover, these figures show that the results illustrated in that approach to a certain limit one with Q. Note that the limit results correspond to the purely elastic plate case, the mechanical constants of which are determined by (53), i.e., to the purely elastic plate with the long-term values of the elastic constants determined by (53). This conclusion is also agree with (60) and (55).

Now we analyze the limit case when $\omega \to 0$. According to (60) and (55), the above mentioned limit must not depend on the values of the parameter Q, but must depend on the values of the parameter d. Therefore, the results illustrated in Figs. 2-4 have various limit values with the decrease in the frequency ω (because the graphs given in these figures are constructed for various values of the parameter d), but the results illustrated in Figs. 5-7 have the same limit with the decreasing frequency ω (because the graphs given in these figures are constructed for the same value of the parameter d=3). According to (60), (61), and (55)-(57), the mentioned limit values with $\omega \to 0$ correspond to the purely elastic plate case with long-term values of the mechanical constants of the plate material. This statement is validated with the results given in the foregoing figures. Moreover, from (60), (61), and (55)-(57), the limit values of the studied quantities could be predicted when $\omega \to \infty$: these limit values must correspond to the purely elastic plate case with instantaneous values of the elastic constants of the plate material. Consequently, the all graphs constructed for various values of the dimensionless rheological parameters d and Q must approach to each other as $\omega \to \infty$. However, within the considered frequency interval, i.e., when $5 \le \omega \le 1000$, the differences between the results obtained for various values of the rheological parameter Q (or d) increase with the frequency. Moreover, the foregoing results show that in the considered range of variation of the frequency, the absolute values of the studied quantities increase monotonically with frequency. At the same time, according to the foregoing results, it can be concluded that the viscosity of the plate material causes to decrease of the absolute values of stress (or pressure) acting on the interface plane and fluid velocities on this plane.

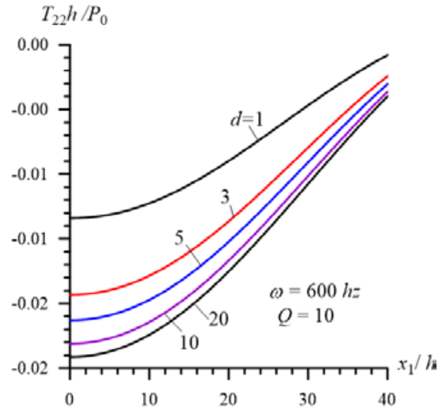

Fig. 8 The influence of the parameter d on the distribution of T_{22} with respect to x_1/h

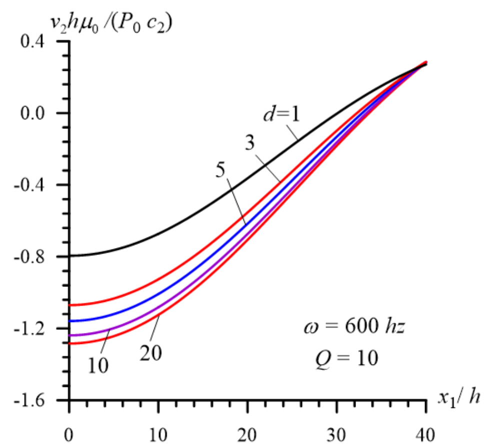

Fig. 9 The influence of the parameter d on the distribution of v_2 with respect to x_1/h

Now we consider the distribution of the studied quantities with respect to x_1/h. The graphs of these distributions are given in Figs. 8-10 for $T_{22}h/P_0$, $v_2 h\mu_0/(P_0 c_2)$, and $v_1 h\mu_0/(P_0 c_2)$, respectively. Note that these curves are constructed for various values of the parameter d at $Q=10$ and $\omega = 600$ Hz.

It follows from the graphs that the stress T_{22} and the velocity v_2 have its absolute maximum values at $x_1/h=0$, but the velocity v_1 at $x_1/h \approx 25$. Therefore, under construction of the foregoing graphs related to the frequency response, the values of T_{22} and v_2 have been calculated at $x_1/h=0$, but the values of v_1 at $x_1/h=25$. Moreover, the graphs presented in Figs. 8-10 show that the values of the studied quantities attenuate with distance from the vibration source. This conclusion for the velocity v_1 occurs in the cases where $x_1/h \ge 25$. The influence of the parameter d on the distribution under consideration has monotone character, i.e., the absolute values of the stress T_{22} and velocities v_2 and v_1 increase monotonically with d for each value of x_1/h.

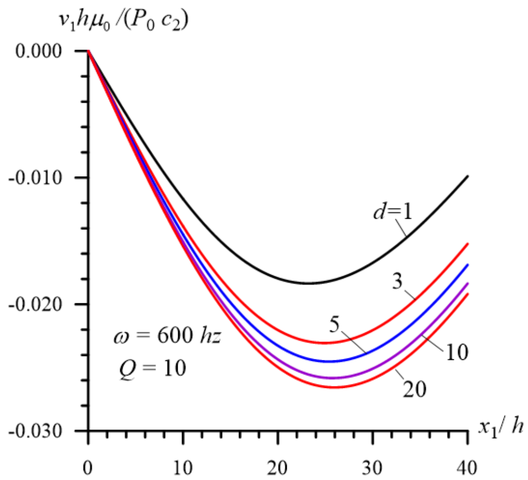

Fig. 10 The influence of the parameter d on the distribution of v_1 with respect to x_1/h

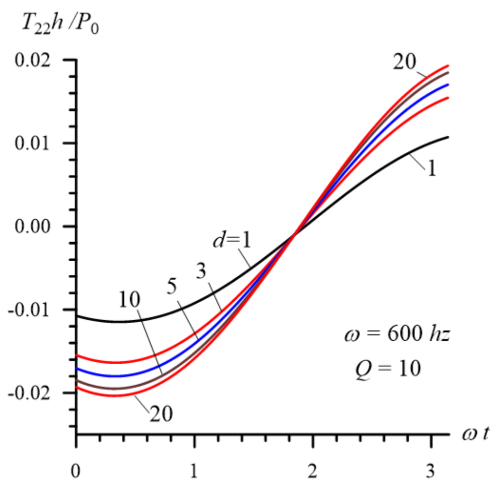

Fig. 11 The graphs of the dependence between T_{22} and ωt constructed for various values of d

We recall that the foregoing results are obtained from the expressions in (62) in the case where $\omega t = 2\pi n$ ($n = 0,1,2,...$). Now we consider numerical results related to the dependencies $T_{22}h/P_0$, $v_2 h\mu_0/(P_0 c_2)$, $v_1 h\mu_0/(P_0 c_2)$ and ωt in the case where $0 \le \omega t \le \pi$. Graphs of these dependencies are given in Figs. 11 (for T_{22}), 12 (for v_2), and 13 (for v_1). These graphs are constructed for various values of the parameter d under $Q = 10$ and $\omega = 600$ Hz, and the values of T_{22} and v_2 are calculated at $x_1/h = 0$, but the values of v_1 at $x_1/h = 25$. It follows from these graphs that the absolute maximum values of the studied quantities arise in the cases where $\omega t \neq 0 + n\pi$ ($n = 0,1,2,...$). In other words, the absolute maximum values of studied quantities arise at $\omega t = (\omega t)_* + n\pi$ and the values of $(\omega t)_*$ can be easily determined from Figs. 11-13 for T_{22}, v_2 and v_1, respectively. However, the absolute maximum values of the external loading arise, namely, at $\omega t = 0 + n\pi$. This means a phase shifting of the studied quantities with respect to the external loading. It follows from the results that this phase shifting is more considerable for the velocities v_2 and v_1 than for the stress T_{22}. Moreover, the results show that the influence of the parameter d on the values of $(\omega t)_*$ is insignificant.

This completes the consideration of the numerical results. The further application of the approach developed here to study the viscoelastic plate and compressible viscous fluid interaction problems will be carried out by the authors.

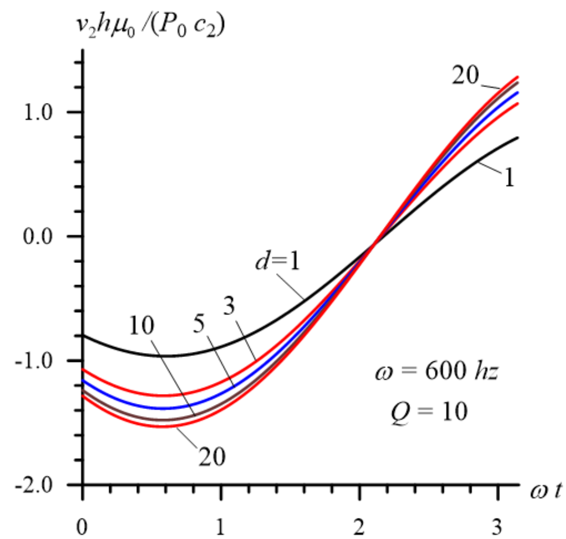

Fig. 12 The graphs of the dependence between v_2 and ωt constructed for various values of d

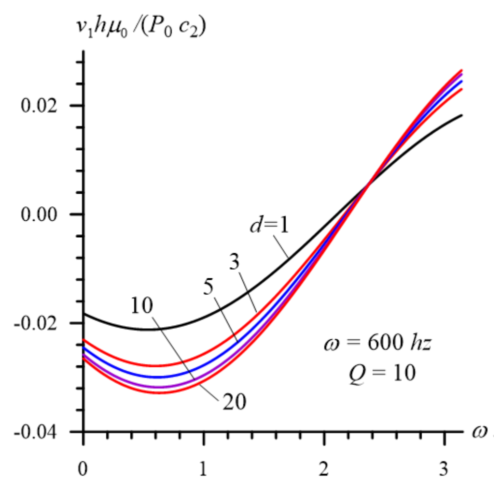

Fig. 13 The graphs of the dependence between v_1 and ωt constructed for various values of d

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