Analysis of energy state of a discrete fractionally damped spherical net of mouse *zona pellucida* before and after fertilization

Andjelka N. Hedrih and Katica (Stevanović) Hedrih

Dedicated to the 100th Anniversary of the Russian Academician Yury Rabotnov

***Abstract*—**Zona pellucida (ZP) is a 3D matrix that surrounds mammalian oocytes and embryo until the stage of early blastocyst. This structure is important for fertilization, polyspermy block, integrity of the growing embryo, guiding the embryo through the oviduct. In the late blastocyst stage, this structure no longer exists. During oocyte maturation, fertilization and embryo development ZP dynamically changes its elasticity. To explain elasticity change of mouse ZP (mZP) during these processes, a discrete fractional order spherical net model of mZP is developed. Elements in the model correspond to ZP glycoproteins that are interconnected with standard light fractional order elements. This model is suitable for modelling different states of the ZP net. Using the part of the ZP spherical net model that still preserves the molar ratio of ZP glycoproteins, we determined characteristic eigen numbers of eigen fractional order oscillations and eigen modes for each chain. Analytical expressions for mechanical energy of the representative part of the ZP net before and after fertilization are defined. Generalised function of energy dissipation for the representative part of the ZP net as well as for the whole ZP net was created. We discussed dynamical change of elasticity of the mZP and dissipation of its energy after fertilisation in biological context.

***Keywords*—**Fractional order model of the Zona Pellucida, generalized function of fractional order dissipation, dissipation of energy, energy state of the Zona Pelucida**.**

I. Introduction

Zona pellucida (ZP) is a 3D acellular matrix that surrounds mammalian oocytes and embryo until the stage of early blastocyst.

It plays an important role in oogenesis, sperm-egg interactions development of embryo in vivo, and during a preimplantation period [1], [2]. Naked eggs transplanted to the oviduct may adhere to its walls, ZP makes it possible for the cleaving eggs to move freely along the oviduct [3].

"The cell association of zona-free embryos at the 4-cell stage influences differentiation of cells" [4]. The mouse ZP consists of three sulfated glycoproteins: *ZP*1, *ZP*2, and *ZP*3. The ratio of *ZP*2/*ZP*3 in mice is close to 1:1, whereas *ZP*1/(*ZP*2-*ZP*3) is 1:5 [1]. These proteins are secreted by the developing oocyte and assembled in the 3D matrix at the outher surface of the cell. Each of these mouse ZP (mZP) glycoproteins has its special function and is essential for normal fertility and function of mZP [2]. ZP dynamically changes its thickness and volume during oocye maturation, fertilization and early embryo development [5]. It is the thickest in fully grown oocyte [1]. Except changing in thickness during these phases, ZP changes its structure [6], [7] and elasticity [8]-[10].

Scanning electron microscopy analysis shows that this porous structure changes a number and a mean diameter of pores before and after fertilization. The maximal mean diameter of the pores is in zygote and minimal in morula. The larger number of the pores is the smallest of the mean diameter of the pores [7]. After the fertilization, the Young modulus of ZP has approximately 2.5 times higher values compare to mature oocyte, no matter with what technique it was obtained [8]-[10]. After the fertilization, until the morula stage of the embryo, the Young modulus of mZP decreases [8]. This so called 'hardening' of mZP of the embryo plays an important role in polispermy block: when one sperm penetrates and fertilize the oocyte, it reacts by the cortical reaction preventing other sperm to penetrate the oocyte. This 'hardening' phenomenon is used in the farm industry for hardenig ZP of unfertilized eggs to reduce the polyspermic fertilization in pigs and cows [11].

Considering the fertilization process as an oscillatory phenomenon, and the mZP as an oscillatory structure, we supposed that the oocyte and embryo are in different oscillatory states [12] and that the whole cell and ZP structure have different energies.

The aim of this study is to analyze the energy state of the mouse *zona pellucida* before and after fertilization. To obtain energy analysis, a discrete fractional order spherical net model of the mouse ZP has been created. It is a modified oscillatory model of mZP [13]. Modification of the model is in way of coupling the material particles (ZP glycoproteins). In the

Parts of this research were supported by the Ministry of Education, Sciences and Technology of the Republic of Serbia through the Mathematical Institute SANU, Belgrade Grant ON174001 "Dynamics of hybrid systems with complex structures", Mechanics of Materials and Faculty of Mechanical Engineering, University of Niš, as well as through the State University of Novi Pazar.

Andjelka N. Hedrih is with the Department for Biomedical Science, State University of Novi Pazar, Vuka karadzica bb, 36 300 Novi Pazar,(e-mail: handjelka@hm.co.rs)

Katica R. (Stevanovic) Hedrih is with the Matematical Institute of SASA, Kneza Mihaila 36, 11001 Beograd, Serbia (corresponding author phone: +381638757599; fax: +381184241663; khedrih@eunet.rs)

fractional order spherical net model of mZP, ZP glycoproteins are interconnected with standard light fractional order elements (SLFOE) forming circular and meridian chains. Each glycoprotein is also connected to the surface of the oocyte with SLFOE (Fig. 1a and b). The net is identical in the circular and meridian directions. Chains are orthogonal. In the model, each ZP molecule can move in 2 directions: in radial and in the direction of the chain - circular or meridian. Exceptions are knot molecules that can move in all three directions.

Using the representative part of the spherical mZP net (Fig. 2), we calculate its mechanical energy as well as for the whole mZP net. Representative part of the mZP net is repeated in the spherical net, and it has been considered the smallest part of the net that still preserves the molar ratio of mZP glycoproteins [1]. This part consists of 4 crossed chains, each of 11 material particles (Fig. 2). ZP 1 (Orange) represents knot molecules in the model.

Using the data of oscillations of fractional order chains [14]-[24] and ideal elastic non-homogeneous chains [25,26], we determined eigen characteristic numbers of eigen fractional order oscillations and corresponding main eigen modes for each chain in the representative part.

The fractional order spherical net model of mZP is a non-conservative oscillatory model. Analytical expressions for potential and kinetic energy were determined. The generalized function of the fractional order dissipation of mechanical energy was defined. This function describes the dissipation of the total mechanical energy of the mZP in different oscillatory states (before and after fertilization).

In order to define the total mechanical energy of the mZP, we defined first the energy of dilatation of a single standard light fractional order element, the energy of a single non-homogeneous chain of 11 material particles and the energy of all four chains in the representative part of the mZP.

In order to model viscoelastic properties in the ZP structure after fertilization with due account for elastic to plastic transition [9], we found the fractional calculus to be a useful tool. Our conclusion is supported by numerous publications utilizing the fractional order calculus in different fields of science. The state-of-the-art articles by Rossikhin and Shitikova [30]-[33] are bright examples of fractional calculus applications in mechanics of solids and structures.

II. Potential Energy of the Representative Part of the mZP Net

The elongation of a standard light fractional order element that interconnects two molecules (material particles) with displacements $x_k(t)$ and $x_{k+1}(t)$ could be described as $\Delta x_{k+1,k}(t) = x_{k+1}(t) - x_k(t)$, $k = 0,1,2,3,...,11,12$, where k and $k+1$ are the orders of material particles in the chain, and $x_0(t) = 0$ and $x_{12}(t) = 0$ are boundary conditions of the chains. The constitutive generalized force – extension relation of the standard light fractional order element is described by the fractional derivative Kelvin-Voigt model:

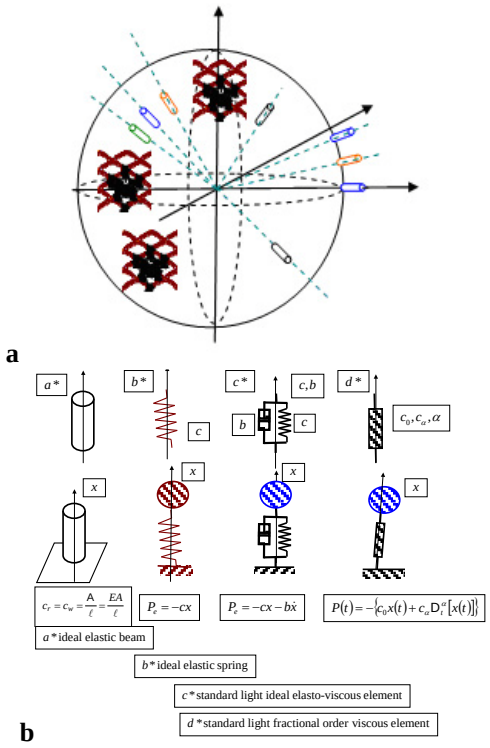


Fig. 1 (a) Part of the ZP spherical net model as a part of the sphere (oocite). Orange (ZP1), blue (ZP2) and green (ZP3) represent ZP proteins. Net is identical in circular and meridian directions. An axis shows the direction of movements of ZP proteins. Each ZP protein is connected to the sphere with standard light visco-elastic elements of fractional order (SLFOE) that can oscillate in the radial direction, and (b) Schematic presentation of the fractional order viscoelastic element as a part of the Zona Pelucida model

$$Q_{\alpha,k}(t) = -\left\{c_{0(k,k+1)}\left[x_{k+1}(t) - x_k(t)\right]\right\} - \left\{c_{\alpha(k,k+1)} D_t^{\alpha}\left[x_{k+1}(t) - x_k(t)\right]\right\} = -Q_{\alpha,k+1}(t),$$
$$x_0(t) = 0 \text{ and } x_{12}(t) = 0,\ k = 0,1,2,...,11,12. \quad (1)$$

where $D_t^{\alpha}[\bullet]$ is the fractional α-order differential operator of with respect to time t of the following form:

$$D_t^{\alpha}[\bullet] = \frac{d^{\alpha}[\bullet]}{dt^{\alpha}} = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{[\bullet]}{(t-\tau)^{\alpha}} d\tau, \quad (2)$$

$\Gamma(1-\alpha)$ is the Euler Gamma function, $0 < \alpha \le 1$, $c_{0(k,k+1)}$ and $c_{\alpha(k,k+1)}$ are rigidity coefficients, in so doing $c_{0(k,k+1)}$ are prolonged moduli of elasticity, $c_{\alpha(k,k+1)} = \tau^{\alpha} c_{0(k,k+1)}$, and τ is the retardation time.

At $\alpha = 1$, the Voigt model (1) with a fractional time-derivative (2), i.e. the generalized Voigt model, goes over into the Voigt model with a conventional time-derivative, i.e. the classical Voigt model, since the Riemann-Liouville fractional derivative (2) goes over into the conventional first-order derivative with respect to time t. At $\alpha = 0$, the generalized Voigt model (1) loses the physical sense (see the paper of the

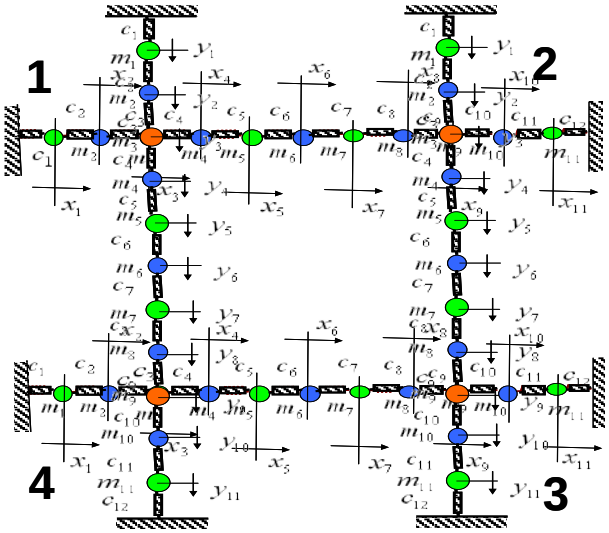


Figure 2. A representative part of the sphere surface net model of mZP: Orange (ZP1)-knot molecules in cross section of the chains, blue (ZP2) - (2,4,6,8, and 10 molecule in chain) and green (ZP3) - (1,5,7, and 11 molecule in chain) represents ZP proteins. ZP proteins are interconnected by standard light visco-elastic elements of fractional order, m_k is a mass of the ZP glycoprotein, c_k is the rigidity, x_k are the displacements of mass particles interconnected by SLFOE in the circular direction, y_k are the displacements of mass particles interconnected of SLFOE in the meridian direction. Numbers in the corners (1-4) denotes position of the knot molecules in the representative part of the mZP net, at cross section between two chains in orthogonal directions.

Guest Editors of this Special Issue [34], as well as their state-of-the-art article [30] for details).

The fractional order element (Fig. 1b) possesses the potential energy $\tilde{\mathbf{E}}_{p,k,k+1,0<\alpha\le 1}$ expressed by

$$\tilde{\mathbf{E}}_{p,k,k+1}=\frac{1}{2}c_{0(k,k+1)}\left[x_{k+1}(t)-x_k(t)\right]^2 \qquad (3)$$

$x_0(t)=0$ and $x_{12}(t)=0$, $k=0,1,2,...,11,12$.

The total potential energy $\mathbf{E}_{p,\text{chain}}$ of one chain from the representative part of the mZP net for $0<\alpha\le 1$ has the form

$$\mathbf{E}_{p,\text{chain}}=\sum_{k=0}^{k=12}\tilde{\mathbf{E}}_{p,k,k+1}=\sum_{k=0}^{k=12}\frac{1}{2}c_{0(k,k+1)}\left[x_{k+1}(t)-x_k(t)\right]^2 \quad (4)$$

Potential energy of the representative part of the mZP net includes the potential energy of *all 4 chains* from the representative part of the mZP net, as well as the potential energy of SLFOE elements that connect ZP molecules and surface of the oocyte. Each such element is oriented in the radial direction.

Using independent generalized coordinates of mass particles in the representative part of the mZP net in the circular direction $u_{k,j}(t)$, in the meridian direction $v_{k,j}(t)$ and in the radial direction $w_{k,j}(t)$, and taking restrictions of the degrees of freedom into account, the total potential energy of the representative part of the mZP spherical net (Fig. 2) $\mathbf{E}_{p,\text{Repr. part},0<\alpha\le 1}$ could be written as expressed as

$$\begin{aligned}\mathbf{E}_{\text{p,Repr. part}}=&\sum_{\substack{j=1\\ j\neq 2}}^{j=3}\sum_{k=0}^{k=12}\frac{1}{2}c_{0(k,k+1),j}\left[u_{k+1,j}(t)-u_{k,j}(t)\right]^2\\ &+\sum_{\substack{j=2\\ j\neq 3}}^{j=4}\sum_{k=0}^{k=12}\frac{1}{2}c_{0(k,k+1),j}\left[v_{k+1,j}(t)-v_{k,j}(t)\right]^2\\ &+\sum_{k=1}^{k=11}\sum_{\substack{j=1\\ j\neq 3\\ j\neq 9}}^{j=11}\tilde{c}_{0(k,k),j}\left[w_{kj}(t)\right]^2\end{aligned} \qquad (5)$$

subjected to the following boundary conditions in chains: $u_{0,j}(t)=0$, $u_{12,j}(t)=0$, $v_{0,j}(t)=0$ and $v_{12,j}(t)=0$, where k denotes the order of a mZP molecule in the corresponding chain, while j labels the order of chain in the representative part of the mZP net (1 and 3 are the chains in the circular direction, and 2 and 4 are the chains in the meridian direction).

Relationship (5) represents the density of potential energy calculated on the surface element dS of the middle spherical surface of Zona Pelucide corresponding to the representative part of the mZP.

The generalized Rayleigh function of the energy dissipation of a viscoelastic fractional order chain Φ_{chain} for one chain from the representative part of the mZP net has the form:

$$\Phi_{\text{chain}}=\sum_{k=0}^{k=12}\frac{1}{2}c_{0<\alpha<1(k,k+1)}\left\langle D_t^{\alpha}\left[x_{k+1}(t)-x_k(t)\right]\right\rangle^2. \qquad (6)$$

Applying the generalized Rayleigh function of viscoelastic fractional order element energy dissipation Φ_{chain} to the whole representative part of the mZP net, it is possible to obtain the following expression in terms of independent generalised coordinates:

$$\begin{aligned}\Phi_{\text{Repr. part}}=&\sum_{\substack{j=1\\ j\neq 2}}^{j=3}\sum_{k=0}^{k=12}\frac{1}{2}c_{0<\alpha<1(k,k+1),j}\left\langle D_t^{\alpha}\left[u_{k+1,j}(t)-u_{k,j}(t)\right]\right\rangle^2\\ &+\sum_{\substack{j=2\\ j\neq 3}}^{j=4}\sum_{k=0}^{k=12}\frac{1}{2}c_{0<\alpha<1(k,k+1),j}\left\langle D_t^{\alpha}\left[v_{k+1,j}(t)-v_{k,j}(t)\right]\right\rangle^2\\ &+\sum_{k=1}^{k=11}\sum_{\substack{j=1\\ j\neq 3\\ j\neq 9}}^{j=11}\tilde{c}_{0<\alpha<1(k,k),j}\left\langle D_t^{\alpha}\left[w_{k,k}(t)\right]\right\rangle^2\end{aligned} \qquad (7)$$

subjected to the following boundary conditions of chains: $D_t^{\alpha}\left[u_{0,j}\right](t)=0$, $D_t^{\alpha}\left[u_{12,j}(t)\right]=0$, $D_t^{\alpha}\left[v_{0,j}(t)\right]=0$, $D_t^{\alpha}\left[v_{12,j}(t)\right]=0$.

The total power of work of the fractional order dissipative forces of the whole mZP net could be written as

$$\Phi=\iint_S\Phi_{\text{Repr. part}}dS. \qquad (8)$$

III. Kinetic Energy of the Representative Part of the mZP Net

According to the model under consideration, each ZP molecule can move in two directions: in the radial and in the direction of the chain, circular or meridian. Only knot molecules could move in all three directions (Fig. 3) with the

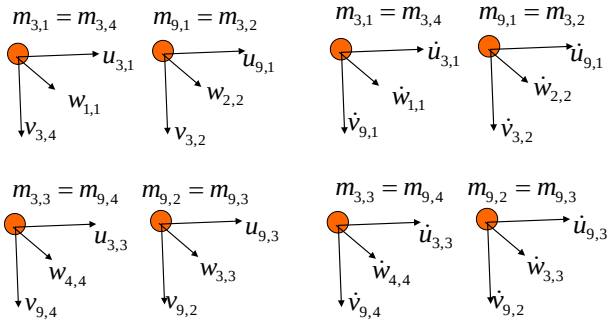


Figure 3. Component displacements and velocities in the circular, meridian and radial directions for knot molecules

velocities $\dot{u}_{k,j}(t)$ $\dot{v}_{k,j}(t)$ and $\dot{w}_{k,j}(t)$, respectively, in the circular, meridian and radial directions.

Expression for the kinetic energy $\mathbf{E}_{\text{Kin,chain},j}$, j=1,3 of non knot molecules in the circular direction is:

$$\mathbf{E}_{\text{Kin, Circ.chain},j}=\sum_{\substack{k=1\\k\neq3\\k\neq9}}^{k=11}\frac{1}{2}m_{k,j}\left\langle\left[\dot{u}_{k,j}(t)\right]^2+\left[\dot{w}_{k,j}(t)\right]^2\right\rangle,\ j=1,3. \tag{9}$$

Expression for kinetic energy $\mathbf{E}_{\text{Kin mer,chain},j}$, j=2,4 for non knot molecules in the meridian direction is:

$$\mathbf{E}_{\text{Kin mer,chain},j}=\sum_{\substack{k=1\\k\neq3\\k\neq9}}^{k=11}\frac{1}{2}m_{k,j}\left\langle\left[\dot{v}_{k,j}(t)\right]^2+\left[\dot{w}_{k,j}(t)\right]^2\right\rangle,\ j=2,4 \tag{10}$$

Expression for kinetic energy $\mathbf{E}_{\text{Kin,chain},j}$, $j=1,2,3,4$ of the first knot molecule in all three directions has in the form:

$$\mathbf{E}_{\text{Kin,knot},1}=\frac{1}{2}m_{3,1}\left[\dot{u}_{3,1}(t)\right]^2+\frac{1}{2}m_{3,4}\left[\dot{v}_{3,4}(t)\right]^2+\frac{1}{2}m_{1,1}\left[\dot{w}_{1,1}(t)\right]^2. \tag{11}$$
$$m_{3,1}=m_{3,4}=m_{1,1}$$

Similar expressions for kinetic energy could be written for other knot molecules in the representative part of the mZP net. Summing the kinetic energy of all material particles in all 4 chains in the representative part of the mZP net, the total kinetic energy has the following form:

$$\mathbf{E}_{\text{Kin,Repr.}}=\sum_{\substack{j=1\\j\neq2}}^{j=3}\sum_{k=1}^{k=11}\frac{1}{2}m_{k,j}\left[\dot{u}_{k,j}(t)\right]^2+\sum_{\substack{j=2\\j\neq3}}^{j=4}\sum_{k=1}^{k=11}\frac{1}{2}m_{k,j}\left[\dot{v}_{k,j}(t)\right]^2+\sum_{k=1}^{k=11}\sum_{\substack{j=1\\j\neq3\\j\neq9}}^{j=11}m_{k,j}\left[\dot{w}_{kj}(t)\right]^2 \tag{12}$$

for $m_{3,1}=m_{3,4}=\tilde{m}_{1,1}$, $m_{9,1}=m_{3,2}=\tilde{m}_{2,2}$, $m_{9,2}=m_{9,3}=\tilde{m}_{3,3}$, $m_{3,3}=m_{9,4}=\tilde{m}_{4,4}$

Masses of the knot molecules have following denotations: $m_{3,1}=m_{3,4}=\tilde{m}_{1,1}$, $m_{9,1}=m_{3,2}=\tilde{m}_{2,2}$, $m_{9,2}=m_{9,3}=\tilde{m}_{3,3}$, $m_{3,3}=m_{9,4}=\tilde{m}_{4,4}$, for the reason that each knot mass particle belongs to two chains, circular and meridian, at the same time.

Use the extended system of Lagrange equations of the second kind (see Refs. [16] and [20]) for the fractional order system and generalized coordinates in all three directions for the boundary conditions $u_{0,j}(t)=0$ and $u_{12,j}(t)=0$ in the circular and $v_{0,j}(t)=0$ and $v_{12,j}(t)=0$ in the meridian direction:

$$\frac{d}{dt}\left[\frac{\partial\mathbf{E}_{\text{Kin,Repr.}}}{\partial\left\langle\dot{u}_{k,j}(t)\right\rangle}\right]-\frac{\partial\mathbf{E}_{\text{Kin,Repr.}}}{\partial\left\langle u_{k,j}(t)\right\rangle}+\frac{\partial\mathbf{E}_{\text{p,Repr. part},\alpha=0}}{\partial\left\langle u_{k,j}(t)\right\rangle}+\frac{\partial\Phi_{\text{Repr.part}}}{\partial\left\langle D_t^{\alpha}\left[u_{k,j}(t)\right]\right\rangle}=0$$
$$k=1,2,3,...,11,\ j=1,3,\ 0<\alpha\leq1 \tag{13}$$

$$\frac{d}{dt}\left[\frac{\partial\mathbf{E}_{\text{Kin,Repr.}}}{\partial\left\langle\dot{v}_{k,j}(t)\right\rangle}\right]-\frac{\partial\mathbf{E}_{\text{Kin,Repr.}}}{\partial\left\langle v_{k,j}(t)\right\rangle}+\frac{\partial\mathbf{E}_{\text{p,Repr. part}}}{\partial\left\langle v_{k,j}(t)\right\rangle}+\frac{\partial\Phi_{\text{Repr. part}}}{\partial\left\langle D_t^{\alpha}\left[v_{k,j}(t)\right]\right\rangle}=0$$
$$k=1,2,3,...,11,\ j=2,4, \tag{14}$$

$$\frac{d}{dt}\left[\frac{\partial\mathbf{E}_{\text{Kin,Repr.}}}{\partial\left\langle\dot{w}_{k,j}(t)\right\rangle}\right]-\frac{\partial\mathbf{E}_{\text{Kin,Repr.}}}{\partial\left\langle w_{k,j}(t)\right\rangle}+\frac{\partial\mathbf{E}_{\text{p,Repr. part}}}{\partial\left\langle w_{k,j}(t)\right\rangle}+\frac{\partial\Phi_{\text{Repr. part}}}{\partial\left\langle D_t^{\alpha}\left[w_{k,j}(t)\right]\right\rangle}=0$$
$$k,j=1,2,3,...,11,\quad k\neq3,\quad k\neq9 \tag{15}$$

Using (13)-(15), as well as expressions for kinetic and potential energy and general function of fractional order energy dissipation of the representative part of the mZP spherical net, we can obtain a system of ordinary differential equations of the fractional order $0<\alpha\leq1$ in the following form:

$$m_{k,j}\ddot{u}_{k,j}(t)+c_{0(k-1,k),j}\left[u_{k,j}(t)-u_{k-1,j}(t)\right]-c_{0(k,k+1),j}\left[u_{k+1,j}(t)-u_{k,j}(t)\right]$$
$$+c_{0<\alpha\leq1(k-1,k),j}D_t^{\alpha}\left[u_{k,j}(t)-u_{k-1,j}(t)\right]-c_{0<\alpha\leq(k,k+1),j}D_t^{\alpha}\left[u_{k+1,j}(t)-u_{k,j}(t)\right]=0$$
$$k=1,2,3,...,11,\ j=1,3 \tag{16}$$

$$m_{k,j}\ddot{v}_{k,j}(t)+c_{0(k-1,k),j}\left[v_{k,j}(t)-v_{k-1,j}(t)\right]-c_{0(k,k+1),j}\left[v_{k+1,j}(t)-v_{k,j}(t)\right]$$
$$+c_{0<\alpha\leq1(k-1,k),j}D_t^{\alpha}\left[v_{k,j}(t)-v_{k-1,j}(t)\right]-c_{0<\alpha\leq(k,k+1),j}D_t^{\alpha}\left[v_{k+1,j}(t)-v_{k,j}(t)\right]=0$$
$$k=1,2,3,...,11,\ j=2,4 \tag{17}$$

$$\tilde{m}_{k,j}\ddot{w}_{k,j}(t)+\tilde{c}_{0(k,j),j}w_{k,j}(t)+c_{0<\alpha\leq1(k,j)}D_t^{\alpha}\left[w_{k,j}(t)\right]=0$$
$$k,j=1,2,3,...,11,\quad k\neq3,\quad k\neq9 \tag{18}$$

The set of ordinary fractional order differential equations (16)-(18) consists of three independent subsystems of the fractional order. Both subsystems (16) and (17) are two independent subsystems of the fractional order each. Each of the last mentioned subsystems corresponds to one chain in the representative part of the mZP net

$$m_k\ddot{x}_k(t)+c_{0(k-1,k)}\left[x_k-x_{k-1}\right]-c_{0(k,k+1)}\left[x_{k+1}-x_k\right]$$
$$+c_{0<\alpha\leq1(k-1,k)}D_t^{\alpha}\left[x_k-x_{k-1}\right]-c_{0<\alpha\leq(k,k+1)}D_t^{\alpha}\left[x_{k+1}-x_k\right]=0$$
$$k=1,2,3,...,11 \tag{19}$$

$$m_k\ddot{y}_k(t)+c_{0(k-1,k)}\left[y_k-y_{k-1}\right]-c_{0(k,k+1)}\left[y_{k+1}-y_k\right]$$
$$+c_{0<\alpha\leq1(k-1,k)}D_t^{\alpha}\left[y_k-y_{k-1}\right]-c_{0<\alpha\leq(k,k+1)}D_t^{\alpha}\left[y_{k+1}-y_k\right]=0$$
$$k=1,2,3,...,11 \tag{20}$$

Subsystem (18) could be rewritten in the following form:

$$\ddot{w}_{k,j}(t)+\tilde{\omega}_{k,j}^2w_{k,j}(t)+\tilde{d}_{0<\alpha\leq1(k,j)}^2D_t^{\alpha}\left[w_{k,j}(t)\right]=0 \tag{21}$$
$$k,j=1,2,3,...,11,\quad k\neq3,\quad k\neq9,$$

where

$$\tilde{\omega}_{k,j}^2 = \frac{\tilde{c}_{0(k,j)}}{\tilde{m}_{k,j}}, \quad \tilde{\omega}_{0<\alpha\le 1(k,j)}^2 = \frac{\tau^\alpha \tilde{c}_{0(k,j)}}{\tilde{m}_{k,j}}. \tag{22}$$

Particular solutions of ordinary fractional order differential equations (21), which are equations of only one coordinate each and which describe displacements of a fractional order oscillator, are known [27]-[31]. According to [27]-[29], particular solutions, $T_{s,\cos}(t,\alpha)$ and $T_{s,\sin}(t,\alpha)$, could be written in the following form:

$$T_{s,\cos}(t,\alpha) = \sum_{k=0}^{\infty}(-1)^k \omega_{(\alpha)s}^{2k} t^{2k} \sum_{m=0}^{k}\binom{k}{m}\frac{\omega_{(\alpha)s}^{-2m} t^{-\alpha m}}{\omega_s^{2m}\Gamma(2k+1-\alpha m)}, \tag{23}$$

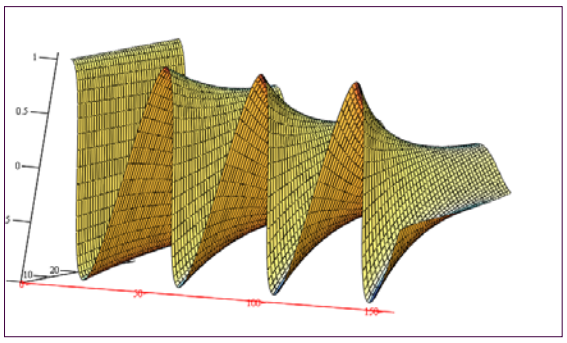
$$T_{s,\sin}(t,\alpha) = \sum_{k=0}^{\infty}(-1)^k \omega_{(\alpha)s}^{2k} t^{2k+1} \sum_{m=0}^{k}\binom{k}{m}\frac{\omega_{(\alpha)s}^{-2m} t^{-\alpha m}}{\omega_s^{2m}\Gamma(2k+2-\alpha m)}. \tag{24}$$

The corresponding particular cases at $\alpha = 1$ are reduced to

$$T_{s,\cos}(t,\alpha)\big|_{\alpha=1} = e^{-\frac{1}{2}\omega_{(\alpha=1)s}^2 t}\cos\left(t\sqrt{\omega_s^2 + \frac{1}{4}\omega_{(\alpha=1)s}^4}\right),$$

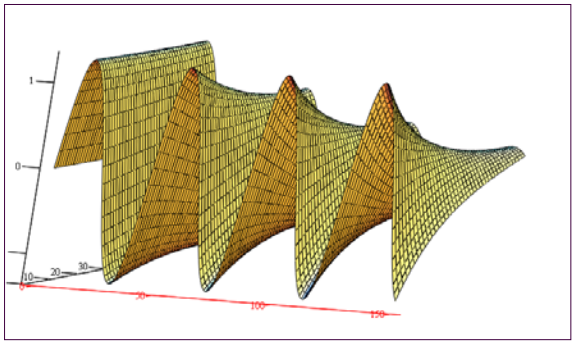
$$T_{s,\sin}(t,\alpha)\big|_{\alpha=1} = e^{-\frac{1}{2}\omega_{(\alpha=1)s}^2 t}\sin\left(t\sqrt{\omega_s^2 + \frac{1}{4}\omega_{(\alpha=1)s}^4}\right).$$

Figures 3 and 4 present two surface modes with the difference in phase.



M

Fig. 3 Main eigen modes of the fractional order: $T_{s,\cos}(t,\alpha)$ is plotted on the ordinate, time t and α $(0<\alpha\le 1)$ are on two abscissa axes.



M

Fig. 4 Main eigen modes of fractional order. $T_{s,\sin}(t,\alpha)$ is plotted on the ordinate, time t and $\alpha\,(0<\alpha\le 1)$ are on two abscissa axes.

Subsystems (19) and (20) are similar ordinary fractional order differential equations. We are going to analyze one subsystem written in a matrix form [20]:

$$\mathbf{A}\{\ddot{x}\} + \mathbf{C}\{x\} + \mathbf{C}_\alpha\left\{D_t^\alpha\{x\}\right\} = \{0\}, \tag{25}$$

where $\mathbf{A} = \left(a_{kj}\right)_{\rightarrow\ j=1,2,3,...,n}^{\downarrow\ k=1,2,3,...,n}$ is the matrix of coefficients of system's mass inertia properties, $\mathbf{C} = \left(c_{kj}\right)_{\rightarrow\ j=1,2,3,...,n}^{\downarrow\ k=1,2,3,...,n}$ is the matrix of coefficients of system's rigidity properties, and $\mathbf{C}_\alpha = \tau^\alpha \mathbf{C}$ is the matrix of coefficients of system's viscoelastic fractional order properties.

Introduce the modal matrix corresponding to the linear set of oscillators (25)

$$\mathbf{R} = \left(\left\{K_{nk}^s\right\}\right) = \left(K_{nk}^s\right)_{\rightarrow\ s=1,2,3,...,n}^{\downarrow\ k=1,2,3,...,n} \tag{26}$$

and apply modal matrix $\mathbf{R}$ to $\mathbf{A}$, $\mathbf{C}$ and $\mathbf{C}_\alpha$, resulting in three diagonal matrices

$$\mathrm{A} = \mathbf{R}'\mathbf{A}\mathbf{R} = \mathrm{diag}(a_{ss}), \quad \mathrm{C} = \mathbf{R}'\mathbf{C}\mathbf{R} = \mathrm{diag}(c_{ss}),$$
$$\mathrm{C}_{\alpha,s} = \mathbf{R}'\mathbf{C}_\alpha\mathbf{R} = \tau^\alpha \mathrm{diag}(c_{ss}) \tag{27}$$

and the following coordinate transformation

$$\{x\} = \mathbf{R}\{\xi_s\} \tag{28}$$

from the generalized coordinates of the linear system to the corresponding eigen coordinates ξ_s $(s=1,2,3,...,11)$ and eigen modes.

Considering (27) and (28) in equations (29) and (30), we obtain subsystems of independent fractional order differential equations in the following form:

$$\ddot{\xi}_s + \omega_s^2\xi_s + \omega_{(\alpha)s}^2 D_t^\alpha[\xi_s] = 0 \quad (s=1,2,3,...,11) \tag{29}$$

The form of ordinary fractional order differential equations (29) is the same as in (21), and thus their particular solutions are similar to (23) and (24) but with the corresponding characteristic eigen values, $\tilde{\omega}_{k,j}^2$ and $\tilde{\omega}_{0<\alpha\le 1(k,j)}^2$.

IV. Conclusion

The energy state of the mouse *zona pellucida* before and after fertilization has been analyzed via the created discrete fractional order spherical net model based on the fractional derivative Voigt model of viscoelasticity. According to this model, after fertilization until the stage of morula the modulus of elasticity decreases [8], and these stages could be modeled as non-conservative systems with viscoelastic properties. In the stage when the mZP has the highest viscosity, its energy is minimal. We can speculate that this minimum of mZP energy is essential (from the mechanical point of view) for embryo to escape from this structure and implant into the uterus.

REFERENCES

[1] P. M. Wassarman, "Zona pellucida glycoproteins," *The Journal of Biological Chemistry*, vol. 283(36), pp. 24285-24289, 2008.

[2] P. M. Wassarman and E. S. Litscher, "Influence of the zona pellucida of the mouse egg on folliculogenesis and fertility," *Int. J. Dev. Biol.*, **vol.** 56, pp. 833-839, 2012.

[3] J. A. Modlinski, "The role of the zona pellucida in the development of mouse eggs *in vivo*," *J. Embryol. Exp. Morph.*, vol. 23(3), pp. 539-547, 1970.

[4] H. Suzuki, M. Togashi, J. Adachi and Y. Toyoda, "Developmental ability of zona-free mouse embryos is influenced by cell association at the 4-cell stage," *Biology of Reproduction*, vol. 53, pp. 78-83, 1995.

[5] S. O. Chung, "The thickness and volume change of the zona pellucida following ovulation and fertilization," *Yonsei Medical Journal*, vol. 12, pp. 28-33, 1971.

[6] G. Vanroose, H. Nauwynck, A. van Soom, M.-T. Ysebaert, G. Charlier, P. van Oostveldt and A. de Kruif, "Structural aspects of the zona pellucida of in vitro-produced bovine embryos: A scanning electron and confocal laser scanning microscopic study," *Biology of Reproduction*, vol. 62, pp. 463-469, 2000.

[7] G. Familiari, R. Heyn, M. Relucenti and H. Sathananthan, "Structural changes of the zona pellucida during fertilization and embryo development," *Front Biosci*., vol. 13, pp. 6730-6751, 2008.

[8] Y. Murayama, J. Mizuno, H. Kamakura, Y. Fueta, H. Nakamura, K. Akaishi, K. Anzai, A. Watanabe, H. Inui and S. Omata, "Mouse zona pellucida dynamically changes its elasticity during oocyte maturation, fertilization and early embryo development," *Human Cell*., vol. 19, pp. 119-125, 2006.

[9] M. Papi, L. Sylla, T. Parasassi, R. Brunelli, M. Monaci, G. Maulucci, M. Missori, G. Arcovito, F. Ursini and M. Spirito, "Evidence of elastic to plastic transition in the zona pellucida of oocytes using atomic force spectroscopy," *App Phy Lett*., vol. 94, pp. 153902, 2009.

[10] M. Khalilian, M. Navidbakhsh, M. R. Valojerdi, M. Chizari and P. E. Yazdi, "Estimating Young's modulus of zona pellucida by micropipette aspiration in combination with theoretical models of ovum," *R. Soc. Interfac*,, vol. 7(45), pp. 687-694, 2010.

[11] P. Coy, L. Grullon, S. Canovas, R. Romar, C. Matas and M. Aviles, "Hardening of the zona pellucida of unfertilized eggs can reduce polyspermic fertilization in the pig and cow," *Reproduction*, vol. 135, pp. 19-27, 2008.

[12] A. Hedrih, "Transition in oscillatory behavior in mouse oocyte and mouse embryo trough oscillatory spherical net model of mouse Zona Pellucida," in: *Applied Non-Linear Dynamical Systems/DSTA 2013, Proc. in Mathematics and Statics* (J. Awrejcewicz, Ed), Springer, 2014, in press.

[13] A. Hedrih, K.(Stevanovic) Hedrih and B. Bugarski, "Oscillatory spherical net model of mouse Zona Pellucida," *Journal of Applied Mathematics and Bioinformatics*, vol. 3(4), pp. 225-268, 2013.

[14] K. (Stevanović) Hedrih, "Modes of the homogeneous chain dynamics," *Signal Processing*, vol. 86, pp. 2678-2702, 2006.

[15] K. (Stevanović) Hedrih, "Fractional order hybrid system dynamics," *PAMM, Proc. Appl. Math. Mech*., vol. 13, pp. 25-26, 2013.

[16] K. (Stevanović) Hedrih and J. T. Machado, "Discrete fractional order system vibrations," (Preprint submitted for review-January 6, 2014).

[17] K. (Stevanović) Hedrih, "The fractional order hybrid system vibrations," *Advances in Nonlinear Sciences*, vol. 2, pp. 226-326, 2008.

[18] K. (Stevanović) Hedrih, "Analytical mechanics of fractional order discrete system vibrations," *Advances in nonlinear sciences*, vol. 3, pp. 101-148. 2011.

[19] K. (Stevanović) Hedrih, "Fractional order differential equations of dynamics of two mass particles, constrained by a fractional order element," in *Abstracts of the 8th Int. Symp. on Classical and Celestial Mechanics* (CCMECH'8), September 25–29, 2013, Siedlce, Poland, pp. 27-28, 2013.

[20] K. (Stevanović) Hedrih, "Generalized function of fractional order dissipation of system energy and extended Lagrange differential equation in matrix form," in *Abstracts of the 13th Int. Conf. Of Tensor Society on Differential Geometry and Its Applications, and Informatics Besides* dedicated to the 86th anniversary of Radu MIRON'S birth**.** September 3-7, 2013. p.3.

[21] K. (Stevanović) Hedrih, "Partial fractional order differential equations of transversal vibrations of creep-connected double plate systems," in *Fractional Differentiation and its Applications* (A. Le Mahaute, J. A. Tenreiro Machado, J. C. Trigeassou and J. Sabatier, Eds), Germany, pp. 289-302, 2005.

[22] K. (Stevanović) Hedrih, "Partial fractional differential equations of creeping and vibrations of plate and their solutions (First Part) ," *Journal of the Mechanical Behavior of Materials*, pp. 305-314, 2005.

[23] A. Hedrih, "Eigen main chain modes of the double DNA fractional order chain helix vibrations (Part I) ," in *Proc. 2nd Int. Congress of Serbian Society of Mechanics*, M1-03, CD, pp. 1-15, 2009.

[24] K. (Stevanović) Hedrih, "Considering Transfer of Signals through Hybrid Fractional Order Homogeneous Structure," in *Proc. of selected AAS 2009 Papers* (G. Dimirovski, Ed), Skopje –Istambul, pp. 19-24, 2009.

[25] Rašković P. Danilo, *Analitička mehanika (Analytical Mechanics)*, Mašinski fakultet Kragujevac, 1974.

[26] D. Rašković, *Teorija oscilacija (Theory of Oscillations*), Naučna knjiga, 1965 (in Serbian).

[27] R. Gorenflo and F. Mainardi, *Fractional Calculus, Integral and Differential Equations of Fractional Order*, CISM Lecture Notes, Udine, Italy, pp. 223-276, 2000.

[28] B. S. Bačlić and T. Atanacković, "Stability and creep of a fractional derivative order viscoelastic Rod," *Bulletin T, CXXI de L'Academie Serbe des Sciences st de Arts - 2000, Class des Sciences mathematques et naturelles Sciences mathematiques*, No. 25, pp. 115-131, 2000.

[29] K. (Stevanović) Hedrih, "A generalization of Lagrange method of variation constants," *Scientific Review*, Special Issue "Nonlinear Dynamics" dedicated to Milutin Milanković (1879- 1958), S2, pp. 37-66, 2013.

[30] Yu. A. Rossikin and M. V. Shitikova, "Applications of fractional calculus to dynamic problems of linear and nonlinear hereditary mechanics of solids," *Applied Mechanics Reviews*, vol 50(1), pp. 15-67, 1997.

[31] Yu. A. Rossikin and M. V. Shitikova, "Application of fractional calculus for dynamic problem of solid mechanics: Novel trends and recent results," *Applied Mechanics Reviews*, vol. 63(1), pp. 010801-1-52, 2010.

[32] Yu. A. Rossikhin, "Reflections on two parallel ways in the progress of fractional calculus in mechanics of solids," *Applied Mechanics Reviews*, Vol. 63(1), pp. 010701-1-12, 2010.

[33] Yu. A. Rossikin and M. V. Shitikova, "Two approaches for studying the impact response of viscoelastic engineering systems: An overview," *Computers and Mathematics with Applications*, vol. 66, pp. 755-773, 2013.

[34] Yu. A. Rossikin and M. V. Shitikova, "The simplest models of viscoelasticity involving fractional derivatives and their connectedness with the Rabotnov fractional order operators," *International Journal of Mechanics*, vol.8, pp. 326-332, 2014.