Because of the unilateral nature of the hinges that are activated alternatively during the motion at the basis corners of the block, uniquely counter-clockwise rotations φ are allowed around the rotation centre C_1 on the right side of the block and clockwise rotations around the rotation centre C_2 on the left side of the block.

By considering positive rotations φ_{C1} around the rotation centre $C_1(x_{C1}, y_{C1})$ with respect to the axes system <x,y> (Fig.7.a), under the hypothesis of large displacements, the generic point P(x,y) of the block undergoes the displacement $s_P(s_{Px}, y_{Py})$ assuming the updated position $P'(x', y')$.

Rotations around $C_2(x_{C2}, y_{C2})$ are then denoted by φ_{C2} . Displacements around the two corners are denoted by $\mathbf{s_{P1}}(s_{P1x}, s_{P1y})$ and $\mathbf{s_{P2}}(s_{P2x}, s_{P2y})$ respectively.

After introducing the Dirac function $\delta(t)$ and the related functions

$$H_1(x) = \int_{-\infty}^{x} \delta(t)dt, \quad H_2(x) = 1 - \int_{-\infty}^{x} \delta(t)dt \tag{1}$$

one may express the displacement vector, as follows

$$\mathbf{s}_P = H_1(\varphi)\cdot \mathbf{s}_{P1} + H_2(\varphi)\cdot \mathbf{s}_{P2} \tag{2}$$

After some developments, one may get the rotational acceleration (denoted by double superimposed dots of the rotational variable φ) in the form

$$\ddot{\varphi} = H_1(\varphi)\ddot{\varphi}_{C_1} + H_2(\varphi)\ddot{\varphi}_{C_2} \tag{3}$$

with

$$\ddot{\varphi}_{C_1} = -2\frac{G(\varphi,\dot{\varphi})\cdot(V_y \cdot s_{P2x} - V_x \cdot s_{P2y}) + (\ddot{v}_g \cdot V_x - \ddot{u}_g \cdot V_y)}{V_y \cdot [2F(\varphi)\cdot s_{P2x} + \cdot s_{P1x}] - V_x \cdot [2F(\varphi)\cdot s_{P2y} + \cdot s_{P1y}]}$$

$$\ddot{\varphi}_{C_2} = -2\frac{G(\varphi,\dot{\varphi})\cdot(S_y \cdot s_{P2x} - S_x \cdot s_{P2y}) + (\ddot{v}_g \cdot S_x - \ddot{u}_g \cdot S_y)}{S_y \cdot [2F(\varphi)\cdot s_{P2x} + \cdot s_{P1x}] - S_x \cdot [2F(\varphi)\cdot s_{P2y} + \cdot s_{P1y}]} \tag{4}$$

where

$$V = \begin{pmatrix} V_x \\ V_y \end{pmatrix} = \begin{pmatrix} s_{Px} + b/2 \\ h + s_{Py} \end{pmatrix}; \quad S = \begin{pmatrix} S_x \\ S_y \end{pmatrix} = \begin{pmatrix} s_{Px} - b/2 \\ h + s_{Py} \end{pmatrix}; \tag{5}$$

and $F(\varphi)$, $G(\varphi, \dot{\varphi})$ two suitably defined functions depending on the rotation and its time derivative.

It is interesting to observe some anomaly in the description of the dynamic behaviour of the unilateral rigid model.

Let superpose an additional term corresponding to a null distribution to Eq.(2) , as follows

$$\mathbf{s}_P = H_1(\varphi)\cdot \mathbf{s}_{P1} + H_2(\varphi)\cdot \mathbf{s}_{P2} + r\,\delta(\varphi)\varphi^n \cdot (\mathbf{s}_{P1} - \mathbf{s}_{P2}) \tag{6}$$

Thereafter, one realizes that it is possible to change Eq. (2) in way that the time-displacement of the system is not formally altered, by adding a null distribution, like in Eq. (6).

This produces some formal change in $F(\varphi)$, $G(\varphi, \dot{\varphi})$ in Eq.(4).

The impact obeys now to a strongly non-linear equation, and numerical solutions must be sought.

So this aim, the distribution $\delta(x)$ and its derivatives can be substituted by functions $\alpha_k(x)$ and their derivatives, that in the limit uniformly converge to $\delta(x)$ and its derivatives.

The results prove that increasing the coefficient r of the null distribution produces an increasing damping and that this effect is rather uniform when k tends to become larger and larger and functions $\alpha_k(x)$ tend to $\delta(x)$ and henceforth it is expected that the numerical solutions of the equilibrium for the displacement, velocity and acceleration fields converge to the solutions of the theoretical case.

So it is expected that the superposition of a null term to the displacement of the model produces a non-null consequence on its motion, an effect that is one more reason to assess that rocking motion can be strongly affected by the way one approaches the problem.

III.3 Towards worst-case forecasts

The low robustness of rocking response of rigid bodies, its randomness and sensitivity to imperfections, make their seismic assessment a big deal especially because of the high degree of uncertainty of the seismic action.

Results may be stabilized through a worst scenario procedure, which is particularly effective for not sharply defined forcing functions, making the response substantially independent on the details of the excitation.

The method was set forth by Drenick in the 70’s [41]-[44] and by Shinozuka in the early 80’s [45] with particular reference to linear structures, and more recently by Elishakoff and Pletner [46] and by Baratta et al. [47].

Worst-scenario results are likewise more severe than probabilistic evaluations based on the statistics of the response process, possibly evaluated by MonteCarlo methods, since the probability measure of the worst-scenario may most of the times be zero, despite of the fact that it is anyway one (15) possible realization and also includes possible instability of the mathematical-numerical model.

As in Drenick's original approach, the worst scenario results in a regular forcing function that overpasses many problems due to the desultory character of the forcing function.

In the specific case, a possible worst scenario approach may be set up aimed at defining the theoretical bounds on the maximum value of the structural response under dynamic loading, for non linear structures behaving like rigid blocks rocking around their base edges under dynamic shaking. (18)

Generally speaking, basic characters of the excitation can be identified in the average power spectrum, in its maximum value, its duration and so on.

So a suitable *class* of forcing functions can be reliably defined, by building up a functional space where it is assumed it must be contained.

All the accelerograms in the class differ from each other by random quantities that cannot be predicted in details. In other words the accelerograms in the class form a *stochastic process* that could be treated by means of the methods and procedures of the relevant theory to produce the probability distribution of the maximum response.

Apart from the difficulty to treat the response of a highly nonlinear system in a stochastic context, probabilistic results may be quite illusory depending on the acquisition of statistical data for the parameters of the stochastic process.

By a worst scenario approach one may search for the worst response produced by accelerograms fitting the basic properties that might be defined on the basis of the investigation of the source of the disturbance, but subject to random realizations of the details, thus looking for the worst situation that can occur for a structure when it is acted on by badly defined forcing time-histories.

To this aim, one may set up a constrained optimization process where the response maximization procedure consists of an iterative process, based on the generation at each step of a new accelerogram compatible with the assigned properties of the disturbance and on the evaluation of the response of the structural model.

IV. Conclusions

The paper focuses on approaches to forecasts about the dynamic response of structural systems in the cases when uncertainty deeply affects the response itself of the structure and make ordinary analyses not effective.

In particular one refers to a large class of structures whose behaviour under seismic motion may be simulated by means of the rigid assumption.

The pure rocking mode, which is the more dangerous as regards to possible structural failure, is focused on.

Some considerations are outlined about the low robustness of the rocking mode response, its randomness and sensitivity to imperfections and errors.

Therefore one finally concludes that the seismic assessment of structural response of structures moving according to the unilateral hinges rigid scheme represents a big deal, especially because of the high degree of uncertainty of the seismic action, but even with reference to any forcing action of any shape.

On this basis, one figures out the need of making recourse to different strategies for vulnerability assessment, mostly based on the forecast of worst-case situations.

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