Supersonic Boundary Layer of Binary Mixture and its Stability

Sergey A. Gaponov, Boris V. Smorodsky

Abstract — Properties of binary-mixture compressible boundary layers are investigated theoretically. Self-similar equations are deduced which describe the boundary-layer of air with distributed injection of a foreign gas from the porous surface. Mach=2 boundary-layer velocity, density, temperature and concentration profiles have been computed for foreign gases with various molecular mass. It has been found that increase of foreign gas injection leads to the monotonous reduction of surface friction and heat transfer. Stability of Mach=2 binary-mixture boundary-layer has been investigated by means of the linear stability theory (LST). It was established that action of heavy gas injection on the boundary-layer is similar to the action of wall cooling and leads to an increase of boundary-layer stability and to the delay of the laminar-turbulent transition. Theoretical estimate of the position of laminar-turbulent transition by means of known e^N-method has been performed. A principal possibility to enlarge the transition Reynolds numbers in approximately two times has been revealed.

Keywords — binary-mixture, boundary-layer, hydrodynamic stability, laminar-turbulent transition.

I. Introduction

This paper is an extended version of a theoretical investigation originally presented at the 7th International Conference on Fluid Mechanics and Heat & Mass Transfer (Prague, Czech Republic, March 18-20, 2016) and published in [1].

Importance to study the binary-mixture boundary-layers is determined by a number of various factors. One of the reasons was to find possibilities to control the skin friction and the wall heat flux in stationary conditions. For the first time a foreign gas injection into the two-dimensional (2D) flat-plate boundary-layer was investigated in [2-4]. It was shown there that in order to reduce the skin friction it is better to inject in the flow a gas lighter than an air. Solutions presented in the mentioned above papers have a lot of limitations. In [2] the skin friction was not calculated explicitly but its value can be deduced from the velocity profiles computed there. However the boundary-layer equations have been solved there with the boundary conditions suitable for the impermeable surface only. Also the foreign gas concentration at the wall was assumed to be large enough whereas the injection velocity was assumed to be zero that is out of logic. In [3,4] the solutions have been obtained in the assumption of the zero heat transfer. In addition in [4] it was supposed that the binary-mixture viscosity does not depend on the foreign-gas concentration and varies linearly with the temperature, while the Schmidt number was taken equal to unity.

The non-zero heat transfer has been considered for the first time in [5], where the influence of an injected gas properties on the skin friction and the wall heat flux variation has been investigated. The author has got a numerical solution to the problem using a simple model of solid spheres to describe the collisional processes, for the case when the injection magnitude varies downstream inversely proportional to the square root of the streamwise coordinate. It has been shown in this paper that the light gas injection leads to the reduction of both – the skin friction and the wall heat transfer. Moreover, the injection of a large specific heat gas reduces the heat flux significantly, but affects the skin friction only weakly. The combination of a large molecular collision diameter and high specific heat of various polyatomic gases can be much more advantageous to reduce the skin friction and the heat transfer in comparison to the injection of such a light monatomic gas as helium. Additional investigations of the laminar boundary layers of binary mixtures can be found in [6-8].

Along with the problem of a thermal protection and drag reduction there is another important problem of the laminar-turbulent transition control. In the framework of our discussion of the binary gas mixture boundary layers this problem is in fact reduced to the investigation of hydrodynamic stability of such boundary layers. Influence of a foreign gas injection on the linear stability of laminar boundary layers has been studied for the first time in [9]. It was shown there that a heavy gas injection can principally lead to a boundary layer stabilization. This possibility has been further investigated in [10], where they have performed a research on the influence of a foreign gas molecular mass on the boundary layer stability. In particular, it has been shown that injection of a light gas leads to a certain reduction of the critical Reynolds number. However the results of [10] have been obtained for Mach number $\mathrm{M}=0$ and it was not possible to solve the problem for the Mach $\mathrm{M}>1.3$ because of the certain limitations inherent to the asymptotic procedure used there.

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S. A. Gaponov is with the Khristianovich Institute of Theoretical and Applied Mechanics, Novosibirsk, 630090, RUSSIA, (corresponding author to provide phone: 7-383-354-3048; fax: 7-383-330-7268; e-mail: gaponov@itam.nsc.ru).

B. V. Smorodsky is with the Khristianovich Institute of Theoretical and Applied Mechanics, Novosibirsk, 630090, RUSSIA, (phone: 7-383-330-1228; fax: 7-383-330-7268; e-mail: smorodsk@itam.nsc.ru)

The complete enough equations of the binary-mixture boundary-layer stability with respect to 2D disturbances in the parallel flow approximation have been obtained in [11]. However those equations have never been used: the author has not published the results of a numerical solution of his equations.

After a while a research on the influence of a gas nonequilibrium dissociation on the boundary-layer stability has been published in [12,13]. After that there was no further systematic investigations of the effect of a foreign gas injection on a boundary-layer properties and its stability.

Present paper fills up this gap in the knowledge of a binary-mixture boundary-layer stability. Some first results of ongoing investigations on possibilities to control the flat-plate supersonic boundary-layer and its linear stability by means of a foreign gas injection from the surface are presented here.

II. Problem Formulation

The time-dependent dynamics of a binary gas mixture is described by the system of partial differential equations [14-16]:

$$\frac{d\rho^*}{dt}+\rho^*\operatorname{div}\mathbf{V}^*=0\ ,\ \frac{d\mathbf{v}^*}{dt}=-\frac{2}{\rho}\operatorname{div}\mathbf{\Pi}\ ,$$

$$\rho^*\frac{dc}{dt}=-\operatorname{div}\mathbf{j}_1^*\ , \qquad (1)$$

$$\rho^*\frac{dh^*}{dt}=\frac{dP^*}{dt}-\operatorname{div}\mathbf{q}^*+2\mu^*\dot{S}^{*2}\ ,\ P^*=\frac{\rho^*RT^*}{m}=\frac{\rho^*R}{m}\frac{h^*}{C_p^*}\ ,$$

where $\frac{1}{m}=\frac{m_1+(m_2-m_1)c}{m_1m_2}$; the enthalpy $h^*=C_p^*T^*$, $C_p^*=(C_{p1}^*-C_{p2}^*)c+C_{p2}^*$; $C_p^*, C_{p2}^*, C_{p1}^*$ – are constant pressure specific heats of the mixture, of the primary and of the foreign gases, m_2, m_1 – molecular masses of primary and foreign gases. In Cartesian coordinates:

$$\dot{S}^{*2}=\sum_{i,j=1}^{3}\dot{S}_{ij}^{*2}\ ,\ \dot{S}_{11}^*=\frac{\partial U^*}{\partial x}-\frac{1}{3}\operatorname{div}\mathbf{V}^*\ ,$$

$$\dot{S}_{22}^*=\frac{\partial V^*}{\partial y}-\frac{1}{3}\operatorname{div}\mathbf{V}^*\ ,\ \dot{S}_{33}^*=\frac{\partial W^*}{\partial z}-\frac{1}{3}\operatorname{div}\mathbf{V}^*\ ,$$

$$\dot{S}_{12}^*=\dot{S}_{21}^*=\frac{1}{2}\left(\frac{\partial U^*}{\partial y}+\frac{\partial V^*}{\partial x}\right)\ ,\ \dot{S}_{13}^*=\dot{S}_{31}^*=\frac{1}{2}\left(\frac{\partial U^*}{\partial z}+\frac{\partial W^*}{\partial x}\right),$$

$$\dot{S}_{23}^*=\dot{S}_{32}^*=\frac{1}{2}\left(\frac{\partial V^*}{\partial z}+\frac{\partial W^*}{\partial y}\right)\ ,$$

$$\Pi_{ij}=P^*\delta_{ij}-\mu^*S_{ij}+\frac{1}{3}\mu^*\operatorname{div}\mathbf{V}^*\delta_{ij}\ ,$$

$$\frac{d}{dt}=\frac{\partial}{\partial t}+U^*\frac{\partial}{\partial x}+V^*\frac{\partial}{\partial y}+W^*\frac{\partial}{\partial z}\ .$$

In the equations above we used the dimensional values for: $\mathbf{V}^*=(U^*,V^*,W^*)$ – velocity components in (x,y,z) – directions respectively; ρ^* – density; P^* – pressure; T^* – temperature; h^* – specific enthalpy per unit mass; μ^* – dynamic viscosity; $\mathbf{q}^*$ – total heat flux; $\mathbf{j}_1^*$ – foreign gas mass flux.

Formulas for the binary-mixture heat flux and for the foreign gas mass flux are taken from [14]:

$$\mathbf{q}^*=-\lambda^*\nabla T^*+\left(h^*_1-h^*_2\right)\mathbf{j}_1^*\ , \qquad (2)$$

$$\mathbf{j}_1^*=-\rho^*D_{12}^*\left[\nabla c+\frac{m_2-m_1}{m}c(1-c)\nabla\left(\ln P^*\right)\right]\ . \qquad (3)$$

III. Boundary-layer Equations

2D stationary supersonic flat-plate boundary-layer of the binary gas mixture without chemical reactions is considered in this paper. Foreign gas with molecular mass m_1 is injected into the main flow of the primary gas with a molecular mass m_2 through a permeable surface of the model. In this case the boundary layer equations can be presented as [15, 16]:

$$\frac{\partial(U\rho)}{\partial x}+\frac{\partial(V\rho)}{\partial y}=0\ ,$$

$$\rho\left(U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\frac{\partial U}{\partial y}\right)\ ,$$

$$\rho\left(U\frac{\partial h}{\partial x}+V\frac{\partial h}{\partial y}\right)=-\frac{\partial q}{\partial y}+\mu\left(\frac{\partial U}{\partial y}\right)^2\ ,$$

$$\rho\left(U\frac{\partial c}{\partial x}+V\frac{\partial c}{\partial y}\right)=-\frac{\partial j}{\partial y}\ ,\ P=\frac{\rho RT}{m}=\rho\tilde{R}T\ .$$

Here x is the streamwise coordinate, y – is normal to the surface,

$$q=-\lambda\frac{\partial T}{\partial y}+(h_1-h_2)\,j+\frac{RT}{m_2}\frac{\alpha_T}{m_1/m_2+(1-m_1/m_2)c}\,j\ ,$$

$j=-\rho D_{12}\left[\frac{\partial c}{\partial y}+\frac{\alpha_T c(1-c)}{T}\frac{\partial T}{\partial y}\right]$, U and V are the velocity components of the mixture in the x- and y-directions; ρ is the density; h – enthalpy per unit mass; P – pressure; T – temperature; q – heat flux of the mixture, j – mass flux of the foreign gas in the y-direction; c – concentration of the foreign gas; R – universal gas constant; μ, λ, D_{12} – coefficients of dynamic viscosity, heat conductivity and binary diffusion; α_T – thermal diffusion coefficient.

Boundary layer equations are solved with the following boundary conditions at the wall:

$$U=0,\ \ V=V_w,\ \ \left(a_1T+a_2\frac{dT}{dy}\right)=0,$$
$$V_w(1-c_w)=D_{12}\left[-\frac{\partial c}{\partial y}+\frac{\alpha_T c(1-c)}{T}\frac{\partial T}{\partial y}\right]. \qquad (4)$$

The last equality is obtained from the condition that the primary gas mass flux $j_2 + (\rho V)(1-c)$ to the permeable wall is zero, [7]. Conditions at the boundary layer outer edge are:

$$U = U_e \ , \ \ T = T_e \ , \ \ c = 0 \ \ \text{at} \ (y \to \infty) \ . \tag{5}$$

Boundary layer equations in self-similar variables and in the absence of thermal diffusion can further be written as follows:

$$\frac{d}{d\bar{y}}\left(\bar{\mu}\frac{d\bar{U}}{d\bar{y}}\right) + F\frac{d\bar{U}}{d\bar{y}} = 0 \ ,$$

$$\frac{d\bar{q}}{d\bar{y}} = F\frac{d\bar{h}}{d\bar{y}} + (\gamma - 1)\mathrm{M}_e^2\bar{\mu}\left(\frac{d\bar{U}}{d\bar{y}}\right)^2 \ , \tag{6}$$

$$\frac{d\bar{j}}{d\bar{y}} = F\frac{dc}{d\bar{y}} \ , \ \bar{q} = -\bar{\lambda}\frac{d\bar{T}}{d\bar{y}} + (\bar{h}_1 - \bar{h}_2)\bar{j} \ , \ \bar{j} = -\bar{\rho}\bar{D}_{12}\frac{dc}{d\bar{y}} \ .$$

Here $\bar{y} = y\big/\sqrt{x\mu_e/U_e\rho_e}$, $\bar{q}(\bar{y}) = \dfrac{\sqrt{x\mu_e/U_e\rho_e}}{\mu_e h_e} q$,

$$\bar{j} = j\frac{\sqrt{x\mu_e/U_e\rho_e}}{\mu_e} \ , \ \bar{U} = \frac{2}{\bar{\rho}}\frac{dF}{d\bar{y}} \ , \ \bar{h} = \frac{h}{C_{p2}T_e} \ , \ \bar{T} = \frac{T}{T_e} \ , \ \bar{\mu} = \frac{\mu}{\mu_e} \ ,$$

$$\bar{\lambda} = \frac{\lambda}{\mu_e C_{p2}} \ , \ \bar{D}_{12} = \frac{\rho_e D_{12}}{\mu_e} .$$

Boundary conditions take the following shape with new variables at the permeable wall

$$F = -f_w \ , \ \ \bar{U} = 0 \ , \ \ \left(b_1\bar{T} + b_2\frac{d\bar{T}}{d\bar{y}}\right) = 0 \ ,$$

$$f_w(1 - c_w) = \bar{\rho}_w\bar{D}_{12}\left[-\frac{\partial c}{\partial\bar{y}} + \frac{\alpha_T c(1-c)}{\bar{T}}\frac{\partial\bar{T}}{\partial\bar{y}}\right] \ ,$$

$$f_w = \bar{\rho}_w\bar{V}_w\,\mathrm{Re} \ ,$$

while at the boundary layer outer edge:

$$(\bar{U}, \bar{T}) = 1 \ , \ \ c = 0 \ \ \text{at} \ (\bar{y} \to \infty) \ .$$

To calculate the viscosity of a binary mixture the relations [17] have been used, which can be written non-dimensionally:

$$\bar{\mu} = \frac{X_1\bar{\mu}_1}{X_1 + G_{12}X_2} + \frac{X_2\bar{\mu}_2}{X_2 + G_{21}X_1} \ , \ X_1 = \frac{cm}{m_1} \ , \ X_2 = \frac{(1-c)m}{m_2} \ ,$$

$$(\bar{\mu}, \bar{\mu}_1, \bar{\mu}_2) = (\mu, \mu_1, \mu_2)/\mu_{2e} \ ,$$

$$G_{12} = \frac{\left[1 + (\mu_1/\mu_2)^{1/2}(m_2/m_1)^{1/4}\right]^2}{8^{1/2}\left[1 + (m_1/m_2)\right]^{1/2}} \ ,$$

$$G_{21} = \frac{[1 + (\mu_2/\mu_1)^{1/2}(m_1/m_2)^{1/4}]^2}{8^{1/2}\left[1 + (m_2/m_1)\right]^{1/2}} \ .$$

The formula for the thermal conductivity coefficient of binary mixtures is looking similar, non-dimensionally:

$$\bar{\lambda} = \frac{X_1\bar{\lambda}_1'}{X_1 + 1.065\tilde{G}_{12}X_2} + \frac{X_2\bar{\lambda}_2'}{X_2 + 1.065\tilde{G}_{21}X_1} \ ,$$

$$\bar{\lambda}_i' = \bar{\lambda}_i\mathrm{Eu} \ , \ \ \mathrm{Eu} = 0.115 + 0.354 C_{pi}/R_i \ ,$$

$$(\bar{\lambda}, \bar{\lambda}_1', \bar{\lambda}_1') = (\lambda, \lambda_1', \lambda_2')/\mu_e c_{pe} \ , \ \lambda_i = 15R\mu_i/m_i \ ,$$

$$\tilde{G}_{12} = \frac{[1 + (\lambda_1/\lambda_2)^{1/2}(m_1/m_2)^{1/4}]^2}{8^{1/2}[1 + (m_1/m_2)]^{1/2}} \ ,$$

$$\tilde{G}_{21} = \frac{\left[1 + (\lambda_2/\lambda_1)^{1/2}(m_2/m_1)^{1/4}\right]^2}{8^{1/2}\left[1 + (m_2/m_1)\right]^{1/2}} \ .$$

Viscosity and thermal conductivity coefficients of a monoatomic gas according to the kinetic theory can be expressed as:

$$\mu_i^* = 2.6693 \cdot 10^{-6}\frac{\sqrt{m_i T^*}}{d_i^2\Omega^{(2,2)^*}} \ , \ \lambda_i^* = \frac{15}{4}\frac{R}{m_i}\mu_i^* \ .$$

The dimensionless coefficient of binary diffusion is

$$\bar{D}_{12} = \frac{\rho_e}{\mu_e}D_{12} = 262.8 \cdot 10^{-5}\frac{\left[T^3(m_1 + m_2)/2m_1m_2\right]^{1/2}}{P\sigma_{12}^2\Omega^{(1,1)^*}}\frac{\rho_e}{\mu_e} ,$$

where D_{12} is in cm^2/s, $\sigma_{12} = (d_1 + d_2)/2$ – in angstroms, P – in bars, T – in $^\circ K$. Collision integrals $\Omega^{(1,1)^*}$ and $\Omega^{(2,2)^*}$ have been calculated using Leonard-Jones potential, according to [14].

IV. Stability Equations

To deduce equations of linear stability we use the relation: $\nabla T^* = \dfrac{\nabla h^*}{C_p} - \dfrac{h^*{}_1 - h^*{}_2}{C_p}\nabla c$, where C_p is the specific heat of a frozen mixture. Neglecting the Dufour effect we can also write:

$$\mathrm{div}\,\mathbf{q}^* = -\mathrm{div}\left[\lambda^*\left(\frac{\nabla h^*}{C_p^*} - \frac{h_1^* - h_2^*}{C_p^*}\nabla c\right)\right] + \mathrm{div}\left[(h_1^* - h_2^*)\,\mathbf{j}_1^*\right].$$

In the case of linear disturbances the dimensionless flow parameters can be written as $\bar{g}^* = \bar{g} + g'$, where $\bar{g}$ are solutions of stationary boundary-layer equations. Neglecting foreign gas thermal diffusion, for a parallel flow approximation and under simplifications used in [18-20] the linearization of the system (1-3) with respect to perturbations and using the variables $dX = dx/\delta$, $dZ = dz/\delta$, $d\tau = U_e dt/\delta$ leads to the following LST equations:

$$\bar{\rho}\left(\frac{du'}{d\tau}\right) = -\frac{1}{\gamma_e\mathrm{M}_e^2}\frac{\partial p'}{\partial X} + \frac{\bar{\mu}}{\mathrm{Re}}\frac{\partial^2 u'}{\partial\bar{y}^2} \ , \ \bar{\rho}\left(\frac{\partial v'}{\partial\tau}\right) = -\frac{1}{\gamma_e\mathrm{M}_e^2}\frac{\partial p'}{\partial\bar{y}} \ ,$$

$$\bar{\rho}\left(\frac{dw'}{d\tau}\right) = -\frac{1}{\gamma_e\mathrm{M}_e^2}\frac{\partial p'}{\partial Z} + \frac{\bar{\mu}}{\mathrm{Re}}\frac{\partial^2 w'}{\partial\bar{y}^2} \ ,$$

$$\bar{\rho}\left(\frac{dh'}{d\tau}\right) = \frac{\gamma_e - 1}{\gamma_e}\left(\frac{dp'}{d\tau}\right) + \bar{\mu}\frac{1}{\mathrm{RePr}}\frac{\partial^2 h'}{\partial\bar{y}^2} + \bar{\mu}\frac{1}{\mathrm{Re}}(\bar{h}_1 - \bar{h}_2)\left(\frac{1}{\mathrm{Sm}} - \frac{1}{\mathrm{Pr}}\right)\frac{\partial^2 c'}{\partial\bar{y}^2}, \tag{7}$$

$$\frac{d\rho'}{d\tau}+\bar\rho\left(\frac{\partial u'}{\partial X}+\frac{\partial v'}{\partial \bar y}+\frac{\partial w'}{\partial Z}\right)=0\ ,$$
$$\left(\frac{dc'}{d\tau}\right)=\frac{\bar\mu}{\mathrm{Sm}\,\mathrm{Re}}\frac{\partial^2 c'}{\partial \bar y^2}\ ,$$
$$\frac{p'}{\bar P}=\frac{\rho'}{\bar\rho}+\frac{h'}{\bar h}+\left[\frac{(R_1-R_2)}{\bar R}-\frac{(C_{p1}-C_{p2})}{C_p}\right]c'\ ,$$

where $\dfrac{d}{dt}=\dfrac{\partial}{\partial\tau}+\bar U\dfrac{\partial}{\partial X}$.

We seek the solution to (7) as a harmonic in space and time perturbation: $q'=\tilde q(\bar y)\exp\left[i\alpha(X-C\tau)+i\beta Z\right]$. Then we come to the system of ODEs:

$$i\alpha(\bar U-C)\tilde\rho+\frac{d\bar\rho}{d\bar y}\tilde v+\bar\rho\left(i(\alpha\tilde u+\beta\tilde w)+\frac{d\tilde v}{d\bar y}\right)=0\ ,$$
$$\bar\rho\left(i\alpha(\bar U-C)\tilde u+\frac{d\bar U}{d\bar y}\tilde v\right)=-\frac{i\alpha\tilde p}{\gamma_e \mathrm{M}_e^2}+\frac{\bar\mu}{\mathrm{Re}}\frac{d^2\tilde u}{d\bar y^2}\ ,$$
$$\bar\rho i\alpha(\bar U-C)\tilde v=-\frac{1}{\gamma_e \mathrm{M}_e^2}\frac{d\tilde p}{d\bar y}\ ,$$
$$\bar\rho i\alpha(\bar U-C)\tilde w=-\frac{i\beta\tilde p}{\gamma_e \mathrm{M}_e^2}+\frac{\bar\mu}{\mathrm{Re}}\frac{d^2\tilde w}{d\bar y^2}\ , \qquad (8)$$
$$i\alpha(\bar U-C)\tilde c+\frac{d\bar c}{d\bar y}\tilde v=\frac{\bar\mu}{\mathrm{Re}\,\mathrm{Sm}}\frac{d^2\tilde c}{d\bar y^2}\ ,$$
$$\bar\rho\left(i\alpha(\bar U-C)\tilde h+\frac{d\bar h}{d\bar y}\tilde v\right)=\frac{\gamma_e-1}{\gamma_e}i\alpha(\bar U-C)\tilde p+$$
$$+\frac{\bar\mu}{\mathrm{Re}\,\mathrm{Pr}}\frac{d^2\tilde h}{d\bar y^2}+\frac{\bar\mu}{\mathrm{Re}}(\bar h_1-\bar h_2)\left(\frac{1}{\mathrm{Sm}}-\frac{1}{\mathrm{Pr}}\right)\frac{d^2\tilde c}{d\bar y^2}.$$

Here $\alpha C=\omega=\omega^*\delta/U_e$, $\omega=2\pi f\delta/U_e=F\,\mathrm{Re}$, $C=\omega/\alpha=F\,\mathrm{Re}/\alpha$, $F=2\pi f\mu_e/\rho_e U_e^2$ – is the reduced frequency, while f is the dimensional frequency in Hertz. System (8) is solved with the following homogeneous boundary conditions:

$$\left(u,\, w,\, h,\, f_w c-\bar\rho_w\bar D_{12}\frac{dc}{dY}\right)=0 \text{ at } Y=0, \qquad (9)$$
$$(u,w,h,c)\to 0 \text{ at } Y\to\infty$$

Numerical integration of the eigenvalue problem (8-9) have been performed by means of method of orthonormalizations [16].

V. Results

In this paper results of the boundary-layer computations performed for the flow of air over the flat-plate at Mach number M=2 are presented. Various foreign gases (such as tetrachloromethane CCl_4, Xenon *Xe*, Helium *He*) have been injected through a permeable surface of the plate. Calculations have also been performed to reveal particularly influence on the boundary layer properties of a ratio of molecular mass of a foreign gas to the main gas at identical specific heat. The described below results are presented in the dimensionless form: all physical quantities are divided to their values at the boundary layer outer edge. Normal-to-the-wall coordinate *y* is referenced to the Blasius length scale $\delta=\sqrt{x\mu_e/U_e\rho_e}$.

Fig.1 shows computed distribution of concentration of tetrachloromethane with $m_1=154$, that is approximately five times heavier than air, across the boundary layer, for various values of the injection factor $-f_w$. One can see that growth of the foreign gas injection leads to the concentration c_1 increase at the wall. It is worth to note here that binary-mixture boundary-layer velocity and temperature profiles are only weakly dependent on the injection factor variation at least in the range $0\le -f_w\le 0.2$.

Influence of an injection and a molecular mass of foreign gases on the local skin friction is presented at Fig.2 for thermally insulated wall. The skin friction coefficient decreases with increasing injection independently of the molecular mass of the injected gas. Since molecular masses of

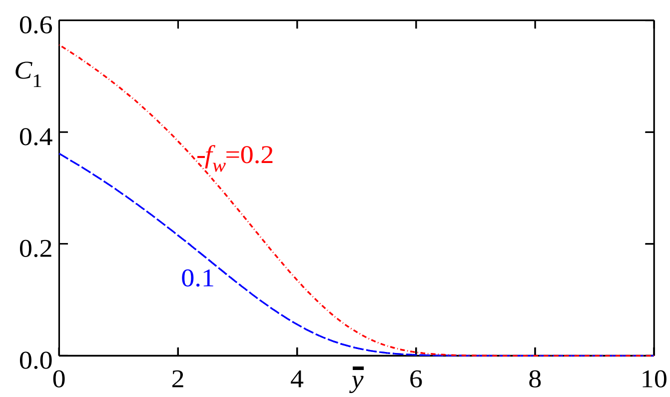

Fig.1: Distribution of the mass concentration of CCl_4 across the boundary layer for two values of $-f_w$; $\mathrm{M}=2$.

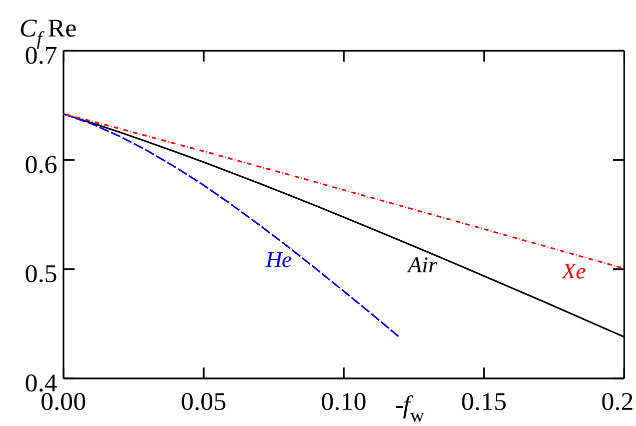

Fig.2: Normalized skin friction coefficient $C_f\,\mathrm{Re}$ versus injection factor for the adiabatic wall; $\mathrm{M}=2$.

He, *Air* and *Xe* are 4, 29 and 131 correspondingly, we see that the skin friction decreases with decreasing molecular mass of a foreign gas. That means that lighter gas gives stronger reduction of the wall friction with increasing injection from the surface.

Fig.3 demonstrates influence of the cooled wall temperature T_w on the local skin friction. Friction drag decreases monotonously with increasing T_w for all considered foreign gases. As before the friction drag increases with increasing foreign gas molecular mass. All presented above results correlate qualitatively with those published in [6], where computations have been performed for Mach number M=0.7.

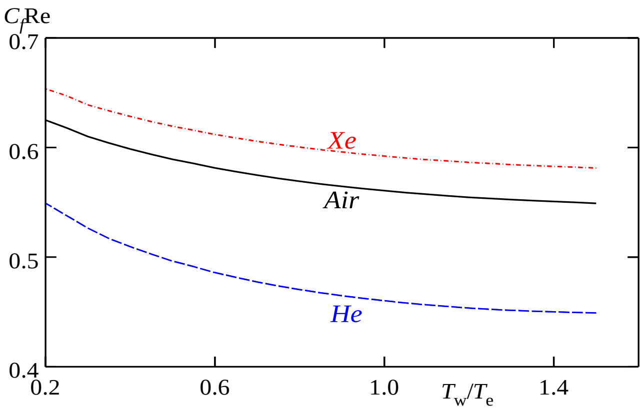

Fig.3: Normalized skin friction coefficient C_f Re versus wall temperature T_w/T_e; $\mathrm{M}=2$, $-f_w=0.1$.

The heat transfer data are not so uncomplicated as friction data. In [1] influence of the wall temperature on the Stanton number with foreign gas injection has been investigated. It has been found that only at $T_w/T_e>0.36$ the results for helium are in conformity with the skin friction results described above. That is the decrease of the foreign gas molecular mass leads to a reduction of the wall heat transfer. However in the range $T_w/T_e<0.3$ injection of helium gives higher heat flux in comparison with injection of air and xenon. So, decrease of the wall heat flux is achieved by the injection of *He* only for not very cold wall, while at lower values of T_w the opposite effect is observed.

Table 1

	m_1	d_1, A	ε/k_1, °K	C_p
H_2	2	2.915	38	14320
He	4	2.576	10	5190
Ne	20	2.789	36	1030
Air	29	3.617	97	1004
Kr	84	3.498	225	248
Xe	131	4.055	229	160
SF_6	146	4.268	233	860
CCl_4	154	5.881	327	866

Stability of compressible boundary layers of binary gas mixtures depends on lots of parameters. The values of the most important of them for various individual gases which have been used for calculation of binary mixture viscosity, thermal conductivity and diffusion on the basis of kinetic theory in the framework of the Leonard-Jones potential are listed in table 1.

We start our discussion of the results of Mach 2 flat plate boundary layer stability by presenting the linear stability diagram depicted at Fig.4 by contours of disturbance spatial amplification rates at zero injection through the surface, on the plane streamwise coordinate *x* – frequency *f*. Our computations have been performed with the goal to give some recommendations concerning parameter selection for planned experiments in the supersonic wind tunnel T-325 of the S.A.Khristianovich Institute of Theoretical and Applied Mechanics, Novosibirsk, Russia. Therefore we choose the dimensional representation of results at this plot. LST computations have been performed for the free-stream unit Reynolds number $\mathrm{Re}_1=5\cdot10^6$ m^{-1}, typical for stability measurements in T-325.

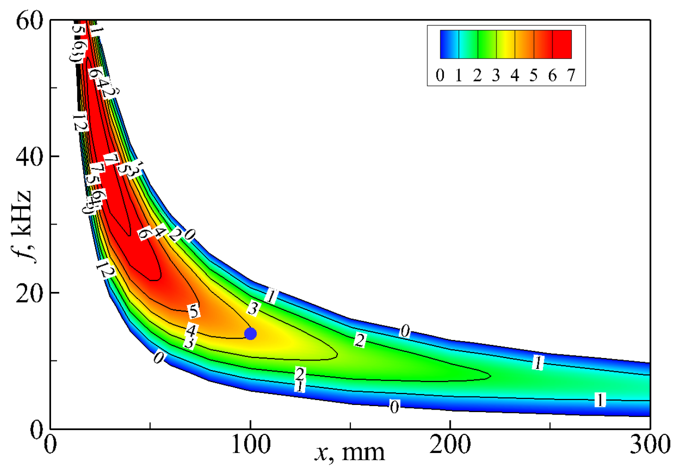

Fig.4: Mach=2 flat-plate boundary-layer zero-injection stability-diagram: dimensional spatial amplification rate contours $-\alpha_i=-\alpha_i(x,f)$ [m^{-1}] for 2D $(\chi=0^\circ)$ perturbations.

Color flooded area at Fig.4 represents the instability region $(-\alpha_i>0)$ where propagating downstream disturbances grow in their amplitudes. For further discussion we take the 2D disturbance with maximal growth rate at $x=100$ mm, which is normally corresponds to the middle of the stability measurement region at T-325 at the selected unit Reynolds number. The selected disturbance ($f=14$ kHz, reduced frequency $F\cdot10^6=35$, $\mathrm{Re}=720$) is depicted by the small circle at Fig.4.

Now we enable injection of foreign gases with definite values of the injection factor $-f_w$ through the permeable surface. Fig.5 shows computed spatial amplification rates $-\alpha_i$ of the selected two-dimensional (2D, $\chi=\arctan(\beta/\alpha_r)=0$) disturbance with mentioned above frequency and Reynolds number on the injection factor $-f_w$ for a number of foreign gases. The primary gas was always air.

One can see that the injection of *Air* or the light gas such as neon (*Ne*) leads to monotonous increase of the perturbation growth rate with increasing injection, due to the formation of more and more unstable mean flow profiles. Injection of heavy inert gas such as xenon (*Xe*) also destabilizes the flow. The reason for that is a low specific heat of xenon in comparison with that of for the air (table 1) that leads to substantial growth of temperature near the wall with increasing $-f_w$. Injection of gases with low molecular weight, such as helium (*He*) or hydrogen (H_2), leads to non-monotonous relation of $-\alpha_i = -\alpha_i(f_w)$. With low injection factor $-f_w < 0.04$ the flow becomes more stable, while further growth of injection leads to the increase of disturbance amplification rate. Such behavior is explained again by the opposite action of the two factors – low molecular weight of the injected foreign gas and its high specific heat. From this diagram one can also see that injection of a heavy gas with the specific heat close to that of the *Air*, tetrachloromethane (CCl_4), or sulfur hexafluoride (SF_6), leads to the monotonous decrease of the perturbation amplification rate, i.e. to the boundary layer stabilization. This result is in agreement with a known earlier stabilizing influence of the injection of a heavy gas. From table 1 we see that CCl_4 and SF_6 is heavier than air in 5.3 and 5.0 times. Greater efficiency in the boundary-layer stabilization by the injection of CCl_4 in comparison with SF_6 is explained by higher molecular mass of the former gas.

The importance of an injected foreign gas specific heat becomes obvious if one compares amplification rate change at injection of krypton and air. Injection of heavy krypton gives faster destabilization of the boundary layer than injection of air. This is because specific heat of krypton is four times smaller than that of for air. Smaller value of the specific heat for *Kr* with increasing injection leads to a certain increase of the mixture temperature near the permeable surface and therefore to a reduction of the mixture density at the wall $\bar{\rho}_w$.

Influence of the foreign gas molecular mass on the boundary layer stability is easier to understand by analyzing the density of a gas mixture near the wall. Fig.6 demonstrates mixture density at the wall $\bar{\rho}_w$ as a function of the injection factor $-f_w$ for the same gases like at Fig.5. Here we see a complete correlation of results presented in Figs.5 and 6. Increase of $\bar{\rho}_w$ due to the injection of a heavy foreign gas causes reduction of perturbation growth rates. Vice versa, injection of a lighter gas, such as *Ne*, causes reduction of $\bar{\rho}_w$ and boundary layer destabilization appearing in the increase of the perturbation growth rates.

Up to now we considered influence of the foreign gas injection on the growth rates of 2D disturbances only. Spatial amplification rates $-\alpha_i$ for 3D perturbations of waves with reduced frequency $F{\cdot}10^6 = 35$ at $\mathrm{Re} = 720$ are shown at Fig.7 for various values of CCl_4 injection factor $-f_w$. We see from the upper curve that at zero injection the most unstable perturbation is a 3D wave with the value of wave vector orientation angle $\chi \approx 60^\circ$, that is usual for the supersonic boundary layer. Growth rate of this wave is almost five times

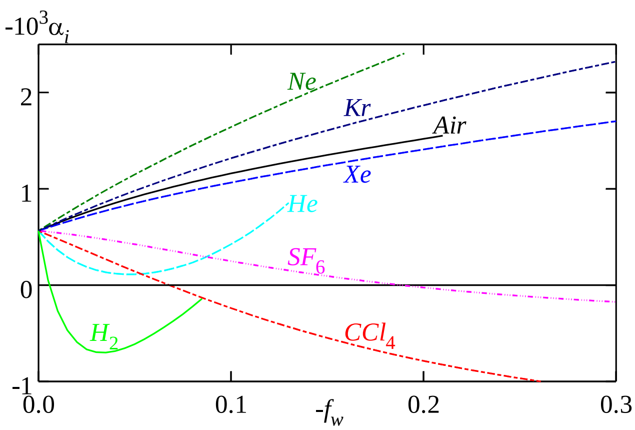

Fig.5: Nondimensional spatial amplification rate $-\alpha_i$ of 2D perturbations versus injection factor $-f_w$ for variety of foreign gases; $\mathrm{Re} = 720$, $F{\cdot}10^6 = 35$.

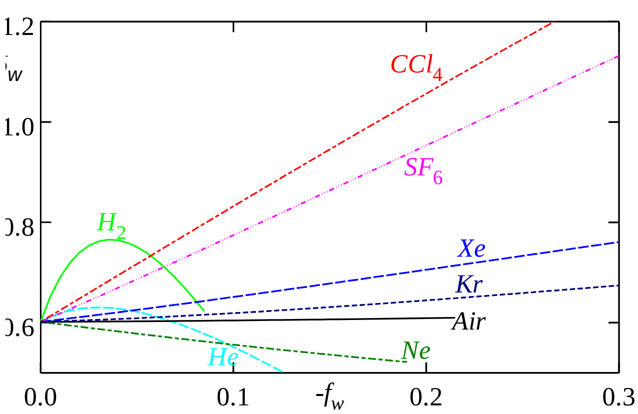

Fig.6: Mixture density at the wall $\bar{\rho}_w$ versus injection factor $-f_w$

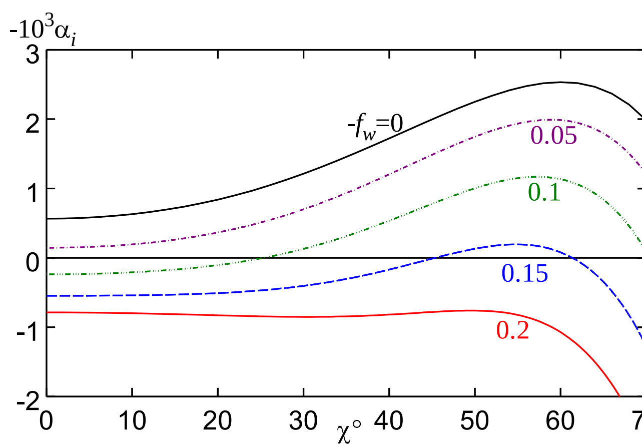

Fig.7: Nondimensional spatial amplification rates $-\alpha_i$ for 3D perturbations versus angle χ; injection of CCl_4, $x = 100$ mm $(\mathrm{Re} = 720)$, $f = 14$ kHz $(F{\cdot}10^6 = 35)$

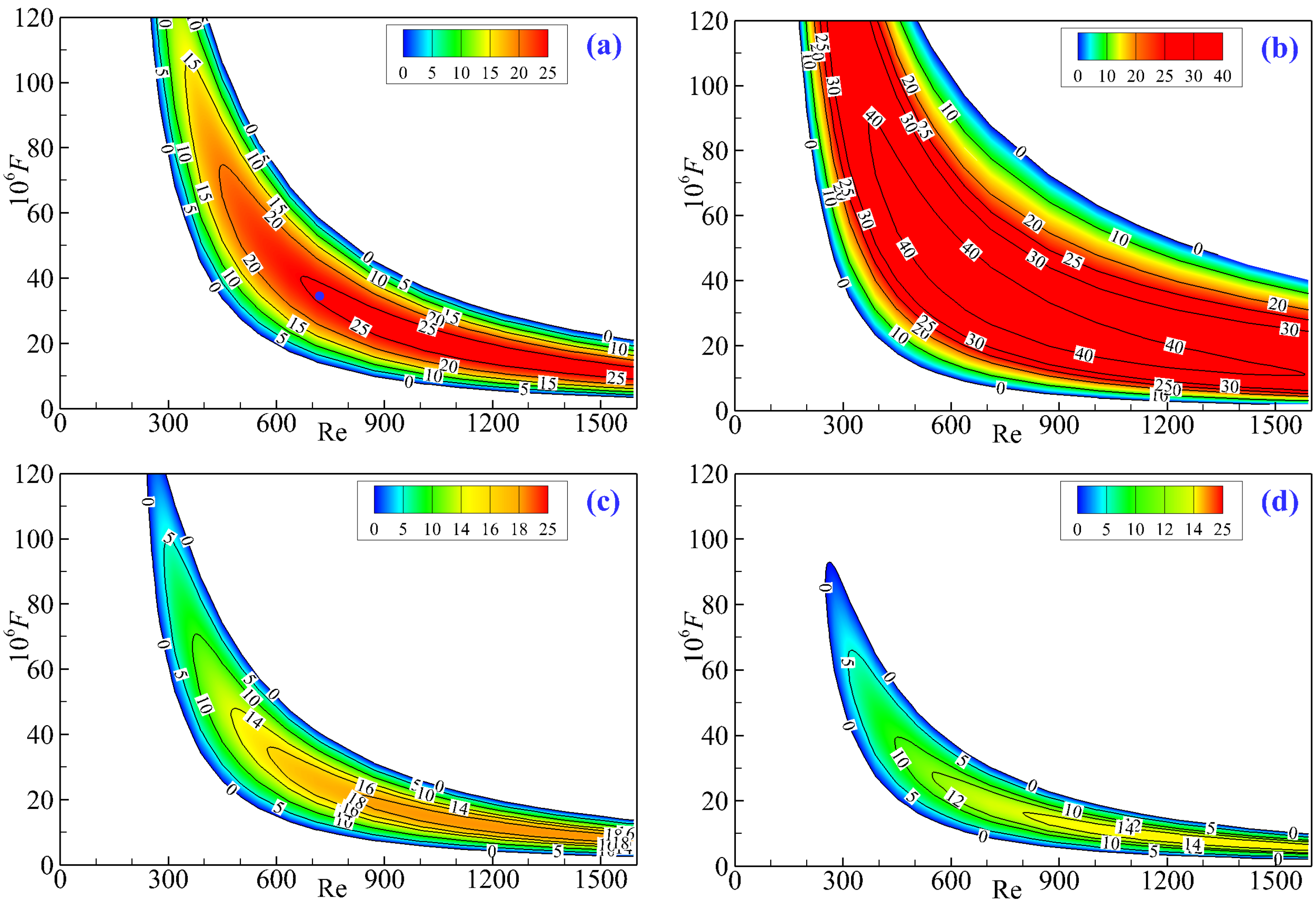

Fig.8: Mach=2 flat-plate boundary-layer stability-diagrams: nondimensional spatial amplification rate contours $-\alpha_i = -\alpha_i\left(\mathrm{Re}, F\right)\cdot 10^4$ for most unstable 3D perturbations with $\chi = 60^\circ$; zero injection (a); injection of *Air* with $-f_w = 0.1$ (b); injection of CCl_4 with $-f_w = 0.1$ (c) and injection of CCl_4 with $-f_w = 0.2$ (d).

higher in comparison to $-\alpha_i$ for 2D disturbance at $\chi = 0^\circ$. Increase of the injection factor $-f_w$ leads to monotonous reduction of disturbance amplification rates for all values of wave-vector orientation angles χ. One can see from Fig.7 that at $-f_w = 0.2$ perturbations with $F\cdot 10^6 = 35$ and with all angles $0 \le \chi \le 70^\circ$ are completely stabilized by the injection of heavy gas tetrachloromethane.

Until now in our discussion of the stability properties we dealt only with a local disturbance with $f = 14$ kHz at $x = 100$ mm. Following figure presents a global overview of the linear stability characteristics of the binary mixture supersonic boundary layer. Fig.8 demonstrates stability diagrams of the supersonic Mach=2 flat-plate boundary-layer as contour lines of nondimensional spatial amplification rates for the most unstable 3D-waves on the plane Reynolds number Re – reduced frequency F: $-\alpha_i = -\alpha_i\left(\mathrm{Re}, F\right)$. Color flooded area corresponds to the region of the linear instability where fluctuations are amplifying downstream. Figs.8 have been produced for injection of *Air* and tetrachloromethane with various factors $-f_w$. Color scheme on all the stability diagrams at Figs.8a,b,c,d is taken the same to make the comparison of computed results easier.

Comparison of Fig.8a and Fig.8b shows that moderate injection of *Air* leads to a considerable destabilization of the boundary layer: maximal growth rate is enlarged from $-\alpha_{i,\max} \approx 0.0027$ at zero injection to $-\alpha_{i,\max} \approx 0.0043$ at injection of the *Air* at $-f_w = 0.1$. Also the unstable frequency range becomes almost double wider with the injection of *Air*. This means that the onset of turbulence in the boundary layer with the *Air* injected from the permeable surface should become much earlier in comparison with the boundary layer with zero injection. At the same time comparison of Fig.8a, Fig.8c and Fig.8d reveals that increasing of the injection factor for tetrachloromethne leads vice versa to a noticeable boundary layer stabilization: maximal growth rate is reduced to $-\alpha_{i,\max} \approx 0.0019$ at $-f_w = 0.1$ (Fig.8c) and further to $-\alpha_{i,\max} \approx 0.0015$ at $-f_w = 0.2$ (Fig.8d). Simultaneously unstable frequency range is appreciably reduced and shifted to lower frequencies. At last, comparison of Fig.8b and Fig.8c

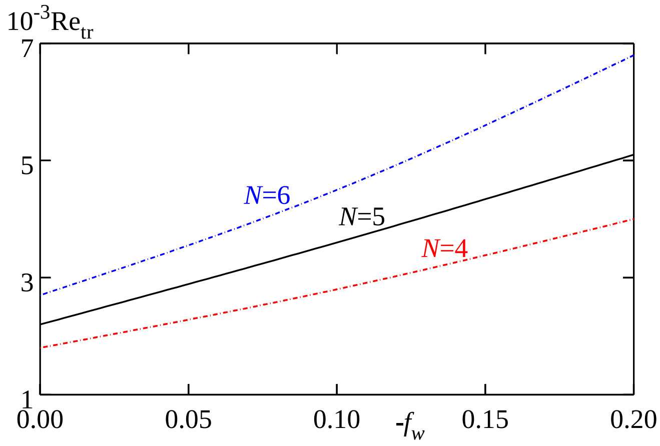

Fig.9: Transition Reynolds number Re_{tr}^{N} versus injection factor $-f_w$ for *CCl*4, as calculated by the e^N-method; $\mathrm{M}=2$.

shows that injection of the *Air* and CCl_4 at the same injection factor $-f_w = 0.1$ leads to the opposite effect: heavy gas stabilizes the flow in the supersonic Mach=2 boundary layer, while injection of the gas with the same molecular weight destabilizes the flow.

Linear stability theory gives a possibility to estimate the position of the laminar- turbulent transition by means of the well known e^N-method. According to this method the transition happens at the streamwise location where the disturbance amplification factor reaches a certain threshold e^N, where the factor $N = -\int_{\mathrm{Re}_0}^{\mathrm{Re}_{tr}} 2\,\mathrm{Im}(\alpha)\,d\,\mathrm{Re}$ has a certain value. Since amplitude and spectrum of external disturbances vary in different wind tunnels and in the flight, the transition appears at different values of *N*. Therefore, in this paper the estimate of the transition Reynolds number Re_{tr} has been performed for different values of the *N*-factor. The results for the injection of CCl_4 are presented at Fig.9. One can see that variation of the injection factor from zero to the value $-f_w = 0.2$ causes more than a double enlargement of the transition Reynolds number Re_{tr} independently of a specific value of the *N*-factor. This corresponds to a fourfold enlargement of the streamwise extent of a laminar region on a model.

VI. Conclusions

Two systems of equations to describe the compressible binary gas mixture boundary-layer and its linear stability in the conditions of a foreign gas injection from a permeable model surface have been developed.

Parametric calculations of the flat-plate boundary-layer profiles have been performed. It has been found that heavy gas injection influences the boundary-layer density profile similar to the influence of surface cooling. Both of them facilitate boundary-layer stabilization and noticeable laminar-turbulent transition delay. It is obtained that increase of a foreign gas injection leads to a reduction of the skin friction and the wall heat flux.

It is established, that injection of a heavy gas leads to the reduction of linear growth rates of disturbances. LST calculations in the framework of e^N-method have been performed to make an estimate of the position of the laminar-turbulent transition. A possibility to enlarge in four times the length of a laminar region in Mach=2 boundary layer by means of a heavy gas injection is found.

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