The effect of skewness and kurtosis on the probability evaluation of fatigue limit states

Z. Kala, A. Omishore, S. Seitl, M. Krejsa, J. Kala

Abstract — The article presents the probabilistic analysis of the limit state function of the fatigue resistance of a steel bridge, which is numerically described as the total number of cycles to failure. An example of the evaluation of the histogram of the fatigue resistance of a steel member using the Latin Hypercube sampling method and linear fracture mechanics based on Paris-Erdogan's law is presented. Probabilistic models of input variables for which the fatigue resistance has a typical log-normal probability density function are described. Differences between the stochastic analyses of one member with one edge crack and the entire steel bridge are discussed. Attention is paid in the limit state function to the effects of skewness and kurtosis of the fatigue resistance on the time-dependent failure probability.

Keywords—Fatigue, steel, bridge, limit state, reliability, fracture mechanics, stochastic, Monte Carlo.

I. Introduction

Steel is the most versatile and effective material for the construction of bridge structures, which is able to carry loads in tension, compression and shear. Designers of steel bridges are familiar with a number of guidelines and requirements needed in order to find the most economic solution for their clients. Modelling and limit states analysis are parts of the design process of effective structural arrangements of load bearing members for various loading conditions including fatigue. Structural mechanics and conventional structural stress analysis provide background material yielding conclusive findings on safety based on expert estimates of traffic intensity over their lifetime.

A well-designed and maintained bridge should serve many generations from its construction. However, this is not always the case. With increasing service life, the actual load intensity and fatigue wear of the load bearing members of bridge structures may more or less differ from the assumptions made in the design documentation. Most bridges are exposed to an ever-increasing number of passages of rail or road vehicles, which are high in intensity and are constantly being repeated. Repeated loading and unloading of a material leads to fatigue. Adverse effects of the environment and load result in slowly progressing and accumulating material damage. Repeated changes in the stress state of load bearing members lead to the initiation and propagation of fatigue cracks. A crack eventually reaches its critical size and propagates suddenly resulting in fracture of the structure. Deterioration of the load bearing structure results in a state of disrepair, degradation of the transport network and threatens traffic safety.

The end of the service life of a bridge results in its shutdown and demolition. When this happens, the girders of the steel bridge can be cut into smaller more manageable sizes in order to facilitate demolition, and transportation to steelworks for recycling. Recycling does not lead to degradation in performance. Around 99% of structural steel is either returned into the steelmaking process where it is used for the production of new steel products or reused. Component parts of steel bridges can also be reused in other structures. Entire bridges have been relocated. Thus, bridges can be designed with the knowledge that they can be easily relocated in the future.

In the future, we can expect further development of lighter and more subtle steel bridges and an increased need for advanced assessment of the remaining life of existing steel bridges. It is therefore necessary to improve the methods of structural analysis and adapt the concept of limits states so that they are applicable in probability analyses of the time-dependent reliability of steel bridges.

II. Fatigue Life Assessment of Steel Bridges

An overview of methods for predicting the fatigue life of steel structures has been published, for e.g. in [1-3]. A number of methods for the estimation of the remaining fatigue life of load bearing steel structures and steel bridges have been proposed in the past [4-6], some are based on probabilistic approaches [7-12]. A review of the state of the art of life cycle analysis of steel bridges, including reliability has recently been presented in [13-15].

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A. S-N curve method

In general, two approaches are used for evaluating the fatigue damage and predicting the life of bridges. The first approach (i) is the stress-life method based on the *S-N* curve, which is determined using appropriate fatigue experiments at constant-amplitude stresses and describes the relationship between the stress range *S* and the number of cycles to failure *N*. Basquin [16] presented the exponential relationship between *S* and *N*, whose graphical representation is the Wöhler curve with log *N* on the abscissa and log *S* on the ordinate. The Basquin function can be expressed mathematically as:

$$NS^{m} = A \tag{1}$$

or

$$\log N = -m \log S + \log A, \tag{2}$$

where *m* and *A* are positive empirical material constants.

Even though the use of *S-N* curves [17] is well established in the field of steel structures [18], information related to time-dependent load, particularly in connection with the detection of cracks from measurements during operation of the bridge, cannot be included in the evaluation of reliability [19]. The principal disadvantage of this approach is that it evaluates all stages of the fatigue process together. Thus, it involves the initiation of a fatigue crack and its propagation up to its critical length ending with a brittle fracture. It does not determine the number of cycles that belong to each stage. This approach cannot be used to determine the critical length of the crack or the critical stress in the structure with an existing crack with respect to other external influences like temperature. Nor can we competently determine the remaining service life of such a structure.

B. Fracture mechanics approach

The second method (ii) is based on fracture mechanics and is dedicated to investigating the features and disciplines pertaining to the initiation and growth of cracks with regard to the stress field at the crack tip. Cumulative damage is characteristic for fatigue. Commonly applied linear elastic fracture mechanics analyses the propagation of initial cracks of size *a* in dependence to the number of fatigue cycles *N*. The growth of fatigue cracks is generally described by Paris's rule, which is expressed by Paris and Erdogan [20]

$$\frac{da}{dN} = C\left(\Delta K\right)^{m}, \tag{3}$$

where *m* and *C* are material-related parameters and the range of stress intensity factor ΔK can be determined by Broek [21].

$$\Delta K = \Delta\sigma\sqrt{\pi a}\,F\left(a\right), \tag{4}$$

where $F(a)$ is the geometric factor (calibration function) describing the course of crack propagation with respect to the geometry of the sample and $\Delta\sigma$ is the quasi–constant stress range.

Approaches (i) and (ii) are applied sequentially, with (i) being increasingly used in the design stage of bridges or the preliminary evaluation of the fatigue life. Method (ii) is used for the analysis and assessment of the remaining fatigue life, for e.g., in connection with applications of decision making methods in civil engineering [22, 23].

C. Probabilistic Fatigue Life of One Structural Member

An example of probabilistic analysis of the fatigue resistance due to uncertainty of input parameters was published in [24]. The number of cycles to failure N_F (fatigue resistance) of the steel member can be evaluated using linear fracture mechanics as the number of cycles leading to the propagation of initial edge crack a_0 into a critical crack a_{cr}.

$$\int_{a_0}^{a_{cr}} \frac{da}{\left[F\left(a\right)\cdot\sqrt{\pi\cdot a}\,\right]^{m}} = C\cdot N_F\cdot\Delta\sigma^{m}, \tag{5}$$

where N_F is the total number of cycles at crack growth from a_0 to a_{cr}. The quasi–constant stress range $\Delta\sigma$ = 50 MPa is considered. *C*, *m* are material constants according (6)

$$\log\left(C\right) = c_1 + c_2 m, \tag{6}$$

where c_1, c_2 can be considered for the steel grade S235 as c_1 = -11.141, c_2 = -0.507 [25]. $F(a)$ is the calibration function evaluated for pure bending in the form [26, 27]:

$$F\left(a\right) = 1.114 - 1.8975\left(\frac{a}{W}\right) + 2.752\left(\frac{a}{W}\right)^{2} - 1.1323\left(\frac{a}{W}\right)^{3}, \tag{7}$$

where *a* is the crack length and *W* is the specimen width in the direction of crack propagation. In accordance with [24] let us consider the parameters of the fatigue process as random variables with probability density functions (pdfs) acc. to Table 1.

TABLE I

INPUT RANDOM QUANTITIES - VARIANT 1

Characteristic	Pdf	Mean value	St. deviation
Initial crack size a_0	log-normal	0.526 mm	0.504 mm
Critical crack size a_{cr}	Gauss	175 mm	14 mm
Specimen width *W*	Gauss	400 mm	20 mm
Parameter *m*	Gauss	3	0.03

The aim of the study is to determine the effects of the pdf of input variables on the output N_F. For this purpose a second variant of input variables acc. to Table 2 is considered.

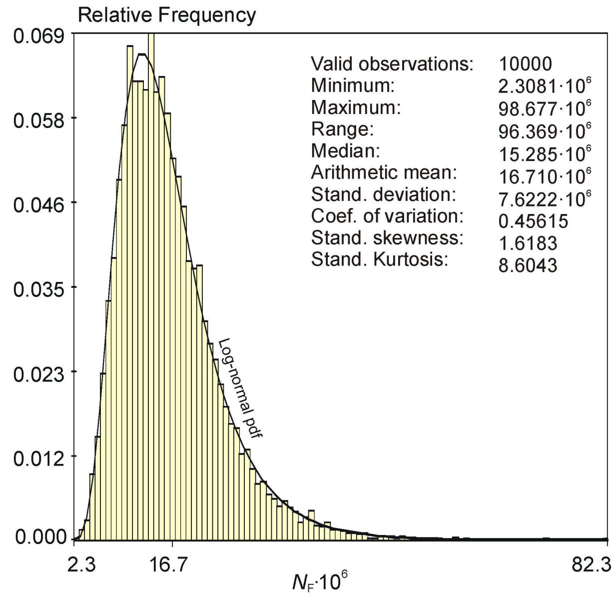

Fig. 1 Lognormal pdf fitting of histogram of N_F for variant 1

TABLE II
INPUT RANDOM QUANTITIES - VARIANT 2

Characteristic	Pdf	Min value	Max value
Initial crack size a_0	Rectangular	0.1 mm	1.1 mm
Critical crack size a_{cr}	Rectangular	151 mm	200 mm
Specimen width W	Rectangular	365.36 mm	434.64 mm
Parameter m	Rectangular	2.94804	3.05196

The second variant considers all input random variables with Rectangular pdf. Histograms of N_F of both variants are evaluated using ten thousand simulation runs of the Latin Hypercube Sampling method [28, 29] and are depicted in Fig. 1 and Fig. 2.

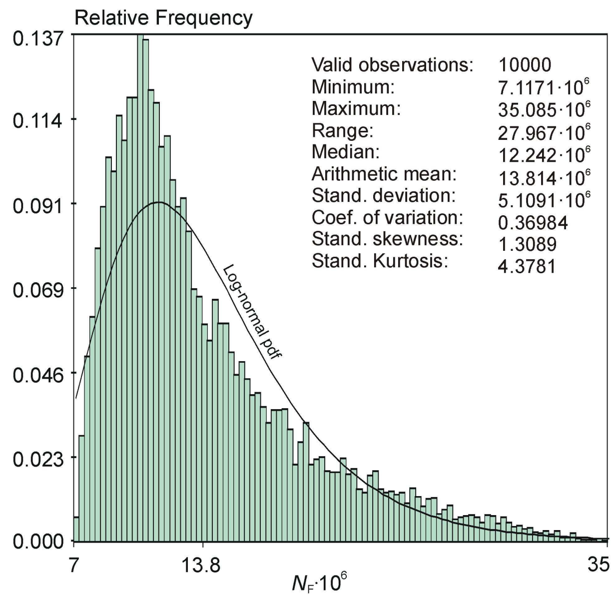

Fig. 2 Lognormal pdf fitting of histogram of N_F for variant 2

The mean value of N_F is 16.7×10^6 for variant 1 and 13.8×10^6 for variant 2, see Fig. 1 and Fig. 2. The standard deviation of N_F is 7.62×10^6 for variant 1 and 5.11×10^6 for variant 2. For clarity, the histogram in Fig.1 is displayed on a shortened interval 2.3×10^6 to 82.3×10^6, even though the maximum value N_F was 98.677×10^6. The Chi-square goodness-of-fit test [30] with the final p-value = 0.05 was carried out to verify the log-normality of the N_F distributions. The hypothesis should not be rejected for variant 1 and should be rejected for variant 2. If a_0 has a log-normal pdf then N_F has a pdf very close to log-normal. However, the result of the study pertains to one structural member under bending. In the general context, it must be assumed that the fatigue resistance of a steel bridge with numerous structural elements involved in serial or parallel interactions will have a different type of pdf from the log-normal.

D. *Probabilistic Fatigue Life Assessment of Steel Bridges*

The development of probabilistic assessment of reliability of existing steel structures and bridges cannot be considered complete in any case. The process of fatigue involves a certain degree of stochastic randomness, which often displays considerable scatter even in seemingly identical samples in well controlled environments. The fatigue performance of steel bridges is dependent on a number of factors, for e.g. material characteristics, stress history, and the environment, which exhibit uncertainty and randomness during the service life of the bridge. On the contrary, when assessment of the fatigue condition is performed using field measurement data, the uncertainties pertinent to the field-measured data and the inaccuracies due to data processing procedures are subsistent and unavoidable. Due to these reasons it is more appropriate to perform the assessment of the fatigue life using probabilistic rather than deterministic methods.

III. The Probabilistic Concepts of Limit States

A. *The basic concept of design reliability conditions*

The basic application of the probabilistic concept is using the reliability index β. This approach is based on expressing the limit-state function in terms of the static resistance R and the load action effect A.

$$G = R - A \geq 0\,, \tag{8}$$

The pdfs of random variables R, A are often introduced as Gauss pdfs [31-33] in the limit states of steel structures exposed to the dominant effects of static load. The random variability of R is examined using analytical [34] or numerical [35, 36] stochastic computational models. The basic characteristics of the reliability condition (8) as a random variables R and A with Gauss pdf in the case of statistical independence of these variables are expressed as:

$$m_{\mathrm{G}} = m_{\mathrm{R}} - m_{\mathrm{A}}\,, \tag{9}$$

$$S_{\mathrm{G}} = \sqrt{S_{\mathrm{R}}^2 + S_{\mathrm{A}}^2}\ , \tag{10}$$

$$a_{\mathrm{G}} = \frac{S_{\mathrm{R}}^3\ a_{\mathrm{R}} - S_{\mathrm{A}}^3\ a_{\mathrm{A}}}{S_{\mathrm{G}}^3}\ , \tag{11}$$

where symbols *m, S, a* represent the mean value, standard deviation and skewness of random variables *G, R, A*. In the case of Gauss pdf of *G*, the failure probability P_f with implementation of normalised random variable

$$T = \frac{G - m_{\mathrm{G}}}{S_{\mathrm{G}}}\ , \tag{12}$$

is expressed as

$$P_{\mathrm{f}} = P(G < 0) = P\left(T < -\frac{m_{\mathrm{G}}}{S_{\mathrm{G}}}\right) = P(T < -\beta) = \Phi(-\beta), \tag{13}$$

where Φ(•) is the normalized Gauss pdf. The so-called reliability index is implemented in expression (13):

$$\beta = \frac{m_{\mathrm{G}}}{S_{\mathrm{G}}}\ , \tag{14}$$

If these assumptions are met the verification of reliability can be performed by comparing the calculated index β with the target value of the reliability index β_d listed by the design rules in standard EN1990 [37].

$$\beta \geq \beta_{\mathrm{d}}\ , \tag{15}$$

Expressing (15) using (14), (9), (10) is obtained:

$$\beta = \frac{m_{\mathrm{R}} - m_{\mathrm{A}}}{\sqrt{S_{\mathrm{R}}^2 + S_{\mathrm{A}}^2}} \geq \beta_{\mathrm{d}}\ , \tag{16}$$

Direct applications using the formula (16) in real tasks are often prevented by the non-fulfillment of the assumption that the random variables of the limit state (8) have Gauss pdfs.

B. Design reliability conditions for fatigue limit state

The fatigue limit state function in steel structures such as steel girder bridges can be expressed in terms of two variables:

$$Q = \frac{N_F}{N_A} \geq 1\ , \tag{17}$$

where N_F denotes the fatigue resistance (number of cycles to failure under the given stress history), and N_A denotes the number of applied load cycles. N_F and N_A are statistical independent random variables with mean values m_{NF}, m_{NA} and standard deviations S_{NF}, S_{NA}. Derivation of the design reliability condition for (17) is based on the logarithmic transformation [38].

$$\ln Q = \ln(N_F) - \ln(N_A) \geq 0\ , \tag{18}$$

where random variables ln(N_F) and ln(N_A) have log-normal pdfs. The mean value and standard deviation of ln(N_F) are defined as:

$$m_{\ln(N_F)} = \ln(m_{NF}) - 0.5 \cdot S_{\ln(N_F)} \approx \ln(m_{NF})\ , \tag{19}$$

$$S_{\ln(N_F)} = \sqrt{\ln\left(1 + V_{NF}^2\right)} \approx V_{NF}\ , \tag{20}$$

where V_{NF} is the coefficient of variation of N_F; the approximate equality ≈ can be applied for small values of V_{NF}.

The mean value and standard deviation of ln(N_A) are defined as:

$$m_{\ln(N_A)} = \ln(m_{NA}) - 0.5 \cdot S_{\ln(N_A)} \approx \ln(m_{NA})\ , \tag{21}$$

$$S_{\ln(N_A)} = \sqrt{\ln\left(1 + V_{NA}^2\right)} \approx V_{NA}\ , \tag{22}$$

where V_{NA} is coefficient of variation of N_A; the approximate equality ≈ can be applied for small values of V_{NA}. The reliability condition based on the comparison of the calculated reliability index β with the target reliability index β_d in [37] can be expressed as:

$$\beta = \frac{m_{\ln(N_F)} - m_{\ln(N_A)}}{\sqrt{S_{\ln(N_F)}^2 + S_{\ln(N_A)}^2}} \geq \beta_{\mathrm{d}}\ . \tag{23}$$

Similarly to (16) the application of (23) is often limited due to the non-fulfillment of the condition that the random variables of the limit state (18) have log-normal pdfs. From a technical point of view, this assumption is approximately fulfilled if the random variables in (17) have a log-normal or at least Gauss pdf with small variation coefficients. As shown in Fig.1, we can consider log-normal pdfs for N_F if a_0 has a log-normal pdf and the other random variables have Gauss pdf as is listed in Table 1. These assumptions are satisfied relatively accurately in the case of one structural element with one initial crack. However, it is questionable if the use of a log-normal pdf is also sufficiently adequate for the analysis of the fatigue limit state of a bridge in which there exist many stochastic interactions between fatigue (or even corrosive) degradation of its structural elements.

In actuality, the propagation of the fatigue crack leads to changes in the stiffness and stress state of the local structural detail, which in turn further affect the crack growth behaviour. Either a global-local crack model of the structure for the implementation of crack propagation analysis or an isolated local crack model with boundary conditions, which are not affected by local stiffness changes resulting from the crack propagation is necessary for this interaction problem [39].

Skewness and kurtosis are two other statistical moments, which should be considered in the probabilistic assessment of limit states, see e.g. [40, 41]. One way to include the effects of skewness and kurtosis in the probabilistic assessment of reliability is to consider N_F and N_A as random variables with Hermite pdf.

C. Hermite probability density function

The design conditions of reliability described in the previous two subchapters have considerable limitations. Probabilistic models of limit states are derived under the assumption of two-parameter Gauss or log-normal pdf, which may not always fit the distributions of probability density functions of the random variables of the limit states with sufficient accuracy. More generally, we encounter random variables whose pdf shapes are approximately bell-shaped with small values of skewness and kurtosis. These random variables are better approximated using a four-parameter Hermite pdf (24), which consists of a Gauss pdf $\varphi_G(x)$ multiplied by the Hermite polynomial.

$$\varphi_H(x) = \varphi_G(x)\left[1 + \frac{a}{6}(x^3 - 3x) + \frac{(k-3)}{24}(x^4 - 6x^2 + 3)\right]. \qquad (24)$$

A detailed derivation of the Hermite pdf is in [42], however, other more sophisticated variants of the Hermite pdf are available, see e.g. software Statrel 3.1.

Parameters a, k in (24) describe the skewness and kurtosis relatively accurately if parameter a is at a small distance from zero and parameter k is at a small distance from three, otherwise they match the skewness and kurtosis only approximately. The function (24) cannot be used for arbitrary parameters a, k, because the condition $\varphi_H(x) \geq 0$ is not automatically fulfilled for $\forall x \in (-\infty, \infty)$. Let interval $[x_L, x_R]$ define the region around the mean value where $\varphi_H(x) \geq 0$. If the interval $[x_L, x_R]$ is sufficiently wide, (24) can be replaced with a truncated Hermite pdf (25) defined on the interval $x \in [x_L, x_R]$.

$$\overline{\varphi}_H(x) = c_6 \varphi_H(x) \quad \begin{cases} \overline{\varphi}_H(x) \geq 0 & \forall\ x \in \langle x_L, x_R \rangle \\ \overline{\varphi}_H(x) = 0 & \text{otherwise} \end{cases} \qquad (25)$$

where parameter c_6 is calculated numerically [42] so that (25) has all the required properties on the interval $[x_L, x_R]$. Dependencies between a, k vs threshold x_L and a, k vs threshold x_R are described and graphically presented in [42].

For practical use of pdf (25) it is useful to have a detailed mapping of parameters a, k, which must be assigned in order for (25) to attain the desired values of skewness and kurtosis, see Fig. 3 to Fig. 6. The normalized Gauss pdf $\varphi_G(x)$ is considered in the present study. Fig. 3 displays 100 discrete points of parameter a realized with constant spacing on the interval -1.1 to 1.1 and 100 discrete points of parameter k realized with constant spacing on the interval 1.4 to 4.6. The generation of pairs a, k can be clearly written in the programming language Pascal:

```
max:=100;                                                    (26)
For i1:=0 to max do
For i2:=0 to max do
begin
 a:=-1.1+2.2*i1/max;
 k:=1.4+3.2*i2/max;
end;
```

The values of skewness and kurtosis shown in Fig. 3 are evaluated for 100^2 pairs of a, k using [42] (26), (27). The results in Fig. 4 are evaluated for max:=200 so that only points for skewness that are approximately equal to -1, -0.9, ..., 1 are displayed. The aim of the depiction in Fig. 4 is the plot of the values of parameter a in dependence on the values of skewness and kurtosis.

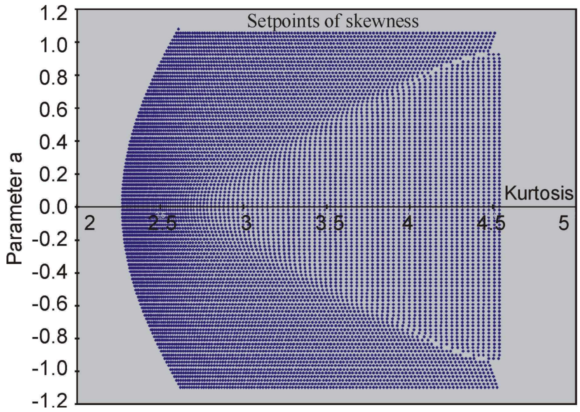

Fig. 3 The set of points of parameter a vs skewness and kurtosis

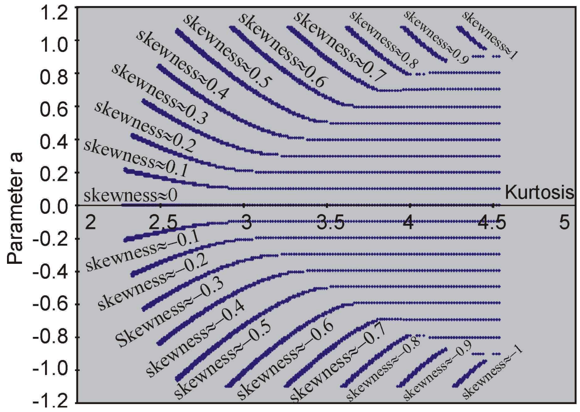

Fig. 4 The set of points of parameter a vs skewness and kurtosis

Similarly, Fig. 5 and Fig. 6 show the values of parameter k in dependence on the values of skewness and kurtosis.

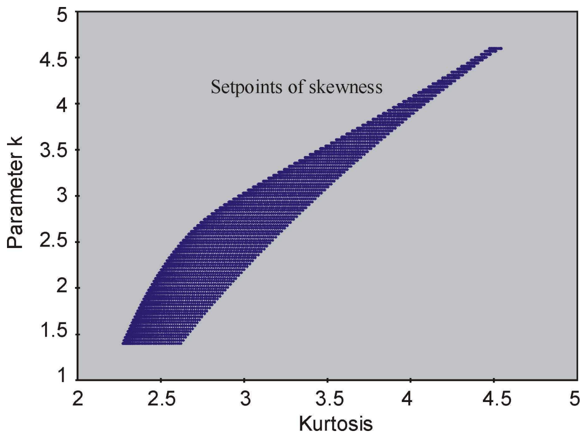

Fig. 5 The set of points of parameter k vs skewness and kurtosis

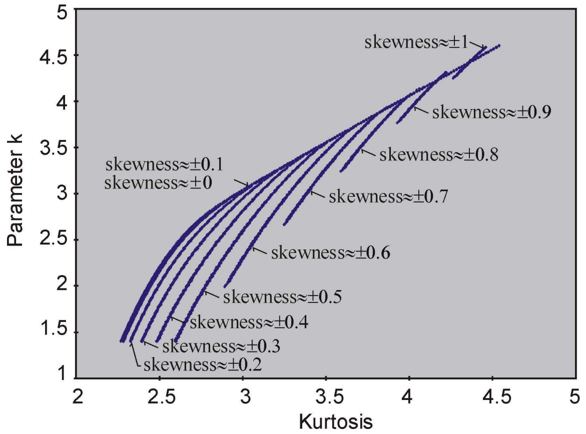

Fig. 6 The set of points of parameter k vs skewness and kurtosis

The same number of pairs a, k that were used for the depictions in Fig. 3 to Fig. 6 were used for Fig. 7 to Fig. 10, which show threshold x_L and x_R.

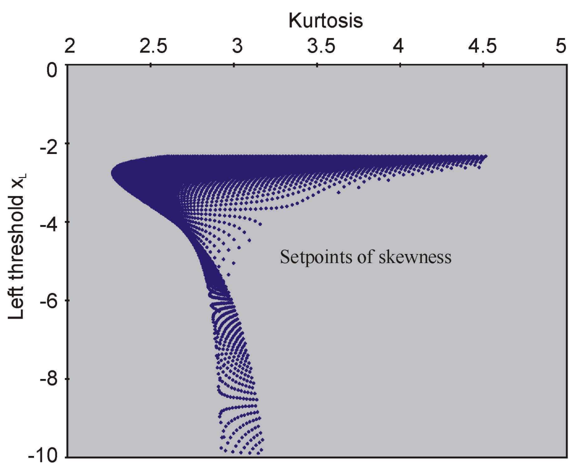

Fig. 7 The set of points of parameter x_L vs skewness and kurtosis

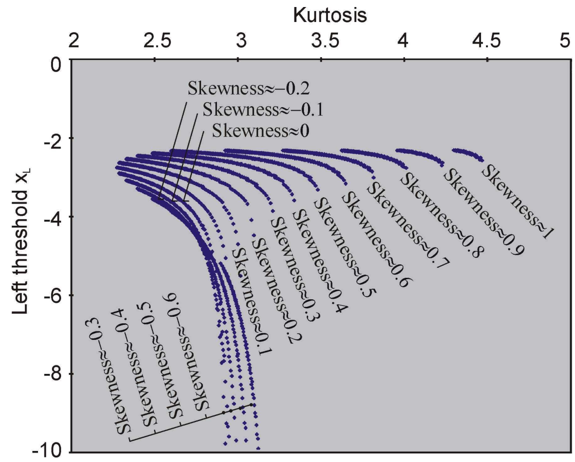

Fig. 8 The set of points of parameter x_L vs skewness and kurtosis

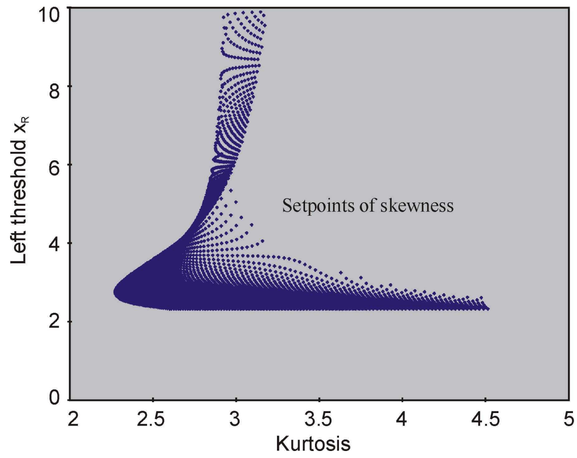

Fig. 9 The set of points of parameter x_R vs skewness and kurtosis

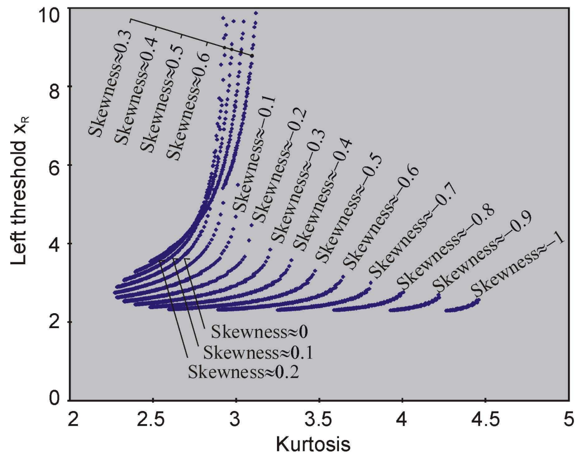

Fig. 10 The set of points of threshold x_R vs skewness and kurtosis

The outputs shown in Fig. 3 to Fig. 10 enable more practical usage of (25) using the input values of skewness and kurtosis. It should be noted that the threshold values shown in Fig. 7 to Fig. 10 are valid only for the normalized Gauss pdf $\varphi_G(x)$ and in the case of the general assignment of the mean value and standard deviation the values of x_L are multiplied by the standard deviation and the mean value is added; and correspondingly for x_R. The outputs shown in Fig. 3to Fig. 6 are generally valid for arbitrary assignment of the mean value and standard deviation.

IV. THE FAILURE PROBABILISTIC ANALYSIS

Let us consider parameters N_F and N_A of the limit state function (17) as random variables with variation coefficient $V_{NF}=V_{NA}=0.2$, where parameter N_F has a Hermite pdf and parameter N_A has a Gauss pdf. Pdfs of the considered N_F, N_A do not allow the assessment of reliability via calculation of the reliability index β in the close form according (23), therefore, the probability assessment is performed using the Monte Carlo method. The average number m_{NF} of cycles to failure is 1.2×10^8 and the average number m_{NA} of applied load cycles is 1200 per day. As the number of applied load cycles increases over time, the probability of the fatigue limit state increases.

The aim of the probabilistic analysis is to quantify the effect of skewness and kurtosis of parameter N_F on the probability of failure P_f. Skewness and kurtosis are not as popular parameters as the first two statistical moments, and the question is, what is their influence on the results of probabilistic analysis of failure?

The probabilistic analysis was evaluated using fifteen million simulation runs of the Monte Carlo method, which enabled the numerical calculation of very small values of the probability of failure. Results of the probabilistic study are shown in Fig. 11 to Fig. 18.

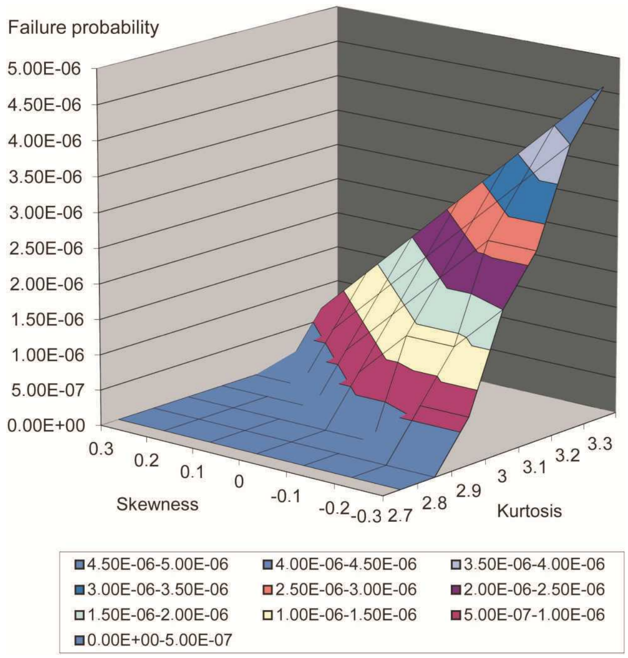

Fig. 11 Failure probability in the 1st year of operation

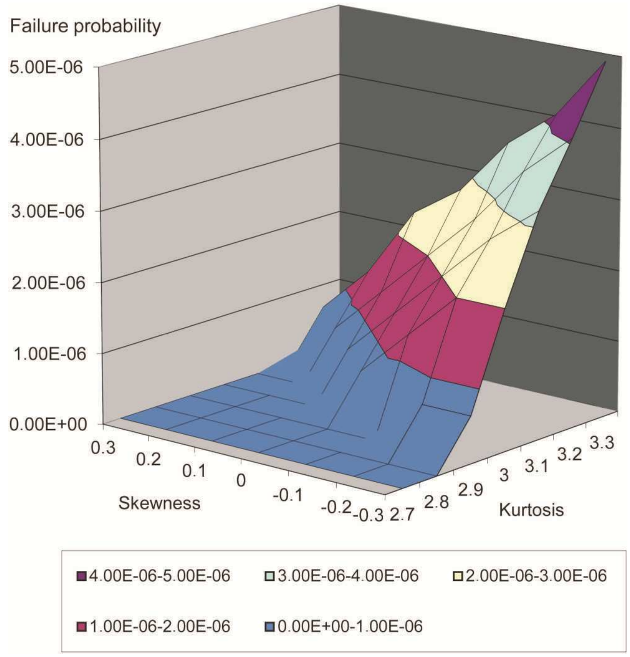

Fig. 12 Failure probability in the 2nd year of operation

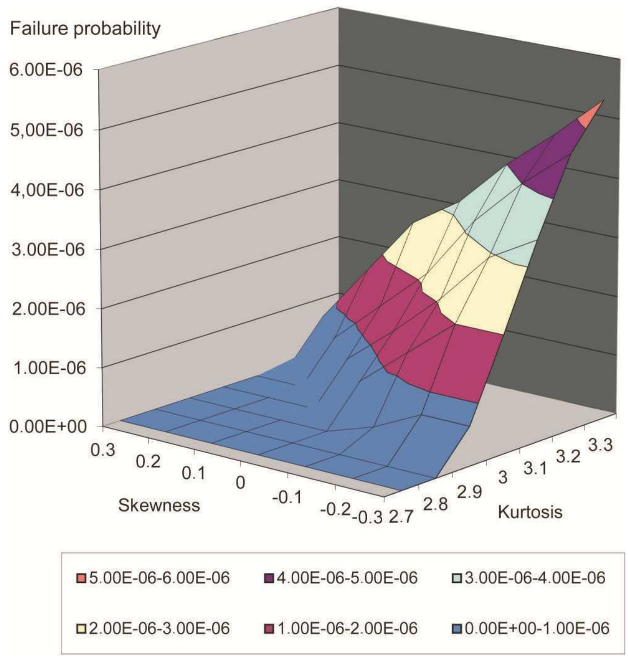

Fig. 13 Failure probability in the 3rd year of operation

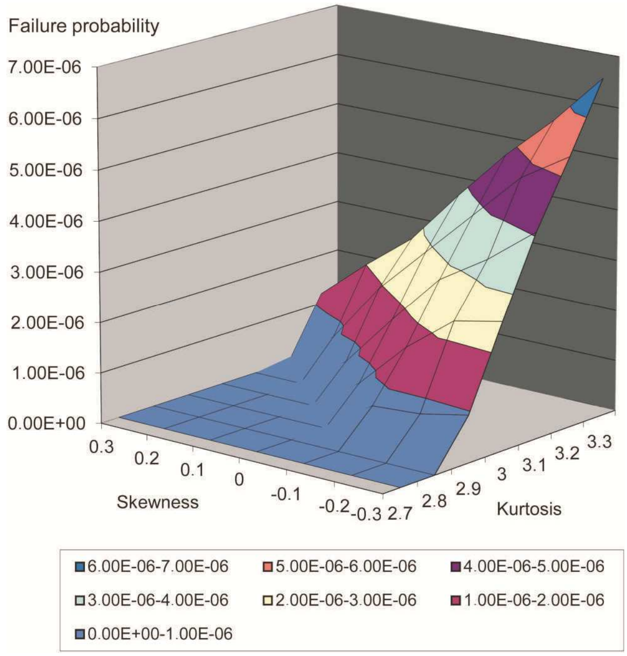

Fig. 14 Failure probability in the 5th year of operation

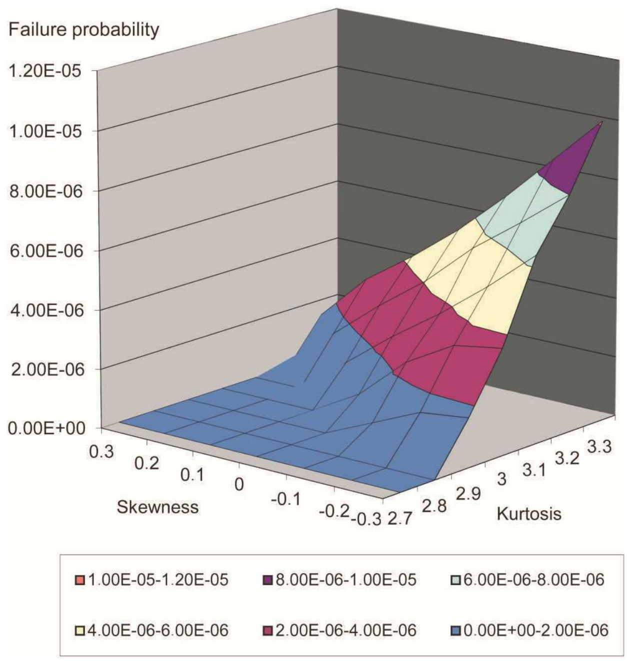

Fig. 15 Failure probability in the 10th year of operation

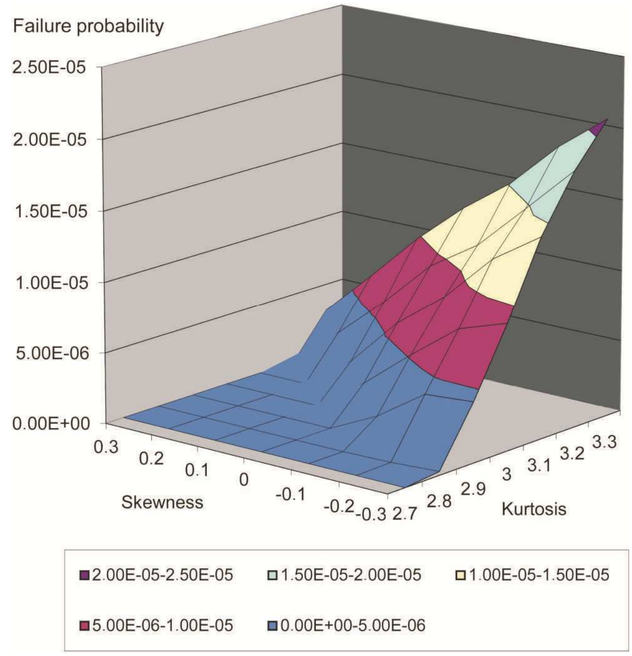

Fig. 16 Failure probability in the 20th year of operation

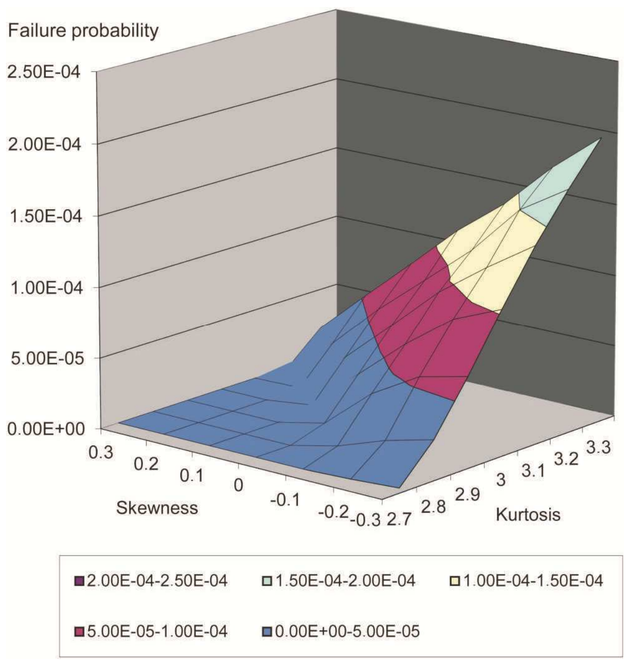

Fig. 17 Failure probability in the 50th year of operation

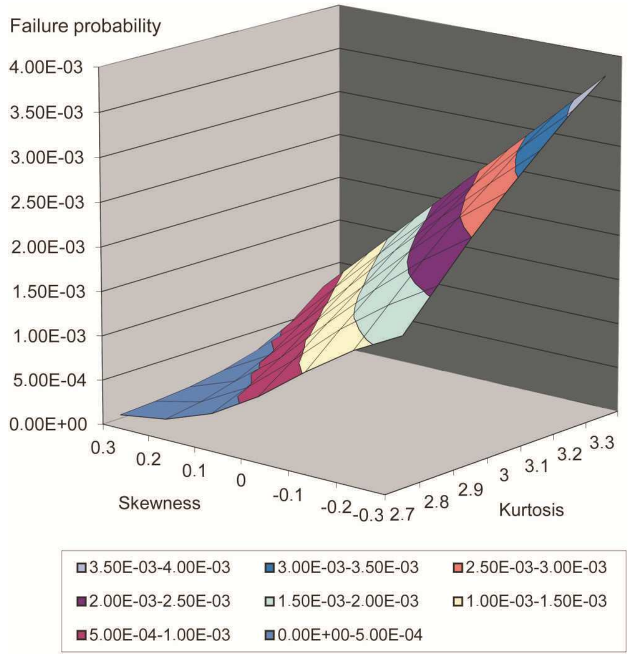

Fig. 18 Failure probability in the 100th year of operation

V. Conclusion

The high effects of skewness and kurtosis of fatigue resistance N_F on the failure probability are apparent from the results of the probabilistic analysis of reliability. Consideration of the influence of skewness and kurtosis of structure resistance and load effects can be an important part of the probabilistic analysis of the limit states of steel structures. The example evaluated using linear fracture mechanics and the LHS method has shown that the fatigue resistance of the steel element with an initial random crack has negligible values of skewness and kurtosis and its pdf may differ from the log-normal pdf. Generally, pdfs of the random variables of fatigue limit states may exhibit significant deviations from Gauss or log-normal pdfs, which can greatly reduce the accuracy of the results obtained from standard design reliability conditions based on reliability index β [37]. The implementation of a four-parameter Hermite pdf (25) together with applications of Monte Carlo simulation methods, as was demonstrated in the present article, offer a more general approach.

It is desirable that the time-dependent change in the reliability of the steel bridge is evaluated using histograms based on detailed design documentation and detailed numerical analysis based on elastic fracture mechanics of the individual load bearing structural elements with respect to their stochastic interactions. Histograms of random variables of limit states can be approximated using appropriate pdfs based on the evaluation of the goodness-of-fit tests. The random number of cycles to failure based on the assumption of a log-normal pdf can be used only when the goodness-of-test does not reject the hypothesis of this pdf. The numerical observations can also be generalized in relation to other limit states that occur in bridges. In addition to the random size of initial cracks, the calculations must also include the random effects of material and geometric characteristics of steel samples whose investigation is the subject of intensive experimental research [43-46] and numerical FEM modelling [45, 47].

From a structural point of view, the fatigue life is significantly influenced by the shape of the structure, for e.g. square holes or sharp corners, which entail increased local stresses and the associated increased risk of the initiation of fatigue cracks. Therefore, round holes and smooth transitions and fillets increase the fatigue strength of structures. Stress states and forms of failure are commonly studied in engineering practice using the finite element method [47, 49], which provides information usable in numerical analyses of linear fracture mechanics.

It is apparent from the numerically obtained results shown in Fig. 11 to Fig. 18 that the failure probability reaches higher values for low values of skewness and high values of kurtosis. With increasing operation time increases the probability of failure. However, the differences between the stochastic analysis of a single element with one crack and the entire steel bridge can be large. With regard to the number of inaccuracies present in probabilistic calculations, targeted time specific inspections of bridge structures, during which the size of already measured cracks are controlled among other things, is significant for the analysis of the propagation of fatigue cracks. The obtained information can be included in the calculation of conditional probabilities, which enriches the evaluation of the fatigue limit state with assumptions, that no failure has occurred before inspection time or that the fatigue damage is within the admissible limits [50].

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