Modeling, regulation and optimization of thermal flux and temperature to a radiation shield between the two planar surfaces

Sabrije F. Osmanaj, Rexhep A. Selimaj *

Abstract - The objective of the paper is the analysis about modeling, regulation and optimization of the radiation thermal flux and temperature between two planar surfaces, considering use of a single shield in a two surface enclosure. To this aim, in the second part we present the main contributions to the radiation heat transfer theory, in general, including equations and other data for planar surfaces without shields, with one and n – shields. In the third part we present the equations that describe modeling of the radiation specific thermal flux and temperature by parallel planes with a radiation shield, including two cases for parallel planes - when their emissivity are the same and different. For comparison in terms of heat radiation transfer, for simulation and analysis are used three types of materials where it is shown the impact of the thermal-physical parameters to the shield radiation, temperature and heat flux over time.

Keywords - Thermal flux, temperature, radiation shield, planar surfaces.

I. INTRODUCTION

There are many influential ways to minimize heat losses when radiation heat transfer is regnant mode. One way of reducing radiant heat transfer between two particular surfaces is to use materials which are highly reflective. Using radiation shield between heat exchange surfaces is other possibility that is open to scientists. Radiation shield can be used to reduce the net radiation transfer between two surfaces [1]. Our goal consists in showing how apparently intractable problems in heat transport by radiation can be easily solved using the concept of net radiation transfer [6].

Nomenclature

σ=5.86·10^{-8}	Stefan–Boltzmann constant, W/(m^{2}K^4)
$C_c=\sigma\cdot 100^4$	Constant, W/(m^{2}K^4)
$J=\dot{Q}/A$	Radiosity, W/m^2
A	Area, m^2
$\dot{Q}$	Radiant flux, W
$\dot{q}$, $\dot{q}_{1,s}$, $\dot{q}_{s,2}$	Radiant specific flux, W/m^2
ε	Emissivity, dimesionless
T, T_{mes}, T_{AB}	Temperature, K
T_s	Radiation shield temperature, K
$E_b=\sigma T^4$	Irradiance, W/m^2
F	View factor or shape factor
B, B_1, B_2	Constant, W/(m^{2}K^4)
$\dot{m}_A$	Specific mass, kg/m^2
c	Specific heat capacity, J/(kgK)
K, K_B	Time constant, s

II. RADIATION TRANSFER BETWEEN TWO PARALLEL SURFACES ENCLOSURES WITHOUT SHIELDS

Simplest enclosure for which radiation exchange is exclusively between two surfaces and a single expression for the rate of radiation transfer may be inferred from a network representation of the exchange [2].

The radiation network of this two-surface enclosure consists of two surface resistances and one space resistance, as shown in Figure 1. In an electrical network, the electric current flowing through these resistances connected in series would be determined by dividing the potential difference between points *1* and *2* by the total resistance between the same two points [4].

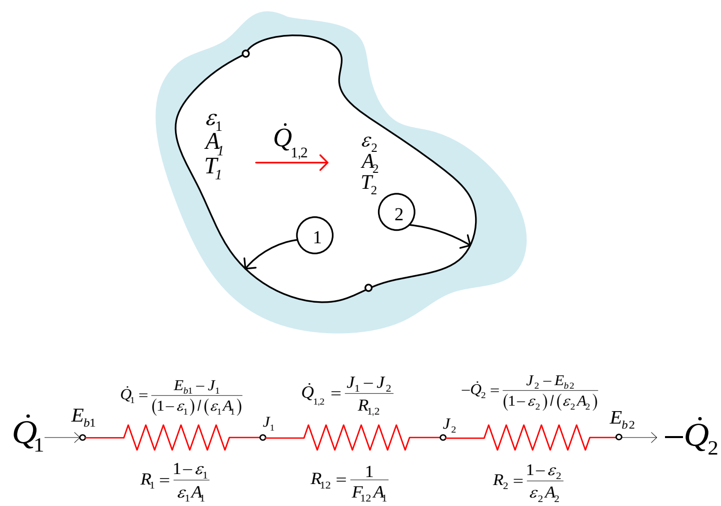

Fig. 1 Radiation network

The net rate of radiation transfer is determined in the same manner and is expressed as:

$$\dot{Q}_{1,2\text{noshield}} = \dot{Q}_1 = -\dot{Q}_2 = \frac{E_{b1}-E_{b2}}{R_1+R_{1,2}+R_2} = \text{, W} \quad (1)$$

$$= \frac{\sigma\left(T_1^4-T_2^4\right)}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}+\frac{1}{A_1F_{12}}+\frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

In case for large (Infinite) parallel planes, Fig. 2, eq. (1) takes the following form:

$$A_1 = A_2 \equiv A,\ F_{1,2} = 1$$

* The authors are with the University of Prishtina "Hasan Prishtina" Faculty of Mechanical Engineering,
Str. Sunny Hill n.n., 10000 Prishtina, Kosovo

$$\dot{Q}_{1,2\,\mathrm{noshield}} = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \quad (2)$$

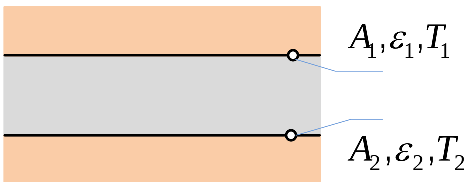

Fig. 2 Parallel planes

The eq. (2) is important and applicable to any two gray, diffuse, opaque surfaces that form an enclosure. The view factor $F_{1,2}$ depends on the geometry and must be determined first [3].

a. Radiation transfer between two parallel surfaces with one shield

Now consider a radiation shield placed between these two plates, as shown in Figure 3. Let the emissivities of the shield facing plates 1 and 2 be $\varepsilon_{1,s}$ and $\varepsilon_{s,2}$, respectively. Note that the emissivity of different surfaces of the shield may be different [5]. The radiation network of this geometry is constructed, as usual, by drawing a surface resistance associated with each surface and connecting these surface resistances with space resistances, as shown in the figure. The resistances are connected in series, and thus the rate of radiation heat transfer is [7]:

$$\dot{Q}_{12\,\mathrm{'oneshield}} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1 F_{1,s}} + \frac{1-\varepsilon_{1,s}}{A_s\varepsilon_{1,s}} + \frac{1-\varepsilon_{s,2}}{A_s\varepsilon_{s,2}} + \frac{1}{A_s F_{s,2}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}} \quad (3)$$

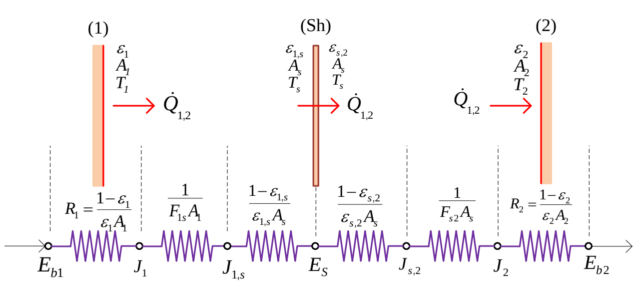

Fig. 3 Radiation shield between parallel plates

Noting that $F_{1,s}=F_{s,2}=1$ and $A_1=A_2=A_3=A$ for infinite parallel plates, Eq. 3 simplifies to:

$$\dot{Q}_{12\,\mathrm{'oneshield}} = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{1,s}} + \frac{1}{\varepsilon_{s,2}} - 1\right)} \quad (4)$$

where the terms in the second set of parentheses in the denominator represent the additional resistance to radiation introduced by the shield.

b. Radiation transfer between two parallel surfaces with n - shields

The appearance of the equation above suggests that parallel plates involving multiple radiation shields can be handled by adding a group of terms like those in the second set of parentheses to the denominator for each radiation shield. Then the radiation heat transfer through large parallel plates separated by *n* radiation shields becomes [8]:

$$\dot{Q}_{12\,\mathrm{'nshields}} = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right) + \left(\frac{1}{\varepsilon_{1,s}} + \frac{1}{\varepsilon_{s,2}} - 1\right) + \ldots + \left(\frac{1}{\varepsilon_{n,1}} + \frac{1}{\varepsilon_{n,2}} - 1\right)} \quad (5)$$

If the emissivities of all surfaces are equal ($\varepsilon_1=\varepsilon_2=\ldots=\varepsilon_{n1},\varepsilon_{n2},\ldots,\varepsilon_{nn}=\varepsilon$), Eq. 5 reduces to:

$$\dot{Q}_{12\,\mathrm{'nshields}} = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{(n+1) + \left(\frac{1}{\varepsilon} + \frac{1}{\varepsilon} - 1\right)} = \frac{1}{n+1}\dot{Q}_{12\,\mathrm{'\,oneshield}} \quad (6)$$

III. MODELING OF THERMAL FLUX AND TEMPERATURE TO THE PARALLEL PLANES WITH A RADIATION SHIELD

We have analyzed two planar surfaces which have unchanged certain temperature. In the middle of the space between the heat exchange surfaces is a radiation shield with given initial temperature [2] (Fig. 4). High reflectivity (low α=ε) surface (A) inserted between two surfaces for which a reduction in radiation exchange is desired. Consider use of a single shield in a two-surface enclosure, such as that associated with large parallel plates. Note that, although rarely the case, emissivities may differ for opposite surfaces of the shield.

By neglecting the heat convection we have found the change of temperature by a radiation shield, and the heat shield which it receives and provides (from the moment of its location) for each of the three cases [3] [5].

Thus, equation (5) can be transformed:

$$\dot{Q}_{1-2}=\frac{AC_c}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{1}{\varepsilon_{1,s}}+\frac{1}{\varepsilon_{s,2}}-1\right)}\left[\left(\frac{T_1}{100}\right)^4-\left(\frac{T_2}{100}\right)^4\right] \qquad (7)$$

The foregoing result may be readily extended to account for multiple shields and may be applied to long, concentric cylinders and concentric spheres, as well as large parallel plates.
The amount of radiant specific flux transmitted by radiation per unit area is:

$$\dot{q}_{1,2}=\frac{\dot{Q}_{1,2}}{A} \qquad (8)$$

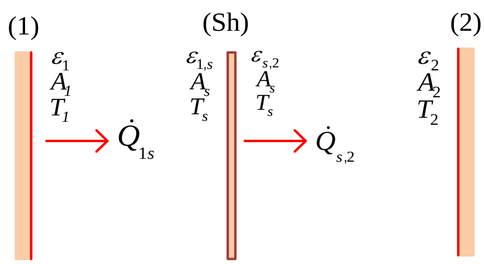

Fig. 4 Radiation shield between the heat exchange surfaces

The heat received in the radiation shield by the planar wall 1 (Fig. 4) is:

$$\dot{q}_{1,s}=\frac{C_c}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_s}-1}\left[\left(\frac{T_1}{100}\right)^4-\left(\frac{T_s}{100}\right)^4\right]=B_1\left[\left(T_1\right)^4-\left(T_s\right)^4\right] \qquad (9)$$

The heat given by a radiation shield to the planar wall 2 is:

$$\dot{q}_{s-2}=\frac{C_c}{\frac{1}{\varepsilon_s}+\frac{1}{\varepsilon_2}-1}\left[\left(\frac{T_s}{100}\right)^4-\left(\frac{T_2}{100}\right)^4\right]=B_2\left[\left(T_s\right)^4-\left(T_2\right)^4\right] \qquad (10)$$

where:

$$B_1=\frac{C_c}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_s}-1}\frac{1}{100^4},\ \text{in}\ \frac{\mathrm{W}}{\mathrm{m^2K^4}};$$

$$B_2=\frac{C_c}{\frac{1}{\varepsilon_s}+\frac{1}{\varepsilon_2}-1}\frac{1}{100^4},\ \text{in}\ \frac{\mathrm{W}}{\mathrm{m^2K^4}} \qquad (11)$$

The difference between these two heat amounts remains in itself radiation shield whose temperature rises:

$$\dot{q}_{1,s}-\dot{q}_{s,2}=\dot{m}_A c\frac{dT_s}{d\tau} \qquad (12)$$

Case 1: Assume that $\varepsilon_1=\varepsilon_2$ so that $B_2=B_1=B$ and:

$$2B\left[\frac{T_1^4+T_2^4}{2}-T_s^4\right]d\tau=\dot{m}_A c dT_s \qquad (13)$$

The term $\frac{T_1^4+T_2^4}{2}=\left(T_{mes}\right)^4$ represents the stationary temperature of the shield.
Then:

$$d\tau=\frac{\dot{m}_A c}{2B}\frac{dT_s}{\left(T_{mes}\right)^4-\left(T_s\right)^4} \qquad (14)$$

This expression is integrated from the time of placing the shield $\tau=0$ till τ, and from the moment the initial temperature of the shield $T_s=T_2$ to the temperature T_s:

$$\tau=\frac{\dot{m}_A c}{2B}\left(\frac{1}{2\left(T_{mes}\right)^3}\operatorname{arctg}\frac{T_s}{T_{mes}}+\frac{1}{4\left(T_{mes}\right)^3}\ln\frac{T_{mes}+T_s}{T_{mes}-T_s}\right)\Bigg|_{T_2}^{T_s} \qquad (15)$$

$$\tau=\frac{\dot{m}_A c}{4B\left(T_{mes}\right)^3}*$$
$$*\left[\left(\operatorname{arctg}\frac{T_s}{T_{mes}}+\frac{1}{2}\ln\frac{T_{mes}+T_s}{T_{mes}-T_s}\right)-\left(\operatorname{arctg}\frac{T_2}{T_{mes}}+\frac{1}{2}\ln\frac{T_{mes}+T_2}{T_{mes}-T_2}\right)\right] \qquad (16)$$

After replacing the following expression we achieve the so-called constant time:

$$K=\frac{\dot{m}_A c}{4B\left(T_{mes}\right)^3},\ \text{in s} \qquad (17)$$

So the final expressions are:

$$\tau=K\left[\left(\operatorname{arctg}\frac{T_s}{T_{mes}}+\frac{1}{2}\ln\frac{T_{mes}+T_s}{T_{mes}-T_s}\right)-\left(\operatorname{arctg}\frac{T_2}{T_{mes}}+\frac{1}{2}\ln\frac{T_{mes}+T_2}{T_{mes}-T_2}\right)\right] \qquad (18)$$

$$\dot{q}_{1,s}=B\left[\left(T_1\right)^4-\left(T_s\right)^4\right] \qquad (19)$$

$$\dot{q}_{s,2}=B\left[\left(T_s\right)^4-\left(T_2\right)^4\right] \qquad (20)$$

Because of the form of the function we take the time in function of temperature, i.e. for the certain time will be reached the certain temperature.

Case 2: For $\varepsilon_1\neq\varepsilon_2\Rightarrow B_1\neq B_2$:

$$B_1\left(T_1^4-T_s^4\right)-B_2\left(T_s^4-T_2^4\right)=\frac{\dot{m}cdT_s}{d\tau} \qquad (21)$$

$$d\tau=\frac{\dot{m}_A c}{B_1+B_2}\frac{dT_s}{\left(T_{AB}\right)^4-\left(T_s\right)^4} \qquad (22)$$

Where: $\left(T_{AB}\right)^4=\frac{B_1T_1^4+B_2T_2^4}{B_1+B_2}\Rightarrow$

$$T_{AB}=\sqrt[4]{\frac{B_1T_1^4+B_2T_2^4}{B_1+B_2}}$$

So the final expressions are:

$$\tau=K_B\left[\left(\operatorname{arctg}\frac{T_s}{T_{AB}}+\frac{1}{2}\ln\frac{T_{AB}+T_s}{T_{AB}-T_s}\right)-\left(\operatorname{arctg}\frac{T_2}{T_{AB}}+\frac{1}{2}\ln\frac{T_{AB}+T_2}{T_{AB}-T_2}\right)\right] \qquad (23)$$

where:

$$K_B = \frac{\dot{m}_A c}{B_1 + B_2} \frac{1}{2\left(T_{AB}\right)^3} \tag{24}$$

$$\dot{q}_{1,s} = B_1\left[\left(T_1\right)^4 - \left(T_s\right)^4\right] \tag{25}$$

$$\dot{q}_{s,2} = B_2\left[\left(T_s\right)^4 - \left(T_2\right)^4\right] \tag{26}$$

IV. REGULATION OF THERMAL FLUX AND TEMPERATURE BY A RADIATION SHIELD BETWEEN THE TWO PLANAR SURFACES

Obtained simulations for regulating and optimization of radiation shield specific thermal flux and temperature are used and analyzed for three types of radiation shield (Table 1), and that in both cases where emissivity for bodies is the same and different. Temperature of parallel planes are considered unchanged $t_1=100^0$C, $t_2=15^0$C and the initial temperature of the shield $t_{s0}=15^0$C (Fig. 4).

The shield can be of materials, with mass, emission coefficient and specific heat coefficient as in the following table 1.

Table 1. The data needed for the three types of materials of a shield Materials	ε_s	c, kJ/(kgK)	$\dot{m}_A$, kg/m^2
a) Alumin	0.09	0.45	0.27
b) Granite	0.45	0.79	0.27
c) Wood	0.95	2	0.27

Case 1: Assume that $\varepsilon_1=\varepsilon_2=0.8$ so that $B_2= B_1=B$, and with the introduction of above equations and numerical data values is achieved:

$$T_{mes} = \sqrt[4]{\frac{T_1^4 + T_2^4}{2}} = 338,43\ K \text{, or } t_{mes} = 65,43^0C$$

Through simulations, for the three above mentioned cases of the shield materials (Table 1), we have achieved results which represent the values of coefficients B and K, and the time for which it reached the shield certain temperature and thermal specific fluxes for certain shield temperature (Tables 2, 3 and 4, Figures 5-10).

Table 2.

Magnitude	Case a	Case b	Case c
ε_s	0.09	0.45	0.95
t_{mes}, ^{0}C	65.43	65.43	65.43
$K = \dfrac{\dot{m}_A \cdot c}{4B\left(T_{mes}\right)^3}$, s	157.1	60.015	80.056
$B = \dfrac{C_c}{\frac{1}{\varepsilon}+\frac{1}{\varepsilon_s}-1}\dfrac{1}{100^4}$, $\dfrac{\text{W}}{\text{m}^2\text{K}^4}$	4.988·10^{-9}	2.292·10^{-8}	4.35·10^{-8}

Table 3.

t_s, ^{0}C	τ, s		
	a	*b*	*c*
15	0	0	0
20	10.16	3.88	5.17
25	21.25	8.12	10.83
30	33.54	12.81	17.09
35	47.37	18.09	24.14
40	63.34	24.19	32.27
45	82.38	31.47	41.98
50	106.24	40.58	54.14
55	138.80	53.02	70.73
60	191.854	73.29	97.76
t_{mes}=65.43	947.035	361.78	482.59

Table 4.

t_s, ^{0}C	$\dot{q}_{1,s}$, W/m^2			$\dot{q}_{s,2}$, W/m^2		
	a	*b*	*c*	*a*	*b*	*c*
15	62.23	286.01	542.81	0	0	0
40	48.67	223.70	424.55	13.55	62.30	118.25
60	35.21	161.84	307.16	27.01	124.16	235.64

Since the shield temperature differs from the initial temperature from 15 ^{0}C until stationary temperature $t_s=t_{mes}$=65,43 ^{0}C, we have represented the time for the different shield temperatures as in the following figures.

Case 2: Assume that $\varepsilon_1=0.08$, ε_2=0.8 so that $B_2\neq B_1$, and by using the introduction of above equations and numerical data values are achieved the Tables 5, 6 and 7 and Figures 11-16 as in following:

Table 5.

Magnitude	Case a	Case b	Case c
ε_s	0.09	0.45	0.95
t_{AB}, ^{0}C	51.238	33.146	26.562
$K_B = \dfrac{\dot{m}_A c}{B_1 + B_2}\dfrac{1}{2\left(T_{AB}\right)^3}$, s	237.805	137.394	209.167
$B_1 = \dfrac{C_c}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_s}-1}\dfrac{1}{100^4}$, $\dfrac{\text{W}}{\text{m}^2\text{K}^4}$	2.506·10^{-9}	4.13·10^{-9}	4.515·10^{-9}
$B_2 = \dfrac{C_c}{\frac{1}{\varepsilon_2}+\frac{1}{\varepsilon_s}-1}\dfrac{1}{100^4}$, $\dfrac{\text{W}}{\text{m}^2\text{K}^4}$	4.988·10^{-9}	2.292·10^{-8}	4.35·10^{-8}

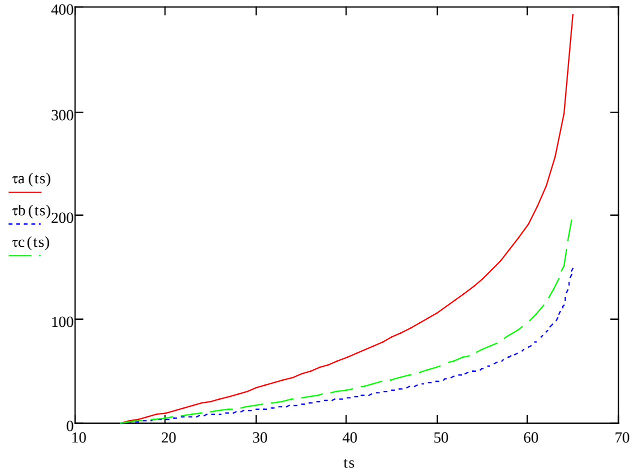

Fig. 5. Shield temperature over time, for three cases

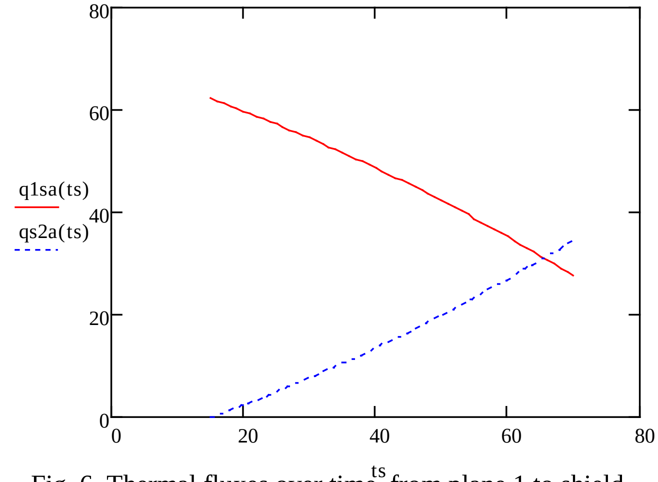

Fig. 6. Thermal fluxes over time, from plane 1 to shield and from shield to plane 2 – case a

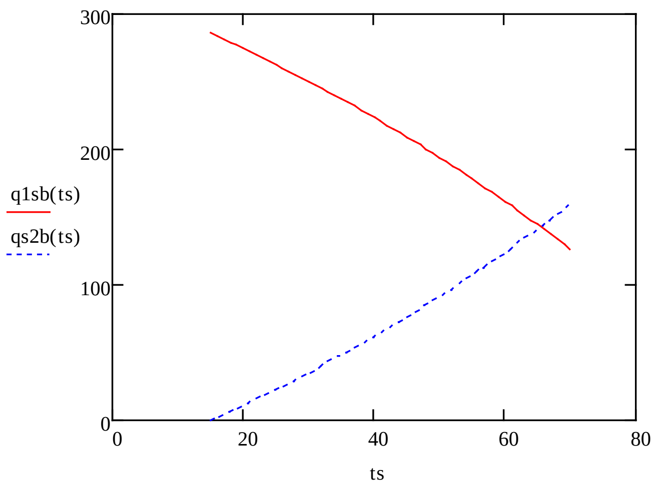

Fig. 7. Thermal fluxes over time, from plane 1 to shield and from shield to plane 2 – case b

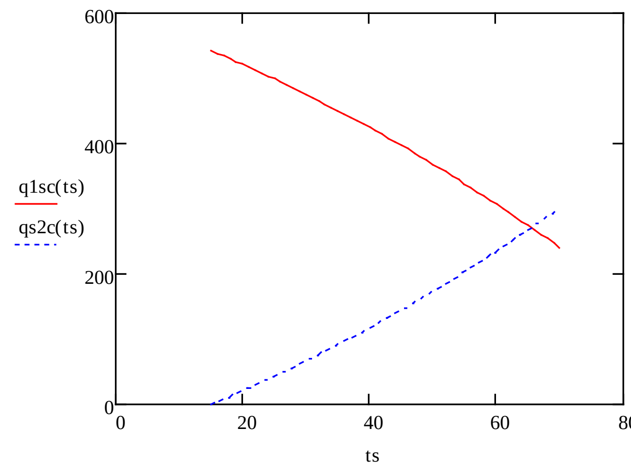

Fig. 8. Thermal fluxes over time, from plane 1 to shield and from shield to plane 2 – case c

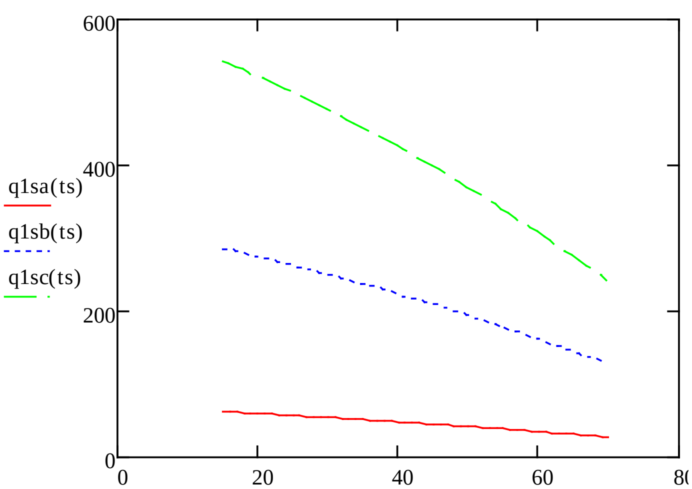

Fig. 9. Thermal fluxes over time, from plane 1 to shield – three cases

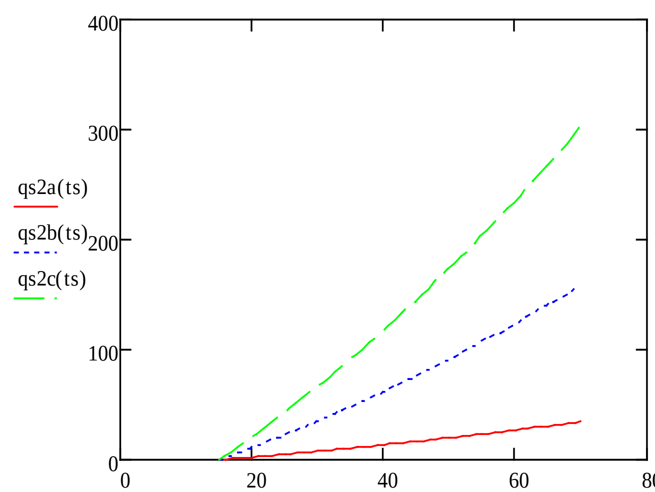

Fig. 10. Thermal fluxes over time, from shield to plane 2 – three cases

Table 6.

t_s, ^{0}C	τ, *s*		
	a	*b*	*c*
15	0	0	0
20	20.65	23.9	61.92
25	44.35	58.51	214.69
t_{ABa}=26.562	52.56	73.66	$1.012\cdot10^3$
30	72.42	125.57	
t_{ABb}=33.146	93.30	848.66	
40	153.83		
45	226.64		
50	421.70		
t_{ABc}=51.238	$1.277\cdot10^3$		

Table 7.

t_s, ^{0}C	$\dot{q}_{1,s}$, W/m^2			$\dot{q}_{s,2}$, W/m^2		
	a	*b*	*c*	*a*	*b*	*c*
15	31.27	51.52	56.33	0	0	0
40	24.45	40.30	44.05	13.55	62.30	118.25
60	17.69	29.15	31.87	27.01	124.16	235.64

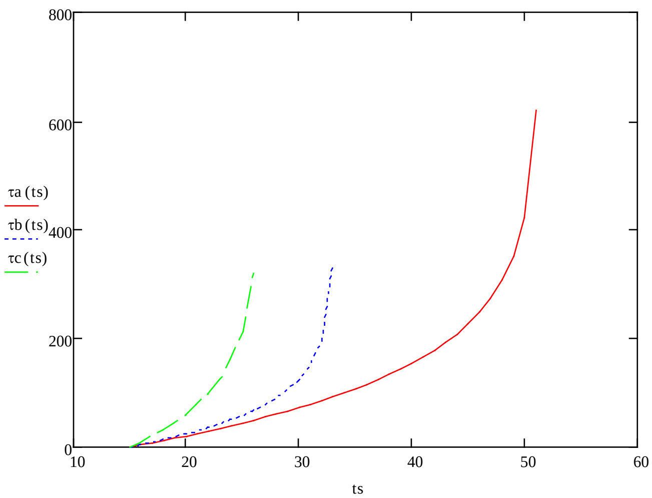

Fig. 11. Shield temperature over time, for three cases

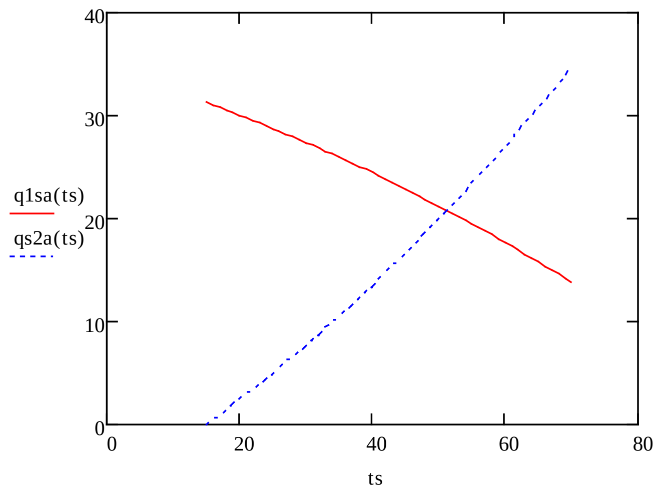

Fig. 12. Thermal fluxes over time, from plane 1 to shield and from shield to plane 2 – case a

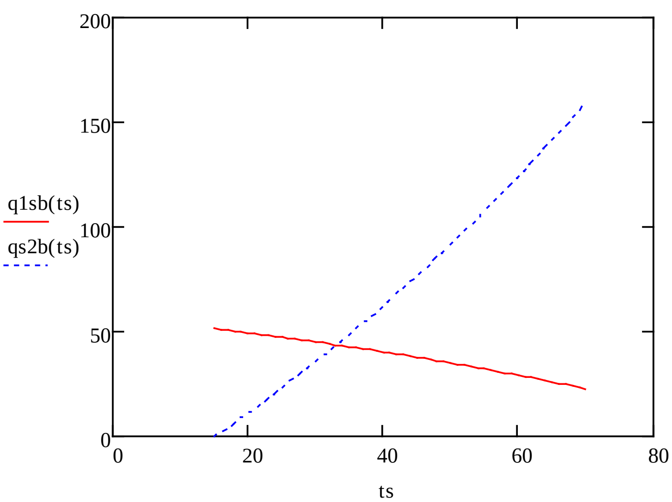

Fig. 13. Thermal fluxes over time, from plane 1 to shield and from shield to plane 2 – case b

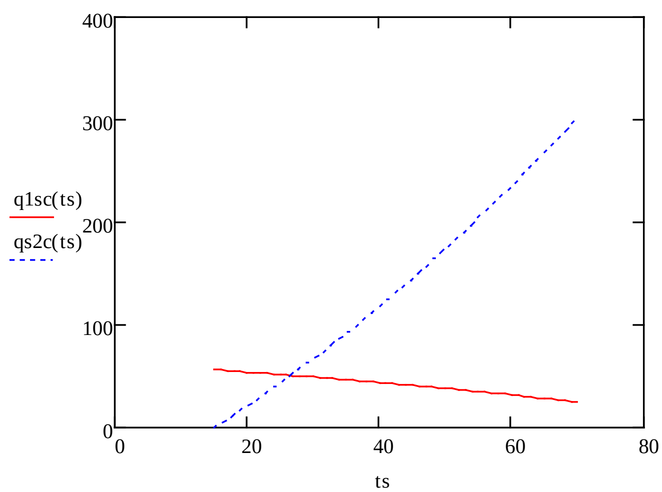

Fig. 14. Thermal fluxes over time, from plane 1 to shield and from shield to plane 2 – case c

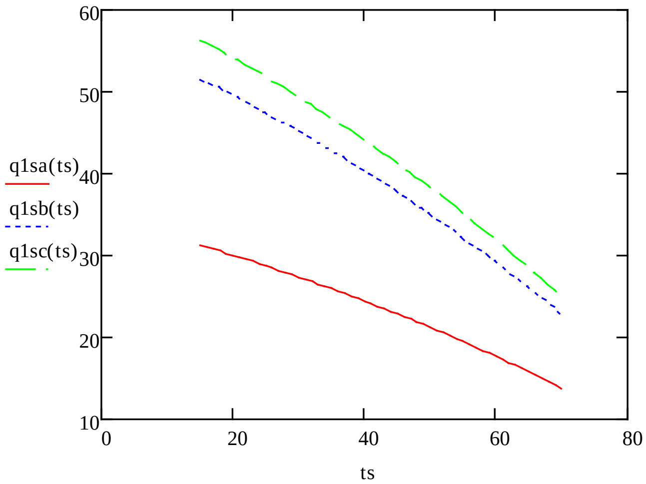

Fig. 15. Thermal fluxes over time, from plane 1 to shield – three cases

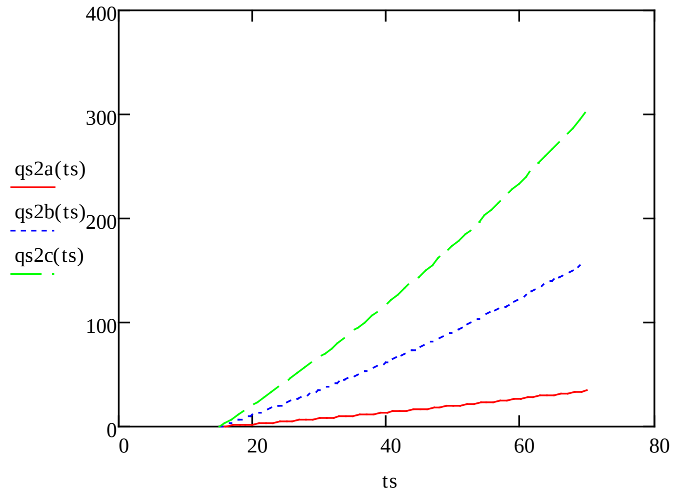

Fig. 16. Thermal fluxes over time, from shield to plane 2 – three cases

V. CONCLUSION

Results show that the shields of materials with low emission coefficient change its temperature slowly. Impact here is the value of thermal capacity ($\dot{m}_A \cdot c$). In above Figures it is shown that the material which has significantly greater specific heat, such is the case c, has longer time to achieve the same set temperature than the case b, although the emission coefficient is greater than the case b. This is also seen from time constants values, where $K_c > K_b$. The foregoing result may be readily extended to account for multiple shields and may be applied to long, concentric cylinders and concentric spheres, as well as large parallel plates.

This modeling and analysis directly affects in the selection of the type of materials that are based on mass, emission coefficients and specific heat coefficients. With this model is reached to the time regulation and the optimization of the radiation temperature and specific heat flux, which is conditioned with total energy savings.

References:

[1] Cengel, Y. (2003). Heat Transfer: A Practical Approach, Chap.12. New York: McGRAW - Hill.
[2] Demneri, I. S. (2003). Termoteknika. Tirane: Pegi.
[3] Lienhard, J. H. (2011). A heat transfer textbook, Chap. 4. Houston: Cambridge MA USA.
[4] Martinez, I. (2015). Heat transfer and thermal radiation modelling. Madrid: http://webserver.dmt.upm.es/~isidoro/.
[5] Rey, G. C. (2006). Numerical methods for radiative heat transfer. Terrassa: University of Politecnica de Catalunya.
[6] Saedodin, S. M. (2010). Application of Net Radiation ransfer Method for Optimization and Calculation of Reduction Heat Transfer, Using Spherical Radiation Shields. World Applied Sciences Journal 11 (4), 457-461.
[7] Sen, M. (2015). Analytical heat transfer. Notre Dame: University of Notre Dame.
[8] Thirumaleshwar, M. (2014). Software solutions to problems on heat transfer - Radiation heat transfer, Part I. India: bookboon.com.