Identification of nonparametric linear systems

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***Abstract*— identifying nonlinear systems is a much more rich and demanding problem area, especially in mechanical systems. In this presentation, some major approaches and concepts are outlined. The aim of the considered problem in this work is the modelling and identification of nonlinear systems based on spectral approach. Presently, the studied nonlinear system can be nonparametric, i.e. not necessarily described by a limited number of parameters and structured by Hammerstein models. These systems consist of nonlinear element followed by a linear block. This latter (the linear subsystem) is not necessarily parametric and the nonlinear function can be nonparametric smooth nonlinearity. The developed identification problem of these nonlinear models is studied in the presence of possibly infinite-order linear dynamics. The determination of linear and nonlinear block can be done using a unique stage.

***Keywords*—** Nonlinear systems, Nonlinear modelling, Fourier expansion, Nonparametric models, Nonlinear systems identification..

I. INTRODUCTION

Linear and nonlinear structured models can represent the most of industrial systems. Hammerstein model structure (Fig. 1) is one of these blocks-oriented nonlinear models. This latter is formed by a series connection of a nonlinear function and a linear dynamical element. Nonlinear system identification has been an active research area, especially over the last two decade [1]-[4]. Parameters determination of black-box nonlinear system is a very wide research area [1]-[4]. Identification have been paid considerable attention due to their benefits such as control and command [5].

In the case where the system is not exactly determined, it is very difficult to control system, waste of extra-energy can be observed. Then, some nonlinearities can destabilize the system or cause undesirable effects if they are not well determined. For these reasons, recent studies have been investigated the problem of modelling and identifying nonlinear systems.

These models (series connection of nonlinear and linear blocks) can describe several industrial systems, e.g. [3] and [5]. The diversity of nonlinear models and structures has led to a large variety of identification problems and identification methods.

The blocks-oriented nonlinear like models are used in a wide range of applications such as identification of BLDC motor [6], PH neutralization process [7].

Nonlinearity effect commonly occurs in hydraulic servo-valves, electric servomotors, bearings and gears [5]. It occurs in transmission systems (e.g. the small gaps) which exist in transmission mechanisms, e.g. the play between the teeth of the drive gear and those of the driven gear. Nonlinear systems parameters identification and modeling is necessary to design a precision controller for these systems [8]- [14]..

In this paper, the problem of modelling and identifying nonlinear systems is addressed. This latter can be structured by a nonlinear block followed by a linear dynamical element (Hammerstein model) (Fig. 1). Presently, the linear element is not necessarily parametric and can be of finite or infinite order. The system nonlinearity is nonparametric but of smooth shape. Furthermore, it is interesting to emphasize that, all the internal signals ($x(t)$, $w(t)$, and $\xi(t)$) are not accessible to measurement.

In view of these difficulties, it is not surprising that few parameter determination methods are available that deal with nonparametric nonlinear models.

Unlike most of previous work, the parameter determination problem is developed with in the continuous-time context. The proposed approach is based on the Fourier expansion using a simple periodic or sine input signal. Here, the determination of linear block as well as the system nonlinearity is done at the same time (in one stage), unlike most of previous papers [15].

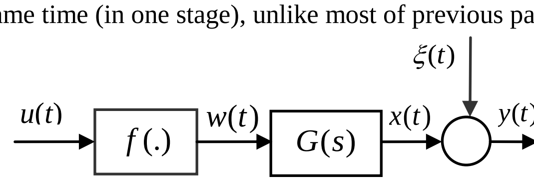

Fig.1. Nonparametric Hammerstein system

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Will Unlike many of previous works e.g. [4], the model structure of the linear block is entirely unknown. Furthermore, the system nonlinearity is of arbitrary-shape and can be noninvertible. In several previous works devoted to modelling and determination parameters of these nonlinear systems, the nonlinear element is supposed to be smooth continuous function. Furthermore, the smoothness assumption implies that the system nonlinearity can be developed within any interval by a limited any function decomposition [5] and [11], e.g. polynomial decomposition.

Different approaches are available that deal with The blocks-oriented nonlinear like models identification. Amongst

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that are: iterative nonlinear optimization procedures, stochastic methods, blind methods, and the frequency approaches [16]-[17]. Most available solutions suggest that, the output nonlinearity is invertible and the linear subsystem is parametric. Generally, the iterative methods necessitates a large amount of data, since computation time and memory usage drastically increase, and have local convergence properties which necessitates that a fairly accurate parameter estimates are available to initialize the search process. The stochastic methods are generally relied on specific assumption (e.g. gaussianity, persistent excitation, MA linear subsystems....).
Several technics can be used, e.g. Fourier decomposition, geometric analysis, etc.

In this paper, a frequency-domain identification scheme is designed for blocks-oriented nonlinear involving linear subsystem of totally unknown structure, and noninvertible nonlinearity. The proposed approach is allowed to concern a wide range of nonlinearity function. The identification problem amounts to obtain an accurate estimate of the complex frequency gain $G(j\omega)$ of linear element, for a set of frequencies $(\omega_1 \dots \omega_m)$, and the nonlinearity parameters. To the author's knowledge, the present identification solution will be performed in one-stage and using simple sine inputs (or periodic).
The paper is organized as follows: The identification problem is formally described in Section 2; relevant mathematical tools are described in Section 3; Section 4 is devoted to the determination of nonlinear element as well as the estimation of system nonlinearity.

II. Modelling problem statement

We are considering nonlinear systems structured by block that can be described by nonlinear block $f(.)$ followed by a linear dynamic element $G(s)$ (Fig. 1). This model is called a standard Hammerstein system and analytically described as follows:
Recall that all the inner signals $w(t)$, $x(t)$ and ξ(t) are not accessible to measurements. The modelling and determination parameters are only based up on the input and output signals ($u(t)$ and $y(t)$ respectively).

Firstly, the input of the system $u(t)$ and the inner signal $w(t)$ are related by a linear relationship:

$$w(t) = f\left(u(t)\right) \tag{1}$$

Be Accordingly, the inner input and output signals of linear element (w(t) and x(t) respectively) are related by a linear relationship:

$$x(t) = g(t) * w(t) = g(t) * f\left(u(t)\right) \tag{2}$$

Finally, the output of the system y(t) can be expressed as follows:

$$y(t) = x(t) + \xi(t) = g(t) * f\left(u(t)\right) + \xi(t) \tag{3}$$

where $g(t) = L^{-1}\left(G(s)\right)$ is the inverse Laplace transform of linear element $G(s)$; the symbol * means the convolution product operator; the inner signal (extra-input) $\xi(t)$ represents the noise signal. Note that, all the inner signal are not accessible to measurement, i.e. only the input $u(t)$ and output $y(t)$ signals are accessible to measurements.

III. Analytical method

The aim of this section is to present some of recipes and tips useful for the parameters determination method. Then, in the proposed modelling and determination of parameters of nonlinear system, the Hammerstein system is excited by periodic input signals, a set of simple sine signals can also be used. To make easy this method, let choose the following input signal:

$$u(t) = U \sin(\omega t) \tag{4}$$

The where the frequency ω belongs to a given set $\{\omega_1, \dots, \omega_n\}$. Then, the output of the system $y(t)$ is recorded in the steady state. This experience is repeated for any frequency $\omega \in \{\omega_1, \dots, \omega_n\}$ (i.e. the nonlinear system is repeatedly excited by the input sine signal (4) for all the given set of frequencies). Accordingly, it is interesting to point out that the inner signals $w(t)$ and $x(t)$ are also periodic of the same period of the system input $u(t)$, i.e. $T = 2\pi / \omega$. Then, this immediately implies that, the inner signals $w(t)$ and $x(t)$ can be developed in the following Fourier expansion (respectively):

$$w(t) = \sum_{i=0}^{\infty} W_i \cos\left(i\omega t + \alpha_i\right) \tag{5}$$

$$x(t) = \sum_{i=0}^{\infty} X_i \cos\left(i\omega t + \beta_i\right) \tag{6}$$

On the other hand, note that this nonlinear modelling problem does not have a unique solution (solutions multiplicity). Specifically, let the couple $\left(f(v), G(s)\right)$ is solution of this problem identification. Then, any model of the form $\left(f(v)/k, kG(s)\right)$ is also a representative model of this nonlinear system, whatever k nonzero real.

The question that currently arises is how to choose the scaling factor k?

In this respect and for convenience, let take the following choice of the scaling factor k:

$$k = \left|G(j\omega_l)\right| \tag{7}$$

where ω_l is any chosen frequency $\omega_l \in \{\omega_1, \ldots, \omega_n\}$.

IV. Nonlinear System Determination Parameters

The aim of this section is to present the identification method of the proposed nonlinear systems. Then, an accurate estimate of the transfer function $G(s)$ (i.e. the phase $\varphi(\omega) = \arg(G(j\omega)) = \angle G(j\omega)$ and the modulus gain $|G(j\omega)|$) can be obtained for any frequency $\omega \in \{\omega_1, \ldots, \omega_n\}$, as well as the parameters of the nonlinear block for any given working interval.
In this respect, recall that the rescaling nonlinear system to be modelled and identified (see section 3) is described by following blocks (using (7)):

$$\left(f^*(v), G^*(s)\right) = \left(f(v)|G(j\omega_l)|, \frac{G(s)}{|G(j\omega_l)|}\right) \quad (8)$$

where ω_l is any chosen frequency $\omega_l \in \{\omega_1, \ldots, \omega_n\}$. For convenience, the nonlinear model (8) will remain noted $(f(v), G(s))$ to avoid multiple notations. It readily follows using (8) that, the modulus gain of the linear element corresponding to the frequency ω_l satisfies the property:

$$|G(j\omega_l)| = 1 \quad (9)$$

On the other hand, it is seen in section 2 that, the considered nonlinear element $f(.)$ is smooth continuous function within any interval. Which means that, the modelling of the system nonlinearity $f(.)$ can be achieved by decomposition into a weighted sum of appropriate basis functions (within any interval), e.g. the function $f(.)$ can be approximated by a polynomial function of a given degree m. Let $\theta = [a_0 \ \cdots \ a_m]^T$ be the vector coefficients of this polynomial decomposition. Specifically, the inner signal $w(t)$ can be written, within any interval, according to the input system $u(t)$ as follows:

$$w(t) = f(u(t)) = \sum_{k=0}^{m} a_k u(t)^k \quad (10)$$

Then, for the input sine signal (4), it follows from (10) that the internal signal $w(t)$ becomes:

$$w(t) = \sum_{k=0}^{m} a_k U^k \sin^k(\omega t) \quad (11)$$

Furthermore, using the trigonometric powers linearization, one immediately gets from (11) that:

$$w(t) = \sum_{k=0}^{m} A_k \sin(k\omega t) \quad (12)$$

where the amplitudes of component A_k depend on the polynomial coefficients a_k and the amplitude of the sine input U (known). Accordingly, it follows using (2) and (12) that the inner signal $x(t)$ can be expressed as:

$$x(t) = \sum_{k=0}^{m} A_k |G(jk\omega)| \sin(k\omega t + \varphi(k\omega)) \quad (13)$$

Therefore, the following results can be easily obtained by comparing the equations (5) and (12) on the one hand, and (6) and (13) on the other hand:

$$W_k \approx 0 \quad \text{for } k = m+1, m+2, \ldots \quad (14a)$$

$$X_k \approx 0 \quad \text{for } k = m+1, m+2, \ldots \quad (14b)$$

Therefore, one immediately gets using (6) and (13):

$$X_k = A_k |G(jk\omega)| \quad \text{for } k = 0, 1, \ldots, m \quad (15a)$$

$$\beta_k = \varphi(k\omega) \quad \text{for } k = 1, 2, \ldots, m \quad (15b)$$

Finally, it follows from (3) and (13) that, the output signal $y(t)$ can be expressed as:

$$y(t) = \sum_{k=0}^{m} A_k |G(jk\omega)| \sin(k\omega t + \varphi(k\omega)) + \xi(t) \quad (16)$$

In this respect, the results given by (15a-b) mean that, if the spectrum (Fourier expansion) is available, then the amplitude $X_k = A_k |G(jk\omega)|$ and the argument $\beta_k = \varphi(k\omega)$ can be obtained. Unfortunately, note that the inner signal $x(t)$ is not accessible. However, given that the undisturbed output $x(t)$ is periodic, with common period T of input $u(t)$, and $\xi(t)$ is a zero-mean ergodic white noise, the effect of the latter can be filtered considering the following trans-period averaging of the output:

$$\hat{x}(t) = \frac{1}{N} \sum_{p=1}^{N} y(t + pT) \quad \text{for } 0 \le t < T \quad (17)$$

for some (large enough) integer N.
Finally, using the estimate of undisturbed output $x(t)$ and getting benefit from the plurality of solution (i.e. using the rescaling of nonlinear system (8) and satisfying the property (9)), this modelling and identification problem can be coped by exciting the system with different frequencies (ω, 2ω, …). Then, the nonlinear function parameters $\theta = [a_0 \ \cdots \ a_m]^T$ as well as the modulus gain $|G(j\omega)|$ and phase $\varphi(\omega)$ of the linear element (for any frequency ω) can be determined.

V. Conclusion

Presently, in this contribution we give an overview and discussion of the basic steps of System Identification. The problem of nonlinear system identification is dealt. The nonlinear system is described by Wiener model.

In this work, nonparametric identification solution is developed with continuous-time Hammerstein systems involving nonparametric smooth nonlinear element. The proposed method is built in one stage using periodic (or sine) input signal. Then, the frequency gain of the linear element is determined at a number of frequencies. The originality of the present study lies in the fact that the linear block and the system nonlinearity are all nonparametric. Accordingly, the linear block is not necessarily finite order and the nonlinearity element may be noninvertible. Another feature of the method is the fact that the exciting signals are easily generated and the estimation algorithms can be simply implemented

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