

Calculation of Collision Probability Matrix of Nuclear Fuel Cell as a Function of Neutron Energy Group Using Flat Flux Model

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Abstract—One of the methods that widely used in solving neutron transport equations in the nuclear fuel cell is the collision probability (CP) method. The neutron transport is very important to solve because the neutron distribution is related to the reactor power distribution. The important thing in the CP method is the CP matrix calculation, better known as P_{ij} has an important role in determining the neutron flux distribution in the reactor core. This study uses a linear flat flux model in each cell region for each energy group with white boundary condition. Although the type of reactor used in this study is a fast reactor, the P_{ij} matrix calculation still carried out in fast and thermal group energy. The P_{ij} matrix depends on the number of mesh in each cell region. The P_{ij} matrix formed from the mesh distribution will produce a 6×6 matrix for each energy group. Because the boundary condition of the system is assumed that there are no contributions neutron source from the outside, the

sum of the P_{ij} matrix must be less than one. In general, the results of the calculations P_{ij} in this study are following the theory.

Keywords—A CP method, P_{ij} matrix, energy group, nuclear fuel cell, linear flat flux model

I. INTRODUCTION

THE calculation of neutron transport and its interaction with matter is a fundamental topic in reactor physics [1]. The most complicated part of the nuclear reactor is to solve the integral neutron transport equation as a function of energy, space, and time. Neutron transport investigates to determine the distribution of neutrons in the reactor core, that is, the motion of the neutrons, the scattering process when its interaction with nuclei and the process of a neutron being absorbed or leaking out of the reactor [2]. One of the methods that widely used in solving neutron transport equations in the

nuclear fuel cell is the collision probability (CP) method. Solving an integral transport equation with the CP method generally uses the linear flat flux model; namely, the neutron flux in each volume region is considered constant [3],[4]. This model allows neutron flux to be maximal in reactor fuel because neutrons move in all directions and will decrease when it reaches shielding limits. The linear flat flux model often used in the CP method to solve neutron transport problems in nuclear fuel cells [5]. CP method has an advantage for simple and symmetrical geometry such as a cylindrical cell.

The review of the flux points in each region is necessary to demonstrate the neutron flux distribution in each region in the nuclear fuel cell. Flux points with certain intervals in the fuel cell are called mesh points. If the distance between the mesh is the same, then the neutron flux in each mesh is considered a flat flux model. Because the total cross-section in each region is considered constant, the neutron flux and neutron source in the region can be considered constant as well. The calculation of the neutron flux distribution cannot be executed directly, because in general, the spatial mesh order is very small; neither is the energy range. Thus, the amount of total mesh that must be done for the entire core becomes very large. For this reason, fuel cell homogenization is done by the CP method [6].

The important thing in the CP method is the CP matrix calculation or better known as P_{ij} matrix. Handling the geometry of nuclear fuel cells is usually done by limiting the choice of cell geometry to be calculated. Furthermore, for each type of geometry, CP matrix calculations are performed by using analytic integration on angles, as well as numerical calculations for complex cases, such as the calculation of the Bickley-Naylor function, and the Dancoff factor [7]. Thus, if a certain computer code does not have the right geometry

choice, to add geometric capabilities to the computer code is a very great effort.

In the previous study [8], the Calculation of P_{ij} matrix in the one-dimensional slab geometry of nuclear fuel cells using the CP method has been carried out. This study uses the CP method is implemented on the one-dimensional cylindrical nuclear fuel cell with the FF approach as a function of 70 energy groups for fast reactor type. The P_{ij} matrix calculated in this study is the P_{ij} matrix for scattering and escape processes. The library data used is JFS-3-J33 from JAEA Japan. The P_{ij} matrix calculation results obtained in this study will then be used in the calculation of neutron flux distribution in the further studies.

II. THEORETICAL BACKGROUND

The neutron probability that is emitted thoroughly and isotropic in the i region and subsequently collides in j region within the cylindrical nuclear fuel cell for each energy group g is following [9]

$$\sum_{jg} V_j \phi_{jg} = \frac{1}{k_{eff}} \sum_i V_i P_{ijg} S_{ig} \quad (1)$$

Where $V_i = \pi(r_i^2 - r_{i-1}^2)$ and Σ_{jg} is a macroscopic cross-section total in the j region. The CP matrix in equation (1) is [10]

$$P_{ij} = \frac{2}{\sum_i V_i} \int_0^{r_i} d\rho \left[K_{i3}(\lambda_j^1) - K_{i3}(\lambda_j^1 + \lambda_i) - K_{i3}(\lambda_j^1 + \lambda_j) + K_{i3}(\lambda_j^1 + \lambda_i + \lambda_j) \right. \\ \left. + K_{i3}(\lambda_j^2) - K_{i3}(\lambda_j^2 + \lambda_i) - K_{i3}(\lambda_j^2 + \lambda_j) + K_{i3}(\lambda_j^2 + \lambda_i + \lambda_j) \right] \quad (2)$$

Where

$$\lambda_k = \sum_k (x_k - x_{k-1}), \lambda_j^2 = \sum_{k=1}^{i-1} \lambda_k + \sum_{k=1}^{j-1} \lambda_k \quad \text{and} \quad \lambda_j^1 = \sum_{k=i-1}^{j-1} \lambda_k .$$

The $K_{ij}(\lambda)$ is the third order of the Bickley-Naylor function. For cases of CP matrix that occur on the i region the same as in the j region (self-collision probability), the CP matrix in equation (2) becomes to [10]

$$P_{ii} = \frac{2}{\sum_i V_i} \int_0^{r_{i-1}} d\rho [2\lambda_i - 2K_{i3}(0) + 2K_{i3}(\lambda_i) + K_{i3}(\lambda_{ii}) - 2K_{i3}(\lambda_{ii} + \lambda_i) + K_{i3}(\lambda_{ii} + 2\lambda_i)] + \frac{2}{\sum_i V_i} \int_{r_{i-1}}^{r_i} d\rho [2\lambda_i - K_{i3}(0) + K_{i3}(2\lambda_i)] \quad (3)$$

The boundary condition used in this study is white boundary conditions which means a reflective condition where all neutron striking the boundary turn back with an isotropic angular distribution. The white boundary condition is usually explained as an isotropic reflection. The computational results are more accurate and time-consuming of the computational process less compared to other boundary conditions. It is assumed that there are no contributions of neutron source from outside. The CP matrix will change because neutrons emitted in the i region will come out of the outer boundary without experiencing any collision (escape probability), so the CP matrix in equation (3) becomes to [10]

$$P_{is} = \frac{2}{\sum_i V_i} \int_0^{r_i} d\rho \{K_{i3}(\lambda_{is}^1) - K_{i3}(\lambda_{is}^1 + \lambda_i) + K_{i3}(\lambda_{is}^2) - K_{i3}(\lambda_{is}^2 + \lambda_i)\}$$

Where

$$\lambda_{is}^1 = \sum_{k=i+1}^N \lambda_k \text{ and } \lambda_{is}^2 = \sum_{k=1}^{i-1} \lambda_k + \sum_{k=1}^N \lambda_k .$$

III. NUCLEAR FUEL DESIGN AND COMPUTATIONAL METHOD

This study only focuses on the completion of the neutron transport process in a nuclear fuel cell. Selected cell geometry is cylindrical, divided into three regions, namely: region 1 is fuel, region 2 is cladding, and region 3 is coolant [11]. Table 1 shows the specification design of the nuclear fuel cell. The nuclear fuel used in this study is a mixture of U-Pu Nitride; the cladding structure material is stainless steel and Pb-Bi as a coolant. Energy group data were selected for 70 groups based on the energy group structure SLAROM JFS-3-J33 [12].

Table 1. Specification design of nuclear fuel cell [6].

Parameters	Specification
Type of nuclear fuel cell	Uranium-Plutonium Nitride
Geometry of cell	Cylindrical 1D
Material Structure	Stainless steel
Coolant	Lead-Bismuth (Pb-Bi)
Diameter of pin cell	1.134 cm
Cladding thickness	0.11 cm
Temperature Average	1183 K
Volume fraction: fuel structure	61.73% 19.40%
coolant	18.87%

The outline of the CP matrix calculation program and its computational aspects are as follows [13]-[15]: After reading the total macroscopic cross-section value calculated from the homogenizing nuclear fuel cells process based on library data. The volume of the cell for each region is determined to run equation (1). The scattering cross-section, neutron fission production, absorption cross-section are calculated for each volume region and energy group. The CP matrix is calculated for all regions divided into amount mesh, and the energy group uses equation (2) and (3). The probability escape matrix is calculated by completing the equation (4). Iteration is done to recalculate the CP matrix and escape probability based on equation (1) to convergence. The computational program has been written in Delphi. The iteration is executed for six meshes and 70 groups of energy.

IV. RESULTS AND DISCUSSION

The calculation of P_{ij} the matrix in this study uses the FF approach in each region on all energy group. When the scattering neutron cross-section is homogeneous in the entire region, the P_{ij} matrix remains constant for the entire i region.

This condition occurs because the neutron flux distribution becomes flat evenly distributed in all cell regions. In most heterogeneity systems, spatial flux is considered to be flat, especially outside resonance. This P_{ij} matrix depends on the number of mesh in each cell region. The matrix P_{ij} formed from the mesh distribution will produce an $n \times n$ matrix, with n is the number of mesh in each region. For example, if the total mesh in the cell is six meshes, so the formed P_{ij} matrix is a 6x6 matrix for each energy group. If the total mesh is raised to 10 meshes, the P_{ij} matrix will form a 10x10 matrix and so on. The calculation results of the P_{ij} matrix for the first three energy groups of fast energy that represent high energy are shown in Figure 1 and the last three energy groups representing thermal energy can be seen in Figure 2. The escape probability matrix will form a $1 \times n$ matrix because in this event neutrons born in the i region do not experience collisions in the j region. The CP matrix produced using the FF approach is used to calculate the neutron flux using equation (1).

In the SLAROM library, the division of the JFS-3-J33-70 group energy group structure for high energy (MeV order) is represented by group 1 to group 19, medium energy (keV order) represented by group 20 to group 37, while low energy (eV order) is represented by groups 38 to group 70. In general, the division of the energy group is divided into two regions,

namely the fast region, which is represented by groups 1 to 37 and thermal regions represented by groups 38 to 70 [12].

From Figures 1 and 2, it can be concluded that in the cylinder geometry, the calculation results of the P_{ij} matrix are very close in a region with the same energy, both in high energy region and in low energy region. Moreover, the matrix values P11, P12, and P13 are greater than P14, P15, and P16, because the composition of nuclear fuels are in the regions 1 to

3 where, in this regions, there will be a lot of nuclear fission processes, while in the region 4 to 6 that contain cladding and coolant do not occur nuclear fission processes. In addition, the macroscopic cross-section of the total reaction in the fuel region, is too large compared to the macroscopic cross-section of the total reaction in the cladding and coolant.

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energy group : 1
escape probability p[esc][i]=1,maxcrg
8.67715935077948E-0001 8.66814081465532E-0001 8.65853415132618E-0001 8.70435891224459E-0001 8.90691753657931E-0001 9.07065312674714E-0001
collision probability. p[ij]
1 5.29097064189061E-0002 1.20341444261998E-0001 1.79348359165704E-0001 4.85268691592724E-0001 7.63452226976876E-0002 8.54749332247084E-0002
2 4.02154379989011E-0002 1.23956935004574E-0001 1.82821083027099E-0001 4.87347512591486E-0001 7.64649414756704E-0002 8.55425181020963E-0002
3 3.5825378090797E-0002 1.10117485565682E-0001 1.88436877832826E-0001 4.95512979723452E-0001 7.68858439279542E-0002 8.58204472641715E-0002
4 3.4328562338805E-0002 1.03880366423737E-0001 1.76134973125419E-0001 5.08686791670922E-0001 7.88213048989180E-0002 8.71479210762052E-0002
5 3.45288081085641E-0002 1.03347061386600E-0001 1.73995951039465E-0001 5.02594553803351E-0001 8.35352616921486E-0002 9.10610137026162E-0002
6 3.48460905715495E-0002 1.04378588193643E-0001 1.74398096608990E-0001 4.99917804976607E-0001 8.24686462926379E-0002 9.49657505063645E-0002
energy group : 2
escape probability p[esc][i]=1,maxcrg
8.93945735315508E-0001 8.92705141304437E-0001 8.91409050482117E-0001 8.95028741440790E-0001 9.10708568127018E-0001 9.24184785253164E-0001
collision prob. p[ij] i! j ->
1 5.06551225836861E-0002 1.21067748572133E-0001 1.84673939440142E-0001 4.92524404424017E-0001 7.12342677387871E-0002 7.96502869842101E-0002
2 4.04379420505886E-0002 1.23974793157863E-0001 1.87345455434418E-0001 4.93884159767049E-0001 7.12824644937011E-0002 7.96557196170242E-0002
3 3.68987460933303E-0002 1.12751035974956E-0001 1.91825875317798E-0001 5.00041719013369E-0001 7.15531315507620E-0002 7.98169887866874E-0002
4 3.57067331075390E-0002 1.07788754807110E-0001 1.81969191331358E-0001 5.11013278795955E-0001 7.29625300045009E-0002 8.07828528069485E-0002
5 3.58724245649713E-0002 1.07356857843361E-0001 1.80272938089463E-0001 5.05739012708890E-0001 7.63298349121537E-0002 8.35531419340928E-0002
6 3.61523520344178E-0002 1.08262976609136E-0001 1.80693572993243E-0001 5.03920322534821E-0001 7.55886536046035E-0002 8.64229847319576E-0002
energy group : 3
escape probability p[esc][i]=1,maxcrg
8.94957669627031E-0001 8.94231031630653E-0001 8.94421373756369E-0001 9.00245910895974E-0001 9.14078290187123E-0001 9.26679449079450E-0001
collision prob. p[ij] i! j ->
1 5.57175198179802E-0002 1.34176346919883E-0001 2.05621001806012E-0001 4.42110782530681E-0001 7.65795492636274E-0002 8.55696166178491E-0002
2 4.48130252088787E-0002 1.37322160762781E-0001 2.08577592487884E-0001 4.43513320740620E-0001 7.66713431277750E-0002 8.562179070607E-0002
3 4.10858085032959E-0002 1.25513317891780E-0001 2.13640433481942E-0001 4.49556741845754E-0001 7.70821668192041E-0002 8.59327550235528E-0002
4 3.99244159844693E-0002 1.20551945719305E-0001 2.03747564854374E-0001 4.60602630577004E-0001 7.87581140718274E-0002 8.71667457003711E-0002
5 4.0062752344373E-0002 1.19974179237503E-0001 2.01746469980559E-0001 4.55335491288093E-0001 8.2109353395557E-0002 8.98917466152422E-0002
6 4.03477303158563E-0002 1.20900645244778E-0001 2.02112261453856E-0001 4.53558564019213E-0001 8.13084084944789E-0002 9.28087366430937E-0002

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Fig. 1 The P_{ij} matrix for the first three energy groups of fast energy.

Fig. 2 The P_{ij} matrix for the last three energy groups of thermal energy.

Because the boundary condition of the system is assumed that there are no contributions from outside, the consequence of this boundary condition is the sum of the P_{ij} matrix must be

less than one $\left(\sum_j P_{ij} < 1\right)$ for all mesh due to leakages from the

system as shown in Figures 1 and 2. In other words, the results of the CP matrix calculation are following the reference [16]-[18]. The results of this P_{ij} matrix are used as the basis for nuclear fuel cell design to estimate the shell temperature of fuel elements in former nuclear fuel storage facilities for the needs of reactor construction and safe operation [19]-[20].

v. CONCLUSIONS

The CP method, which is based on integral transport, has proven to be very effective and suitable in solving the problem of neutron transport in nuclear reactors, especially for calculating the CP matrix in each region in nuclear fuel cells. The description of the CP matrix is very close in a region with the same energy, both in high energy regions and in low energy region. Because the boundary condition of the system is assumed that there are no contributions from outside, the sum of the P_{ij} matrix must be less than one that is following the reference. The CP matrix calculation has an important role in determining the neutron flux distribution in the reactor core.

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