Numerical and Analytical Modeling of Permanent Deformations in Panels Made of Nanomodified Carbon Fiber Reinforced Plastic with Asymmetric Packing

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Abstract—In this paper, a mathematical model of a multilayer panel made of nanomodified carbon fiber reinforced plastic with asymmetric packing is proposed. The introduction of nanosized particles into the composition of the composite or its components (fiber or binder) allows not only to increase its physical and mechanical properties, but also to improve the picture of the residual stress-strain state.

The paper investigates the effect of nanomodification of carbon fiber reinforced plastic on the residual stress-strain state after molding using numerical and analytical methods. Numerous results of computational experiments have been obtained. The results of numerical and analytical modeling are compared with experimental data. Conclusions are drawn about the possibility of reducing the residual stress-strain state in structures with asymmetric reinforcement schemes when using a matrix containing carbon nanoparticles. A mathematical model of a multilayer panel made of nano-modified carbon fiber with asymmetric packing has been built.

Investigation of the residual stress-strain state of structural elements made of carbon fiber reinforced plastic made it possible to reveal the possibility of reducing the residual stress-strain state and leash in structures with asymmetric reinforcement schemes when using a matrix containing carbon nanoparticles.

Keywords—Composite material, mathematical modeling, nanomodified carbon fiber, stress-strain state, nanoparticles.

I. INTRODUCTION

Currently composite materials which have significant advantages over traditional materials such as metals are widely used in various industries in the design of structures for aviation and space technology, mechanical engineering, etc. [1]-[11]. Composite materials have increased specific strength and rigidity, resistance to temperature and vibration loads, as well as low specific gravity, which is especially important in aerospace engineering [12]-[18]. Therefore, the development of modern technology requires the creation of new structural materials with high elastic-strength characteristics, and on their basis, designs with more effective weight data. The creation of polymer composites based on nanomodified binders has recently become one of the priority research areas in the field of composite materials manufacturing technologies. Significant progress has been made in this area [19]-[31]. The development of materials that improve their operational limits is based on the reinforcement of two or more fibers into a single polymer matrix, which leads to an improved material system called hybrid composites with a wide variety of material properties [32]-[41]. When creating nanocomposites, the key tasks are the development of efficient, reliable, and affordable production technologies for mass production, which make it possible to obtain materials with stable characteristics. In this work the study of the influence of nanomodification of carbon fiber on the residual stress-strain state after molding is carried out. As a rule, composites are formed at elevated temperatures, after which they are cooled to the operating temperature. Due to the high anisotropy of physical and mechanical properties during cooling, shrinkage of the composite layers is uneven in thickness and directions. This leads to the appearance of residual deflections and internal...
stresses in composite parts [42]-[52]. One of the ways to reduce residual stresses and strains is nanomodification [53]-[60]. The introduction of nanosized particles into the composition of the composite or its components (fiber or binder) allows not only to increase its physical and mechanical properties, but also to improve the picture of the residual stress-strain state [61]-[77]. The main task in the work was to determine the degree of influence of nanomodification parameters on the residual stress-strain state. For this purpose, a study was carried out to determine the residual stress-strain state using the proposed mathematical model of a multilayer panel made of nanomodified carbon fiber reinforced plastic with asymmetric packing.

II. STATEMENT OF THE TASK

Let us consider a multilayer panel made of a polymer composite, which has anisotropy due to the asymmetry of the properties of the structure of the package along the thickness. The panel in question is free from stress and fastening.

Let us introduce the coordinate system 1,2,3 (Fig. 1) associated with the direction of reinforcement. For unidirectional material, axis 1 is aligned with the direction of the fibers, for woven material, axis 1 is aligned with the direction of the warp thread. Axis 2 is perpendicular to axis 1 and lies in the plane of reinforcement. Axis 3 is directed along the layer thickness and is orthogonal to the layer plane. For the panel, we introduce the x, y, z coordinate system so that the x, y axes lie in the reinforcement plane, and the z axis is directed along the thickness of the package.

In the general case, six internal force factors arise in the panel: Nx, Ny, Nxy, Mx, My, Mxy (Fig. 2).

III. SOLUTION METHOD

Let us write down the connection between the panel deformation in the reduction plane and the displacements u0, v0:

\[ \varepsilon_x = \frac{\partial u_0}{\partial x} \quad \varepsilon_y = \frac{\partial v_0}{\partial y} \quad \varepsilon_{xy} = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \]  

(2)

Relationship of the curvature of the panel to the angles of rotation of the normal θx, θy:
\[ \kappa_x = \frac{\partial \theta_x}{\partial x}; \kappa_y = \frac{\partial \theta_y}{\partial y}; \kappa_{xy} = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}. \]

The relationship between the angles of rotation of the normal and the deflection \( w \) is as follows:

\[ \theta_x = \psi_x - \frac{\partial w}{\partial x}; \theta_y = \psi_y - \frac{\partial w}{\partial y}, \]

\( \psi_x, \psi_y \) - lateral shears.

The generalized package stiffnesses are defined as follows:

\[ B_{mn} = I^{(0)}_{mn}, \]
\[ C_{mn} = I^{(1)}_{mn} - eI^{(0)}_{mn}, \]
\[ D_{mn} = I^{(2)}_{mn} - 2eI^{(1)}_{mn} + e^2 I^{(0)}_{mn}, \]
\[ I^{(r)}_{mn} = \frac{1}{r!} \int b_{mn} Z' dt = \frac{1}{r+1} \sum_{k=1}^{N} b_{mn}^{(k)} (Z_k - Z_{k-1}), \]

where \( e \) - coordinate of the reference plane (for an asymmetric packet, it is chosen arbitrarily);

\( m, n = 1, 2, 3, r = 0, 1, 2 \).

Let us write expressions for the forces and moments caused by the temperature fields:

\[ N_x^T = \sum_{j=1}^{3} N_{1j}^T, \quad N_y^T = \sum_{j=1}^{3} N_{2j}^T, \quad N_{xy}^T = \sum_{j=1}^{3} N_{3j}^T, \]
\[ M_x^T = \sum_{j=1}^{3} M_{1j}^T, M_y^T = \sum_{j=1}^{3} M_{2j}^T, M_{xy}^T = \sum_{j=1}^{3} M_{3j}^T, \]

where

\[ N_{ij}^T = \Delta T \sum_{k=1}^{N} [b_{ij}^{(k)} \alpha_j^{(k)} (Z_k - Z_{k-1})]; \]
\[ M_j^T = \Delta T \sum_{k=1}^{N} b_{ij}^{(k)} \alpha_j^{(k)} \left[ \frac{1}{2} (Z_k^2 - Z_{k-1}^2) - e(Z_k - Z_{k-1}) \right]; \]

\( Z_k \) - coordinate of the \( k \)-th layer, measured from the reference plane; \( N \) - number of layers; \( b_{ij}^{(k)} \) - linear stiffness of the \( k \)-th layer reduced to the axes of the panel \((x, y)\) \((i, j = 1, 2, 3)\); \( \Delta T \) - temperature drop due to cooling;

\( \alpha_1^{(k)}, \alpha_2^{(k)}, \alpha_3^{(k)} \) - coefficients of linear thermal expansion of the \( k \)-th layer in the axes of the panel.

Similarly, we write down the efforts and moments from the initial tension:

\[ N_x^H = \sum_{j=1}^{3} N_{1j}^H, \quad N_y^H = \sum_{j=1}^{3} N_{2j}^H, \quad N_{xy}^H = \sum_{j=1}^{3} N_{3j}^H, \]
\[ M_x^H = \sum_{j=1}^{3} M_{1j}^H, M_y^H = \sum_{j=1}^{3} M_{2j}^H, M_{xy}^H = \sum_{j=1}^{3} M_{3j}^H, \]

where

\[ N_{ij}^H = \sum_{k=1}^{N} [b_{ij}^{(k)} \bar{e}^{(k)} (Z_k - Z_{k-1})]; \]
\[ M_j^H = \sum_{k=1}^{N} b_{ij}^{(k)} \bar{e}^{(k)} \left[ \frac{1}{2} (Z_k^2 - Z_{k-1}^2) - e(Z_k - Z_{k-1}) \right]; \]

\( \bar{e}^{(k)} \) - initial deformations of layers in the axes of the panel.

Linear stiffnesses for the \( k \)-th layer in the case of an asymmetric packet have the following form:

\[ b_{11}^{(k)} = [E_1 m^4 + E_2 n^4 + 2(E_1 v_{12} + 2G_{12}) m^2 n^2]^k; \]
\[ b_{22}^{(k)} = [E_2 m^4 + E_2 n^4 + 2(E_1 v_{12} + 2G_{12}) m^2 n^2]^k; \]
\[ b_{12}^{(k)} = b_{21}^{(k)} = [E_1 v_{12} + (E_2 + 2E_1 v_{12} + 2G_{12}) m n]^k; \]
\[ b_{13}^{(k)} = b_{31}^{(k)} = [mm[E_2 m^2 - E_2 n^2 - (E_1 v_{12} + 2G_{12}) m n]^k; \]
\[ b_{23}^{(k)} = b_{32}^{(k)} = [mm[E_2 n^2 - E_2 m^2 + (E_1 v_{12} + 2G_{12}) m n]^k; \]

\( E_1^{(k)} = \frac{E_1}{1 - v_{12}^{(k)} v_{21}^{(k)}}, \)
\[ E_2^{(k)} = \frac{E_2}{1 - v_{12}^{(k)} v_{21}^{(k)}}, \]

The coefficients of linear thermal expansion for the \( k \)-th layer, as well as the layer deformations caused by the initial tension, in the axes of the panel are determined by the appropriate transformation:
The stresses in the layers are determined from Hooke's law using the curvature and deformation components obtained from (1), i.e.

\[
\begin{align*}
\sigma_x^{(k)} &= \left( b_{11} \bar{b}_{11} + b_{12} \bar{b}_{12} - b_{31} \bar{b}_{31} \right) \left( \varepsilon_1 + \kappa \cdot \varepsilon_2 - \Delta T \cdot \varepsilon_{11}^{(l)} \right) - \Delta T \cdot \varepsilon_{22}^{(l)}, \\
\sigma_y^{(k)} &= \left( b_{21} \bar{b}_{21} + b_{22} \bar{b}_{22} - b_{31} \bar{b}_{31} \right) \left( \varepsilon_1 + \kappa \cdot \varepsilon_2 - \Delta T \cdot \varepsilon_{11}^{(l)} \right) - \Delta T \cdot \varepsilon_{22}^{(l)}, \\
\tau_{12}^{(k)} &= \left( b_{12} \bar{b}_{12} + b_{11} \bar{b}_{12} - b_{31} \bar{b}_{32} \right) \left( \varepsilon_1 + \kappa \cdot \varepsilon_2 - \Delta T \cdot \varepsilon_{11}^{(l)} \right) - \Delta T \cdot \varepsilon_{22}^{(l)}.
\end{align*}
\]  

In this case \( z_k \) is the coordinate of the middle surface of the layer, that is \( z_k = (Z_k - e) - h_k / 2 \).

To pass to stresses \( \sigma_1, \sigma_2, \tau_{12} \) in the axes of the layer, it is necessary to use the transformation formulas when rotating the coordinate axes:

\[
\begin{bmatrix}
\sigma_x^{(k)} \\
\sigma_y^{(k)} \\
\tau_{12}^{(k)}
\end{bmatrix} = \begin{bmatrix}
m^2 & n^2 & 2\text{mn} \\
n^2 & m^2 & -2\text{mn} \\
2\text{mn} & -2\text{mn} & m^2 - n^2
\end{bmatrix}^{(k)} \begin{bmatrix}
\sigma_x^{(k)} \\
\sigma_y^{(k)} \\
\tau_{12}^{(k)}
\end{bmatrix}.
\]  

As can be seen from the physical relationships (1), in multilayer panels with an asymmetric stacking, the layers subjected to tension-compression will cause the panel to bend. Such complex behavior of panels in highly loaded structures can lead to a decrease in their efficiency.

In addition, the formation of such panels will be accompanied by the occurrence of residual thermal bending deformations (warpage).

Panels with a symmetrical arrangement of layers are devoid of this drawback, i.e. when layer \((k)\) corresponds to the same layer \((N-k)\), where \(N\) is the total number of layers in the package. With this in mind, we write down expressions for generalized stiffnesses in the following simplified form:

\[
B_{mn} = 2 \sum_{k=1}^{N/2} b_{mn}^{(k)} \left( Z_k - Z_{k-1} \right),
\]

\[
D_{mn} = \frac{2}{3} \sum_{k=1}^{N/2} b_{mn}^{(k)} \left( Z_k^3 - Z_{k-1}^3 \right),
\]

\[
C_{mn} = 0.
\]

In this case, the \( Z_k \) coordinate is measured from the median plane \( e = h / 2 \), and the sum is calculated only for half of the packet. Physical relations (1) for an orthotropic panel with free from load edges then take the form:

\[
\frac{\varepsilon_{H1}^{(k)}}{\varepsilon_{H2}^{(k)}} = \left( \frac{m^2}{n^2} \right)^{(k)} \left( \varepsilon_H^{(k)} \right),
\]  

\[
\begin{align*}
\varepsilon_{H1}^{(k)} &= \left( \varepsilon_{1} + \kappa \cdot \varepsilon_2 - \Delta T \cdot \varepsilon_{11}^{(l)} \right) - \Delta T \cdot \varepsilon_{22}^{(l)}, \\
\varepsilon_{H2}^{(k)} &= \left( \varepsilon_{1} + \kappa \cdot \varepsilon_2 - \Delta T \cdot \varepsilon_{11}^{(l)} \right) - \Delta T \cdot \varepsilon_{22}^{(l)}. \\
\end{align*}
\]  

In the formulas above, \( m(k) \) and \( n(k) \) – are trigonometric functions of the orientation angle of the layers \( \varphi^{(k)} \) relative to the x-axis of the panel:

\[
m^{(k)} = \cos(\varphi^{(k)}),
\]

\[
n^{(k)} = \sin(\varphi^{(k)}).
\]  

We will consider a flat panel without initial curvature with edges free from load and fastening, subject to the action of a temperature field uniformly distributed over the thickness. The reference surface coincides with the middle surface. For the orthotropic structure of the composite \( B_{13} = B_{31} = 0 \), and the coefficients \( C_{13}, C_{31}, C_{23}, C_{32}, D_{13}, D_{31} \) are small and can be neglected. Taking this into account, the physical relations for the orthotropic structure of the composite in expanded form take the form:

\[
0 = B_{11} \varepsilon_x + B_{12} \varepsilon_y + C_{13} \kappa_x + C_{12} \kappa_y - N_x T - N_y H,
\]

\[
0 = B_{21} \varepsilon_x + B_{22} \varepsilon_y + C_{23} \kappa_x + C_{22} \kappa_y - N_x T - N_y H,
\]

\[
0 = C_{13} \varepsilon_x + C_{12} \varepsilon_y + C_{33} \kappa_x + D_{32} \kappa_y - M_x T - M_y H,
\]

\[
0 = C_{23} \varepsilon_x + C_{22} \varepsilon_y + C_{32} \kappa_x + D_{33} \kappa_y - M_x T - M_y H,
\]  

\[
0 = B_{35} \varepsilon_{xy} + C_{33} \kappa_{xy} - N_{xy} T - N_{xy} H,
\]

\[
0 = C_{35} \varepsilon_{xy} + D_{33} \kappa_{xy} - M_{xy} T - M_{xy} H.
\]  

Thus, it is easy to see that in the case of an orthotropic structure of a composite material, the definition of the deformed state is split into two independent problems of finding deflections (curvature component \( \kappa_{xx}, \kappa_{yy} \)) and twists (\( \kappa_{xy} \)).
It is important to note, that mathematical model applied allows to obtain reliable results for various materials with different physical and mechanical characteristics.

For each installation option, 4 calculations were carried out using the properties of a CFRP monolayer from table. 2. Similarly, 4 calculations were carried out using the properties of a nanomodified carbon fiber monolayer from Table 2. In total, 8 calculations were carried out for the laying option [010 / 4510] and 8 calculations for [010 / 9010].

We considered two options for stacking the composite (Tables 2 and 3):

Table 2. Stacking layers [010 / 4510]

<table>
<thead>
<tr>
<th>№</th>
<th>Material</th>
<th>Number of layers</th>
<th>Angle, degrees</th>
<th>Layer thickness, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Carbon fiber</td>
<td>10</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>Carbon fiber</td>
<td>10</td>
<td>45</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**Total thickness: 4.1**

Table 3. Laying of layers [010 / 9010]

<table>
<thead>
<tr>
<th>№</th>
<th>Material</th>
<th>Number of layers</th>
<th>Angle, degrees</th>
<th>Layer thickness, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Carbon fiber</td>
<td>10</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>Carbon fiber</td>
<td>10</td>
<td>90</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**Total thickness: 4.1**

Table 4. Comparison of deflections for options 1-4 and laying [010 / 4510] with experiment.

<table>
<thead>
<tr>
<th></th>
<th>Long side deflection, mm</th>
<th>Short side deflection, mm</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>With nano 19.9</td>
<td>With nano 10.7</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>No nano 13.3</td>
<td>No nano 7.2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>With nano 12.4</td>
<td>With nano 5.7</td>
<td>5.85</td>
</tr>
<tr>
<td>4</td>
<td>No nano 12.5</td>
<td>No nano 4.3</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Table 5 - Comparison of deflections for options 1-4 and laying [010 / 9010] with experiment.

<table>
<thead>
<tr>
<th></th>
<th>Long side deflection, mm</th>
<th>Short side deflection, mm</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
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<td>5.85</td>
</tr>
<tr>
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<td>No nano 13.3</td>
<td>No nano 7.2</td>
<td>3.2</td>
</tr>
<tr>
<td>3</td>
<td>With nano 12.4</td>
<td>With nano 5.7</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>No nano 12.5</td>
<td>No nano 4.3</td>
<td></td>
</tr>
</tbody>
</table>

Tables 4 and 5 show a comparison of deflections for two layering options. Figures 3 and 4 show the distribution of normal stresses over the layers σ1 and σ2 for two variants of laying.
Fig. 3 Results of distribution of normal stresses $\sigma_1$ (a,b) and $\sigma_2$ (c,d) by layers, MPa for options 1-4 and stacking [010 / 4510].

Fig. 4 Results of distribution of normal stresses $\sigma_1$ (a,b) and $\sigma_2$ (c,d) by layers, MPa for options 1-4 and stacking [010 / 9010].

The results obtained by analytical and numerical methods are identical. To verify the data obtained, a comparison was made between the calculated and the deflections obtained on experimental samples. The greatest similarity with experimental data is provided by method 4 for determining the effective properties of a monolayer.

The results obtained show the possibility of reducing the residual stress-strain rate and lean in structures with asymmetric reinforcement schemes when using a matrix containing carbon nanoparticles. This means that the application of nanomaterials as nanofiller can substantially improve the characteristics of the resulting material. This finding opens prospective for practical application of the model obtained for design and calculation of construction composite materials.

V. CONCLUSION

It was shown, that the introduction of nanosized particles into the composition of the composite or its components (fiber or binder) allows not only to increase its physical and mechanical properties, but also to improve the picture of the residual stress-strain state.

The following conclusions are drawn about the possibility of reducing the residual stress-strain state in structures with asymmetric reinforcement schemes when using a matrix containing carbon nanoparticles.

1. A mathematical model of a multilayer panel made of nano-modified carbon fiber with asymmetric packing has been built.

2. A study of the influence of nanomodification of carbon fiber reinforced plastic on the residual stress-strain state after molding was carried out using numerical and analytical methods.

3. Comparison of the results of numerical and analytical modeling with experimental data.

4. Investigation of the residual stress-strain state of structural elements made of carbon fiber reinforced plastic made it possible to reveal the possibility of reducing the residual stress-strain state and lean in structures with asymmetric reinforcement schemes when using a matrix containing carbon nanoparticles.

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