Bearing Capacity Analysis of Reinforced Concrete Beams

Oldřich Sucharda, Jiří Brožovský

***Abstract*—**This paper deals with a non-linear analysis of reinforced concrete beams. The goal is to determine the total bearing capacity of the structure. For purposes of the analysis, an elastic-plastic model of concrete has been chosen. In some calculations, other constituent models of concrete have been used. These models of the concrete are based on fracture mechanics, model of smeared cracks or plasticity. The paper also compares the model of smeared reinforcement and that of discrete reinforcement. When analysing a specific example of a T-beam, stochastic modelling has been used.

***Keywords*—**Non-linear analysis, concrete, plasticity, reinforcement, bearing capacity.

I. Introduction

Concrete structures are among the most common types of civil engineering structures. Use of concrete has numerous advantages but it also introduces numerous problems which are often related to its non-linear constitutive behaviour. There have been developed numerous theories and computational models for concrete and for reinforced concrete [6, 7, 31, 38, 40].

There have been developed models for reinforcement itself (for example [12, 14, 15, 26]). In concrete structures, special types of reinforcements are also used [33].

Many of these approaches have been used for design or for assessment of full-size objects [23].The non-linear analysis is also combined with the stochastic modelling [22]. During the non-linear analysis of reinforced concrete structures, it is essential to make experiments, guess or calculate a series of special properties of concrete [3, 4, 24, 27]. Recent experimental research is described in [17], for example.

The non-linear analysis of building structures is important for assessment of emergency cases [37] such as fire [10].

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Most of these models use elastic–plastic behaviour [6] and fracture mechanics approaches [11]. There are numerous problems with practical utilization of advanced numerical models for reinforced concrete.

The task of non-linear modelling of concrete under mechanical load is non-trivial because of irregular and non-homogenous nature of the material. In other hand mechanical behaviour of concrete is highly non-linear so its correct interpretation needs advanced constitutive models with precise input data.

In this paper we discuss a constitutive model for reinforced concrete which requires a relatively small set of input data. This constitutive model is based on older works [18, 29] and it is purely elastic–plastic model but it still can provide very good results in many cases. The paper includes several examples of remodelled laboratory experiments.

One of the examples is also compared with results of more advanced analysis which has been done with use of "Fracture Plastic Constitutive Model" or model SBETA in the Atena software [11] or model CONCRETE in the ANSYS software [30].

II. Constitutive model

The proposed computational model of concrete [34, 35] is based on the framework of the Chen-Chen [4] condition and on the modified hardening rule by Ohtani [29].

A. Elastic–plastic model

The elastic–plastic behaviour is expected. The relation between increments of stress $\boldsymbol{\sigma}$ and strain $\boldsymbol{\varepsilon}$ are

$$\sigma = \mathbf{D}_{ep}\varepsilon , \tag{1}$$

where $\mathbf{D_{ep}}$ is elastic–plastic material stiffness matrix:

$$\mathbf{D_{ep}} = \mathbf{D_e} - \frac{\mathbf{D_e}\left\{\dfrac{d\mathbf{f}}{d\boldsymbol{\sigma}}\right\}\left\{\dfrac{d\mathbf{f}}{d\boldsymbol{\sigma}}\right\}^T\mathbf{D_e}}{\left\{\dfrac{d\mathbf{f}}{d\boldsymbol{\sigma}}\right\}^T\mathbf{D_e}\left\{\dfrac{d\mathbf{f}}{d\boldsymbol{\sigma}}\right\}+\mathbf{H}} , \tag{2}$$

where $\boldsymbol{H}$ is a hardening function and f is the elastic condition function.

B. Chen-Chen condition

The Chen-Chen condition [6] has been created especially for concrete. The Kupfer's experimental data [25] were used as a foundation for creation of this condition.

The Chen-Chen plasticity condition [6] can be defined by elastic limit in uniaxial compression f_{yc}, by elastic limit in biaxial compression f_{ybc} and by elastic limit in uniaxial tension f_{yt}.

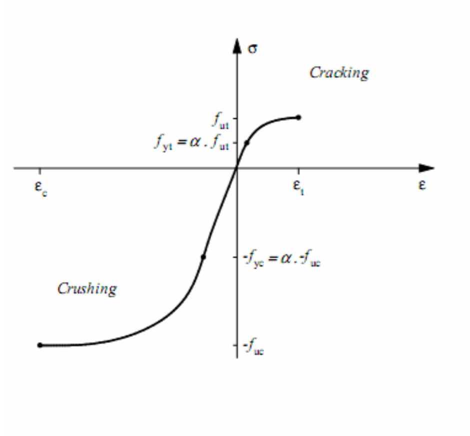

Fig.1 The working diagram of concrete

The condition can be written for purely compression state as

$$f_c = J_2 + \frac{Ayc}{3} I_1 - \tau_{yc} \leq 0. \quad (3)$$

For stress state with tension a different equation cane be used

$$f_t = J_2 - \frac{1}{6} I_1^2 + \frac{A_{yt}}{3} I_1 - \tau_{yt}^2 \leq 0. \quad (4)$$

The I_1 represents thefirst stress invariant and the J_2 represents the second stress deviator invariant. The A_i and τ_1^2 incorporate the elastic limit stresses f_{yc} ,f_{ybc} and f_{yt}. The equations (3 and 4) represent only a special case of the original Chen work [5] and they have been chosen as the most matching for the purposes of the discussed constitutive model. The Chen-Chen condition can also serve as a failure condition. In this case it is defined by strengths in uniaxial compression f_{uc}, in biaxial compression f_{ubc} and in uniaxial tension f_{ut}, respectively. The plasticity condition in the plane of principal stresses are shown in Figure 2. Instead of the Chen-Chen condition the CEB-FIP failure condition [7] is used. The idealized working chart of concrete for a uniaxial state of stress is shown in Figure 1.

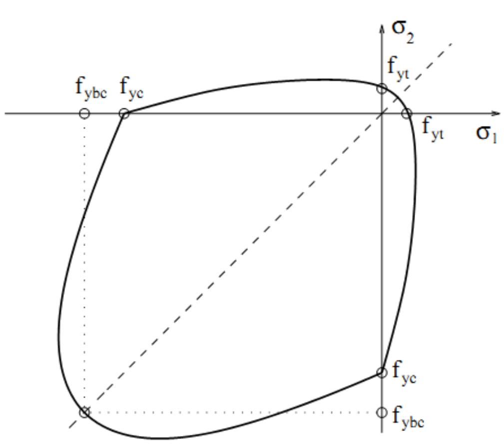

Fig.2 Chen–Chen condition.

C. Hardening rule

The hardening rule is based on the work of Ohtani and Chen [29]. This rule has been designed for use with Chen-Chen plasticity condition. It allows to model materials with different hardening behaviour in compression and in tension. The hardening rule can be written as

$$\psi = \alpha_1 Q_1 H_c + \alpha_2 Q_2 H_{bc} + \alpha_3 Q_3 H_t . \quad (5)$$

The α_i members are coefficients of influence. The Q_i members are derivatives of plastic potential function. It is assummed here that the plastic potential function has the same form as the plasticity condition and can be written as

$$Q_c = \left\{ \frac{\partial \sigma_c}{\partial \varepsilon_{pc}} \right\},.$$
$$Q_{bc} = \left\{ \frac{\partial \sigma_{bc}}{\partial \varepsilon_{pbc}} \right\}, \quad (6)$$
$$Q_t = \left\{ \frac{\partial \sigma_c}{\partial \varepsilon_{pt}} \right\},$$

The Q_i parameters depend on actual stress state of material. They are related to equivalent stresses in uniaxial and biaxial stress states which have to be computed iteratively.

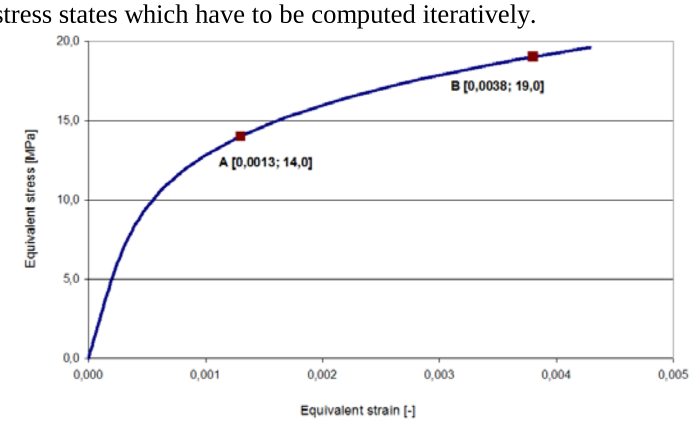

Fig.3 Ramberg–Osgood approximation.

The H_i members represent particular hardening functions and they can be written as

$$H_c = \frac{\partial \sigma_c}{\partial \varepsilon_{pc}},.$$
$$H_{bc} = \frac{\partial \sigma_{bc}}{\partial \varepsilon_{pbc}}, \quad (7)$$
$$H_t = \frac{\partial \sigma_c}{\partial \varepsilon_{pt}},$$

The partial hardening functions 8 have to be obtained from results of experimental testing. An initial Young modulus and two points of an experimentally obtained stress–strain relation are needed here.

The H_i s can be approximated by suitable function, the Ramberg–Osgood function (Figure 3) [18, 34] is used here. The Ramberg–Osgood function can be defined

$$\varepsilon = \frac{\sigma}{E_0} + k\left(\frac{\sigma}{E_0}\right)^n \tag{8}$$

where the E_o is initial Young modulus of material and the parameters can be defined as

$$n = \frac{\ln\left(\dfrac{E_0\varepsilon_a - \sigma_a}{E_0\varepsilon_b - \sigma_b}\right)}{\ln\left(\dfrac{\sigma_a}{\sigma_b}\right)}, \tag{9}$$

$$k = \left(\frac{\sigma_a}{E_0}\right)^{1-n}\left(\frac{E_0\varepsilon_a}{\sigma_a} - 1\right). \tag{10}$$

The ε_i are relative deformation and the σ_i are stresses in certain points of the stress–strain relation.

D. Models of reinforcement

Two basic ways of reinforcement modelling have been used here [36]. There is a smeared reinforcement approach which includes an average stiffness of the reinforcement into the material stiffness matrix. In this case it is possible to write the material stiffness matrix of concrete as

$$\mathbf{D}_{s,i} = \begin{bmatrix} pE_{s,i} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{11}$$

where E_s is Young modulus of the reinforcement and p is ratio between volume of reinforcement and the total volume of material.

The matrix of stiffness of the material is transformed for the respective direction using the equation

$$\mathbf{D}_s = \mathbf{T}_\sigma^{-1}\mathbf{D}_{S,x}\mathbf{T}_\varepsilon, \tag{12}$$

where $\mathbf{T}_\sigma^{-1}$ and $\mathbf{T}_\varepsilon$ are transformation matrix. Shape transformation matrixes are

$$\mathbf{T}_\sigma = \begin{bmatrix} c^2 & s^2 & 2cs \\ s^2 & c^2 & -2cs \\ -cs & cs & c^2 - s^2 \end{bmatrix}, \tag{13}$$

$$\mathbf{T}_\varepsilon = \begin{bmatrix} c^2 & s^2 & cs \\ s^2 & c^2 & -cs \\ -2cs & 2cs & c^2 - s^2 \end{bmatrix}. \tag{14}$$

This approach is easy to use but it is unable to target two important properties of the reinforcement in concrete: the inelastic behaviour and the slippage between reinforcement and concrete.

As an alternative the discrete reinforcement is used (Figure 4). It utilises one-dimensional finite elements for modelling of reinforcement in concrete. This approach also allows utilisation of special connecting elements which can simulate slippage between reinforcement and concrete [26, 19, 36].

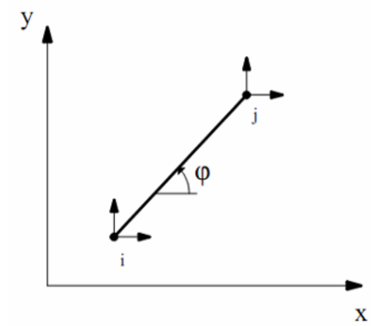

Fig.4 Discrete reinforcement model.

III. Experiments

This section briefly describes the experimental tests which have bee remodelled with use of discussed constitutive model. All these experimental setups are simple supported reinforced concrete beams of rectangular cross-section and they are loaded by on or two forces.

There are three experiments labelled A, B, C and D. Input data are given in Tables I and II and experimental setups are schematically shown in Figure 5.

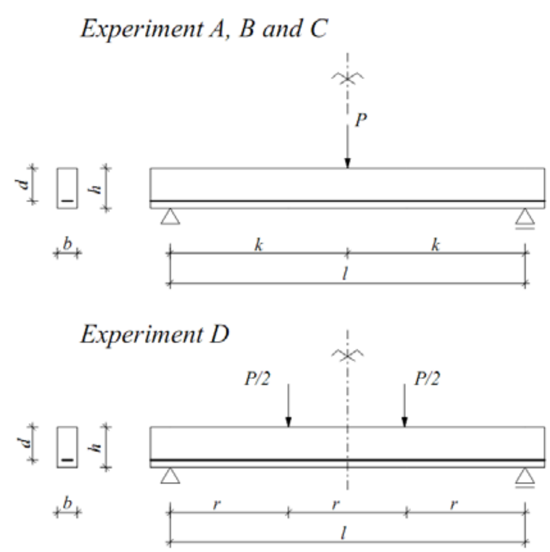

Fig.5 Scheme of experiments

The recommendations in [6, 7, 36] have been used to compute input data not provided by original authors of experiments.

The A experiment has been conducted by Bresler and Scordelis [2]. The beam was reinforced by two levels of reinforcement in bottom half of the cross-section. The radius of four reinforcing bars is 28.7 mm. The authors provided the compression strength of concrete f_{uc} = 22.6 MPa.

The B experiment was conducted by Vecchio [39] on basis of the previous experiment of Bresler and Scordelis. It was reinforced by two levels of reinforcing bars. The diameter of the upper bars was 29.9 mm and diameter of the lower ones was 25.2 mm.

Exp.	f_y	f_u	E_s
	[MPa]	[MPa]	[MPa]
A	555.0	933.0	218 000.0
B (25,2)	445.0	680.0	220 000.0
B (29,9)	436.0	700.0	210 000.0
C	309.4	-	203 255.0
D	323.6	-	198 000.0

Tab. I Input data - reinforcement

The author provided the compression strength of concrete f_{uc} = 22.6 MPa and the initial Young modulus E_o = 36.5 GPa.

The C experiment was published in [18]. There was an one layer of reinforcing bars of diameter 25.5 mm. The author provided an initial Young modulus E_o = 26.2 MPa.

The D was published by Gaston Sleiss and Newmark [16]. Additional data have been obtained from [26]. The authors provided the compression strength f_{uc} = 32.3 MPa and the initial Young modulus E_o = 27.1 GPa.

The model E in the second type experiment is a beam with a T-shaped cross-section which is loaded with two forces [1].

The total length of the beam is 7.04 m. The height of the beam incl. the slab is e 0.625 m. The height of the upper slab is 0.11 m and the width of the upper slab is 1.00 m.

The rib is 0.20 m wide. Geometry of the tested beam, distribution of forces and shape of the cross-section are shown in Figure 6. The load is 600 kN. The position of the reinforcement is shown in Figure. 7. The reinforcement consists of the main load-carrying reinforcement with six rods, dia. 30 mm. Table III shows properties of the materials specified by the author [1] which were used for the calculations.

Exp.	***B***	***h***	***D***	***l***	***k (r)***	**Load**
	[m]	[m]	[m]	[m]	[m]	[kN]
A	0,310	0,556	0,461	3,66	1,83	400
B	0,305	0,552	0,457	3,66	1,83	400
C	0,203	0,508	0,456	3,66	1,83	200
D	0,152	0,305	0,272	2,7	0,9	58

Tab. II Input data – Experiments

The Poisson coefficient for the concrete was chosen to be 0.15. An ideally elastic-plastic reinforcement was assumed for the calculation. For the reinforcing inserts, dia. 6 and 8 mm, the yield point of steel was f_y = 482.45 MPa. As for the other reinforcing inserts (dia. 12 and 30 mm), the yield point was f_y = 567.77 MPa. Figure 8 shows a planar calculation model.

In.	Unit	Exp.	ANSYS	SBETA	BSA
E	GPa	31.38	31.38	31.38	31.38
F_c	MPa	23.4	40.0	24.5	23.4
F_t	MPa	2.16	2.5	2.45	2.16

Tab. III Input data – reinforcement

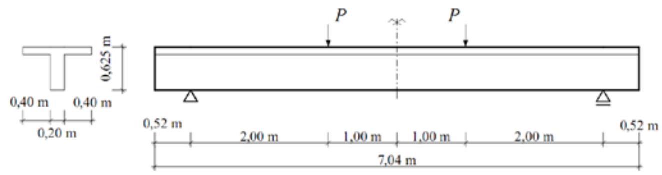

Fig.6 Scheme of experiment: T-beams.

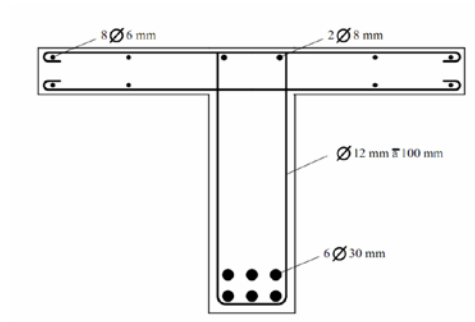

Fig.7 Scheme of experiment: T-beams (cross-section).

IV. Numerical modelling

The numerical modelling has been conducted to verify the discussed constitutive model and its computer code implementation in the BSA software [34]. In this section load–deformation diagrams comparison of experimental and computational results is given. The computations have been done with use of two-dimensional four-node isoparametric finite elements [32, 41]. Plane stress state has been assumed. A 3D visualisation of 2D models is shown in Figure 8 and 9.

The non-linear solution has been done with use of the Newton–Raphson procedure.

The schemes of computational models are shown in Figure 8 and 9.

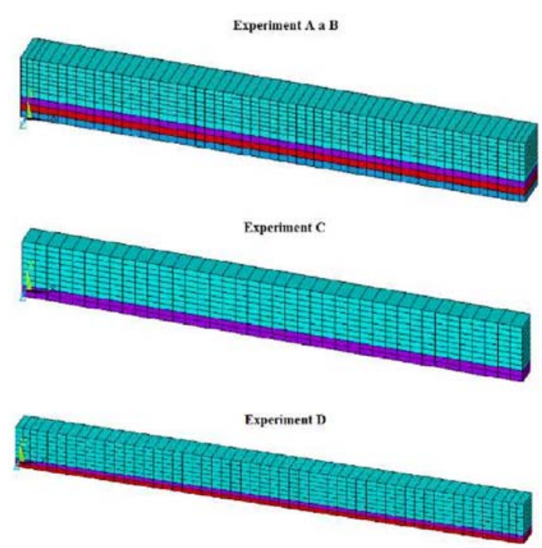

Fig.8 Finite element models.

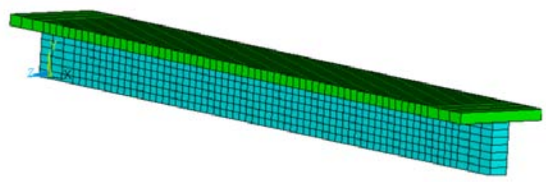

Fig.9 Finite element models – Experiment E.

V. RESULTS

A. Experiment A

The load–displacement diagram obtained for the model A is shown in Figure 10. It can be noted that results of experimental testing are very close to both results of the BSA software (which uses the discussed constitutive model) and to the commercial Atena software which uses more advanced ”Fracture Plastic Constitutive Model”. The cracks in concrete computed with the Atena are shown in Figure 11.

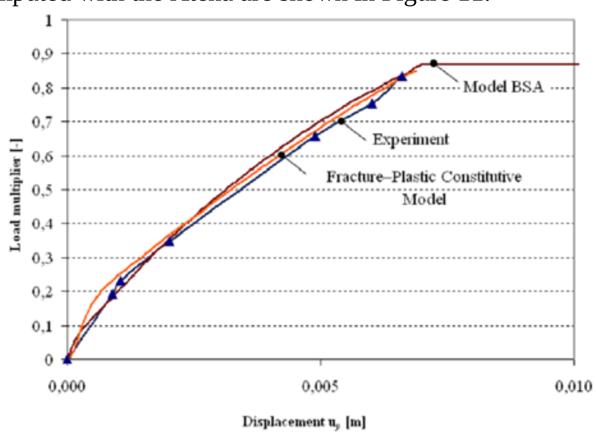

Fig.10 Load-displacement diagrams for case A.

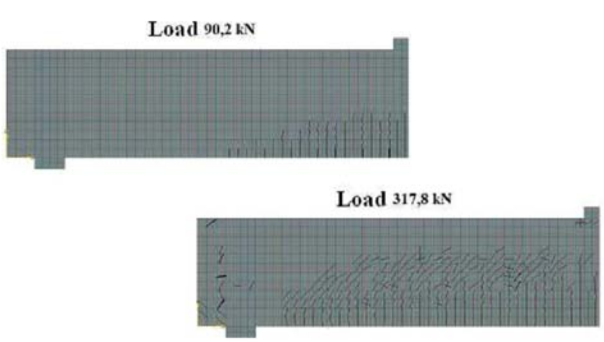

Fig.11 Crack patterns for case A (Atena).

B. Experiment B

The load–displacement diagram obtained for the model B is shown in Figure 12. Also in this case the global behaviour of the model is very close to the experimental data.

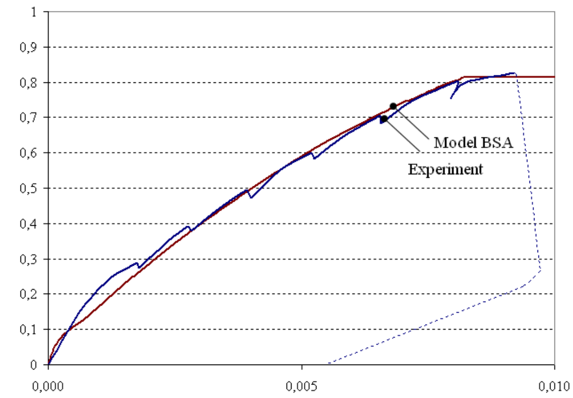

Fig.12 Load-displacement diagrams for case B.

C. Experiment C

The computations of model C have included two alternatives of reinforcement: the smeared one and the discrete one (the link elements). The Figure 13 shows the obtained load-displacement curve.

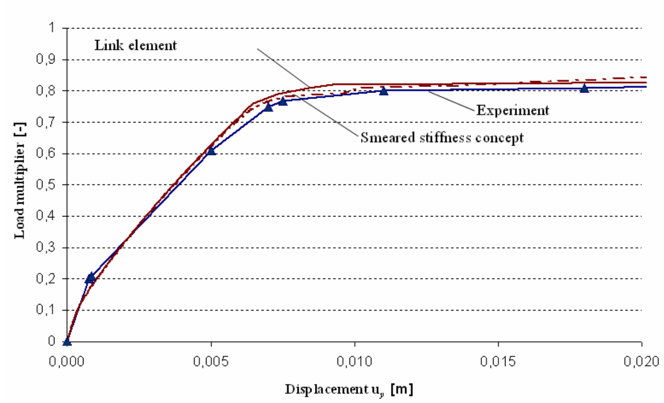

Fig.13 Load-displacement diagrams for case C.

D. Experiment D

It is visible that there are slightly bigger differences between the experimental data and the computed ones. The elastic–plastic model is more conservative than the real behaviour (Figure 14). Figures 15-17 show stress σ_x during loading.

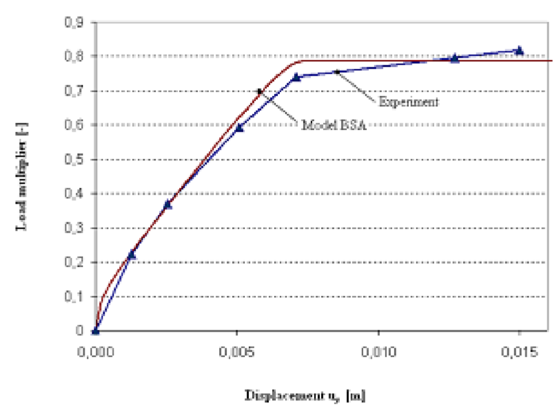

Fig.14 Load-displacement diagrams for case D.

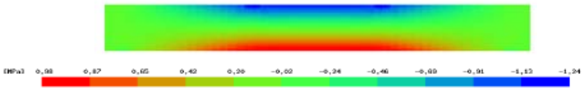

Fig.15 Stress σ_x during loading [MPa], load multiplier= 0.10 (experiment: D).

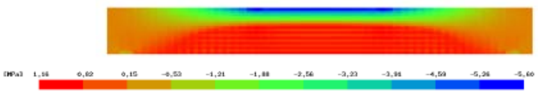

Fig.16 Stress σ_x during loading [MPa], load multiplier= 0.30 (experiment: D).

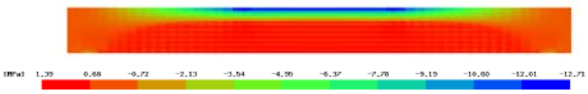

Fig.17 Stress σ_x during loading [MPa], load multiplier= 0.62 (experiment: D).

E. *Experiment E*

In this case, the numerical solution correlates well to the experiment. The load-displacement diagrams (Figure 18) prove good correlation, in particular, in case of lower levels of loads. The SBETA [13] model shows better compliance in the required loading level, while the elastic-plastic model shows better compliance in the loading curve working diagram. But the calculated values are overestimated, if compared with the experiment.

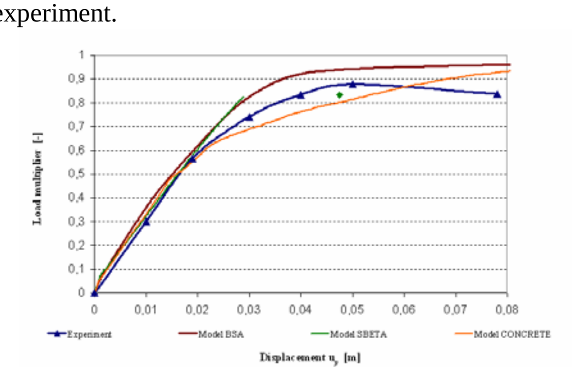

Fig.18 Load-displacement diagrams for case D

In the chart, there is a green dot which ranks among the results acquired in the SBETA model [13]. But for the specified state, the convergence criteria are not fulfilled anymore. Figure 19 and 20 indicate occurrence of initial cracks inside the span at the lower edge and occurrence of the shear crack for higher loads. For the position of the cracks in ANSYS [30] see Figure 21 and 22. The initial development of cracks is shown in Figure 21, while Figure 22 shows development of the cracks for the loading step prior to the maximum load.

Fig.19 Crack in concrete – initial crack (load multiplier = 0.08): ATENA.

Fig.20 Crack in concrete – shear crack (load multiplier = 0.72): ATENA.

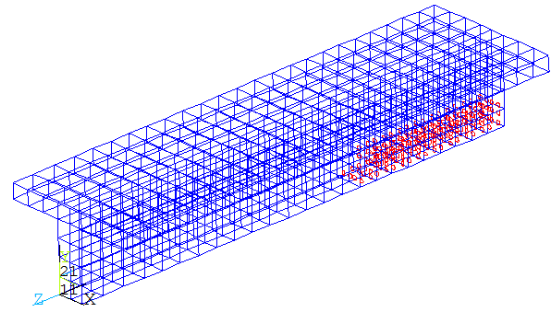

Fig.21 Crack in concrete – initial crack (load multiplier = 0.08): ANSYS.

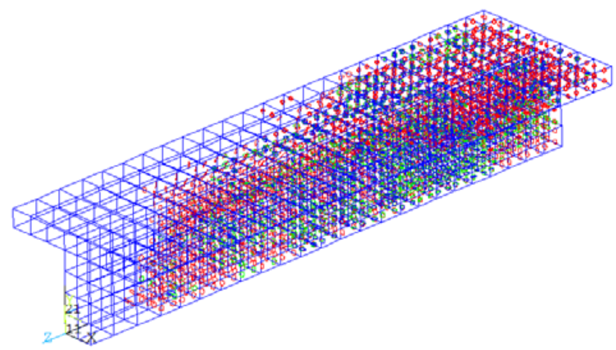

Fig.22 Crack in concrete (násobitel zatížení = 0,91): ANSYS.

F. *Comparison of results*

Table IV summarises differences between load capacities of beams which were obtained from experimental tests and from computations. These results are compared in the forms of maximum loads. The computed value mean the end of convergence of the Newton–Raphson method which is usually near the peak load capacity of the model.

The overall behaviour of model in all of the discussed cases shows that it is possible to use this model for modelling of such types of reinforced concrete beams.

Exp.	A	B	C	D
$P_{max,\ exp.}/\ P_{max,\ BSA}$	1.04	0.98	1.07	0.96

Tab. IV Bearing capacity.

VI. Reliability Analysis Of The Structure

In order to analyse reliability of the structure, it is necessary to determine the size and location of the expected load and to calculate resistance of the structure. When calculating resistance, it is possible to calculate responses of the structure for individual sets of stochastic input parameters which describe uncertainty of input data [20], [21] and [28]. Random quantities are described by the type of distribution and related properties which may include the medium value and standard deviation.

The software FReET [8, 9], we have used, includes more than 25 pre-defined distributions including the possibility to enter a custom one.

LHS [8, 28] (Latin Hypercube Sampling) method can be used for calculation of static characteristics of response of the structure. This method is suitable for tasks which required much time and high-performance systems for calculation. When computing random quantities, a sophisticated approach is used to select values from the predefined intervals of probability distributions, the goal being to reach the maximum efficiency. Details about LHS are given in the theoretical documentation [8] and in the User Manual [9]. An advantage of the FReET is that the software include databases of stochastic parameters for many building materials.

It is also possible to use JCSS [21] and [20] for selection of random quantities. For random inputs which are being modelled, it is recommended to create a correlation matrix.

The number of simulations depends on complexity of the tasks – it is possible to perform several calculations with the increasing number of simulations and to monitor how the results are becoming more accurate. The shape of the final histogram, for instance, the deflection, can be calculated using Monte Carlo which is also available in the FReET [28].

The process of calculation of the reliability of reinforced concrete structures can be divided into following steps:

- selection of the structure to be analysed
- selection of a suitable constitutive model of concrete and statistic tools
- a non-linear analysis for the deterministic model
- preparation of the specified random input data
- stochastic modelling
- evaluation of results.

VII. Stochastic modelling example

A T-shaped beam has been chosen for the analysis. It was loaded with two forces [1]. First, a non-linear analysis for a deterministic model has been carried out. See the experiment E. The non-linear analysis of the reinforced concrete structure was carried out using an elastic-plastic model of the concrete and the BSA software. The stochastic modelling was carried out using the sophisticated method LHS [28] and FReET [8]. Auxiliary subroutines were developed for the computational task. They process the generated input values for FReET to create a format of data suitable for the BSA.

Seven random quantities were chosen for the stochastic modelling. The objective is to monitor sensitivity and dispersion of maximum deformation and maximum load which can be used then for calculation of safety and load-carrying capacity of the structure.

An available probabilistic distribution has been used in order to describe probability distribution of the chosen random quantities. Then, statistic parameters were employed.

In.	Unit	Mean	Var. coof.	Type
E_c	GPa	31.380	0.15	Lognormální
f_t	MPa	24.5	0.18	Weibullovo
f_c	MPa	2.45	0.1	Lognormální
E_{s1}	GPa	210.0	0.03	Lognormální
f_{y1}	MPa	482.45	0.05	Lognormální
E_{s2}	GPa	210.0	0.03	Lognormální
F_{y2}	MPa	567.77	0.05	Lognormální

Tab. V Input data

Input	E_c	f_t	f_c
E_c	1	0.7	0.9
f_t	0.69882	1	0.8
f_c	0.90007	0.8011	1

Tab. VI Correlation matrix

Table V shows the chosen parameters of the random values. Selection of the statistic parameters was made using the integrated database of building materials [20] and recommendations specified in JCSS [21]. In case of the stochastic input values of the concrete, the correlation matrix was created - see Table VI.

6.

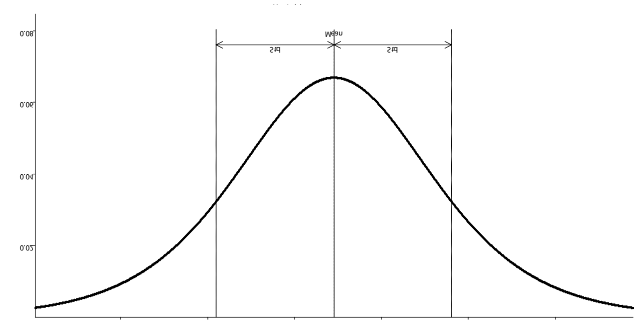

Fig.23 Estimated probabilistic distribution of the load [kN] for deformation of 0.05 m in an elastic-plastic model.

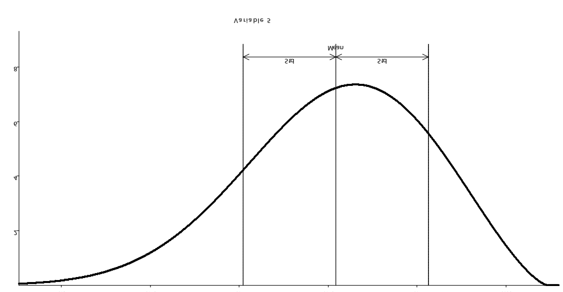

Fig.24 Estimated probabilistic distribution for maximum deformation in an elastic-plastic model.

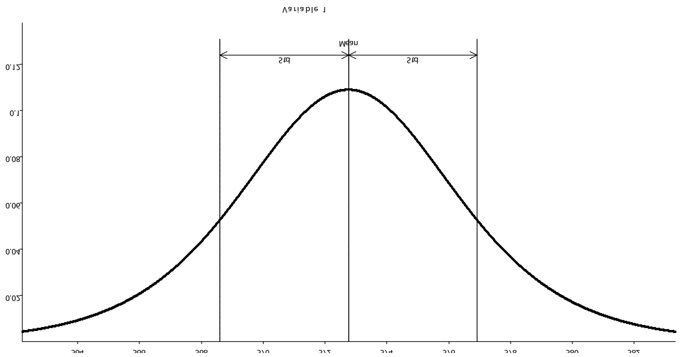

Fig.25 Estimated probabilistic distribution for maximum load [kN] in an elastic-plastic model of concrete.

About 30 simulations were used for the stochastic modelling. Further increase in the number of simulations would result, in particular, in more time and complexity of the calculation. The stochastic results monitored maximum deformation in the middle of the span, magnitude of maximum load and magnitude of load for deformation of 0.05 m. The resulting probabilistic distribution is shown in Figure 23 through Figure 25. Statistic characteristics of the results are summarised in Table VII.

Input	Mean.	Std.	COV	Histogram
F [kN] for 0,05 m	572,75	4,1642	0,0072	Logistic
F [kN]	587,24	6,7808	0,011547	Logistic
u_{max} [m]	-0,14585	0,052232	0,35811	Weibull (3 par)

Tab. VII Output data

VIII. Conclusion

The results discussed in the previous sections show that the proposed elastic–plastic model can be used for numerical modelling of reinforced concrete beams with good precision. The model is based on a combination of rather traditional approaches (the Chen-Chen condition, multi–parameter hardening rule, Ramberg–Osgood approximation of hardening functions).

The main advantage of the model is the low demand for input data and its low computational requirements. Both of these advantages can be very important: experimental testing of concrete is quite complicated and obtaining of data that are often required for more advanced numerical models (the fracture energy, for example) is usually quite tricky. The elastic–plastic model when combined with the Newton–Raphson procedure requires much smaller number of computational operations (and usually also much smaller number of computational steps or iterations) than advanced models based on the fracture mechanics.

In order to analyse the concrete structures in detail and in order to take into account uncertainties in input materials of the material, the non-linear analysis and stochastic modelling were carried out for the T-beam which was loaded with two forces. The calculations indicate that a correlation matrix could be a good solution for this type of the task. The resulting estimated probabilistic distribution is very close to Gaussian distribution.

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