Investigation on Heat Transfer in Quadrangular Plate under Vertical or Horizontal Crack Using FDM

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Abstract: In this study an efficient method for distribution of temperature in quadrangular plate is developed. Heat transfer is transient which by that one can gain distribution of temperature of plate in various times. Temperature of plate is calculated using FDM. Applying transient heat transfer equation and comparing with results of ANSYS Software, excellent agreement is shown. Then equations to calculate temperature in cracked plate in all of regions of plate were derived. The transient temperature distribution of finite plate with insulated cracks are compared with an non-cracked plate and discussed in detail.

Keyword : Temperature, crack , heat transfer , FDM

Manuscript received January 2, 2007.
Revised received May 7, 2007

1 Introduction

Although the most of the progresses in solid mechanic science has been achieved in the last fifty years, but its life is very long. One branch of this science with lots of application and complicate analyze is fatigue mechanic. The safety assessments of structures in harsh thermal environments are increasing design engineer considerations. It has been also recognized that subject to a large variation of thermal conditions, the heat flow would increase the cracks. Almost eighty percent of failure of structure is from expanding of cracks, hence the development of analyzing methods that can accurately estimate the temperature distribution of structures with crack is needed. When a noticeable deformation is created in material theories of mechanical fracture such as linear elastic fracture mechanics (LEFM) are not valid. Irwin [1] with use LEFM presented correction of plastic region. Likewise Rice [2] with proposing non-linear elastic for material with plastic deformation succeeded expand conception energy release rate for material with non-linear treatment. Emery et al.[3] computed transient thermal stress intensity (TSIF) for an edge crack in a finite plate subjected to heat flow by finite element method (FEM). Nied [4] studied the case of edge–cracked strip with the crack and insulated on the other side. It was shown that surface heating might induce compressive, transient stress in the plate surface, which will force the crack surface contact together over a certain length. Similar problem was also studied by Rizk [5] and unique results were obtained. A ramp function that is more realistic than a step function was assumed at the boundary by Rizk [5]. Rizk and Radwan [6] studied a cracked semi- infinite plate subjected to a sudden cooling on the surface in the form of a ramp Function. KoKini and Reynolds [7] analyzed the transient behavior of interface crack located at the center or edge of two finite different materials under thermal boundary conditions by FEM. Lee and Hong [8] computed the transient TSIF For a finite plate with a cusp Crack by BEM. Magalhaes and Emery [9] studied the transient effect of thermal boundary condition on the propagation of crack in a brittle substrate caused by residual tension in a brittle film using a finite element approach.

2 Fracture of plate

Energy of joint is $E_b = \int_{x_0}^{\infty} p\,dx$ which x_0 is atomic distance in balance state and p is applied force. Strength of adhesion in atomic surface considering ideal condition is assumed by relation between force and displacement to a half wave of sinus:

$$p = pc\sin\left(\frac{\pi x}{\lambda}\right) \qquad (1)$$

for small displacement relation between force and displacement is linear.

$$p = pc\left(\frac{\pi x}{\lambda}\right) \qquad (2)$$

3 Effect of concentration crack stress

Theoretically strength adhesion of material is almost E/π which E is elastic module. So surface energy approximately equal to:

$$\gamma_s = \frac{1}{2}\int_0^{\lambda} \sigma_c \sin\left(\frac{\pi x}{\lambda}\right)dx = \sigma_c \frac{\lambda}{\pi} \qquad (3)$$

Which σ_c is fracture strength. Surface energy divide surface unique surface is half energy of fracture when material is broken two surface is created. Strength of Fracture is three to four times less than E/π. The experiments showed that difference between actual strength of brittle material and the theoretical prediction due to existence of very small crack increases local stresses cause increase of the total strength of material. Inglis [10] studied concentration effects of small crack by analyzing it with in elliptical hole with length of 2a and width (2b) for plate under vertical Stresses. Stress in head of principal axial is :

$$\sigma_A = \left(1 + \frac{2a}{b}\right) \qquad (4)$$

Inglis [10] also found stress as defined to radius of bending was more sufficient

$$\sigma_A = 2\sigma\sqrt{\frac{a}{x_0}} \qquad (5)$$

Which σ is energy of fracture can be found as:

$$\sigma = \sqrt{\frac{E\,\gamma_s}{x_0}} \qquad (6)$$

When $\sigma_A = \sigma_c$, the fracture would occur. Therefore fracture can be found :

$$\sigma_A = \left[\frac{E\gamma_s}{4a}\right]^{1/2} \qquad (7)$$

The high intensification of transient temperature gradient will induce thermal stress that may cause rapid linkage of several small cracks into a large Crack. Although each individual crack may be considered safe within the damage tolerance of the structure damaged to the structure can be caused by the interaction of multiple cracks in the structure. If temperature of a body varies from T_0 to T with continuum mechanic equation:

$$\sigma_{ij} = C_{ijkl}\varepsilon_k - B_{ij}(T - T_0) \qquad (8)$$

Following relation is found:

$$\varepsilon_{ij} = \frac{1+\gamma}{E}\varepsilon_{ij} - \frac{\gamma}{E}\varepsilon_{kk}\delta_{ij} - \frac{1-2\gamma}{E}(T - T_0)\delta_{ij} \qquad (9)$$

Equation of transient heat transfer is:

$$a^2\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] = \frac{\partial u}{\partial t} \qquad (10)$$

The following boundary conditions can be applied to (10):

$$\begin{cases} a^2(u_{xx} + u_{yy}) = ut & \\ u(x,0,t) = u(a,h,t) = 0 & 0 < x < L \\ u(0,y,t) = u(l,y,t) = 0 & 0 \le y \le H \\ u(x,y,0) = F(x,y) & \end{cases} \qquad (11)$$

Where exact solutions is :

$$\lambda_{mn} = X_{mn} + \gamma_n = \left(\frac{m\pi}{H}\right)^2 + \left(\frac{n\pi}{L}\right)^2 - \begin{cases} m = 1,2,000 \\ n = 1,2,000 \end{cases} \qquad (12)$$

$$u(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} \sin\frac{n\pi}{L}x \sin\frac{ma}{lt} Exp(-a^2\lambda mnt) \qquad (13.a)$$

$$B_{mn} = \frac{4}{HL}\int_0^H\int_0^L F(x,y)\sin\frac{n\pi}{L}x\sin\frac{n\pi}{lt}y\,dxdy \qquad (13.b)$$

4 Numerical Method

Equation transient heat transfer equation can be written as:

$$k\left[\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2}\right] = C\frac{\partial T}{\partial t} \qquad (14\text{-a})$$

$$\frac{\partial^2 t}{\partial x^2} = \frac{1}{(\Delta x)^2}\left[T^p_{m+1,n} + T^p_{m-1,n} - 2T^p_{m,n}\right] \qquad (14\text{-b})$$

$$\frac{\partial^2 t}{\partial y^2} = \frac{1}{(\Delta y)^2}\left[T^p_{m+1,n} + T^p_{m-1,n} - 2T^p_{m,n}\right] \qquad (14\text{-c})$$

Deriving of temperature respect to time nearly equal to :

$$\frac{\partial T}{\partial t} = \frac{T^{p+1}_{m.n} - T^p_{m,n}}{\Delta t} \qquad (15)$$

$$\frac{\partial T}{\partial t} = \frac{T^{p+1}_{m.n} - T^p_{m,n}}{\Delta t} \qquad (16)$$

In order to establish the second thermodynamic law (16) must be written as:

$$\left(\frac{\Delta x}{\alpha\Delta t}\right)^2 \ge \frac{1}{4} \qquad (17)$$

$$\frac{\alpha\Delta T}{(\Delta x)^2} = F_0(\pi,a)(17.a),\ \frac{h}{k}\Delta x = Bi$$

Where F_o is Fourier coefficient and Bi is Bio number. Environment temperature is considered T. All of region in plate is shown in Fig.2.

In region*(1):*

$$T_{m,n}^{P+1} = F_o\left(T_{m-1,n}^p + T_{m-1,n}^p + 2T_{m,n-1}^p + 2Bit_\infty\right) + \left(1 - 4F_0 - 2BiF_o\right)T_{m,n}^p \quad (18)$$

In region *(2,3,4)*

$$T_{m,n}^{P+1} = F_o\left(2T_{m,n+1}^p + T_{m-1,n}^p + T_{m+1,n}^p + 2Bit_\infty\right) + \left(1 - 4F_o - 2BiF_o\right)T_{m,n}^p \quad (19)$$

In region (5)

$$T_{m,n}^{P+1} = 2F_o\left(T_{m+1,n}^p + T_{m,n-1}^p + 2BiT_\infty\right) + \left(1 - 4F_o - 4BiF_o\right)T_{m,n}^p \quad (20.a)$$

In region (6)

$$T_{m,n}^{P+1} = 2F_o\left(T_{m-1,n}^p + T_{m,n-1}^p + 2BiT_\infty\right) + \left(1 - 4F_o - 4BiF_o\right)T_{m,n}^p \quad (20.b)$$

In region *(7)*

$$T_{m,n}^{P+1} = 2F_o\left(T_{m-1,n}^p + T_{m,n-1}^p + JBiT_\infty\right) + \left(1 - 4F_o - 4BiF_o\right)T_{m,n}^p \quad (20.c)$$

In region (8)

$$T_{m,n}^{P+1} = 2F_o\left(T_{m-1,n}^p + T_{m,n+1}^p + 2BiT_\infty\right) + \left(1 - 4F_o - 4BiF_o\right)T_{m,n}^p \quad (20.d)$$

All of equation are find to calculation temperature point (m,n) at moment of (p+1)

$$i) F_0 \le 1/4 \qquad ii) F_0(1+Bi) \le 1/4 \qquad iii)\ f_0(3+Bi) \le 1/2 \qquad (21)$$
$$iv) F_o(3+Bi) \le 3/4$$

In this research whole of cracks must be horizontal or vertical cracks. So, all of cracks are simulated to rectangular shape (Fig.3).

Region(1)

$$T_{m,n}^{P+1} = \frac{2}{3}F_o\left(2T_{m-1,n}^p + 2T_{m,n+1}^p + T_{m,n-1}^p + T_{m+1,n}^p\right) + \left(1 - 4F_o\right)T_{m,n}^p$$

Region(2)

$$T_{m,n}^{P+1} = \frac{2}{3}F_o\left(2T_{m,n+1}^p + 2T_{m+1,n}^p + T_{m-1,n}^p + T_{m,n-1}^p\right) + \left(1 - 4F_o\right)T_{m,n}^p$$

Region(3)

$$T_{m,n}^{P+1} = \frac{2}{3}F_o\left(2T_{m-1,n}^p + 2T_{m,n11}^p + T_{m,n+1}^p + T_{m+1,n}^p\right) + \left(1 - 4F_o\right)T_{m,n}^p$$

Region(4)

$$T_{m,n}^{P+1} = \frac{2}{3}F_o\left(2T_{m,n-1}^p + 2T_{m+1,n}^p + T_{m-1,n}^p + T_{m,n+1}^p\right) + \left(1 - 4F_o\right)T_{m,n}^p$$

Region(5)

$$T_{m,n}^{P+1} = F_o\left(2T_{m,n+1}^p + 2T_{m-1,n}^p + T_{m+1,n}^p\right) + \left(1 - 4F_o\right)T_{m,n}^p$$

Region(6)

$$T_{m,n}^{P+1} = F_o\left(2T_{m,n+1}^p + 2T_{m-1,n}^p + T_{m+1,n}^p\right) + \left(1 - 4F_o\right)T_{m,n}^p$$

Region(7)

$$T_{m,n}^{P+1} = F_o\left(2T_{m+1,n}^p + 2T_{m,n-1}^p + T_{m,n-1}^p\right) + \left(1 - 4F_o\right)T_{m,n}^p$$

Region(8)

$$T_{m,n}^{P+1} = F_o\left(2T_{m,n-1}^p + 2T_{m-1,n}^p + T_{m+1,n}^p\right) + \left(1 - 4F_o\right)T_{m,n}^p$$

The temperature of whole points of plate were found with following specification:

Ti=300 ^oC ; h=50 $\frac{w}{m^2C}$;

$T_\infty = 25^0 C$; $\alpha = 1/172 \times 10^{-5}\,\frac{m^2}{s}$

$\Delta t = 0.01\sec$; $k = 41/06\,\frac{W}{m^0 c}$; $\Delta x = 2mm$

and material of plate are selected steel (C$\cong$1/0%). For stability it must be :

$$F_o = \frac{\alpha \Delta t}{\left(\Delta x\right)^2} \qquad b = 0.012403;$$

$$F_o \le \frac{1}{4} \qquad F_o(2 + Bi) \le \frac{1}{2} \Rightarrow 0.0586 \le \frac{1}{2}$$

$$F_o\left(1 + Bi\right) \le \frac{1}{4} \Rightarrow 0.02933 \le \frac{1}{4} \quad ;$$

$$F_o\left(3 + Bi\right) \le \frac{3}{4} \Rightarrow 0.0881 \le \frac{3}{4}$$

In the following tables the output of the developed model were verified by the results of ANSYS Software, and also exact solution. From table (1) it can be found that results with FEM is comparable and the error is in the acceptable range.

Table (2) also compare the results from the exact solution with the developed model for T=1 min and T=5 min. It can be seen that with passing the time the difference is decreased. In the theoretical solution temperature at border considered as zero, but in a small distance from the border the temperature is more than 20 ^oC. with passing time and reduce of disconnectivity the error is decreased.

The following table (Table 3) temperature of the plate with two crack is shown. Increase in temperature is not like the plate without crack or with a unit crack.

5 Conclusion

An efficient numerical method is developed in this study to investigate transient thermal conduction in a finite plate with and without insulated cracks. The results showed good agreement with other numerical methods such as FEM as well as exact solution specially after passing time. With passing (lapse) of time and deduction of disconectivity, consequently error is decreased. Insulated cracked cause increase temperature around cracks as shown in table (2). Temperature at cracked plate is more than without cracked plate and insulated crack can increase temperature around crack.

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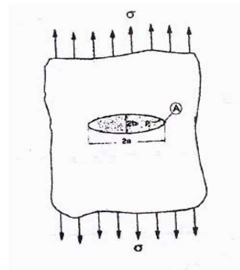

Fig1. Plate with elliptical hole under tension

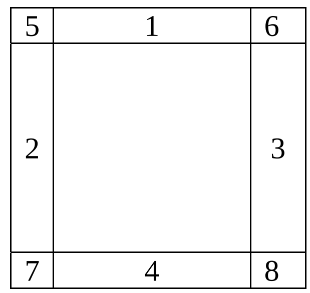

Fig2. All regions in a rectangular plate

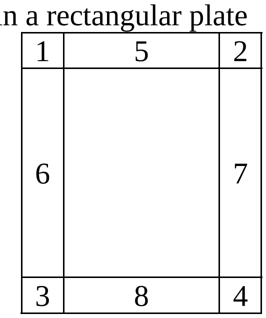

Fig. 3. Simulated crack region

solution / Location	**Presented Solution T=1min**	**Ansys Solution T=1min**	**Err. %**	**Presented Solution T=5min**	**Ansys Solution T=5min**	**Err %**	**Presented Solution T=10min**	**Ansys Solution T=10min**	**Err. %**
T(1,1)	39.3648	38.067	3.29	70.3916	69.904	0.69	107.5585	104.018	3.29
T(1,25)	32.0036	30.402	5	65.9172	62.849	4.65	101.858	98.009	3.78
T(25,1)	32.0036	30.402	5	65.9172	62.849	4.65	101.858	98.009	3.78
T(25,25)	24.4352	24.271	0.67	58.9956	57.205	3.03	95.9864	93.202	2.9

Table 1:Difference between FDM solution and FEM solution

time / location	**With crack T=1min**	**With crack T= 1min**	**Without crack T=5 min**	**Without crack T=5 min**
T(1,1)	39.3648	36.8392	72.6392	74.1576
T(2,1)	38.7516	36.3128	72.1038	73.62
T(3,1)	38.1692	35.8088	71.5928	73.0992
T(48,1)	38. 1692	35.512	72.1028	71.2848
T(49,1)	38.7516	36.0132	72.1038	71.6
T(5,1)	39.3648	36.5396	72.6392	72.3348
T(13,12)	28.5092	28.008	62.896	64.7132
T(14,12)	28.2163	27.672	62.6288	64.212
T(15,12)	27.9520	26.7396	62.3668	62.896
T(13,42)	29.6292	28.2936	63.9264	64.6376
T(15,42)	29.072	27.784	63.4	63.9824

Table 2: Plate with a crack and without Crack

time / location	**T=1min**	**T=5 min**	**T=10 min**
T(1,1)	56.148	80.1524	75.2272
T(2,1)	54.9944	76.2912	74.4404
T(3,1)	56.4456	75.804	73.9616
T(48,1)	54.9412	80.7236	86.1164
T(49,1)	55.504	81.236	86.1176
T(50,1)	56.092	81.768	86.6428
T(13,12)	45.4464	65.7156	64.8532
T(14,12)	45.3624	65.8724	62.6972
T(15,12)	38.676	38.6676	62.5821
T(13,42)	47.2663	69.896	69.7224
T(14,42)	37.976	60.7112	69.7336
T(15,42)	35.54	41.56084	69.7924

Table 3: Plate with two crack