# Experimental and theoretical analysis of the dynamical behavior of the technological equipment foundation

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Abstract: The aim of the work presented here was to investigate the possibility of using hangs in the non-linear vibration characteristics to detect damage in mechanical structure. The nonlinearities were detected by examining the changes in time and frequency response, in over time (and hence over amplitude of vibration). These analyses were made by comparison for the two considerate hypotheses: linear and nonlinear elastic characteristic of the viscous-elastic system.

**Keywords:** vibration, non-linear, linear, detect, damage, frequency

### **I INTRODUCTION**

There is a permanent worldwide interest to identify new control and effect reducing methods due to undesirable actions generated by vibrations and shocks over the generic and constructions' environment. There is a major economical and social interest concerning the development of reliable vibration protection systems. Developing and implementing the equipment and procedure to increase the monitoring equipment quality of vibration protection systems represents the main goal of this project (it could prevent the undesirable consequences of vibrations and shocks).

There are many categories of vibrations and shocks generating technological equipments (working at specified level) in Romania. Undesirable effects over the generic and constructions' environment have been noticed (vibrations and structure noise in the nearby buildings). Every machine in function, becomes a source of vibration generator, able to disseminate vibration in the environment either in structural shape (through connections), or in radial shape.



Fig. 1 Forging hammer

Also, the negative effects of the vibration or the human factor felt along with overtaking of the limit level both below the appearance of exposure duration, and of vibration values, which appearances leads to professional decays. The knowledge and evaluation of the shock and vibration influences on environment become a priority in European society sustainable development. In this way, European Directive 44/2002 establishes minimal requirements, in order to limit the level exposure of transmitted vibration on human or environment.

A multitude of technological equipments utilize shocks and vibration in the production process [4], such as forging hammer (figure 1) or press with eccentric (figure 2).

Every machine in function, becomes a source of vibration generator, able to propagate vibration in the environment either in structurall shape (through connections), or in radial shape.



Fig. 2 Press with eccentric

Due to peculiarity of production process, this equipment propagates shocks and vibration to environment, wherefore is necessary to implement a vibration protection system (figure 3), able to decrease the effect impact on the environment. The whole system is placed in a vat foundation with protection aim against agents like water, fire or dross goal. In time the vibration protection system, that is based on viscous-elastic type, suffering damage that produce system malfunction. These damages are recognized by non-linear characteristics of the viscous-elastic vibration protection system. The

nonlinearities are identified through monitoring in time the fundamental frequency of the viscous-elastic system.





Fig. 3 Mounting viscous-elastic system under technological equipment foundation

### II THEORETICAL PROBLEM FORMULATIONS

In this chapter, will be develop a generalized theoretical model, capable to characterized both linear and non-linear characteristics of the viscous-elastic system in the mathematical approach. In this way will consider that foundation of the technological equipment like forging hammer is placed on the four identically viscous-elastic elements and it has one plane of symmetry, figure 3. The mathematical model was developed for two cases of rigidity characteristic: linear and non-linear expressions.

# A Generalized model

It is considered a rigid body in the inertial system OXYZ that is considered fix and a reference system attached on rigid [1], with the origin placed in its mass centre Cxyz, figure 4.

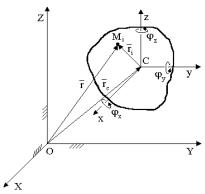


Fig. 4 The rigid in the inertial system OXYZ

The translational movements of the mass centre C are determinate by X, Y, Z coordinate toward fixed system OXYZ, and the rotary movements are describe by angular movements  $\phi_x$ ,  $\phi_y$  and  $\phi_z$  of the Oxyz system.

In order, to calculate the movement of point A of the rigid toward Cxyz system when the rigid make an instantaneous rotation, as the case from figure 5. The rigid rotation  $\Delta \phi$  can be the result of infinitesimal rotation sum.

Using the second kind Lagrange equation is obtained the differential equation system for the movement. The general form of the second kind Lagrange [1] equation is:

$$\frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}_i} \right) - \frac{\partial E}{\partial q_i} = Q_i^P + Q_i^F + Q_i^R, i=1..6$$
 (1)

where

 $Q_i^P = -\frac{\partial V}{\partial q_i}$  are generalized forces on potential kind,

 $Q_i^R = -\frac{\partial D}{\partial \dot{q}_i}$  are generalized forces on viscous kind,

 $Q_i^F = \frac{\partial L_{q_i}}{\partial q_i}$  are generalized forces on perturbation,

 $\delta L_{q_i}$  - virtual mechanical work on perturbation which corresponds  $\mathbf{q}_i$  coordinate.

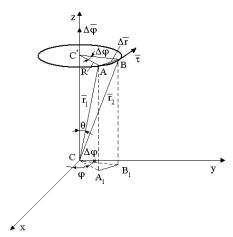


Fig. 5 Instantaneous rotation of the rigid

The metrical expression on the second kind Lagrange equation is like:

$$\underline{A}\underline{\ddot{q}} + \underline{B}\underline{\dot{q}} + \underline{C}\underline{q} = \underline{f} \tag{2}$$

Where:

 $\underbrace{q}_{=} \begin{bmatrix} q_1, q_2, q_3, q_4, q_5, q_6 \end{bmatrix}^T = \begin{bmatrix} X, Y, Z, \varphi_x, \varphi_y, \varphi_z \end{bmatrix}^T \text{ - the vector of generalized coordinates;}$ 

 $\ddot{\underline{q}} = \begin{bmatrix} \ddot{q}_1, \ddot{q}_2, \ddot{q}_3, \ddot{q}_4, \ddot{q}_5, \ddot{q}_6 \end{bmatrix}^T = \begin{bmatrix} \ddot{X}, \ddot{Y}, \ddot{Z}, \ddot{\varphi}_x, \ddot{\varphi}_y, \ddot{\varphi}_z \end{bmatrix}^T$  - the vector of generalized accelerations;

# f - the vector of generalized forces;

$$\underbrace{f}_{=} = \begin{cases}
f_{1} \\
f_{2} \\
f_{3} \\
f_{4} \\
f_{5} \\
f_{6}
\end{cases} = \begin{cases}
Q_{X}^{F} \\
Q_{Y}^{F} \\
Q_{Z}^{F} \\
Q_{\varphi_{x}}^{F} \\
Q_{\varphi_{y}}^{F} \\
Q_{\varphi_{z}}^{F}
\end{cases} = \begin{cases}
\sum_{k=1}^{p} F_{kx} \\
\sum_{k=1}^{p} F_{kz} \\
\sum_{k=1}^{p} \left(y_{k} F_{kz} - z_{k} F_{ky}\right) + \sum_{l=1}^{q} M_{lx} \\
\sum_{k=1}^{p} \left(z_{k} F_{kx} - x_{k} F_{kz}\right) + \sum_{l=1}^{q} M_{ly} \\
\sum_{k=1}^{p} \left(x_{k} F_{ky} - y_{k} F_{kx}\right) + \sum_{l=1}^{q} M_{lz}
\end{cases} \tag{3}$$

## A - inertial matrix;

$$\underline{A} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & J_y & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_z \end{bmatrix}$$

$$(4)$$

### B - damping matrix;

$$B = \begin{bmatrix} \sum c_{jk} & 0 & 0 & 0 & \sum c_{jk}z_j & -\sum c_{jk}y_j \\ 0 & \sum c_{jk} & 0 & -\sum c_{jk}z_j & 0 & \sum c_{jk}x_j \\ 0 & 0 & \sum c_{jk} & \sum c_{jk}y_j & -\sum c_{jk}x_j & 0 \\ 0 & -\sum c_{jk}z_j & \sum (c_{jk}z_j^2 + c_{jk}y_j^2) & -\sum c_{jk}x_jy_j & -\sum c_{jk}z_jx_j \\ \sum c_{jk}z_j & 0 & -\sum c_{jk}x_j & -\sum c_{jk}x_jy_j & \sum (c_{jk}x_j^2 + c_{jk}z_j^2) & -\sum c_{jk}y_jz_j \\ -\sum c_{jk}y_j & \sum c_{jk}x_j & 0 & -\sum c_{jk}x_jx_j & -\sum c_{jk}y_jz_j & \sum (c_{jk}y_j^2 + c_{jk}x_j^2) \end{bmatrix}$$

$$(5)$$

# C - rigidity matrix;

$$\underline{C} = \begin{bmatrix} \sum_{k_{jk}} & 0 & 0 & 0 & \sum_{k_{jk}} z_j & -\sum_{k_{jk}} y_j \\ 0 & \sum_{k_{jk}} & 0 & -\sum_{k_{jk}} z_j & 0 & \sum_{k_{jk}} x_j \\ 0 & 0 & \sum_{k_{jk}} & \sum_{k_{jk}} y_j & -\sum_{k_{jk}} x_j & 0 \\ 0 & -\sum_{k_{jk}} z_j & \sum_{k_{jk}} y_j & \sum_{k_{jk}} (k_{jk} z_j^2 + k_{jk} y_j^2) & -\sum_{k_{jk}} x_j y_j & -\sum_{k_{jk}} x_j x_j \\ \sum_{k_{jk}} z_j & 0 & -\sum_{k_{jk}} x_j & -\sum_{k_{jk}} x_j y_j & \sum_{k_{jk}} (k_{jk} x_j^2 + k_{jk} x_j^2) & -\sum_{k_{jk}} x_j y_j z_j \\ -\sum_{k_{jk}} x_j & \sum_{k_{jk}} x_i & 0 & -\sum_{k_{jk}} x_j x_i & -\sum_{k_{jk}} x_j x_j & \sum_{k_{jk}} (k_{jk} y_j^2 + k_{jk} x_j^2) \end{bmatrix}$$

In this way will consider that foundation of the technological equipment like forging hammer is placed on the four identically viscous-elastic elements [4], and it has one plan of symmetry, figure 6.

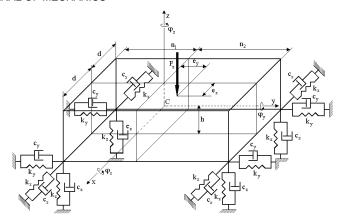


Fig. 6 The physical model

This presented model has a general character, and the possible rigid movements are: on direction OX – forcing lateral vibration, on direction OY - forcing longitudinal vibration, on direction OZ - forcing vertical vibration,  $\phi_x$  - forcing pitching vibration,  $\phi_y$  - forcing rolling vibration,  $\phi_z$  - forcing turning vibration.

The principal axes of the elastic supports are parallel with the references axis. In this case, the movements corresponding to the six degree of freedom are decoupled in two possibilities: coupled movements that are characterized by the coordinate Y, Z and  $\phi_x$  variations and coupled movement that are characterized by the coordinate Y,  $\phi_v$  and  $\phi_z$  variations.

# B The coupled mode " $YZ\varphi_x$ "

Forwards, will be analyzed the coupled model characterized by the coordinate Y, Z and  $\phi_x$  variations because the movement on OZ direction is a very important factor in propagation vibration from technological equipment.

# C The linear elastic characteristic hypothesis

The rigidity on OZ direction of the viscous-elastic element on which is the foundation placed of the technological equipments, have constant value.

The shape of linear elastic characteristic of the viscouselastic system is presented in figure 7.

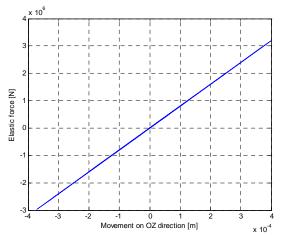


Fig. 7 The shape of elastic characteristic

nonlinearities are identified through monitoring in time the fundamental frequency of the viscous-elastic system.





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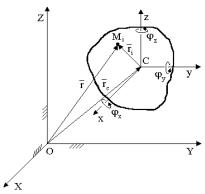


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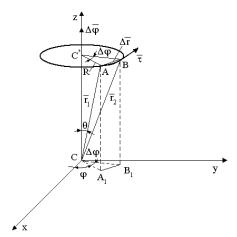


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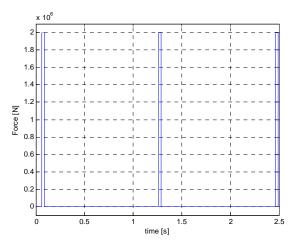


Fig. 9 The shape of rectangle shock

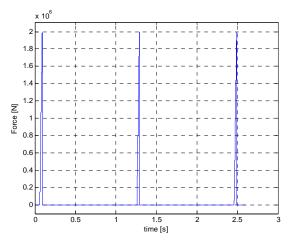


Fig. 10 The shape of triangle shock

The solution of the system (7) leads to the evolution determination in time for three cinematically parameters: acceleration, velocity and movement - on OZ direction [3], figures 11, 12 and 13.

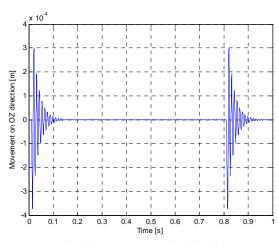


Fig. 11 Movement on OZ direction

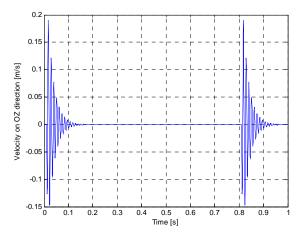


Fig. 12 Velocity on OZ direction

These three cinematically measures are quantitative criteria for evaluating the vibration effects on the human structure or on environment.

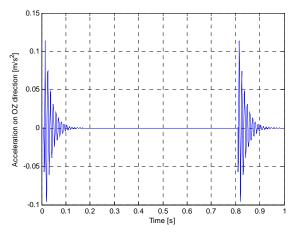


Fig. 13 Acceleration on OZ direction

Eliminating the time between movement and velocity expressions, it is obtained the characteristically curve or movement trajectory (figure 8).

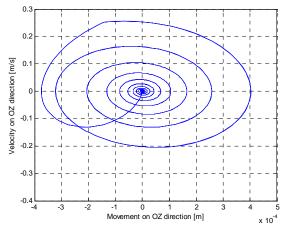


Fig. 14 The phase plan representation

From the figure 14 we observe that movement is damping and stabilized because the amplitude of movement don't have an increasing infinite value.

In the figure 15 is presented the movement on OZ direction in the frequency representation. From this representation we observe that dominant frequency domain is around on 97Hz value.

Another analyze in frequency response is power spectral density, figure 16. The goal of spectral estimation is to describe the distribution (over frequency) of the power contained in a signal, based on a finite set of data.

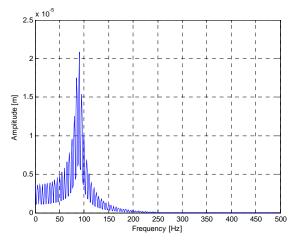


Fig. 15 The system response in frequency domain

Observe a diminution of the dissipate energy in the non-linear elastic characteristic of the viscous-elastic system by comparison with the linear case, this fact is explained by the decrease of the movement on OZ direction in for the non-linear case.

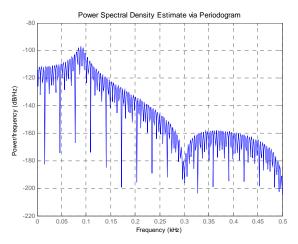


Fig. 16 Power spectral density

Because the elements on which is the foundation placed have viscous-elastic characteristic; these elements dissipate hysteretic energy with W=5093 J, figure 17.

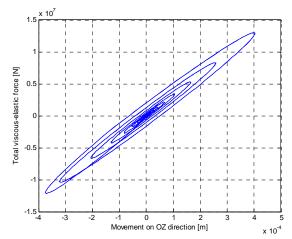


Fig. 17 The hysteretic characteristic

# D The non-linear elastic characteristics hypothesis

The rigidity on OZ direction of the viscous-elastic element on which is placed the foundation of the technological equipments, have the nonlinear expression [2] followed:

$$k_z = k_0 (1 + \beta \cdot x_{OZ}^2)$$
 (18)

The shape of non-linear elastic characteristic of the viscous-elastic system is presented in figure 18.

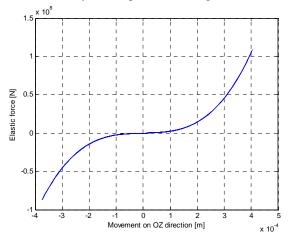


Fig. 18 The shape of elastic characteristic

The mathematical model can be writing, as follow:

$$\begin{cases} m\ddot{Y} + 4c_{y}\dot{Y} + 4c_{y}h\dot{\varphi}_{x} + 4k_{y}Y + 4k_{y}h\varphi_{x} = 0 \\ m\ddot{Z} + 4c_{z}\dot{Z} + 2c_{z}(n_{2} - n_{1})\dot{\varphi}_{x} + 4k_{0}(1 + \beta \cdot Z^{2}) \cdot Z + \\ + 2k_{0}(1 + \beta \cdot Z^{2})(n_{2} - n_{1})\varphi_{x} = -F_{z} \\ J_{x}\ddot{\varphi}_{x} + 4hc_{y}\dot{Y} + 2c_{z}(n_{2} - n_{1})\dot{Z} + 2\left[2c_{y}h^{2} + (n_{2}^{2} + n_{1}^{2})\right]\dot{\varphi}_{x} + \\ + 4hk_{y}Y + 2k_{0}(1 + \beta \cdot Z^{2})(n_{2} - n_{1})Z + \\ + 2\left[2k_{y}h^{2} + k_{0}(1 + \beta \cdot Z^{2})(n_{2}^{2} + n_{1}^{2})\right]\varphi_{x} = -e_{y}F_{z} \end{cases}$$

$$(19)$$

The solving system was made in the next numerical value hypothesis: P=900·10<sup>4</sup>N;  $k_0$ =2.5·10<sup>9</sup> N/m;  $c_y$ =2.5·10<sup>6</sup> Ns/m; m=100·10<sup>3</sup> kg;  $k_z$ =8·10<sup>9</sup> N/m;  $c_z$ =2.1·10<sup>6</sup> Ns/m; J=77·10<sup>4</sup> kgm²; e=0.02 m;  $n_1$ =3m;  $n_2$ =3m; h=1.5m;  $\beta$ =2·10<sup>8</sup>m²².

The solution of the system (19) leads to the evolution determination in time for three kinematics parameters:

acceleration, velocity and movement - on OZ direction [3], figures 19, 20 and 21.

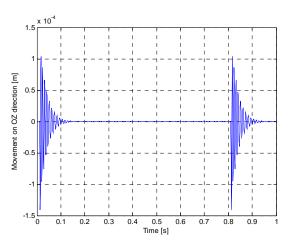


Fig. 19 Movement on OZ direction

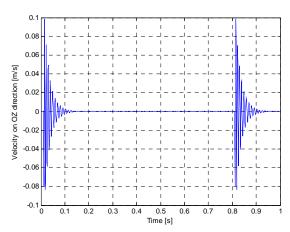


Fig. 20 Velocity on OZ direction

Time history of kinematics parameters enable effects characterization of transmitted vibration to the environment, comparative with established limit of effectual standard.

Eliminating time between velocity and movement permits obtaining - movement trajectory (figure 22), that shows the movement is damped and stabilized.

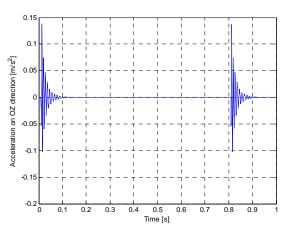


Fig. 21 Acceleration on OZ direction

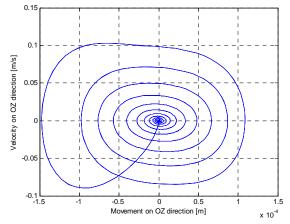


Fig. 22 The phase plane representation

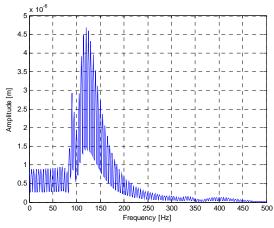


Fig. 23 The system response in frequency domain

So, frequency responses have spectral components around of 97 Hz value (as in the case of linear rigidity), but appear dominant spectral components around of 120Hz value, figure 23. Distribution of the energy of the shock on spectral components is noticed by plotting the power spectral density (figure 24). Like in the linear case, the goal of spectral estimation is to describe the distribution (over frequency) of the power contained in a signal, based on a finite set of data.

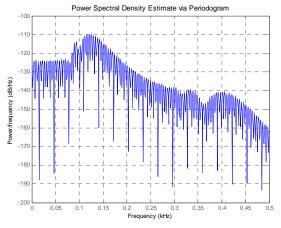


Fig. 24 Power spectral density

The energy dissipation is made by viscous amortization that is emphasizing by plotting the total forces viscous-elastic function of movement (figure 25).

The value of dissipate energies on a loop of movement is of W= 1341J. Towards the case of linear rigidity we observe a diminution of dissipate energy, explained by the diminution of movement amplitudes on OZ direction.

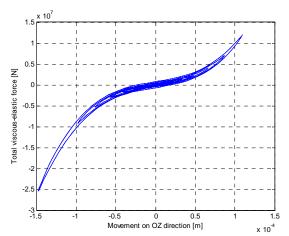


Fig. 25 The hysteretic characteristic

### III EXPERIMENTAL RESEARCHES

This chapter presents the result of the experimental determinations made on forging hammer (2000 kg capacity) at Workshop in IUS – Brasov. The measurements were made simultaneous on anvil block and foundation vat between are placed the viscous-elastic systems for isolating and damping generated vibration during the technological process.

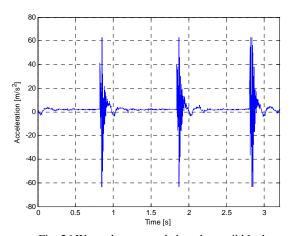


Fig. 26 Wave shape recorded on the anvil block

Wave shape recorded on the anvil block and the spectral density are represented in figures 26 and 27, and wave shape recorded on the foundation vat and the spectral density are represented in figures 28 and 29.

From the frequency response representation observe a diminution of acceleration amplitude in the non-linear case beside the linear case, this fact explained by the isolating and damping properties of viscous-elastic system.

Also, the difference between two cases is obvious by the dominant spectral component.

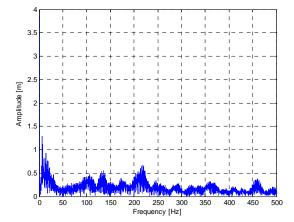


Fig. 27 Frequency response - anvil block

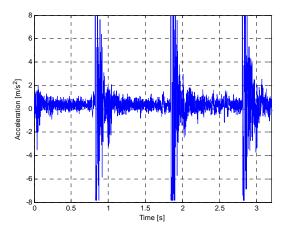


Fig. 28 Wave shape recorded on the foundation vat

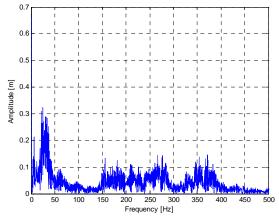


Fig. 29 Frequency response – foundation vat

The magnitude squared coherence estimate is a function of frequency with values between 0 and 1 that indicates how well x corresponds to y at each frequency. In this instance, x represent signal recorded on anvil block and y represent signal recorded on foundation vat.

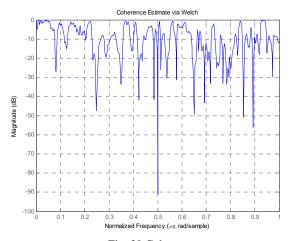


Fig. 30 Coherence

From figure 30, observe the presence of the important nonlinear character of transmitted signal from anvil block to foundation vat.

This experimental study represents a reference point in monitoring activities of the technological equipment dynamical behavior. After on interspaces, by experimental measurements, well be analyzed by comparison the obtained dates with the initial dates. In this way, well be determining the level of nonlinearities of the viscous-elastic vibration protection system, corresponding with a level of damage.

### IV CONCLUSIONS

This paper presents a theoretical model to characterize dynamically, a much diversified field of real technological situations in which equipments utilize shocks and vibration in the production process.

In time representation, observe small difference between cinematically parameters variations for two considerate cases: linear and non-linear characteristic.

It's clearly that in the case of the non-linear elastic characteristic on OZ direction, the dynamical response system is different comparing to linear elastic characteristic on OZ direction case. Theoretically, the presence of nonlinearities characteristic in the viscous-elastic protection system, conducts inevitable to a dynamical response modification (frequency response).

Practically, based on monitoring the frequency response of the technological equipment and detecting its modification, can determine level of elastic characteristic nonlinearities. In the same time, this study represents the beginning of experimentally research development regarding detection of damage in structure of viscous-elastic systems, based on the non-linear vibration technique.

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