Interaction of External Vortical and Thermal Disturbances with Boundary Layer

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Abstract. Longitudinal structures generated by external vortical and thermal waves in subsonic and supersonic boundary layers are studied in the paper. Particular attention is paid to the boundary conditions at the boundary layer outer edge. It was established that longitudinal velocity and mass flow disturbances inside the boundary layer can exceed the amplitude of external vortical wave in several times. Excitation efficiency decreases with increasing Mach number. Influence of thermal external waves on the flow structure in the boundary layer is much weaker.

Key words: Mach number, turbulence, supersonic boundary layers, disturbances, waves.

1 INTRODUCTION

At research of a problem of originating of turbulence in a boundary layer the special attention give to the excitation disturbances in a boundary layer by external waves. Morkovin was the first who discussed this phenomenon, which is called now as a problem of a receptivity of a boundary layer [1]. There are a lot of experimental and theoretical papers on the receptivity of a subsonic boundary layer. One can find the detailed review of these investigations in [2,3]. Much less papers are dedicated to a case of a supersonic boundary layer. Mainly, the interaction of external acoustic waves with a supersonic boundary layer was studied [4-6]. In [7] the interaction of hydrodynamic vortex-free waves with a supersonic boundary layer was investigated. At the same time, at supersonic flow together with acoustic and vortex-free hydrodynamic waves there are vortical and thermal ones. Unfortunately, even in case of subsonic speeds there are few papers, in which the interaction of vortical disturbances with a boundary layer was studied numerically. Papers [8,9] are the most interesting for us. But their results differ among themselves. Nobody considered the interaction of thermal external waves with the boundary layers. This paper is dedicated to research of the disturbances excitation in subsonic and supersonic boundary layers by external vortical and thermal waves.

2 FORMULATION AND BASIC EQUATIONS

The linear statement is considered. Disturbances in a boundary layer we shall consider in orthogonal coordinate system \((\xi, \psi, z)\) [9,10] connected with stream-surfaces of basic flow and look like \(a(\xi, \psi) \exp(ia\xi + i\beta z - i\omega t)\).

Here \(\psi\) - flow function; for a plate \(\zeta = x + O(Re^{-2})\)

\(Re = \sqrt{u_\infty x / \nu_e}\); \(u_\infty, v_\infty\) - speed and kinematical viscosity of a ram airflow; \(x, y, z\) - longitudinal, normal to a wall and transversal co-ordinates of the Cartesian system with the beginning on an edge of a plate. Gas is perfect with a constant Prandtl number \(Pr\). Using estimates in integer degrees of \(Re\), taking into account the properties of the critical layer, and omitting the terms of order \(Re^{-2}\) in linearized Navier - Stokes equations, one can obtain the set of governing equations [7]:

\[
\begin{align*}
\frac{\partial}{\partial t}(\rho \psi) &= \frac{\partial}{\partial x}(\rho u \psi) \\
\frac{\partial}{\partial t}(\rho u) &= \frac{\partial}{\partial x}(\rho u^2 + \rho \psi) + \frac{1}{Re} \frac{\partial p}{\partial x} + \frac{1}{Pr} \frac{\partial}{\partial x}(\rho \psi) - \frac{1}{RePr} \frac{\partial}{\partial x}(\rho \psi^2) \tag{1}
\end{align*}
\]

...
∞ corresponds to values in the ram airflow). In this case: \( g_n = \gamma M^2 \), \( g_{n0} = (\gamma - 1)M^2 \), where \( \gamma = c_p/c_v \) - relation of heat capacities; \( M \) – Mach number.

Entering independent variables \( \text{Re} = \frac{\sqrt{\rho}}{S}, \) \( d\eta = d\psi \div u \text{Re} \) and using notations: \( \ddot{\alpha} = (1/R\psi)(\ddot{\alpha} + f\dot{\alpha}), \) \( \ddot{\alpha} = \rho a^2/R \), where \( \delta = 0,5\delta \div \delta \text{Re} \); the prime means a derivative on \( \eta \), \( d\eta = (d\psi/u)/\text{Re}, \) \( f_i = -\psi/(2\text{Re}^2 u) \), equations (1) are led to a view:

\[
\begin{align*}
\ddot{\psi} &= -g_n u T \dot{\psi} (\ddot{\psi} + \rho T \dot{\psi} - (f, u' + \dot{\psi}) - \nu u - i, T \ddot{T} - (f, u' - \nu T) \ddot{T} - f, T \ddot{u} + \\
\tilde{p}' &= (i + \frac{r}{u} u' + \nu i, \ddot{T} + i, \ddot{u} + i, \ddot{i} T - 2 \mu u', \nu) , \\
\ddot{\tilde{T}} &= \dot{\psi} + (i + \frac{r}{u} u' + \nu i, \ddot{T} + i, \ddot{u} + \nu i, \ddot{\psi} + \nu u' \ddot{\psi} + f, u' \ddot{u} - i, T \ddot{T} - f, u' \ddot{u} + \\
\tilde{u}' &= -i, T \dot{u} - \nu u' \dot{T} + \ddot{\alpha} / \mu , \nu, \frac{\alpha}{\beta} = - + + + + + \\
\ddot{\alpha}' &= -i, T \dot{\alpha} + f, \ddot{u} , \\
\dot{\psi}' &= i, 0, \psi' - \nu u' \ddot{\psi} + f, \ddot{u} - i, T \ddot{T} - f, \ddot{u} - i, \ddot{T} \ddot{u} + \\
\dot{\psi}' &= -i, \dot{\psi} + i, T \ddot{T} + f, \ddot{u} .
\end{align*}
\]

The necessary solutions of a homogeneous set of equations on the edge of a boundary layer we obtain from analytic solutions of a locally-parallel approach at 1-th step. 

In a free stream \( u' = T_0 \), \( u = T_1 \). Therefore there are four vectors converging to decreasing solutions on the infinity.

\[
\begin{align*}
Z_1 &= (0, 0, 0, 0, -k, 0, 0, -k) , \\
Z_2 &= (0, -i\beta, 0, 0, 0, 0, 0, -k^2), \\
Z_3 &= (0, 0, 0, -i\beta, 0, 0, 0, -k^2), \\
Z_4 &= (0, 1, 0, -1, 0, 2i\beta, 0, 1).
\end{align*}
\]

4 RESULTS

The calculations were conducted for a boundary layer on a flat plate for Mach numbers \( M = 0 \) and 2.0 and frequency \( \omega = 10^{-6} \). The adopted frequency satisfies to steady conditions. Viscosity-temperature relation, adopted in calculations, was determined by the Sutherland formula, Prandtl number \( \text{Pr} = 0.72 \).

The obtained results were set norms on an amplitude of a velocity disturbance in a free stream \( u' = (\dot{\psi}^2 + \ddot{\alpha}^2 + \ddot{\psi}^2) / 2 \) nearly by to choosing position \( x_0 \). Parameters of the problem were \( \alpha, \beta, \text{Re}, x_0 \), where \( \alpha \) – damping intensity of external disturbances along longitudinal coordinate, \( x_0 \) - dimensionless spacing interval from a choosing position up to a leading edge of a plate, \( \text{Re} = (x_0 / \alpha)^{1/2} \), and \( x \) - dimensionless spacing interval from a leading edge of a plate. The value of \( x_0 \) is oriented on papers [12,13], in which the grid was located 1.6 m and 1 m from the plate leading edge. The maximum stream velocity was equal to 12 ms\(^{-1}\) and minimum – 2 ms\(^{-1}\), thus \( 0.80 \times 10^{-5} \leq x_0 \leq 1.28 \times 10^{-6} \). For a given value \( \alpha \) wave numbers \( \beta \) and \( k \) in z u y-directions were taken real, satisfied to the ratio \( \alpha = \beta^2 + k^2 \) for vortical disturbances and \( \alpha = (\beta^2 + k^2) \) for parallel flow approach. Inside of a boundary layer they are satisfied to a system of homogeneous equations. The fifth solution is agreed with the external wave, and inside a boundary layer it is satisfied to an inhomogeneous set of equations. The common solution is constructed as superposition, \( Z = \sum Z_n(x)Z_n(z) + Z_s \), \( Z_m(x) \) are determined from boundary conditions on the plate.

Disturbances in the free stream are proportional to \( \exp[i(kx + i\beta z + i\alpha \pi - \omega t)] \), where \( k, \beta, \omega \) - real. As it is established in [11], for vortical and thermal waves the values \( \alpha \) is determined from the equation \( i(\omega - \alpha) = \alpha^2 + \beta^2 + k^2 \) or \( i(\omega - \alpha) = (\alpha^2 + \beta^2 + k^2) / \text{Pr} \) accordingly. Numerous experiments and the analytical investigations at subsonic speeds demonstrate, that under the influence of external turbulence in boundary layer the longitudinal structures develop. It means, that stationary disturbances with a longitudinal vorticity \( (u = 0) \) with \( Z^2 = (0, -i\beta, 0, 0, 0, 0, 0, -k^2) \) are the most important.

Vector, basically of thermal disturbances \( Z^2 = (0, 0, 0, 0, 0, 0, 0, 0, 0, -k^2, 0, 0, -k^2) \) is determined by the Sutherland formula, Prandtl number \( \text{Pr} = 0.72 \).
\( + \frac{k^2}{Pr} \) for thermal waves. All calculations are conducted for a boundary layer thickness \( \delta = \eta = 6. \)

Let's discuss, first of all, results obtained for a Mach number \( M=0. \)

In a Fig.1 the distributions of disturbances amplitudes of pressure (\( A_p \)), velocities (\( A_v, A_u, A_w \)) and enthalpy (\( A_h \)) are shown at \( Re = 760, \alpha_i = 10^{-8}, \beta = 3 \times 10^{-4}. \) It is necessary to note, that the view of distribution of the longitudinal velocity disturbance is conservative to change of the problem parameters. After normalization on a dependence maximum it is resulted practically to the view, which is coincided with dependences of other papers including [8].

In a Fig. 2 the ratio of the calculation results to analytical values \( \beta Re^2 \) of the paper [14, 16] is shown. It is necessary to note, that data [14, 16] were obtained in the supposition that \( \beta Re << 1 << \beta Re^2. \) Analytical value - (1), results of present calculations (2, 3, 4) at \( \beta = (1; 5; 10) \times 10^{-5}. \) respectively. U_{max} at change of \( \beta. \) The checkmark ♥ corresponds to experimental value of the paper [15], the checkmark ✶ to [13]. Normalization in [8] differs from ours on value \( \sqrt{2}A_v, \) where \( A_v \) corresponds (on an order of values) to amplitude of disturbances of an external flow. So, in order to result in conformity data of [8] to ours they were divided by \( \sqrt{2}A_v. \)

It is visible, that present data agree very well with analytical results at \( \beta = 10^{-5}. \) There is a transient region \( Re < 300, \) where the divergence of data is watched. It is explained by a violation of an inequality \( \beta Re^2 >> 1. \) Even at \( Re=300 \) the value \( \beta Re^2 = 0.9. \) At the same value \( Re=300 \) for \( \beta = 5 \times 10^{-5} \) and \( 10^4 \) differences of predicted data from analytical value are 5% and 10% respectively though \( \beta Re^2 \) is great enough. Apparently it is connected with a violation the second ine-

quality, \( \beta Re << 1. \) At \( Re=300 \) and \( \beta = 10^{-4} \) value \( \beta Re = 0.03. \) At the same \( \beta = 10^{-4} \) and \( Re=10^3, \beta Re=0.1, \) and the deviation of calculations from analytical values exceeds 30%. This analysis demonstrates that the calculation results agreed with analytical values at fulfillment of the corresponding inequalities. Moreover a strong inequality \( \beta Re^2 >> 1 \) can be changed on the simple inequality \( \beta Re^2 > 1. \)

In Fig. 3 the comparison our results with data of [8, 16] is shown. ( \( U_{max}= [\bar{u}_{max}] \).) The main results were obtained for \( k=\beta/3: \) The line 1 - data [8], lines 2,3 - our results (obtained in a locally-parallel approaching and on the basis of parabolized equations 5 respectively ), the line 4 is obtained on the basis of analytical expression of the paper[16]. The line 5 – our data at \( k=\beta, \) conforming to maximum values

The comparison of our data with theoretical results [8] (k=\beta/3) for two values of a Reynold's number Re=500 (1, 2); Re=1000 (3, 4) is given in Fig.4, where data [8] (1, 3) and present results (2, 4). There are present data (5) and results [9] (6) for Re=500: k=\beta. The experiments result [13, 15] is marked by the checkmark ♥. It is possible to see that
our results are agreed with theoretical [9] and experimental [13, 15] data.

Fig. 5
In Fig. 5 the dependences $U_{\text{max}}$ on the wave vector $\beta$ are shown for $\text{Re}=600$, $-\alpha_i=10^{-6}$, and $x_0 = 80 \times 10^3$ (1); $320 \times 10^3$ (2); $640 \times 10^3$ (3). It is visible that $U_{\text{max}}$ increases with reduction $x_0$. It is apparent because of low damping of external disturbances on more short spacing intervals $x_0$. At the same time it is watched some displacement of a maximum of the dependence in the region of larger values of $\beta$. However this displacement is not strong and the wave number conforming to a maximum of a curve is approximately equal to $0.7 \times 10^{-3}$ and it is agreed with data obtained in [13].

Fig. 6
In Fig. 6 the values $U_{\text{max}} = |\Omega_{\text{max}}|$ in depending on Reynolds number at $\alpha_i=10^{-6}$, $\beta=10^4$ = 2.0; 4.0; 6.0; 8.0; 10 for $x_0=6.4 \times 10^6$ are shown.

The dependence of a phase velocity $Cr$ on Reynolds number is adduced in Fig. 7. The data are obtained at $M=0.0$, $-\alpha_i=10^{-6}$, $\omega=10^{-5}$, $x_0=64\times10^6$ and three values $\beta$. The small change of $Cr$ from a wave number $\beta$ is visible. On the other hand, $Cr$ essentially depends on a Reynolds number and varies within the limits $0.5 < Cr < 1.0$ at change $\text{Re}$ from 250 up to 700. It is interesting to address to experiments (see [3]) on the turbulent spots originating. It was established there, that the leading front of a spot, located in the field of large numbers $\text{Re}$, is propagated with speed 0.9 while back one — with the speed equal 0.5. These results are in good, at least, qualitative conformity with our data.

Fig. 7
Let’s proceed to the data for a case of supersonic speeds. Our results demonstrate that the distributions of disturbances amplitudes of pressure, velocities and enthalpy on a boundary layer at $M=2.0$ are similar to the case of Mach number $M=0$ (Fig. 1).
Fig. 8
In Fig. 8 the dependence $U_{\text{max}}$ on Reynolds number are shown for $M=2.0$ ($\alpha_{i}=10^{-6}$; $x_{0}=6.4\times10^{5}$; $b=\beta\times10^{4}=2.0, 4.0, 6.0, 8.0, 10$). This dependence is similar qualitatively to the case of Mach number $M=0$ (Fig. 6). But the value $U_{\text{max}}$ for $M=2$ is less than at $M=0$.

Fig. 9
By calculations it is established that the value $U_{\text{max}}$ is decreasing monotonically with Mach number increasing. This concluding is demonstrated in Fig. 9 ($\Re = 600, \alpha_{i}=10^{-6}, x_{0}=6.4\times10^{5}$). At last, we shall consider excitation of disturbances inside a boundary layer by external thermal disturbances. In a Fig. 10 the distributions of amplitudes of the disturbances of pressure, velocity and enthalpy ($Ap$, $Av$, $Au$, $Ah$), normalized on amplitude of a disturbance of an enthalpy in a free flow are shown ($M=2$, $Re = 600, \alpha_{i}=10^{-6}, x_{0}=6.4\times10^{5}$). The comparison of these results with data in a Fig. 1 indicates that the shape of a dependence $Au$ on the normal coordinate is similar to a case of external vortical disturbances. However maximum of $Au$ is much lower than in a Fig. 1. Nevertheless the velocity shape deformation is watched in this case too.

Fig. 10
In Fig. 11 the dependence $U_{\text{max}}$ on Reynolds number is shown at $-\alpha_{i}=10^{-6}$ and different values of a wave number $\beta$ for $x_{0}=6.4\times10^{5}$. The main feature of this dependence is the fast decreasing the disturbances amplitude of the longitudinal velocity, at least, in area $Re < 800$. However at large values of a Reynolds number the increase of disturbances inside a boundary layer can be seen.

Fig.11

CONCLUSIONS
Thus the conducted researches demonstrate that the external vortical wave can excite disturbances of the longitudinal velocity inside a boundary layer. Their intensity depends on the wave spectrum of disturbances and Mach number. At the given Reynolds's number there are a reference value $\beta^{*}$ at which the amplitude of a longitudinal velocity disturbance inside a boundary layer is maximum. It explains the appearance of longitudinal structures with the conforming periodicity in a lateral direction, observed in experiments [13, 15]. The phase velocity of the maximum disturbances inside a boundary layer varies from $0.5 u_{\infty}$ (at $Re=250$) up to $0.95 u_{\infty}$ (at $Re=700$). With increase of a Mach number the intensity of the longitudinal velocity disturbances inside a boundary layer excited by the external vortical waves decreases. The efficiency of the flow deformation inside a boundary layer by external thermal waves is lower in comparison with a case of vortical ones. The results of the present paper are agreed satisfactorily with the experiments [13, 15] and theoretical papers [9, 14, 16] but they differ quantitatively from data of Bertolotti [8]. Reasons of this difference remain are unknown.

REFERENCES


