Interaction of External Vortical and Thermal Disturbances with Boundary Layer

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Abstract. Longitudinal structures generated by external vortical and thermal waves in subsonic and supersonic boundary layers are studied in the paper. Particular attention is paid to the boundary conditions at the boundary layer outer edge. It was established that longitudinal velocity and mass flow disturbances inside the boundary layer can exceed the amplitude of external vortical wave in several times. Excitation efficiency decreases with increasing Mach number. Influence of thermal external waves on the flow structure in the boundary layer is much weaker.

Key words: Mach number, turbulence, supersonic boundary layers, disturbances, waves.

1 Introduction

At research of a problem of originating of turbulence in a boundary layer the special attention give to the excitation disturbances in a boundary layer by external waves. Morkovin was the first who discussed this phenomenon, which is called now as a problem of a receptivity of a boundary layer [1]. There are a lot of experimental and theoretical papers on the receptivity of a subsonic boundary layer. One can find the detail review of these investigations in [2,3]. Much less papers are dedicated to a case of a supersonic boundary layer. Mainly, the interaction of external acoustic waves with a supersonic boundary layer was studied [4-6]. In [7] the interaction of hydrodynamic vortex-free waves with a supersonic boundary layer was investigated. At the same time, at supersonic flow together with acoustic and vortex-free hydrodynamics waves there are vortical and thermal ones. Unfortunately, even in case of subsonic speeds there are few papers, in which the interaction of vortical disturbances with a boundary layer was studied numerically. Papers [8,9] are the most interesting for us. But their results differ among themselves. Nobody considered the interaction of thermal external waves with the boundary layers. This paper is dedicated to research of the disturbances excitation in subsonic and supersonic boundary layers by external vortical and thermal waves.

2 Formulation and Basic Equations

The linear statement is considered. Disturbances in a boundary layer we shall consider in orthogonal coordinate system (ξ,ψ,z) [9,10] connected with stream-surfaces of basic flow and look like $\tilde{a}(\xi,\psi)\exp(i\alpha\xi+i\beta z-i\omega t)$.

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Here ψ - flow function; for a plate $\xi = x + O(\mathrm{Re}^{-2})$ $\mathrm{Re}=\sqrt{u_\infty x/\nu_\infty}$; u_∞,ν_∞ - speed and kinematical viscosity of a ram airflow; x,y,z - longitudinal, normal to a wall and transversal co-ordinates of the Cartesian system with the beginning on an edge of a plate. Gas is perfect with a constant Prandtl number Pr. Using estimates in integer degrees of Re, taking into account the properties of the critical layer, and omitting the terms of order Re^{-2} in linearized Navier - Stokes equations, one can obtain the set of governing equations [7]:

$$\partial_2\tilde{v} = -(\partial_2 \ln\rho)\tilde{v} - [i\alpha - (\partial_1 \ln u) + \partial_1]\tilde{u} - i\beta\tilde{w} - u_c\,\tilde{\rho}/\rho - g_m u\partial_1\tilde{p} + u\partial_1(\tilde{T}/T),$$

$$\partial_2[\tilde{p} + 2\mu(i\alpha\tilde{u} + i\beta\tilde{w} - 2\tilde{e}_0/3)] = -\rho(h_1 u + d_t)\tilde{v} + i\alpha\tilde{\tau}_{12} + i\beta\tilde{\tau}_{23},$$

$$\partial_2\tilde{\tau}_{12} = (i\alpha + \partial_1)\tilde{p} + \rho(\partial_2 u)\tilde{v} + \rho(\partial_1 u + d_t)\tilde{u} + u(\partial_1 u)\tilde{\rho} - i\alpha\tilde{\tau}_{11} - i\beta\tilde{\tau}_{13},$$

$$\partial_2\tilde{u} = -(i\alpha + \partial_1)\tilde{v} - (\partial_2 u)\tilde{\mu}/\mu + \tilde{\tau}_{12}/\mu,$$

$$\partial_2\tilde{\tau}_{23} = i\beta\,\tilde{p} + \rho d_t\tilde{w} - i\alpha\tilde{\tau}_{13} - i\beta\tilde{\tau}_{33}, \quad (1)$$

$$\partial_2\tilde{w} = -i\beta\,\tilde{v} + \tilde{\tau}_{23}/\mu,$$

$$\partial_2\tilde{q} = i\omega\tilde{p} + [\rho(\partial_2 H) - i\alpha\mu(\partial_2 u)]\tilde{v} + (\partial_1 H)(\rho\tilde{u} + u\tilde{\rho}) + (\alpha^2+\beta^2)\mu\tilde{h}/\mathrm{Pr} + \rho d_t\tilde{H} - u(i\alpha\tilde{\tau}_{11} + i\beta\tilde{\tau}_{13}),$$

$$\partial_2\tilde{h} = -\mathrm{Pr}(\partial_2 u)\tilde{u} - (\partial_2 h)\tilde{\mu}/\mu + \mathrm{Pr}(\tilde{q} - u\tilde{\tau}_{12})/\mu,$$

where: $\partial_1 = \partial/\partial\xi$; $\partial_2 = \rho u\,\partial/\partial\psi$; $d_t = u_c + u\partial_1$; $u_c = i\alpha u - i\omega$; $h_1 = -\partial_1 \ln(\rho u)$; $\tilde{\tau}_{11} = 2\mu(i\alpha\tilde{u} - \tilde{e}_0/3)$; $\tilde{\tau}_{13} = \mu(i\alpha\tilde{w} + i\beta\tilde{u})$; $\tilde{\tau}_{33} = 2\mu(i\beta\tilde{w} - \tilde{e}_0/3)$; $\tilde{e}_0 = -(\partial_2 \ln\rho)\tilde{v} - u_c\tilde{\rho}/\rho$; u –velocity; T –temperature; ρ - density; p – pressure; $H = h + u^2/2$ - full enthalpy; μ - viscosity; $\tilde{u}$, $\tilde{v}$, $\tilde{w}$ - complex amplitudes of stream-wise, normal to a surface and transversal components of velocity disturbances; $\tilde{\rho}/\rho = g_m\,\tilde{p} - \tilde{T}/T$; $\tilde{T} = g_{m1}\tilde{h}$; $\tilde{H} = \tilde{h} + u\tilde{u}$; $g_m = 1/p$; $g_{m1} = 1/c_p$; c_p - specific heat of gas at constant pressure. The view of equations will not change after normalizing with the help of following scales: ν_∞/u_∞ - length, ν_∞/u_∞^2 - time , μ_∞ - viscosity and flow function, u_∞ - velocity and its disturbances, T_∞ - temperature, ρ_∞ - density , u_∞^2 - enthalpy, $\rho_\infty u_\infty^2$ - pressure and disturbances of viscous stresses, $\rho_\infty u_\infty^3$ - value $\tilde{q}$, u_∞^2/T_∞ - specific heat (the index

∞ corresponds to values in the ram airflow). In this case: $g_m = \gamma M^2$, $g_{m1} = (\gamma - 1)M^2$, where $\gamma = c_p/c_V$ - relation of heat capacities; M – Mach number.

Entering independent variables $\mathrm{Re} = \sqrt{\xi}$, $d\eta = d\psi / u\,\mathrm{Re}$ and using notations: $\partial_1 \tilde{a} = (1/\mathrm{Re})(\partial\tilde{a} + f_1\tilde{a}')$, $\partial_2 \tilde{a} = \rho\tilde{a}'/\mathrm{Re}$, where $\partial = 0{,}5\partial/\partial\mathrm{Re}$; the prime means a derivative on η, $d\eta = (d\psi/u)/\mathrm{Re}$, $f_1 = -\psi/(2\mathrm{Re}^2 u)$), equations (1) are led to a view:

$$\tilde{v}' = -g_m uT\partial\tilde{p} + \rho T'\tilde{v} - T(f_0 u' + \partial)\tilde{u} - \\ -\tilde{u}_w - i_c T\tilde{r} - (f_2\rho T' - u\partial)\tilde{T} - f_1 T\tilde{u}' + \\ + f_2\tilde{T}',$$

$$\tilde{p}' = -(i_c + r_h u)\tilde{v} + i_x\tilde{\tau}_{12} + i_z\tilde{\tau}_{23} - 2\mu_r\tilde{u}_w',$$

$$\tilde{\tau}_{12}' = (i_x + T\partial)\tilde{p} + (i_c + f_1 u' + u\partial)\tilde{u} + \\ f_2 u'\tilde{r} - \tilde{i}_T + f_2\tilde{u}' + \rho u'\tilde{v}$$

$$\tilde{u}' = -i_x\tilde{v} - u'\mu_t\tilde{T} + \tilde{\tau}_{12}/\mu_r,$$

$$\tilde{\tau}_{23}' = i_z\tilde{p} + (i_c - \mu_a + u\partial)\tilde{w} - \\ -i_z\mu_r\tilde{u}_w + f_2\tilde{w}', \qquad \tilde{w}' = -i_z\tilde{v} + \tilde{\tau}_{23}/\mu_r, \quad (2)$$

$$\tilde{q}' = i\omega RT\,\tilde{p} + \rho H'\tilde{v} + f_2 H'\tilde{r} - u\tilde{i}_t + \\ + (i_c u + f_1 H' + f_2 u' + u^2\partial)\tilde{u} + f_2\tilde{h}' + \\ + f_2 u\tilde{u}' + (i_c - \mu_a/\mathrm{Pr} + u\partial)\tilde{h},$$

$$\tilde{h}' = -\mathrm{Pr}\,u'\tilde{u} - h'\mu_t\tilde{T} + \mathrm{Pr}(\tilde{q} - u\tilde{\tau}_{12})/\mu_R, \text{ where}$$

$$\tilde{u}_w = i_x\tilde{u} + i_z\tilde{w};\ \tilde{i}_t = i_x\mu_r\tilde{u}_w + \mu_a\tilde{u};\ \mu_a = (i_x^2 + i_z^2)\mu_r;$$

$$\tilde{p} = \tilde{\pi} - 2\mu(i\alpha\tilde{u} + i\beta\tilde{w} - 2\tilde{e}_0/3);\ \tilde{r} = \tilde{\rho}/\rho = g_m\tilde{p} - \rho\tilde{T};$$

$$i_c = \mathrm{Re}\,u_c = i\,\mathrm{Re}(u\alpha - \omega);\ i_x = i\alpha\,\mathrm{Re}\,T;\ i_z = i\beta\,\mathrm{Re}\,T;$$

$$r_h = \mathrm{Re}\,h_1 = f_0 u' + f_1\rho T';\ f_0 = -f_1/u;\ f_2 = f_1 u;$$

$$\mu_r = \mu\rho/\mathrm{Re};\ \mu_T = d\ln\mu/dT.$$

The equations (2) can be written in a view:

$$\mathbf{Z}' = (A + D\partial)\mathbf{Z}.$$

Here $\partial = 0{,}5\partial/\partial\mathrm{Re}$, $\mathbf{Z} = (\tilde{p}, \tilde{v}, \tilde{u}, \tilde{w}, \tilde{h}, \tilde{\tau}_{12}, \tilde{\tau}_{23}, \tilde{q})$, A, D—quadratic matrixes of given functions of main flow parameters. The parabolized set of equations is solved at the following boundary conditions. The disturbances of speeds and temperature on a surface are equals to zero, $\tilde{v}(0) = \tilde{u}(0 = \tilde{w}(0) = \tilde{T}(0) = 0$. Outside of a boundary layer the disturbances are determined by the correspondent values in free (in a model absence) flow.

3 Numerical scheme and boundary conditions

Using approximation $\partial\tilde{a}/\partial R \approx (\tilde{a} - \tilde{a}_0)/\Delta R$ ($\Delta R = R - R_0$ - step of the marching scheme, the index 0 here and below correspond to the previous step) we transform a parabolized set of equations into a system of the ordinary differential equations: $Z' = AZ + B(Z - Z_0)$. The common solution of a system is constructed as follows. At the boundary layer edge four solutions are selected, which correspond to damping disturbances outside of a boundary layer in a parallel flow approach. Inside of a boundary layer they are satisfied to a system of homogeneous equations. The fifth solution is agreed with the external wave, and inside a boundary layer it is satisfied to an inhomogeneous set of equations. The common solution is constructed as superposition, $Z = \sum_{m=1}^{4} C_m(x)Z_m + Z_5$, $C_m(x)$ are determined from boundary conditions on the plate.

Disturbances in the free stream are proportional to $\exp[iky + i\beta z + i\alpha x - i\omega t]$, where k, β, ω - real. As it is established in [11], for vortical and thermal waves the values α is determined from the equation $i(\omega - \alpha) = \alpha^2 + \beta^2 + k^2$ or $i(\omega - \alpha) = (\alpha^2 + \beta^2 + k^2)/\mathrm{Pr}$ accordingly. Numerous experiments and the analytical investigations at subsonic speeds demonstrate, that under the influence of external turbulence in boundary layer the longitudinal structures develop. It means, that stationary disturbances with a longitudinal vorticity ($\tilde{u} = 0$) with $Z_5^2 = (0, -i\beta, 0, ik, 0, 0, -k^2 + \beta^2, 0)$ are the most important. Vector, basically of thermal disturbances $Z_5^3 = (0, ikB_1, 0, i\beta B_1, \mathrm{Pr}, 0, -2\beta kB_1, ik)$ [7], where $B_1 = (\gamma - 1)M^2$, $A_1 \approx M^2$.

The necessary solutions of a homogeneous set of equations on the edge of a boundary layer we obtain from analytic solutions of a locally - parallel approach at $\eta \gg 1$.

In a free stream $u' = T' = 0$, $u = T = 1$. Therefore there are four vectors conforming to decreasing solutions on the infinity.

$$Z_1 = (0, 0, -ik, 0, 0, -k^2, 0, -k^2).$$

$$Z_2 = (0, -i\beta, 0, -ik, 0, 0, -k^2 + \beta^2, 0)$$

$$Z_3 = (0, -ikB_1, 0, i\beta B_1, \mathrm{Pr}, 0, -2\beta kB_1, -ik)$$

$$Z_4 = (0, 1, 0, -1, 0, 2i\beta, 0, 1)$$

4 Results

The calculations were conducted for a boundary layer on a flat plate for Mach numbers M=0 and 2.0 and frequency $\omega = 10^{-6}$. The adopted frequency satisfies to steady conditions. Viscosity-temperature relation, adopted in calculations, was determined by the Sutherland formula, Prandtl number Pr=0.72.

The obtained results were set norms on an amplitude of a velocity disturbance in a free stream u= $(\tilde{v}^2 + \tilde{w}^2)^{1/2}$ nearly by to choosing position x_0. Parameters of the problem were α_i, Re, x_0, where α_i –damping intensity of external disturbances along longitudinal coordinate, x_0 - dimensionless spacing interval from a choosing position up to a leading edge of a plate, Re = $(x)^{1/2}$, and x- dimensionless spacing interval from a leading edge of a plate. The value of x_0 is oriented on papers [12.13], in which the grid was located 1.6 m and 1 m from the plate leading edge. The maximum stream velocity was equal to 12 ms^{-1} and minimum – 2 ms^{-1}, thus $0.80\cdot10^5 \le x_0 \le 1.28\cdot10^6$. For a given value α_i wave numbers β and k in z и y-directions were taken real, satisfied to the ratio $\alpha_i = \beta^2 + k^2$ for vortical disturbances and $\alpha_i = (\beta^2$

+ k^2)/Pr for thermal waves. All calculations are conducted for a boundary layer thickness $\delta = \eta_1 = 6$.

Let's discuss, first of all, results obtained for a Mach number M=0.

In a Fig.1 the distributions of disturbances amplitudes of pressure (Ap), velocities (Av, Au, Aw) and enthalpy (Ah) are shown at Re = 760, $\alpha_i = 10^{-8}$, $\beta = 3\cdot 10^{-4}$. It is necessary to note, that the view of distribution of the longitudinal velocity disturbance is conservative to change of the problem parameters. After normalization on a dependence maximum it is resulted practically to the view, which is coincided with dependences of other papers including [8].

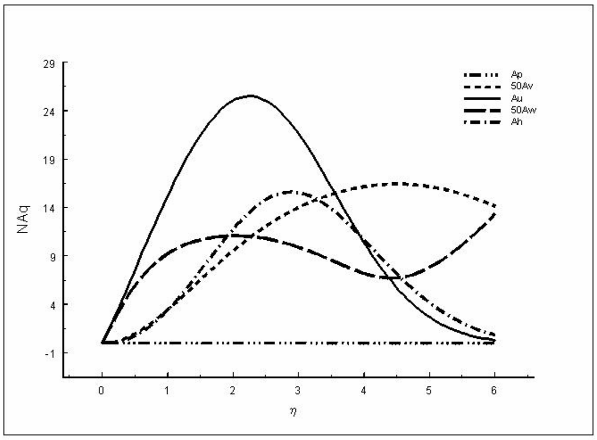

Fig.1

In a Fig. 2 the ratio of the calculation results to analytical values βRe^2 of the paper [14, 16] is shown. It is necessary to note, that data [14, 16] were obtained in the supposition that $\beta Re \ll 1 \ll \beta Re^2$. Analytical value - (1), results of present calculations (2, 3, 4) at $\beta = (1; 5; 10)\cdot 10^{-5}$ respectively.

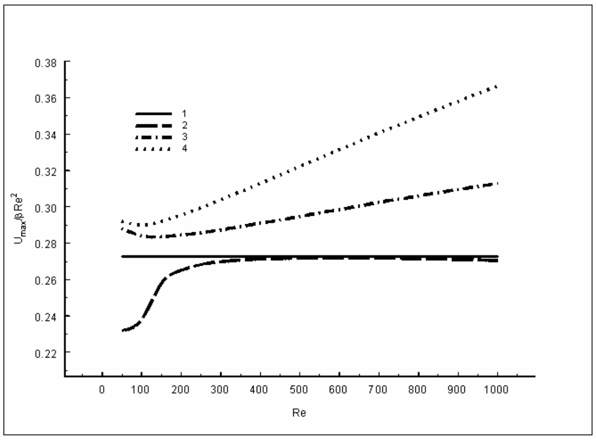

Fig.2

It is visible, that present data agree very well with analytical results at $\beta = 10^{-5}$. There is a transient region Re < 300, where the divergence of data is watched. It is explained by a violation of an inequality $\beta Re^2 \gg 1$. Even at Re=300 the value $\beta Re^2 = 0.9$. At the same value Re=300 for $\beta = 5\cdot 10^{-5}$ and 10^{-4} differences of predicted data from analytical value are 5% and 10% respectively though βRe^2 is great enough. Apparently it is connected with a violation the second inequality, $\beta Re \ll 1$. At Re=300 and $\beta = 10^{-4}$ value $\beta Re = 0.03$. At the same $\beta = 10^{-4}$ and $Re = 10^3$, $\beta Re = 0.1$, and the deviation of calculations from analytical values exceeds 30 %. This analysis demonstrates that the calculation results agreed with analytical values at fulfillment of the corresponding inequalities. Moreover a strong inequality $\beta Re^2 \gg 1$ can be changed on the simple inequality $\beta Re^2 > 1$.

In Fig. 3 the comparison our results with data of [8, 16] is shown. ($U_{max} = |\tilde{u}|_{max}$). The main results were obtained for $k = \beta/3$: The line 1 - data [8], lines 2,3 - our results (obtained in a locally-parallel approaching and on the basis of parabolized equations [5] respectively), the line 4 is obtained on the basis of analytical expression of the paper[16]. The line 5 – our data at $k = \beta$, conforming to maximum values

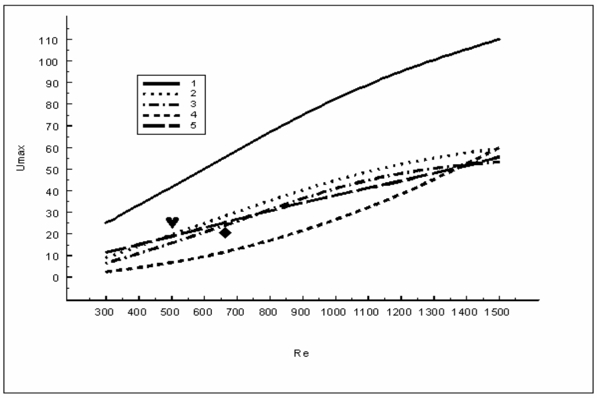

Fig.3

U_{max} at change of β. The checkmark ♥ corresponds to experimental value of the paper [15], the checkmark ◆ to [13]. Normalization in [8] differs from ours on value $\sqrt{2A_v}$, where A_v corresponds (on an order of values) to amplitude of disturbances of an external flow. So, in order to result in conformity data of [8] to ours they were divided by $\sqrt{2A_v}$.

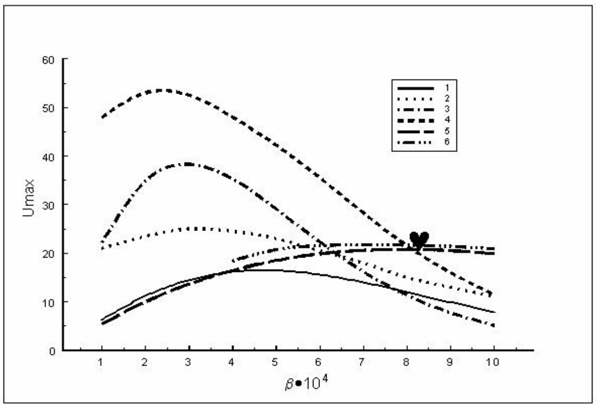

Fig.4

The comparison of our data with theoretical results [8] ($k = \beta/3$) for two values of a Reynold's number Re=500 (1, 2); Re=1000 (3, 4) is given in Fig.4, where data [8] (1, 3) and present results (2, 4). There are present data (5) and results [9] (6) for Re=500: $k = \beta$. The experiments result [13, 15] is marked by the checkmark ♥. It is possible to see that our results are agreed with theoretical [9] and experimental [13, 15] data.

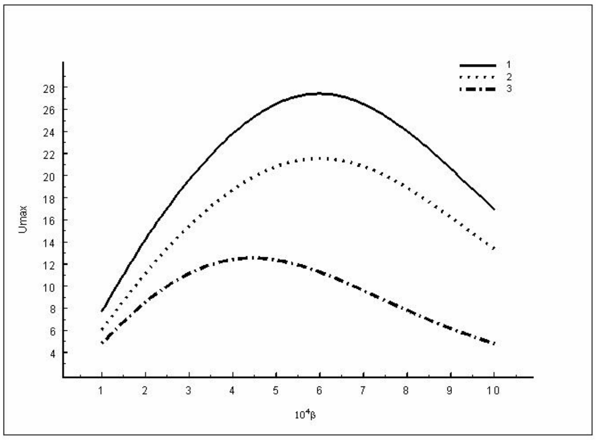

Fig.5

In Fig.5 the dependences U_{max} on the wave vector β are shown for Re=600, $-\alpha_i=10^{-6}$ and $x_0 = 80\cdot10^3$ (1); $320\cdot10^3$ (2); $640\cdot10^3$ (3). It is visible that U_{max} increases with reduction x_0. It is apparent because of low damping of external disturbances on more short spacing intervals x_0. At the same time it is watched some displacement of a maximum of the dependence in the region of larger values of β. However this displacement is not strong and the wave number conforming to a maximum of a curve is approximately equal to $0.7\bullet10^{-3}$ and it is agreed with data obtained in [13].

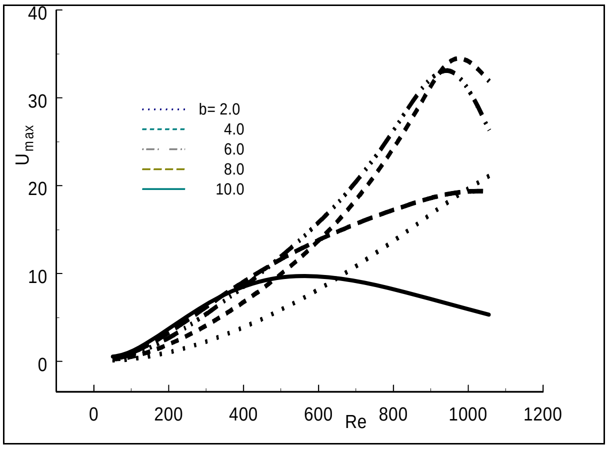

Fig.6

In Fig.6 the values $U_{max}= |\tilde{u}|_{max}$ in depending on Reynolds number at $\alpha_i=10^{-6}$, $b=\beta\cdot10^4$ =2.0; 4.0; 6.0; 8.0; 10 for $x_0= 6.4\cdot10^5$ are shown.

The dependence of a phase velocity Cr on Reynolds number is adduced in Fig. 7. The data are obtained at M=0.0, $-\alpha_i=10^{-6}$, $\omega=10^{-5}$, $x_0=.64\cdot10^6$ and three values β. The small change of Cr from a wave number β is visible. On the other hand, Cr essentially depends on a Reynolds number and varies within the limits 0.5<Cr < 1.0 at change Re from 250 up to 700. It is interesting to address to experiments (see [3]) on the turbulent spots originating. It was established there, that the leading front of a spot, located in the field of large numbers Re, is propagated with speed 0.9 while back one — with the speed equal 0.5. These results are in good, at least, qualitative conformity with our data.

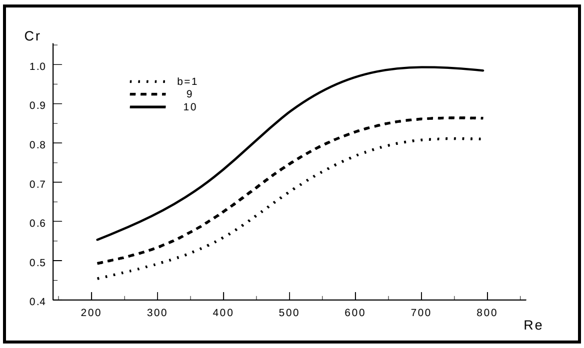

Fig.7

Let's proceed to the data for a case of supersonic speeds. Our results demonstrate that the distributions of disturbances amplitudes of pressure, velocities and enthalpy on a boundary layer at M = 2.0 are similar to the case of Mach number M=0 (Fig, 1).

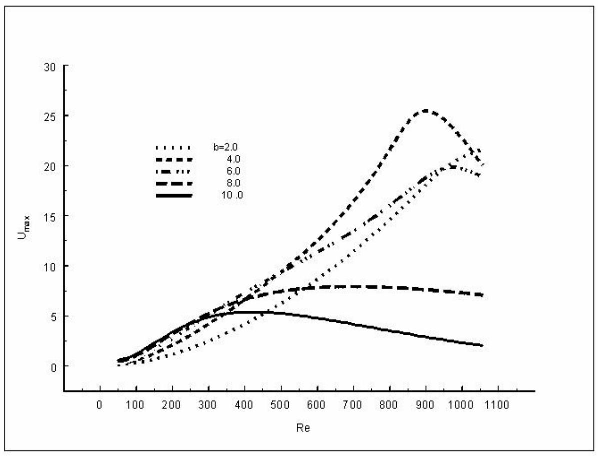

Fig.8

In Fig.8 the dependence U_{max} on Reynolds number are shown for M=2.0 (α_i=10^{-6}; x_0= $6.4\cdot10^5$; b=$\beta\cdot10^4$ =2.0, 4.0, 6.0, 8.0, 10). This dependence is similar qualitatively to the case of Mach number M=0 (Fig.6). But the value U_{max} for M = 2 is less than at M = 0.

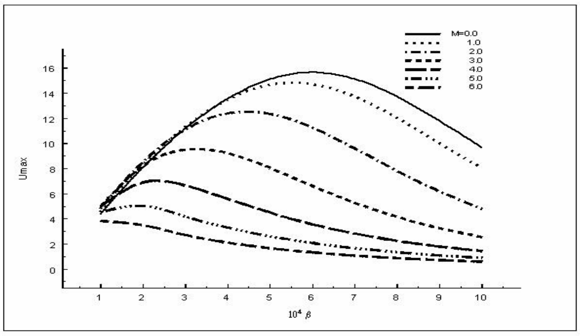

Fig.9

By calculations it is established that the value U_{max} is decreasing monotonically with Mach number increasing. This concluding is demonstrated in Fig.9 (Re=600, α_i=10^{-6}, x_0= $6.4\cdot10^5$).

At last, we shall consider excitation of disturbances inside a boundary layer by external thermal disturbances. In a Fig. 10 the distributions of amplitudes of the disturbances of pressure, velocity and enthalpy (Ap, Av, Au, Aw, Ah), normalized on amplitude of a disturbance of an enthalpy in a free flow are shown (M=2, Re = 600, -α_i =10^{-6}/Pr, β= 10^{-4}, x_0 = $6.4\cdot10^5$). The comparison of these results with data in a Fig. 1 indicates that the shape of a dependence Au on the normal coordinate is similar to a case of external vortical disturbances. However maximum of Au is much lower than in a Fig. 1. Nevertheless the velocity shape deformation is watched in this case too.

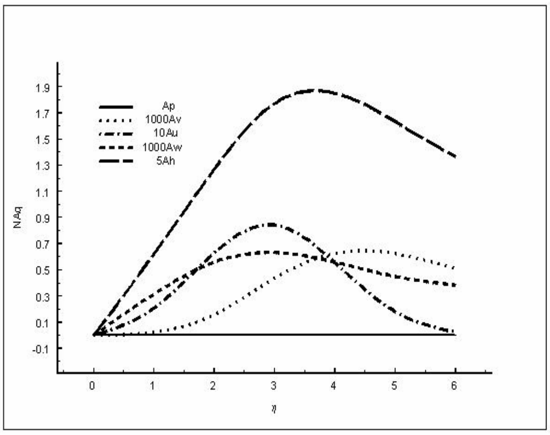

Fig. 10

In Fig.11 the dependence U_{max} on Reynold's number is shown at -α_i=10^{-6} and different values of a wave number β for x_0= $6.4\cdot10^5$. The main feature of this dependence is the fast decreasing the disturbances amplitude of the longitudinal velocity, at least, in area Re < 800. However at large values of a Reynolds number the increase of disturbances inside a boundary layer can be seen.

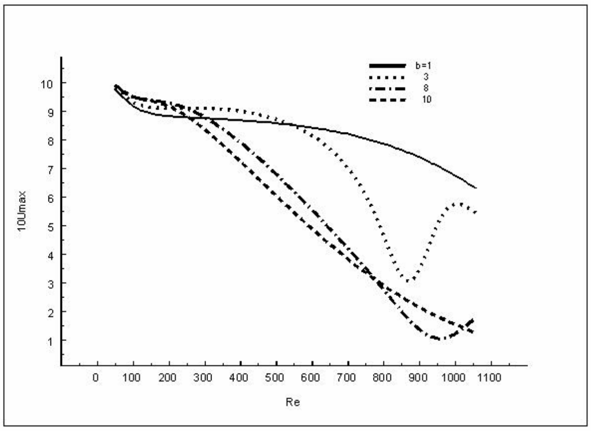

Fig.11

CONCLUSIONS

Thus the conducted researches demonstrate that the external vortical wave can excite disturbances of the longitudinal velocity inside a boundary layer. Their intensity depends on the wave spectrum of disturbances and Mach number. At the given Reynold's number there are a reference value β= β* at which the amplitude of a longitudinal velocity disturbance inside a boundary layer is maximum. It explains the appearance of longitudinal structures with the conforming periodicity in a lateral direction, observed in experiments [13, 15]. The phase velocity of the maximum disturbances inside a boundary layer varies from $0.5\,u_\infty$ (at Re=250) up to $0.95\,u_\infty$ (at Re=700). With increase of a Mach number the intensity of the longitudinal velocity disturbances inside a boundary layer excited by the external vortical waves decreases. The efficiency of the flow deformation inside a boundary layer by external thermal waves is lower in comparison with a case of vortical ones. The results of the present paper are agreed satisfactorily with the experiments [13, 15] and theoretical papers [9,14, 16] but they differ quantitatively from data of Bertolotti [8]. Reasons of this difference remain are unknown.

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