Infra –α- Compact and Infra –α- Connected Spaces

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***Abstract:* - In 2016 Hakeem A. Othman and Md. Hanif Page introduced a new notion of set in general topology called an** $\text{infra}-\alpha-\text{open}$ **set and investigated its fundamental properties and studied the relationship between** $\text{infra}-\alpha-\text{open}$ **set and other topological sets. The objective of this paper is to introduce the new concepts called** $\text{infra}-\alpha-\text{compact}$ **space, countably** $\text{infra}-\alpha-\text{compact}$ **space,** $\text{infra}-\alpha-\text{Lindel\"of}$ **space,** almost $\text{infra}-\alpha-\text{compact}$ space, mildly $\text{infra}-\alpha-\text{compact}$ space **and** $\text{infra}-\alpha-\text{connected}$ **space in general topology and investigate several properties and characterizations of these new concepts in topological spaces.**

***Key-Words:-* Topological space, open set, generalized open set, $\text{infra}-\alpha-\text{open}$ set, $\text{infra}-\alpha-\text{compact}$ space, countably $\text{infra}-\alpha-\text{compact}$ space, $\text{infra}-\alpha-\text{Lindel\"of}$ space, almost $\text{infra}-\alpha-\text{compact}$ space, mildly $\text{infra}-\alpha-\text{compact}$ space, $\text{infra}-\alpha-\text{connected}$ space.**

I. INTRODUCTION

The concept of supra topology was introduced by A. S. Mashhour et al [12] in the year 1983. They studied about $s-\text{continuous}$ functions and $s^*-\text{continuous}$ functions. In 2008, R. Devi et al [5] introduced the concept of supra $\alpha-\text{open}$ sets and supra $\alpha-\text{continuous}$ maps. Jamal. M. Mustafa [14] studied about supra $b-\text{compact}$ and supra $b-\text{Lindelof}$ spaces. Vidyarani et al in [26] introduced the concept of supra $N-\text{compact}$, countably supra $N-\text{compact}$, supra $N-\text{Lindelof}$ and supra $N-\text{connectedness}$ and investigated about their relationships using the concept of continuity. In 2013, Missier and Rodrigo introduced new class of set in general topology called an $\alpha-\text{open}$ $(\text{supra }\alpha-\text{open})$ set. In 2016, Hakeem A. Othman and Md. Hanif Page defined a new class of sets in general topology called an $\mathit{infra}-\alpha-\mathit{open}$ set and investigated its fundamental properties and studied the relation between $\mathit{infra}-\alpha-\mathit{open}$ set and other topological sets. In this paper we introduce the new concepts called $\mathit{infra}-\alpha-\mathit{compact}$ space, countably $\mathit{infra}-\alpha-\mathit{compact}$ space, $\mathit{infra}-\alpha-\mathit{Lindeloff}$ space, almost $\mathit{infra}-\alpha-\mathit{compact}$ space, mildly $\mathit{infra}-\alpha-\mathit{compact}$ space and $\mathit{infra}-\alpha-\mathit{connected}$ space in general topology and investigate several properties and characterization of these new concepts.

Throughout this paper (X,τ) or simply by X we denote topological space on which no separation axioms are assumed unless explicitly stated and

$f:(X,\tau)\longrightarrow(Y,\sigma)$ means a mapping f from a topological space X to a topological space Y. If U is a set and x is a point in X, then $N(x)$, $Int(U), Cl(U)$ and U^c denote respectively, the neighbourhood system of x, the interior of U, the closure of U and complement of U.

II. PRELIMINARIES

Definition 2.1. A subset A of topological space (X,τ) is called a generalized closed set (briefly, $g-closed$) if $Cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in X and generalized open if A^c is $g-closed$ set in X.

We characterize $g-closed$ sets.

Theorem 2.2. A set A in a topological space (X,τ) is $g-closed$ if and only if $Cl(A)-A$ contains no non empty closed set.

Definition 2.3. Let (X,τ) be a topological space. Let $A\subseteq X$. Then we define $closure^*$ and $interior^*$.

$Cl^*(A)=\bigcap\{G: A\subseteq G \,\&\, G \text{ is generalized closed set}\}$ is called $closure^*$.

$Int^*(A)=\bigcup\{G: G\subseteq A \,\&\, G \text{ is generalized open set}\}$ is called $interior^*$.

Lemma 2.4. Let (X,τ) be a topological space and suppose A be any subset of X. Then

$(1).A\subseteq Cl^*(A)\subseteq Cl(A)$.

$(2).Int(A)\subseteq Int^*(A)\subseteq A$.

Definition 2.5. A subset A of space X is called $infra-\alpha-open$ $(infra-\alpha-closed)$ set if $A\subseteq Int\lfloor Cl^*(Int(A))\rfloor$ $\left(Cl\left[Int^*(Cl(A))\right]\subseteq A\right)$. The class of all $infra-\alpha-open$ $(infra-\alpha-closed)$ sets in X will be denoted as $I\alpha-O(X)$ $(I\alpha-C(X))$.

Definition 2.6. Let (X,τ) be a topological space and let A be a subset of X. Then we have,

*. $I\alpha-Cl(A)=\bigcap\{F: A\subseteq F, F\in I\alpha-C(X)\}$ is called an $infra-\alpha-closure$.

**. $I\alpha-Int(A)=\bigcup\{U: U\subseteq A, U\in I\alpha-O(X)\}$ is called an $infra-\alpha-intetrier$.

Theorem 2.7. Let (X,τ) be a topological space. Then a set $A\in I\alpha-O(X)$ if and only if there exists an open set U such that $U\subseteq A\subseteq Int\lfloor Cl^*(U)\rfloor$.

Proof. Necessity: Suppose that $A\in I\alpha-O(X)$. Then $A\subseteq Int\lfloor Cl^*(Int(A))\rfloor$. Put $U=Int(A)$, then U is an open set and $U\subseteq A\subseteq Int\lfloor Cl^*(U)\rfloor$.

Sufficiency: Let U be an open set such that $U\subseteq A\subseteq Int\lfloor Cl^*(U)\rfloor$, this implies that $Int\lfloor Cl^*(U)\rfloor\subseteq Int\lfloor Cl^*(Int(A))\rfloor$, then $A\subseteq Int\lfloor Cl^*(Int(A))\rfloor$.

Theorem 2.8. A set $A\in I\alpha-C(X)$ if and only if there exists a closed set F such that $Cl\lfloor Int^*(F)\rfloor\subseteq A\subseteq F$.

Proof. Necessity: If $A\in I\alpha-C(X)$, then $Cl\lfloor Int^*(Cl(A))\rfloor\subseteq A$. Put $F=Cl(A)$, then F is a closed set and $Cl\lfloor Int^*(F)\rfloor\subseteq A\subseteq F$.

Sufficiency: Let F be a closed set such that $Cl\lfloor Int^*(F)\rfloor\subseteq A\subseteq F$, this implies that $Cl\lfloor Int^*(Cl(A))\rfloor\subseteq Cl\lfloor Int^*(F)\rfloor$, then $Cl\lfloor Int^*(Cl(A))\rfloor\subseteq A$.

Theorem 2.9. Let A be a subset of a space X. Then the following statements hold.

(i) If $A\subseteq B\subseteq Int\lfloor Cl^*(A)\rfloor$ and $A\in I\alpha-O(X)$, then $B\in I\alpha-O(X)$.

(ii) $Cl\lfloor Int^*(A)\rfloor\subseteq B\subseteq A$ and $A\in I\alpha-C(X)$, then $B\in I\alpha-C(X)$,

Proof. (i) Let $A\in I\alpha-O(X)$, then there exists U an open set such that $U\subseteq A\subseteq Int\lfloor Cl^*(U)\rfloor$, this implies that $U \subseteq B$ and $A \subseteq Int\lfloor Cl^*(U)\rfloor$. Therefore, $Int\lfloor Cl^*(A)\rfloor \subseteq Int\lfloor Cl^*(U)\rfloor$ and $U \subseteq B \subseteq Int\lfloor Cl^*(U)\rfloor$, then $B \in I\alpha - O(X)$,

(ii) Easy to prove by using the same technique of proof (i).

Proposition 2.10. Let A and B be the sets in X and $A \subseteq B$. Then, the following statements hold:

1. $I\alpha - Int(A)$ is the largest $infra-\alpha-open$ set contained in A.
2. $I\alpha - Int(A) \subseteq A$.
3. $I\alpha - Int(A) \subseteq I\alpha - Int(B)$.
4. $I\alpha - Int(I\alpha - Int(A)) = I\alpha - Int(A)$.
5. $A \in I\alpha - O(X) \Leftrightarrow I\alpha - Int(A) = A$.

Proposition 2.11. Let A and B be the sets in X and $A \subseteq B$. Then, the following statements hold:

1. $I\alpha - Cl(A)$ is the smallest $infra-\alpha-closed$ set containing A.
2. $A \subseteq I\alpha - Cl(A)$.
3. $I\alpha - Cl(A) \subseteq I\alpha - Cl(B)$.
4. $I\alpha - Cl(I\alpha - Cl(A)) = I\alpha - Cl(A)$.
5. $A \in I\alpha - C(X) \Leftrightarrow I\alpha - Cl(A) = A$.

Theorem 2.12. Let A be a set of X. Then, the following properties are true:

(a) $[I\alpha - Int(A)]^c = I\alpha - Cl(A)$.

(b) $[I\alpha - Cl(A)]^c = I\alpha - Int(A)$.

(c) $I\alpha - Int(A) \subseteq A \cap Int\lfloor Cl^*(Int(A))\rfloor$.

(d) $I\alpha - Cl(A) \supseteq A \cup Cl\lfloor Int^*(Cl(A))\rfloor$.

Corollary 2.13. Let A be a set of X. Then, the following properties are true:

(a) If A is an open set, then $I\alpha - Int(A) \subseteq Int\lfloor Cl^*(Int(A))\rfloor$.

(b) $I\alpha - Cl(A) \supseteq Cl\lfloor Int^*(Cl(A))\rfloor$.

Theorem 2.14. Let (X, τ) be a topological space. Then the following assertions are true:

(a) The arbitrary union of $infra-\alpha-open$ sets is an $infra-\alpha-open$ set.

(b) The arbitrary intersection of $infra-\alpha-closed$ sets is an $infra-\alpha-closed$ set.

Proof. Let $\{U_i : i \in I\}$ be a family of $infra-\alpha-open$ sets. Then, for each $i \in I$, $U_i \subseteq Int\lfloor Cl^*(Int(U_i))\rfloor$ and

$$\bigcup_{i\in I} U_i \subseteq \bigcup_{i\in I} Int[Cl^*(Int(U_i))] \subseteq Int\left\lfloor Cl^*\left(Int\left(\bigcup_{i\in I} U_i\right)\right)\right\rfloor.$$

Hence $\cup\{U_i : i \in I\}$ is an $infra-\alpha-open$ set.

(b) Obvious.

Theorem 2.15. Let A be a set of X. Then the following statement holds:

$Int^*(A) \subseteq I\alpha - Int(A) \subseteq A \subset I\alpha - Cl(A) \subseteq Cl^*(A)$.

Proof. We know that $Int^*(A) \subseteq A$, this implies that $I\alpha - Int\lfloor Int^*(A)\rfloor \subseteq I\alpha - Int(A)$. Then, $I\alpha - Int\lfloor Int^*(A)\rfloor = Int^*(A)$ and so, $Int^*(A) \subseteq I\alpha - Int(A) \longrightarrow (*)$.

Also, we know that $A \subseteq Cl^*(A)$, this implies that $I\alpha - Cl(A) \subseteq I\alpha - Cl\lfloor Cl^*(A)\rfloor$. Then, $I\alpha - Cl\lfloor Cl^*(A)\rfloor = Cl^*(A)$ and so, $I\alpha - Cl(A) \subseteq Cl^*(A) \longrightarrow (**)$.

From $(*$ and $(**)$,it follows that $Int^*(A) \subseteq I\alpha - Int(A) \subseteq A \subset I\alpha - Cl(A) \subseteq Cl^*(A)$.

Definition 2.16. A set $A \subseteq X$ is called an $\alpha-open$ [15] A $(Semiopen[10])$ set if $A \subseteq Int\lfloor Cl(Int(A))\rfloor$ $(A \subseteq Cl[Int(A)])$. The collection of all $\alpha-open$ $(semi\ open)$ sets of X is denoted as $\alpha O(X)$ $(SO(X))$.

Theorem 2.17. Let A be a set of a topological space X. Then the following statements hold:

(a) If A is an open (closed) set, then A is an $infra-\alpha-open$ $(infra-\alpha-closed)$ set.

(b) If A is an $infra-\alpha-open$ $(infra-\alpha-closed)$ set, then A is an $\alpha-open$ $(\alpha-closed)$ set.

Remark 2.18. Let (X,τ) be a topological Space. Then the following relation holds for subsets of X.

$Open\ Set \rightarrow Infra-\alpha-Open \rightarrow \alpha-Open \rightarrow Semi-Open$

Definition 2.19. A mapping $f:(X,\tau)\longrightarrow(Y,\sigma)$ is said to be an $infra-\alpha-continuous$ if $f^{-1}(V)$ is an $infra-\alpha-open$ $(infra-\alpha-closed)$ set in X for each open (closed) set V in Y.

Definition 2.20. A mapping $f:(X,\tau)\longrightarrow(Y,\sigma)$ is said to be an $infra-\alpha-irresolute$ if $f^{-1}(V)$ is an $infra-\alpha-open$ $(infra-\alpha-closed)$ set in X for X each $infra-\alpha-open$ $(infra-\alpha-closed)$ set V in Y.

Definition 2.21. A mapping $f:(X,\tau)\longrightarrow(Y,\sigma)$ is said to be an $infra-\alpha-open$ $(infra-\alpha-closed)$ if $f(U)$ is an $infra-\alpha-open$ $(infra-\alpha-closed)$ set in Y for each open (closed) set U in X.

Definition 2.22. A set $A\subseteq X$ is said to be $infra-\alpha-connected$ if A cannot be written as the union of two $infra-\alpha-separated$ sets.

Definition 2.23. Let X be any nonempty set and $\tau\subseteq P(X)$. We say that τ is a supra topology on X if $\phi, X\in\tau$ and τ is closed under arbitrary union. The pair (X,τ) is called supra topological space. The elements of τ are called supra open sets in (X,τ) and complement of a supra open set is called a supra closed set.

Definition 2.24. A supra topological space is called supra compact (S – compact) if and only if every supra open cover of X has a finite sub cover.

Definition 2.25. A function $f:(X,\tau)\longrightarrow(Y,\sigma)$ is called perfectly $infra-\alpha-continuous$ if the inverse image $f^{-1}(V)$ of every $infra-\alpha-open$ set V of Y is both open and closed in X.

Definition 2.26. A function $f:(X,\tau)\longrightarrow(Y,\sigma)$ is called strongly $infra-\alpha-continuous$ if the inverse image $f^{-1}(V)$ of every $infra-\alpha-open$ V in Y is open in X.

Definition 2.27. Let X be a non-empty set. The subfamily $\mu\subseteq P(X)$ is said to be a supra topology on X if $\phi, X\in\mu$ and μ is closed under arbitrary unions. The pair (X,μ) is called a supra topological space. The elements of μ are said to be supra open in (X,μ). Complement of supra open sets are called supra closed sets.

III. INFRA - α -COMPACT SPACES

Definition 3.1. A collection $\{A_i : i\in I\}$ of $infra-\alpha-open$ sets in a topological space (X,τ) is called an $infra-\alpha-open$ cover of a subset B of X if $B\subseteq \cup\{A_i : i\in I\}$ holds.

Definition 3.2. A topological space (X,τ) is called $infra-\alpha-compact$ if every $infra-\alpha-open$ cover of X has a finite sub cover.

Definition 3.3. A subset B of a topological space (X,τ) is said to be $infra-\alpha-compact$ relative to (X,τ) if, for every collection $\{A_i : i\in I\}$ of $infra-\alpha-open$ subsets of X such that $B\subseteq \cup\{A_i : i\in I\}$ there exists a finite subset I_0 of I such that $B\subseteq \cup\{A_i : i\in I_0\}$.

Definition 3.4. A subset B of a topological space (X,τ) is said to be $infra-\alpha-compact$ if B is $infra-\alpha-compact$ as a subspace of X.

Theorem 3.5. Every $infra-\alpha-compact$ space is compact.

Proof. Let $\{A_i : i\in I\}$ be an open cover of (X,τ). Since every open set in X is $infra-\alpha-open$ in

X.So $\{A_i : i \in I\}$ is an $infra-\alpha-open$ cover of (X,τ). Since (X,τ) is $infra-\alpha-compact$, $infra-\alpha-open$ cover $\{A_i : i \in I\}$ of (X,τ) has a finite sub cover say $\{A_i : i = 1,2,3,...,n\}$ for X. Hence (X,τ) is a compact space.

Theorem 3.6. Every $infra-\alpha-closed$ subset of an $infra-\alpha-compact$ space (X,τ) is $infra-\alpha-compact$ relative to X.

Proof. Let A be an $infra-\alpha-closed$ closed subset of a topological space (X,τ). Then A^c is $infra-\alpha-open$ in (X,τ). Let $\Gamma = \{A_i : i \in I\}$ be an $infra-\alpha-open$ cover of A by $infra-\alpha-open$ subsets of (X,τ). Then $\Gamma^* = \Gamma \cup \{A^c\}$ is an $infra-\alpha-open$ cover of (X,τ). That is $X = \left(\bigcup_{i\in I} A_i\right) \cup A^c$. By hypothesis (X,τ) is an $infra-\alpha-compact$ space and hence Γ^* is reducible to a finite sub cover of (X,τ) say $X = \left(\bigcup_{i\in I_0} A_i\right) \cup A^c$ for some finite subset I_0 of I. But A and A^c are disjoint. Hence $A \subseteq \cup\{A_i : i \in I_0\}$. Thus $infra-\alpha-open$ cover $\Gamma = \{A_i : i \in I\}$ of A contains a finite sub cover. Hence A is $infra-\alpha-compact$ relative to (X,τ).

Theorem 3.7. An $infra-\alpha-continuous$ image of an $infra-\alpha-compact$ space is compact.

Proof. Let $f:(X,\tau) \longrightarrow (Y,\sigma)$ be an $infra-\alpha-continuous$ map from an $infra-\alpha-compact$ (X,τ) onto a topological space (Y,σ). Let $\Gamma = \{A_i : i \in I\}$ be an open cover of Y. Therefore $f^{-1}(\Gamma) = \{f^{-1}(A_i) : i \in I\}$ is an $infra-\alpha-open$ cover of X, as f is $infra-\alpha-continuous$. Since X is $infra-\alpha-compact$, the $infra-\alpha-open$ cover $f^{-1}(\Gamma) = \{f^{-1}(A_i) : i \in I\}$ of X, has a finite sub cover say $\{f^{-1}(A_i) : i = 1,2,3,...,n\}$. Therefore $X = \bigcup_{i=1}^{n} f^{-1}(A_i)$, which implies $Y = f(X) = \bigcup_{i=1}^{n} A_i$. That is $\{A_i : i = 1,2,3,...,n\}$ is a finite sub cover of $\Gamma = \{A_i : i \in I\}$. Hence (Y,σ) is compact.

Theorem 3.8. Suppose that a function $f:(X,\tau) \longrightarrow (Y,\sigma)$ is $infra-\alpha-irresolute$ and a subset S of X is $infra-\alpha-compact$ relative to (X,τ), then the image $f(S)$ is $infra-\alpha-compact$ relative to (Y,σ).

Proof. Let $\Gamma = \{A_i : i \in I\}$ be a collection of $infra-\alpha-open$ cover of (Y,σ), such that $f(S) \subseteq \cup\{A_i : i \in I\}$. Since f is $infra-\alpha-irresolute$. So $S \subseteq \cup\{f^{-1}(A_i) : i \in I\}$, where $\{f^{-1}(A_i) : i \in I\} \subseteq I\alpha-O(X,\tau)$. Since S is $infra-\alpha-compact$ relative to (X,τ), there exists a finite sub collection $\{f^{-1}(A_1), f^{-1}(A_2), \ldots, f^{-1}(A_n)\}$ such that $S \subseteq \cup\{f^{-1}(A_1), f^{-1}(A_2), \ldots, f^{-1}(A_n)\}$. That is $f(S) \subseteq \cup\{A_1, A_2, ..., A_n\}$. Hence $f(S)$ is $infra-\alpha-compact$ relative to (Y,σ).

Theorem 3.9. Suppose that a map $f:(X,\tau) \longrightarrow (Y,\sigma)$ is strongly $infra-\alpha-continuous$ map from a compact space (X,τ) onto a topological space (Y,σ), then (Y,σ) is $infra-\alpha-compact$.

Proof. Let $\{A_i : i \in I\}$ be an $infra-\alpha-open$ cover of (Y,σ). Since f is strongly $infra-\alpha-continuous$, $\{f^{-1}(A_i) : i \in I\}$ is an open cover of (X,τ). Again, since (X,τ) is compact, the open cover $\{f^{-1}(A_i) : i \in I\}$ of (X,τ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1,2,3,...,n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1,2,3,...,n\}$, which implies $f(X) = \cup\{A_i : i = 1,2,3,...,n\}$, so that

$Y = \cup\{A_i : i = i = 1,2,3,...,n\}$. That is $\{A_1, A_2, ..., A_n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y, σ). Hence (Y, σ) is $infra-\alpha-compact$.

Theorem 3.10. Suppose that a map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is perfectly $infra-\alpha-continuous$ map from a compact space (X, τ) onto a topological space (Y, σ). Then (Y, σ) is $infra-\alpha-compact$.

Proof. Let $\{A_i : i \in I\}$ be an $infra-\alpha-open$ cover of (Y, σ). Since f is perfectly $infra-\alpha-continuous$, $\{f^{-1}(A_i) : i \in I\}$ is an open cover of (X, τ). Again, since (X, τ) is compact, the open cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1,2,3,...,n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1,2,3,...,n\}$, which implies $f(X) = \cup\{A_i : i = 1,2,3,...,n\}$, so that $Y = \cup\{A_i : i = 1,2,3,...,n\}$. That is $\{A_1, A_2, \ldots, A_n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y, σ). Hence (Y, σ) is $infra-\alpha-compact$.

Theorem 3.11. Suppose that a function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is $infra-\alpha-irresolute$ map from an $infra-\alpha-compact$ space (X, τ) onto a topological space (Y, σ). Then (Y, σ) is $infra-\alpha-compact$.

Proof. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be an $infra-\alpha-irresolute$ map from an $infra-\alpha-compact$ space (X, τ) onto a topological space (Y, σ). Let $\{A_i : i \in I\}$ be an $infra-\alpha-open$ cover of (Y, σ). Then $\{f^{-1}(A_i) : i \in I\}$ is an $infra-\alpha-open$ cover of (X, τ), since f is $infra-\alpha-irresolute$. As (X, τ) is $infra-\alpha-compact$, the $infra-\alpha-open$ cover $\{f^{-1}(A_i) : i \in I\}$ of (X, τ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1,2,3,...,n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1,2,3,...,n\}$, which implies $f(X) = \cup\{A_i : i = 1,2,3,...,n\}$, so that $Y = \cup\{A_i : i = 1,2,3,...,n\}$. That is $\{A_1, A_2, \ldots, A_n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y, σ). Hence (Y, σ) is $infra-\alpha-compact$.

Theorem 3.12. If (X, τ) is compact and every $infra-\alpha-closed$ set in X is also closed in X, then (X, τ) is $infra-\alpha-compact$.

Proof. Let $\{A_i : i \in I\}$ be an $infra-\alpha-open$ cover of X. Since every $infra-\alpha-closed$ set in X is also closed in X. Thus $\{X - A_i : i \in I\}$ is a closed cover of X and hence $\{A_i : i \in I\}$ is an open cover of X. Since (X, τ) is compact. So there exists a finite sub cover $\{A_i : i = 1,2,3,...,n\}$ of $\{A_i : i \in I\}$ such that $X = \cup\{A_i : i = 1,2,3,...,n\}$. Hence (X, τ) is $infra-\alpha-compact$.

Theorem 3.13. A topological space (X, τ) is $infra-\alpha-compact$ if and only if every family of $infra-\alpha-closed$ sets of (X, τ) having finite intersection property has a non empty intersection.

Proof. Suppose (X, τ) is $infra-\alpha-compact$, Let $\{A_i : i \in I\}$ be a family of $infra-\alpha-closed$ sets with finite intersection property. Suppose $\bigcap_{i \in I} A_i = \phi$, then $X - \bigcap(\{A_i : i \in I\}) = X$. This implies $\bigcup\{(X - A_i) : i \in I\} = X$. Thus the cover $\{(X - A_i) : i \in I\}$ is an $infra-\alpha-open$ cover of (X, τ). Then as (X, τ) is $infra-\alpha-compact$, the $infra-\alpha-open$ cover $\{(X - A_i) : i \in I\}$ has a finite sub cover say $\{(X - A_i) : i = 1,2,3,...,n\}$. This implies that $X = \cup\{(X - A_i) : i = 1,2,3,...,n\}$ which implies $X = X - \cap\{A_i : i = 1,2,3,...,n\}$, which implies $X - X = X - \lfloor X - \cap\{A_i : i = 1,2,3,...,n\}\rfloor$, which implies $\phi = \bigcap \{A_i : i = 1,2,3,...,n\}$. This disproves the assumption. Hence $\bigcap \{A_i : i \in I\} \neq \phi$.

Conversely, suppose (X, τ) is not $infra-\alpha-compact$. Then there exits an $infra-\alpha-open$ cover of (X, τ) say $\{G_i : i \in I\}$ having no finite sub cover. This implies for any finite sub family $\{G_i : i = 1,2,3,...,n\}$ of $\{G_i : i \in I\}$, we have $\bigcup\{G_i : i = 1,2,3,...,n\} \neq X$, which implies $X - (\bigcup\{G_i : i = 1,2,3,...,n\}) \neq X - X$, therefore $\bigcap \{X - G_i : i = 1,2,3,...,n\} \neq \phi$. Then the family $\{X - G_i : i \in I\}$ of $infra-\alpha-closed$ sets has a finite intersection property. Also by assumption $\bigcap \{X - G_i : i \in I\} \neq \phi$ which implies $X - (\bigcup\{G_i : i \in I\}) \neq \phi$, so that $\bigcup\{G_i : i \in I\} \neq X$. This implies $\{G_i : i \in I\}$ is not a cover of (X, τ). This disproves the fact that $\{G_i : i \in I\}$ is a cover for (X, τ). Therefore an $infra-\alpha-open$ cover $\{G_i : i \in I\}$ of (X, τ) has a finite sub cover $\{G_i : i = 1,2,3,...,n\}$. Hence (X, τ) is $infra-\alpha-compact$.

Theorem 3.14. Let A be an $infra-\alpha-compact$ set relative to a topological space X and be an $infra-\alpha-closed$ subset of X. Then $A \cap B$ is $infra-\alpha-compact$ relative to X.

Proof. Let A be $infra-\alpha-compact$ relative to X. Let $\{A_i : i \in I\}$ be a cover of $A \cap B$ by $infra-\alpha-open$ sets in X. Then $\{A_i : i \in I\} \cup \{B^C\}$ is a cover of A by $infra-\alpha-open$ sets in X, but A is $infra-\alpha-compact$ relative to X, so there exists a finite subset $I_0 = \{i_1, i_2, i_3, \ldots, i_n\} \subseteq I$ such that $A \subseteq (\bigcup\{A_{i_k} : k = 1, 2, 3, \ldots, n\}) \cup B^C$. Then $A \cap B \subseteq \bigcup\{A_{i_k} \cap B : k = 1, 2, 3, \ldots, n\} \subseteq \bigcup\{A_{i_k} : k = 1,2,3,...,n\}$. Hence $A \cap B$ is $infra-\alpha-compact$.

Theorem 3.15. Suppose that a function $f : (X, \tau) \longrightarrow (Y, \sigma)$ is $infra-\alpha-irresolute$ and a subset B of X is $infra-\alpha-compact$ relative to X. Then $f(B)$ is $infra-\alpha-compact$ relative to Y.

Proof. Let $\{A_i : i \in I\}$ be a cover of $f(B)$ by $infra-\alpha-open$ subsets of Y. Since f is $infra-\alpha-irresolute$. Then $\{f^{-1}(A_i) : i \in I\}$ is a cover of B by $infra-\alpha-open$ subsets of X. Since B is $infra-\alpha-compact$ relative to X, $\{f^{-1}(A_i) : i \in I\}$ has a finite sub cover say $\{f^{-1}(A_1), f^{-1}(A_2), \ldots, f^{-1}(A_n)\}$ for B. Then it implies that $\{A_i : i = 1,2,3,...,n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for $f(B)$. So $f(B)$ is $infra-\alpha-compact$ relative to Y.

Definition 3.16. Let (X, τ) be a topological space and let E be a subset of X. Let $\tau_E^{i\alpha} = \{A \cap E : A \in I\alpha - O(X, \tau)\}$. Then $(E, \tau_E^{i\alpha})$ is a supra topological space.

Theorem 3.17. Let (X, τ) be a topological space and let E be a subset of X. Then $(E, \tau_E^{i\alpha})$ is supra compact if and only if for any $infra-\alpha-open$ cover Γ of E has a finite sub cover of E.

Proof. Suppose E is supra compact. Let $\Gamma \subseteq I\alpha - O(X, \tau)$ such that $E \subseteq \bigcup\Gamma$. Let $\Gamma_E = \{A \cap E : A \in \Gamma\}$. Then $E = \bigcup\Gamma_E$ and $\Gamma_E \subseteq \tau_E^{i\alpha}$. By hypothesis there exists a finite subset $\Gamma_E^* = \{A_i \cap E : i = 1,2,3,...,n\} \subseteq \Gamma_E$ such that $E = \bigcup\Gamma_E^*$. Then $\Gamma^* = \{A_i : i = 1,2,3,...,n\} \subseteq \Gamma$ and $E \subseteq \bigcup\Gamma^*$.

Conversely, let $\Upsilon = \{A_i \cap E : i \in I\} \subseteq \tau_E^{i\alpha}$ such that $E = \bigcup\Upsilon$. Then $\Upsilon^* = \{A_i : i \in A\}$ is an $infra-\alpha-open$ covering of E. By hypothesis there exists $\Upsilon^{**} = \{A_i : i = 1,2,3,...,n\}$ a finite subset of Υ^* such that $E \subseteq \bigcup\Upsilon^{**}$. Then

$\Upsilon^{\#} = \{A_i \cap E : i = 1,2,3,...,n\}$ is a finite subset of Υ such that $E = \cup\Upsilon^{\#}$. This proves that $(E, \tau_E^{i\alpha})$ is supra compact.

IV. COUNTABLY INFRA - α - COMPACT SPACES

In this section, we present the concept of countably $infra-\alpha-compactness$ and its properties.

Definition 4.1. A topological space (X, τ) is said to be countably $infra-\alpha-compact$ if every countable $infra-\alpha-open$ cover of X has a finite sub cover.

Theorem 4.2. If (X, τ) is a countably $infra-\alpha-compact$ space, then (X, τ) is countably compact.

Proof. Let (X, τ) be a countably $infra-\alpha-compact$ space. Let $\{A_i : i \in I\}$ be a countable open cover of (X, τ). Since $\tau \subseteq I\alpha-O(X, \tau)$. So $\{A_i : i \in I\}$ is a countable $infra-\alpha-open$ cover of (X, τ). Since (X, τ) is countably $infra-\alpha-compact$, therefore countable $infra-\alpha-open$ cover $\{A_i : i \in I\}$ of (X, τ) has a finite sub cover say $\{A_i : i = 1, 2, 3, ..., n\}$ for X. Hence (X, τ) is a countably compact space.

Theorem 4.3. If (X, τ) is countably compact and every $infra-\alpha-closed$ subset of X is closed in X, then (X, τ) is countably $infra-\alpha-compact$.

Proof. Let (X, τ) be a countably compact space. Let $\{A_i : i \in I\}$ be a countable $infra-\alpha-open$ cover of (X, τ). Since every $infra-\alpha-closed$ subset of X is closed in X. Thus every $infra-\alpha-open$ set in X is open in X. Therefore $\{A_i : i \in I\}$ is a countable open cover of (X, τ). Since (X, τ) is countably compact, so countable open cover $\{A_i : i \in I\}$ of (X, τ) has a finite sub cover say $\{A_i : i = 1,2,3,...,n\}$ for X. Hence (X, τ) is a countably $infra-\alpha-compact$ space.

Theorem 4.4. Every $infra-\alpha-compact$ space is countably $infra-\alpha-compact$.

Proof. Let (X, τ) be an $infra-\alpha-compact$ space. Let $\{A_i : i \in I\}$ be a countable $infra-\alpha-open$ cover of (X, τ). Since (X, τ) is $infra-\alpha-compact$, so $infra-\alpha-open$ cover $\{A_i : i \in I\}$ of (X, τ) has a finite sub cover say $\{A_i : i = 1,2,3,...,n\}$ for (X, τ). Hence (X, τ) is countably $infra-\alpha-compact$ space.

Theorem 4.5. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be a $infra-\alpha-continuous$ onjective mapping. If X is countably $infra-\alpha-compact$ space, then (Y, σ) is countably compact.

Proof. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be an $infra-\alpha-continuous$ map from a countably $infra-\alpha-compact$ space (X, τ) onto a topological space (Y, σ). Let $\{A_i : i \in I\}$ be a countable open cover of Y. Then $\{f^{-1}(A_i) : i \in I\}$ is a countable $infra-\alpha-open$ cover of X, as f is $infra-\alpha-continuous$. Since X is countably $infra-\alpha-compact$, the countable $infra-\alpha-open$ cover $\{f^{-1}(A_i) : i \in I\}$ of X has a finite sub cover say $\{f^{-1}(A_i) : i = 1,2,3,...,n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1,2,3,...,n\}$, which implies $Y = f(X) = \cup\{A_i : i = 1,2,3,...,n\}$. That is $\{A_i : i = 1,2,3,...,n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for Y. Hence Y is countably compact.

Theorem 4.6. Suppose that a map $f : (X, \tau) \longrightarrow (Y, \sigma)$ is perfectly $infra-\alpha-continuous$ map from a countably compact space (X, τ) onto a topological space (Y, σ). Then (Y, σ) is countably $infra-\alpha-compact$.

Proof. Let $\{A_i : i \in I\}$ be a countable *infra* $-\alpha-$ *open* cover of (Y,σ). Since f is perfectly *infra* $-\alpha-$ *continuous*, $\{f^{-1}(A_i) : i \in I\}$ is a countable open cover of (Y,σ). Again, since (X,τ) is countably *infra* $-\alpha-$ *compact*, the countable open cover $\{f^{-1}(A_i) : i \in I\}$ of (X,τ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1,2,3,...,n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1,2,3,...,n\}$, which implies $f(X) = \cup\{A_i : i = 1,2,3,...,n\}$, so that $Y = \cup\{A_i : i = 1,2,3,...,n\}$. That is $\{A_1, A_2, \ldots, A_n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y,σ). Hence (Y,σ) is countably *infra* $-\alpha-$ *compact*.

Theorem 4.7. Suppose that a map $f : (X,\tau) \longrightarrow (Y,\sigma)$ is strongly *infra* $-\alpha-$ *continuous* map from a countably compact space (X,τ) onto a topological space (Y,σ). Then (Y,σ) is countably *infra* $-\alpha-$ *compact*.

Proof. Let $\{A_i : i \in I\}$ be a countable *infra* $-\alpha-$ *open* cover of (Y,σ). Since f is strongly *infra* $-\alpha-$ *continuous*, $\{f^{-1}(A_i) : i \in I\}$ is a countable open cover of (X,τ). Again, since (X,τ) is countably compact, the countable supra open cover $\{f^{-1}(A_i) : i \in I\}$ of (X,τ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1,2,3,...,n\}$. Therefore $X = \cup\{f^{-1}(A_i) : i = 1,2,3,...,n\}$, which implies $f(X) = \cup\{A_i : i = 1,2,3,...,n\}$, so that $Y = \cup\{A_i : i = 1,2,3,...,n\}$. That is $\{A_1, A_2, ..., A_n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y,σ). Hence (Y,σ) is countably *infra* $-\alpha-$ *compact*.

Theorem 4.8. The image of a countably *infra* $-\alpha-$ *compact* space under an *infra* $-\alpha-$ *irresolute* map is countably *infra* $-\alpha-$ *compact*.

Proof. Suppose that a map $f : (X,\tau) \longrightarrow (Y,\sigma)$ is an *infra* $-\alpha-$ *irresolute* map from a countably *infra* $-\alpha-$ *compact* space (X,τ) onto a topological space (Y,σ). Let $\{A_i : i \in I\}$ be a countable *infra* $-\alpha-$ *open* cover of (Y,σ). Then $\{f^{-1}(A_i) : i \in I\}$ is a countable *infra* $-\alpha-$ *open* cover of (X,τ), since f is *infra* $-\alpha-$ *irresolute*. As (X,τ) is countably *infra* $-\alpha-$ *compact*, the countable *infra* $-\alpha-$ *open* cover $\{f^{-1}(A_i) : i \in I\}$ of (X,τ) has a finite sub cover say $\{f^{-1}(A_i) : i = 1,2,3,...,n\}$. Then it follows that $X = \cup\{f^{-1}(A_i) : i = 1,2,3,...,n\}$, which implies $f(X) = \cup\{A_i : i = 1,2,3,...,n\}$, so that $Y = \cup\{A_i : i = 1,2,3,...,n\}$. That is $\{A_1, A_2, \ldots, A_n\}$ is a finite sub cover of $\{A_i : i \in I\}$ for (Y,σ). Hence (Y,σ) is countably *infra* $-\alpha-$ *compact*.

Definition 4.9. Let (X,τ) be a topological space and $x \in X$. A point x is said to be *infra* $-\alpha-$ *limit* point of $A \subseteq X$ provided that every *infra* $-\alpha-$ *neighborhood* of x contains at least one point of A different from x.

Theorem 4.10. Every infinite subset of an *infra* $-\alpha-$ *compact* space has an *infra* $-\alpha-$ *limit* point.

Proof. Let A be an infinite subset of an *infra* $-\alpha-$ *compact* space (X,τ). Suppose that A has not an *infra* $-\alpha-$ *limit* point. Then for each $x \in X$, there exists an *infra* $-\alpha-$ *open* set G_x containing at most one point of A. Now, the collection $\Lambda = \{G_x : x \in X\}$ forms an *infra* $-\alpha-$ *open* cover of X. As X is *infra* $-\alpha-$ *compact*, then there exist $x_1, x_2, \ldots, x_n$ in X such that $X = \bigcup_{i=1}^{i=n} G_{x_i}$. Therefore X has at most n points of A. This implies that A is finite.

But this contradicts that A is infinite. Thus A has an $infra-\alpha-limit$ point.

V. INFRA - α - LINDELÖF SPACES

In this section, we concentrate on the concept of $infra-\alpha-Lindelöf$ space and its properties.

Definition 5.1. A topological space (X,τ) is said to be $infra-\alpha-Lindelöf$ space if every $infra-\alpha-open$ cover of X has a countable sub cover.

Theorem 5.2. Every $infra-\alpha-Lindelöf$ space (X,τ) is $Lindelöf$ space.

Proof. Let (X,τ) be an $infra-\alpha-Lindelöf$ space. Let $\{A_i : i\in I\}$ be an open cover of (X,τ). Since $\tau \subseteq I\alpha-O(X,\tau)$. Therefore $\{A_i : i\in I\}$ is an $infra-\alpha-open$ cover of (X,τ). Since (X,τ) is $infra-\alpha-Lindelöf$ space. So there exists a countable subset I_0 of I such that $\{A_i : i\in I_0\}$ is an $infra-\alpha-open$ sub cover of (X,τ). Hence (X,τ) is a $Lindelöf$ space.

Theorem 5.3. Every $infra-\alpha-compact$ space is $infra-\alpha-Lindelöf$.

Proof. Let (X,τ) be an $infra-\alpha-compact$ space. Let $\{A_i : i\in I\}$ be an $infra-\alpha-open$ cover of (X,τ). Since (X,τ) is $infra-\alpha-compact$ space. Then $\{A_i : i\in I\}$ has a finite sub cover say $\{A_i : i=1,2,3,...,n\}$. Since every finite sub cover is always countable sub cover and therefore $\{A_i : i=1,2,3,...,n\}$. is countable sub cover of $\{A_i : i\in I\}$. Hence (X,τ) is $infra-\alpha-Lindelöf$ space.

Theorem 5.4. Every $infra-\alpha-closed$ subset of an $infra-\alpha-Lindelöf$ space is $infra-\alpha-Lindelöf$.

Proof. Let F be an $infra-\alpha-closed$ subset of X and $\{G_i : i\in I\}$ be $infra-\alpha-open$ cover of F. Then F^c is $infra-\alpha-open$ and $F\subseteq \cup\{G_i : i\in I\}$. Hence $X=(\cup\{G_i : i\in I\})\cup F^c$. Since X is $infra-\alpha-Lindelöf$, then $X=(\cup\{G_i : i\in I_0\})\cup F^c$ for some countable subset I_0 of I. Therefore $F\subseteq \cup\{G_i : i\in I_0\}$. Thus F is $infra-\alpha-Lindelöf$.

Theorem 5.5. Let A be an $infra-\alpha-Lindelöf$ subset of X and B be an $infra-\alpha-closed$ subset of X. Then $A\cap B$ is $infra-\alpha-Lindelöf$.

Proof. Let $\{G_i : i\in I\}$ be an $infra-\alpha-open$ cover of $A\cap B$. Then $A\subseteq\left(\bigcup_{i\in I} G_i\right)\cup B^c$. Since A is $infra-\alpha-Lindelöf$, then there exists a countable subset I_0 of I such that $A\subseteq\left(\bigcup_{i\in I_0} G_i\right)\cup B^c$. Therefore $A\cap B\subseteq\bigcup_{i\in I_0} G_i$. Thus $A\cap B$ is $infra-\alpha-Lindelöf$.

Theorem 5.6. A topological space (X,τ) is $infra-\alpha-Lindelöf$ if and only if every collection of $infra-\alpha-closed$ subsets of X satisfying the countable intersection property, has, itself, a non-empty intersection.

Necessity: Let $\Lambda=\{F_i : i\in I\}$ be a collection of $infra-\alpha-closed$ subsets of X which has the countable intersection property. Assume that $\bigcap_{i\in I} F_i=\phi$. Then $X=\bigcup_{i\in I} F_i^c$. Since X is $infra-\alpha-Lindelöf$, then there exists a countable subset I_0 of I such that $X=\bigcup_{i\in I_0} F_i^c$. Therefore, $\bigcap_{i\in I_0} F_i=\phi$ contradicts that Λ has the countable intersection property. Thus Λ has, itself, a non-empty intersection.

Sufficiency: Let $\{G_i : i\in I\}$ be an $infra-\alpha-open$ cover of X. Suppose $\{G_i : i\in I\}$ has no countable sub cover. Then $X-\bigcup_{i\in J} G_i\neq\phi$, for any countable subset J of I. Now, $\bigcap_{i\in J} G_i^c\neq\phi$ implies that $\{G_i^c : i\in I\}$ is a collection of $infra-\alpha-closed$ closed subsets of X which has the countable intersection property. Therefore $\bigcap_{i\in I} G_i^c\neq\phi$. Thus $X\neq\bigcup_{i\in I} G_i$ contradicts that $\{G_i : i\in I\}$ is an $infra-\alpha-open$ cover of X. Hence X is $infra-\alpha-Lindelöf$.

Theorem 5.7. An $infra-\alpha-continuous$ image of an $infra-\alpha-Lindelöf$ space is a $Lindelöf$ space.

Proof. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be an $infra-\alpha-continuous$ map from an $infra-\alpha-Lindelöf$ space X onto a topological space Y. Let $\{A_i : i\in I\}$ be an open cover of Y. Then $\{f^{-1}(A_i): i\in I\}$ is an $infra-\alpha-open$ cover of X, as f is $infra-\alpha-continuous$. Since X is $infra-\alpha-Lindelöf$ space, the $infra-\alpha-open$ cover $\{f^{-1}(A_i): i\in I\}$ of X has a countable sub cover say $\{f^{-1}(A_i): i\in I_0\}$ for some countable set $I_0\subseteq I$. Therefore $X=\cup\{f^{-1}(A_i): i\in I_0\}$, which implies $f(X)=\cup\{A_i : i\in I_0\}$, then $Y=\cup\{A_i : i\in I_0\}$. That is $\{A_i : i\in I_0\}$ is a countable sub cover of $\{A_i : i\in I\}$ for Y. Hence (Y,σ) is a $Lindelöf$ space.

Theorem 5.8. The image of an $infra-\alpha-Lindelöf$ space under an $infra-\alpha-irresolue$ map is $infra-\alpha-Lindelöf$ space.

Proof. Suppose that a map $f:(X,\tau)\longrightarrow(Y,\sigma)$ is an $infra-\alpha-irresolue$ map from an $infra-\alpha-Lindelöf$ space (X,τ) onto a topological space (Y,σ). Let $\{B_i : i\in I\}$ be an $infra-\alpha-open$ cover of (Y,σ). Since f is $infra-\alpha-irresolue$. Therefore $\{f^{-1}(B_i): i\in I\}$ is an $infra-\alpha-open$ cover of (X,τ). As (X,τ) is $infra-\alpha-Lindelöf$ space. the $infra-\alpha-open$ cover $\{f^{-1}(B_i): i\in I\}$ of (X,τ) has a countable sub cover say $\{f^{-1}(B_i): i\in I_0\}$ for some countable set $I_0\subseteq I$. Therefore $X=\cup\{f^{-1}(B_i): i\in I_0\}$, which implies $f(X)=\cup\{B_i : i\in I_0\}$, so that $Y=\cup\{B_i : i\in I_0\}$. That is $\{B_i : i\in I_0\}$ a countable sub cover of $\{B_i : i\in I\}$ for Y. Hence (Y,σ) is an $infra-\alpha-Lindelöf$ space.

Theorem 5.9. If (X,τ) is $infra-\alpha-Lindelöf$ space and countably $infra-\alpha-compact$ space, then (X,τ) is $infra-\alpha-compact$ space.

Proof. Suppose (X,τ) is $infra-\alpha-Lindelöf$ space and countably $infra-\alpha-compact$ space. Let $\{A_i : i\in I\}$ be an $infra-\alpha-open$ cover of (X,τ). Since (X,τ) is $infra-\alpha-Lindelöf$ space, $\{A_i : i\in I\}$ has a countable sub cover say $\{A_i : i\in I_0\}$ for some countable set $I_0\subseteq I$. Therefore $\{A_i : i\in I_0\}$ is a countable $infra-\alpha-open$ cover of (X,τ). Again, since (X,τ) is countably $infra-\alpha-compact$ space, $\{A_i : i\in I_0\}$ has a finite sub cover and say $\{A_i : i=1,2,3,\ldots,n\}$. Therefore $\{A_i : i=1,2,3,\ldots,n\}$ is a finite sub cover of $\{A_i : i\in I\}$ for (X,τ). Hence (X,τ) is an $infra-\alpha-compact$ space.

Theorem 5.10. If a function $f:(X,\tau)\longrightarrow(Y,\sigma)$ is $infra-\alpha-irresolue$ and a subset A of X is $infra-\alpha-Lindelöf$ relative to X, then $f(A)$ is $infra-\alpha-Lindelöf$ relative to Y.

Proof. Let $\{B_i : i\in I\}$ be a cover of $f(A)$ by $infra-\alpha-open$ subsets of Y. By hypothesis f is $infra-\alpha-irresolue$ and so $\{f^{-1}(B_i): i\in I\}$ is a cover of A by $infra-\alpha-open$ subsets of X. Since A is $infra-\alpha-Lindelöf$ relative to X, $\{f^{-1}(B_i): i\in I\}$ has a countable sub cover say $\{f^{-1}(B_i): i\in I_0\}$ for A, where I_0 is a countable subset of I. Now $\{B_i : i\in I_0\}$ is a countable sub cover of $\{B_i : i\in I\}$ for $f(A)$. So $f(A)$ is $infra-\alpha-Lindelöf$ relative to Y.

VI. ALMOST INFRA -α -COMPACT SPACES

Definition 6.1. A topological space (X,τ) is called almost $infra-\alpha-compact$ $(infra-\alpha-Lindel\ddot{o}f)$ provided that every $infra-\alpha-open$ cover of X has a finite (countable) sub collection, the $infra-\alpha-closure$ of whose members cover X.

The proofs of the following four propositions are straightforward and therefore will be omitted.

Proposition 6.2. Every almost $infra-\alpha-compact$ space is almost $infra-\alpha-Lindel\ddot{o}f$ space.

Proposition 6.3. Every $infra-\alpha-compact$ space $(infra-\alpha-Lindel\ddot{o}f\ space)$ is almost $infra-\alpha-compact$ $(almost\ infra-\alpha-Lindel\ddot{o}f)$.

Proposition 6.4. Any finite (countable) topological space (X,τ) is almost $infra-\alpha-compact$ $(almost\ infra-\alpha-Lindel\ddot{o}f)$.

Proposition 6.5. A finite (countable) union of almost $infra-\alpha-compact$ $(almost\ infra-\alpha-Lindel\ddot{o}f)$ subsets of (X,τ) is almost $infra-\alpha-compact$ $(almost\ infra-\alpha-Lindel\ddot{o}f)$.

Definition 6.6. A subset E of (X,τ) is called $infra-\alpha-clopen$ provided that it is $infra-\alpha-open$ and $infra-\alpha-closed$.

Theorem 6.7. Let F be an $infra-\alpha-clopen$ subset of an almost $infra-\alpha-compact$ $(almost\ infra-\alpha-Lindel\ddot{o}f)$ space (X,τ). Then F is almost $infra-\alpha-compact$ $(almost\ infra-\alpha-Lindel\ddot{o}f)$.

Proof. Let F be an $infra-\alpha-clopen$ subset of an almost $infra-\alpha-compact$ space X and $\{G_i : i\in I\}$ be an $infra-\alpha-open$ cover of F. Then F^c is $infra-\alpha-open$ and $X\subseteq(\cup\{G_i : i\in I\})\cup F^c$. Since X is almost $infra-\alpha-compact$, then there exists a finite subset I_0 of I such that $X=(\cup\{I\alpha-Cl(G_i) : i\in I_0\})\cup F^c$. Thus it follows that $F\subseteq\cup\{I\alpha-Cl(G_i) : i\in I_0\}$. Hence F is almost $infra-\alpha-compact$.

The proof is similar in case of almost $infra-\alpha-Lindel\ddot{o}f$.

Theorem 6.8. If A is an almost $infra-\alpha-compact$ $(almost\ infra-\alpha-Lindel\ddot{o}f)$ subset of (X,τ) and B is an $infra-\alpha-clopen$ subset of X, then $A\cap B$ is almost $infra-\alpha-compact$ $(almost\ infra-\alpha-Lindel\ddot{o}f)$.

Proof. Let $\Lambda=\{G_i : i\in I\}$ be an $infra-\alpha-open$ cover of $A\cap B$. Then $A\subseteq(\cup\{G_i : i\in I\})\cup B^c$. Since A is almost $infra-\alpha-compact$, then there exists a finite subset I_0 of I such that $A\subseteq(\cup\{I\alpha-Cl(G_i) : i\in I_0\})\cup B^c$. Therefore $A\cap B\subseteq\cup\{I\alpha-Cl(G_i) : i\in I_0\}$. Thus $A\cap B$ is almost $infra-\alpha-compact$.

The proof is similar in case of almost $infra-\alpha-Lindel\ddot{o}f$.

Theorem 6.9. Let a map $f:(X,\tau)\longrightarrow(Y,\sigma)$ be $infra-\alpha-irresolute$. Suppose that A is almost $infra-\alpha-compact$ $(almost\ infra-\alpha-Lindel\ddot{o}f)$ subset of X. Then $f(A)$ is almost $infra-\alpha-compact$ $(almost\ infra-\alpha-Lindel\ddot{o}f)$.

Proof. Suppose that $\{G_i : i\in I\}$ is $infra-\alpha-open$ cover of $f(A)$. Then $f(A)\subseteq\cup\{G_i : i\in I\}$. Now, $A\subseteq\cup\{f^{-1}(G_i) : i\in I\}$. Since f is $infra-\alpha-irresolute$, then $\{f^{-1}(G_i) : i\in I\}$ is an $infra-\alpha-open$ cover of A. By hypothesis, A is almost $infra-\alpha-compact$, then there exists a finite subset I_0 of I such that $A\subseteq\cup\{I\alpha-Cl[f^{-1}(G_i)] : i\in I_0\}$. Since f is $infra-\alpha-irresolute$, then $I\alpha-Cl(f^{-1}(G_i))\subseteq f^{-1}[I\alpha-Cl(G_i)]$, for all $i\in I_0$. Hence it follows that $f(A)\subseteq\bigcup_{i\in I_0} f\lfloor f^{-1}(I\alpha-Cl(G_i))\rfloor\subseteq \bigcup_{i\in I_0} I\alpha-Cl(G_i)$, which implies that $f(A)\subseteq\bigcup_{i\in I_0} I\alpha-Cl(G_i)$. Thus $f(A)$ is almost $infra-\alpha-compact$.

The proof is similar in case of almost $infra-\alpha-Lindel\ddot{o}f$.

Theorem 6.10. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be an $infra-\alpha-open$ bijective map and (Y,σ) is almost $infra-\alpha-compact$. Then (X,τ) is almost compact.

Proof. Let $\{G_i : i\in I\}$ be an open cover of X. Then $f(X)=f(\bigcup_{i\in I} G_i)$. Therefore $Y=\bigcup_{i\in I} f(G_i)$. Now, Y is almost $infra-\alpha-compact$, then there exists a finite subset I_0 of I such that $Y=\bigcup_{i\in I_0} I\alpha-Cl\lfloor f(G_i)\rfloor$. Since f is $infra-\alpha-open$ bijective map, then f is $infra-\alpha-closed$ map. Therefore, we have $I\alpha-Cl\lfloor f(G_i)\rfloor\subseteq f\lfloor Cl(G_i)\rfloor$, for all $i\in I_0$. Thus $Y\subseteq\bigcup_{i\in I_0} f\lfloor Cl(G_i)\rfloor\subseteq f\lfloor\bigcup_{i\in I_0} Cl(G_i)\rfloor$, which implies that $X=f^{-1}(Y)\subseteq\bigcup_{i\in I_0} Cl(G_i)$. Thus $X=\bigcup_{i\in I_0} Cl(G_i)$. Hence X is almost compact.

Theorem 6.11. If every collection of $infra-\alpha-closed$ subsets of (X,τ), satisfying the finite (countable) intersection property, has, itself, a non-empty intersection, then X is almost $infra-\alpha-compact$ $(almost\ infra-\alpha-Lindel\ddot{o}f)$.

Proof. Let $\{G_i : i\in I\}$ be an $infra-\alpha-open$ cover of X. Suppose $\{G_i : i\in I\}$ has no finite sub-collection such that the $infra-\alpha-closure$ of whose members cover X. Then $X-\bigcup_{i=1}^{i=n} I\alpha-Cl(G_i)\neq\phi$, for any $n\in N$. Therefore $X-\bigcup_{i=1}^{i=n} G_i\neq\phi$. Now, $\bigcap_{i=1}^{n} G_i^c\neq\phi$ implies $\{G_i^c : i\in I\}$ is a collection of $infra-\alpha-closed$ subsets of X which has the finite intersection property. Thus $\bigcap_{i\in I} G_i^c\neq\phi$ implies $X\neq\bigcup_{i\in I} G_i$. But this is a contradiction. Hence X is almost $infra-\alpha-compact$.

A similar proof is given in a case of $almost\ infra-\alpha-Lindel\ddot{o}f$.

VII. MILDLY INFRA - α -COMPACT SPACES

Definition 7.1. A topological space (X,τ) is called mildly $infra-\alpha-compact$ $(mildly\ infra-\alpha-Lindel\ddot{o}f)$ provided that every $infra-\alpha-clopen$ cover of X has a finite (countable) sub cover.

Theorem 7.2. Every mildly $infra-\alpha-compact$ space is mildly $infra-\alpha-Lindel\ddot{o}f$.

Proof. It is straight forward.

Theorem 7.3. Every almost $infra-\alpha-compact$ $(almost\ infra-\alpha-Lindel\ddot{o}f)$ space (X,τ) is mildly $infra-\alpha-compact$ ($mildly\ infra-\alpha-Lindel\ddot{o}f$).

Proof. Let $\Lambda=\{H_i : i\in I\}$ be an $infra-\alpha-clopen$ cover of (X,τ). Since (X,τ) is almost $infra-\alpha-compact$, then there exists a finite subset I_0 of I such that $X=\bigcup_{i\in I_0} I\alpha-Cl(H_i)$. Now, $I\alpha-Cl(H_i)=H_i$. Thus (X,τ) is mildly $infra-\alpha-compact$.

A similar proof is given when (X,τ) is $almost\ infra-\alpha-Lindel\ddot{o}f$.

Corollary 7.4. Every $infra-\alpha-compact$ $(infra-\alpha-Lindel\ddot{o}f)$ space is mildly $infra-\alpha-compact$ $(mildly\ infra-\alpha-Lindel\ddot{o}f)$.

Theorem 7.5. If F is an $infra-\alpha-clopen$ subset of a mildly $infra-\alpha-compact$ $(mildly\ infra-\alpha-Lindel\ddot{o}f)$ space X, then F is mildly $infra-\alpha-compact$ ($mildly\ infra-\alpha-Lindel\ddot{o}f$).

Proof. Let F be an $infra-\alpha-clopen$ subset of X and $\{G_i : i\in I\}$ be an $infra-\alpha-clopen$ cover of F. Then F^c is an $infra-\alpha-clopen$ and

$F \subseteq \bigcup_{i\in I} G_i$. Therefore $X = \left(\bigcup_{i\in I} G_i\right) \cup F^c$. Since X is mildly *infra* $-\alpha-$ *compact*, then there exists a finite subset I_0 of I such that $X = \left(\bigcup_{i\in I_0} G_i\right) \cup F^c$. So $F \subseteq \left(\bigcup_{i\in I_0} G_i\right)$. Hence F is mildly *infra* $-\alpha-$*compact*.

The proof is similar in a case of mildly *infra* $-\alpha-$ *Lindelöf*.

Theorem 7.6. If A is a mildly *infra* $-\alpha-$ *compact* (*mildly infra* $-\alpha-$ *Lindelöf*) subset of X and B is an *infra* $-\alpha-$ *clopen* subset of X, then $A \cap B$ is mildly *infra* $-\alpha-$ *compact* (*mildly infra* $-\alpha-$ *Lindelöf*).

Proof. Let $\Lambda = \{G_i : i \in I\}$ be an *infra* $-\alpha-$ *clopen* cover of $A \cap B$. Then $A \subseteq \left(\bigcup_{i\in I} G_i\right) \cup B^c$. Since A is mildly *infra* $-\alpha-$ *compact*, then there exists a finite subset I_0 of I such that $A \subseteq \left(\bigcup_{i\in I_0} G_i\right) \cup B^c$. Therefore $A \cap B \subseteq \bigcup_{i\in I_0} G_i$. Thus $A \cap B$ is mildly *infra* $-\alpha-$ *compact*.

The proof is similar in case of mildly *infra* $-\alpha-$ *Lindelöf*.

Theorem 7.7. If $f : (X, \tau) \longrightarrow (Y, \sigma)$ is an *infra* $-\alpha-$ *open* bijective map and (Y, σ) is mildly *infra* $-\alpha-$ *compact*, then (X, τ) is mildly compact.

Proof. Let $\{G_i : i \in I\}$ be a clopen cover for X. Then $f(X) = f\left(\bigcup_{i\in I} G_i\right)$. Hence $Y = \bigcup_{i\in I} f(G_i)$. Since f is *infra* $-\alpha-$ *open* bijective map, then f is *infra* $-\alpha-$ *closed*. Therefore $\{f(G_i) : i \in I\}$ is an *infra* $-\alpha-$ *clopen* cover of X. Since Y is mildly *infra* $-\alpha-$ *compact*, then there exists a finite subset I_0 of I such that $Y = \bigcup_{i\in I_0} f(G_i)$. Therefore $X = \bigcup_{i\in I_0} G_i$. Thus X is mildly compact.

Proposition 7.8. A subset A of (X, τ) is mildly compact (*mildly Lindelöf*) if and only if (X, τ_A) is mildly compact (*mildly Lindelöf*).

VIII. INFRA - α - CONNECTED SPACES

Definition 8.1. A topological space (X, τ) is said to be connected if X cannot be written as a disjoint union of two non empty open sets. A subset of (X, τ) is connected if it is connected as a subspace.

Definition 8.2. A topological space (X, τ) is said to be *infra* $-\alpha-$ *connected* if X cannot be written as a disjoint union of two non empty *infra* $-\alpha-$ *open* sets. A subset of (X, τ) is *infra* $-\alpha-$ *connected* if it is *infra* $-\alpha-$ *connected* as a subspace.

Theorem 8.3. Every *infra* $-\alpha-$ *connected* space (X, τ) is connected.

Proof. Let A and B be two non empty disjoint proper open sets in X. Since every open set is *infra* $-\alpha-$ *open* set. Therefore A and B are non empty disjoint proper *infra* $-\alpha-$ *open* sets in X and X is *infra* $-\alpha-$ *connected* space. Hence $X \neq A \cup B$. Therefore X is *infra* $-\alpha-$ *connected*.

Theorem 8.4. Let (X, τ) be a topological space. Then the following statements are equivalent

(i) (X, τ) is *infra* $-\alpha-$ *connected*.

(ii) The only subsets of (X, τ) which are both *infra* $-\alpha-$ *open* and *infra* $-\alpha-$ *closed* are the empty set ϕ and X.

(iii) Each *infra* $-\alpha-$ *continuous* map of (X, τ) into a discrete space (Y, σ) with at least two points is a constant map.

Proof. ***(i)*** $\Rightarrow$ ***(ii)***: Let G be a non empty proper *infra* $-\alpha-$ *open* and *infra* $-\alpha-$ *closed* subset of (X, τ). Then $X - G$ is also both *infra* $-\alpha-$ *open* and *infra* $-\alpha-$ *closed*. Then $X = G \cup (X - G)$ is a disjoint union of two non empty *infra* $-\alpha-$ *open* sets, which contradicts the fact that (X, τ) is *infra* $-\alpha-$ *connected*. Hence $G = \phi$ or $G = X$.

$(ii)\Rightarrow(i)$**:** Suppose that $X = A\cup B$ where A and B are disjoint non empty $infra-\alpha-open$ subsets of (X,τ). Since $A = X - B$, then A is both $infra-\alpha-open$ and $infra-\alpha-closed$. By assumption $A=\phi$ or $A=X$, which is a contradiction. Hence (X,τ) is $infra-\alpha-connected$.

$(ii)\Rightarrow(iii)$**:** Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be an $infra-\alpha-continuous$ map, where (Y,σ) is discrete space with at least two points. Then $f^{-1}(y)$ is $infra-\alpha-closed$ and $infra-\alpha-open$ for each $y\in Y$. Thus (X,τ) is covered by $infra-\alpha-closed$ and $infra-\alpha-open$ covering $\{f^{-1}(y): y\in Y\}$. By assumption, $f^{-1}(y)=\phi$ or $f^{-1}(y)=X$ for each $y\in Y$. If $f^{-1}(y)=\phi$ for each $y\in Y$, then f fails to be a map. Therefore their exists at least one point say $y^*\in Y$ such that $f^{-1}(\{y^*\})\neq\phi$. Since $f^{-1}(\{y^*\})$ is also both $infra-\alpha-open$ and $infra-\alpha-closed$ set. Therefore by hypothesis $f^{-1}(\{y^*\})=X$. This shows that f is a constant map.

$(iii)\Rightarrow(ii)$**:** Let G be both $infra-\alpha-open$ and $infra-\alpha-closed$ set in (X,τ). Suppose $G\neq\phi$. Let $f:(X,\tau)\longrightarrow(Y,\sigma)$ be an $infra-\alpha-continuous$ map defined by $f(G)=\{a\}$ and $f(X-G)=\{b\}$ where $a\neq b$ and $a,b\in Y$. By assumption, f is constant so $G=X$.

Theorem 8.5. If $f:(X,\tau)\longrightarrow(Y,\sigma)$ is an $infra-\alpha-continuous$ surjection and (X,τ) is $infra-\alpha-connected$, then (Y,σ) is connected.

Proof. Suppose (Y,σ) is not connected. Let $Y = A\cup B$, where A and B are disjoint non empty open subsets of (Y,σ). Since f is $infra-\alpha-continuous$, $X = f^{-1}(A)\cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non empty $infra-\alpha-open$ subsets of X. This disproves the fact that (X,τ) is $infra-\alpha-connected$. Hence (Y,σ) is connected.

Theorem 8.6. If $f:(X,\tau)\longrightarrow(Y,\sigma)$ is an $infra-\alpha-irresolute$ surjection and X is $infra-\alpha-connected$, then Y is $infra-\alpha-connected$.

Proof. Suppose that Y is not $infra-\alpha-connected$. Let $Y = A\cup B$, where A and B are disjoint non empty $infra-\alpha-open$ sets in Y. Since f is $infra-\alpha-irresolute$ map and onto, $X = f^{-1}(A)\cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non empty $infra-\alpha-open$ sets in (X,τ). This contradicts the fact that (X,τ) is $infra-\alpha-connected$. Hence (Y,σ) is $infra-\alpha-connected$.

Theorem 8.7. If every $infra-\alpha-closed$ set in X is closed in X and X is connected, then X is $infra-\alpha-connected$.

Proof. Suppose that X is connected. Then X cannot be expressed as disjoint union of two nonempty proper open subset of X. Let X be not $infra-\alpha-connected$ space. Let A and B be any two non empty $infra-\alpha-open$ subsets of X such that $X = A\cup B$, where $A\cap B=\phi$. Since every $infra-\alpha-closed$ set in X is closed in X. Therefore every $infra-\alpha-open$ set in X is open in X. Hence A and B are open subsets of X, which contradicts that X is connected. Therefore X is $infra-\alpha-connected$.

Theorem 8.8. Every $infra-\alpha-connected$ space (X,τ) is mildly $infra-\alpha-compact$.

Proof. Since (X,τ) is $infra-\alpha-connected$, then the only $infra-\alpha-clopen$ subsets of (X,τ) are X and ϕ. Therefore (X,τ) is mildly $infra-\alpha-compact$.

Theorem 8.9. If two $infra-\alpha-open$ sets C and D form a separation of X and if Y is

$infra-\alpha-connected$ subspace of X, then Y lies entirely within C or D.

Proof. By hypothesis C and D are both $infra-\alpha-open$ sets in X. The sets $C \cap Y$ and $D \cap Y$ are $infra-\alpha-open$ in Y, these two sets are disjoint and their union is Y. If they were both non empty, they would constitute a separation of Y. Therefore, one of them is empty. Hence Y must lie entirely in C or D.

Theorem 8.10. Let A be an $infra-\alpha-connected$ subspace of X. If $A \subseteq B \subseteq I\alpha-Cl(A)$, then B is also $infra-\alpha-connected$.

Proof. Let A be $infra-\alpha-connected$. Let $A \subseteq B \subseteq I\alpha-Cl(A)$. Suppose that $B = C \cup D$ is a separation of B by $infra-\alpha-open$ sets. Thus by previous theorem A must lie entirely in C or D. Suppose that $A \subseteq C$, then it implies that $I\alpha-Cl(A) \subseteq I\alpha-Cl(C)$. Since $I\alpha-Cl(C)$ and D are disjoint, B cannot intersect D. This disproves the fact that D is non empty subset of B So $D = \phi$ which implies B is $infra-\alpha-connected$.

IX. CONCLUSIONS

We have used $\text{infra}-\alpha-\text{open}$ sets to introduce the new concepts of notions in topological spaces namely $infra-\alpha-compact$ space, countably $infra-\alpha-compact$ space, $infra-\alpha-Lindelöf$ space, almost $infra-\alpha-compact$ space, mildly $infra-\alpha-compact$ space and $infra-\alpha-connected$ space and have investigated several properties and characterization of these new concepts.

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