A Fundamental Conception to Formulate Image Data Hiding Scheme Based on Error Diffusion from Stochastic Viewpoint

Masakazu Higuchi, Shuji Kawasaki, Jonah Gamba, Atsushi Koike and Hitomi Murakami

***Abstract*—Image data hiding schemes are techniques to embed secret image data into several images. The embedded data can be extracted with some procedure. On the other hand, visual cryptographic techniques break up a secret image into several shares so that only someone with all shares can decrypt the secret image by superposing all shares together. Image data hiding schemes based on error diffusion have the feature of visual cryptography with respect to extracting of embedded data. They embed secret image data into several halftone images without affecting their perceptual qualities and the embedded data can be restored with apparently high quality when the halftone images are overlaid without any special electronic calculation. In this paper, we consider to formulate an image data hiding scheme based on error diffusion. We propose a formulation for the scheme in the view of a stochastic analysis. The idea is very basic, but theoretical studies by formulating is important trial in this field.**

***Keywords*—Image data hiding, Visual cryptography, Halftoning, Error diffusion, Probability theory.**

I. Introduction

THERE are techniques that embed digital data into various multimedia data (music, image, video and so on). The embedded data are visible or invisible, and they can be extracted with some procedure. In this paper, we consider the case that visual patterns are embedded into some images and the embedded patterns are invisible to humans, i.e., this case is image data hiding. The flowchart of image data hidings is shown in Fig.1.

On the other hand, visual cryptography is a technique which encrypts a secret image into plural share images such that when some of the share images are overlaid, the secret image will be revealed. The decryption can be performed by human visual system without any special electronic calculation for decryption. The flowchart of visual cryptography is shown in Fig.2. It is the case that share images have binary tones. The first visual cryptographic technique has been developed by Noar and Shamir in [3]. They have developed the scheme generates share images with not meaningful random dot pattern.

In recent years, image data hiding techniques with the feature of visual cryptography have been studied actively. The techniques embed secret visual patterns into some halftone images such that when the halftone images are overlaid, the embedded patterns can be viewed directly on the halftone images without any special electronic calculation. The process of this data extraction is the same as it of visual cryptography. In these studies, Fu and Au have dealt with binary or ternary images like text as secret visual patterns in [6], while other many researchers have studied about natural gray-scale images like photographs as secret visual patterns in [5], [9], [11], [13]–[17]. Also, Koga and Yamamoto have challenged to handle color secret images in [4].

In [5], [6], [9], [11], [13]–[17], halftonings have been used in order to embed secret data. Halftoning is a process of converting a gray scale image into a binary image. It is well-known that error diffusion is one of halftoning techniques and generates a halftone image with apparently high quality from a multitone image (see in [1]). Error diffusion has been used as halftoning in [5], [6], [9], [13]–[17]. Applying error diffusion makes noise less noticeable which arises by embedding information of secret data into halftone images. So, secret data are embedded into halftone images without

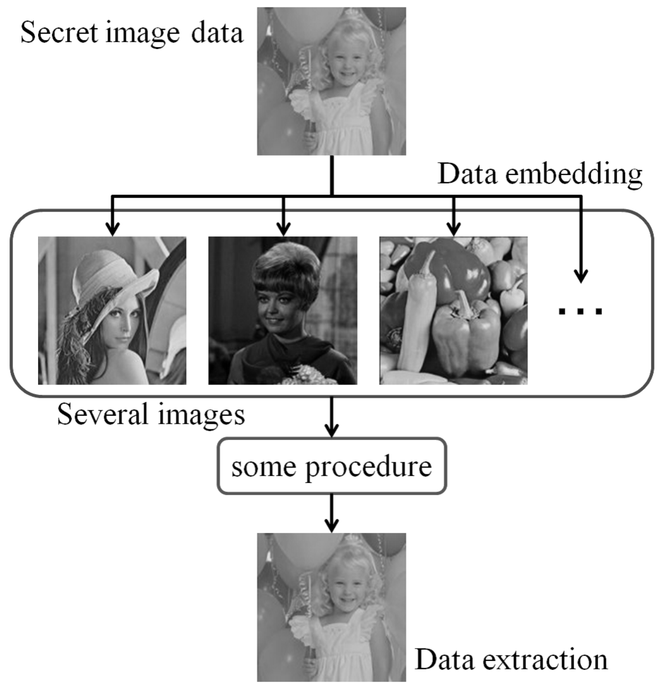

Fig. 1. The flowchart of image data hidings

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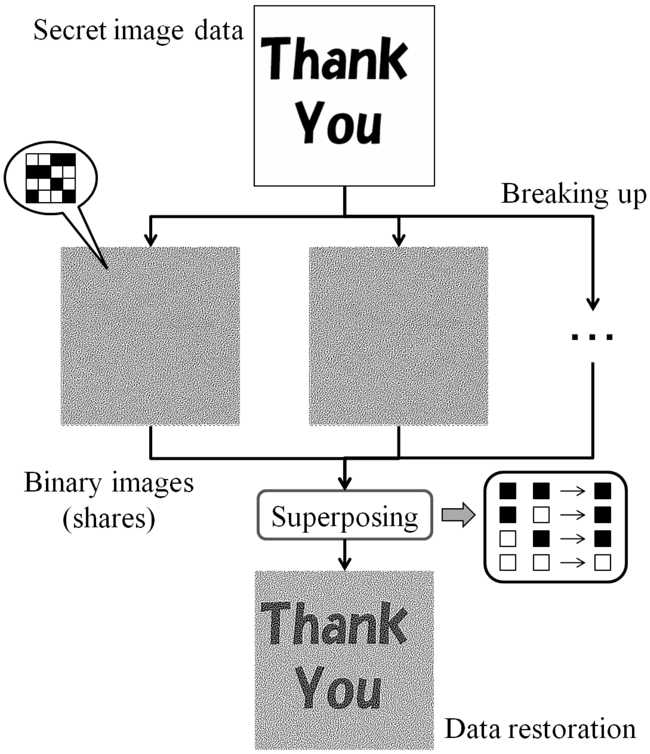

Fig. 2. The flowchart of visual cryptography

affecting their perceptual qualities.

In [9], [13]–[17], Myodo et al. and authors have achieved to extract secret data with apparently high quality by modifing Fu's and Au's method in [6] and placing the process of giving appropriate transformations to original input data in the modified method. Myodo et al. have improved Fu's and Au's method and changed intensities of each pixel of input images in the first step of the method by affine transformations in [9]. In Myodo's method, authors have applied the histogram equalization to input secret data and adjusted parameters of affine transformation to input images according to the properties of those images in [13]–[17].

The main theme of studies in the field are to embed secret visual patterns into halftone images without affecting their perceptual qualities such that the embedded patterns can be restored with apparently high quality when the halftone images are overlaid. In order to study the theme, it is necessary to theoretically investigate behavior of data embedding algorithm. However, it is difficult because of nonlinearity of halftoning, change of pixel values of images depending on various data embedding conditions, difficulty of representing variations in intensities of images as functions, and so on. In the previous works, parameters affect the performance of data embedding algorithm have been determined experimentally. Authors have tried formulating Fu's and Au's data hiding scheme using a stochastic method in [18] and [19]. Fu and Au have dealt with binary or ternary images like text as secret data. Their method also demonstrates relatively good performance for secret data like natural gray-scale images by giving appropriate transformations to original input data.

In this paper, we review the formulation in [18] and [19]. It is difficult to represent variations in intensities of an image as a function. So we represent it as the probability density function based on the relative frequency distribution of pixel intensities for the image. Also we use the similar technique to formulate errors arise in error diffusion. To model dynamic behaviors and indeterminate factors, it is effective to give a stochastic formulation. Formulating data embedding algorithms would allow us to investigate behavior of the algorithm theoretically. By this work, for example, we might be able to find easily pairs of input image such that secret data can be embedded into halftone images without affecting their perceptual qualities and the embedded data can be restored with apparently high quality when the halftone images are overlaid. Moreover, we might be able to optimize parameters affect the performance of data embedding algorithm.

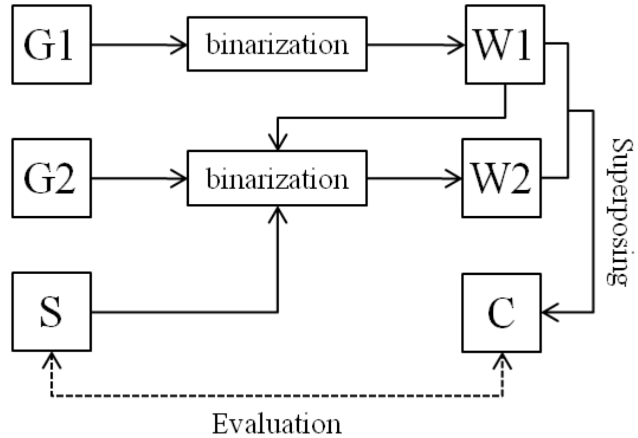

Fig. 3. The flowchart of Fu's and Au's method

The organization of this paper is as follows: In Section II, we explain an image data hiding scheme based on error diffusion, which is Fu's and Au's method. In Section III, we formulate the target scheme by using a stochastic method. Finally, we conclude this paper in Section IV.

II. Image data hiding scheme based on error diffusion

A. The flow of hiding scheme

We focus on Fu's and Au's method in [6]. The method is as follows: It takes three images as input and generates two output halftone images which correspond to two of the three input images. The other one is a secret image. This is reconstructed as a halftone image by stacking (superposing) the two output images together. This operation corresponds to logical product in Boolean algebra.

The flowchart of the method is shown in Fig.3. G1, G2 and S in Fig.3 are input images and have the same size. G1 and G2 are gray-scale images like photographs. S is a secret image with binary or ternary tones, but may be a gray-scale image by giving appropriate transformations to input data. W1 and W2 are output halftone images. W1 is produced from G1 by using regular error diffusion with high quality. Then, W2 is generated with high quality by binarizing G2 depending on information of W1 and S by using error

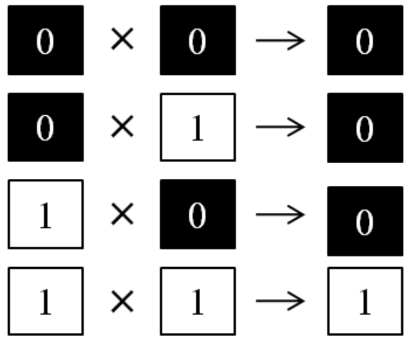

Fig. 4. Superposing two binary pixels (which is the same as logical product of two pixels in Boolean algebra)

diffusion. It means that pixel arrangements of pixel domains in intermediate image of producing W2 change depending on information of W1 and S. C is obtained by superposing W1 and W2. This is the halftone image corresponding to S The qualities of W2 and C are evaluated by, for example, PSNRs (Peak Signal-to-Noise Ratio) of pairs (G2, W2) and (S, C), respectively. In this method, secret data are embedded into only W2, and W1 is the key image to restore the secret data on halftone images.

In this paper, the intensity of a pixel in an image is a continuous value on from 0 to 1. If the intensity is 0, the pixel is black pixel. On the other hand, if the intensity is 1, the pixel is white pixel. Superposing two binary pixels corresponds to calculating logical product of these pixels in Boolean algebra (see in Fig.4).

B. *Error diffusion*

Error diffusion is one of halftoning techniques, which can generate a high quality halftone image from a multitone, like gray-scale, image. It is a causal single-pass sequential algorithm. A multitone image is halftoned line-by-line sequentially. In this algorithm, the past errors are diffused back to the current pixel.

The relationship between input and output of error diffusion can be described by the following equations,

$$u_k = x_k + ER_k, \quad ER_k = \sum_{i \in J_k} j_i e_i, \tag{1}$$

$$u_k \to w_k = \begin{cases} 0 & (u_k < T) \\ 1 & (u_k \geq T) \end{cases}, \tag{2}$$

$$e_k = u_k - w_k, \tag{3}$$

where x_k denotes the intensity of k-th pixel in an image, ER_k denotes the accumulative error at k-th pixel in an image, and e_i denotes the error which arises at a pixel before k-th pixel, and j_i denotes a weight of diffusing error, and J_k denotes a pixel domain in an image corresponding to k-th pixel, and $k \geq 1$. Note that $ER_1 = 0$ when $k = 1$. Equation (2) expresses the operation of quantization on a threshold T.

C. *Data embedding process*

In Fu's and Au's method, secret data are embedded into W2 when binarizing G2. Then, if u_k in Equation (1) satisfies

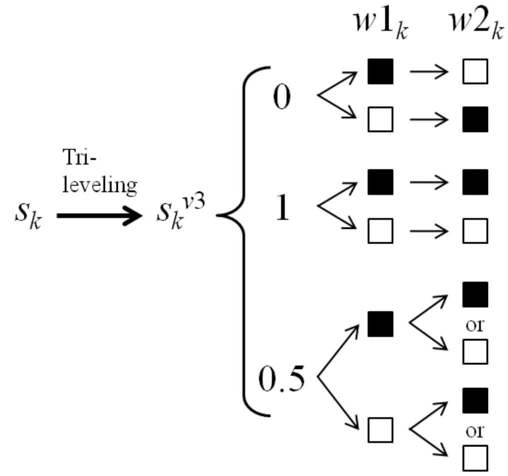

Fig. 5. The rule of embedding data

the following inequality,

$$T - \Delta u < u_k < T + \Delta u, \tag{4}$$

where Δu denotes a positive constant, then information of k-th pixel in S is embedded at k-th pixel position in W2. Let $w1_k$, $w2_k$ and s_k be intensities of k-th pixel in W1, W2 and S, respectively. The data embedding rule is as follows:

- if $s_k^{v3} = 0$ (black),

$$w2_k = \overline{w1_k}, \tag{5}$$

- if $s_k^{v3} = 1$ (white),

$$w2_k = w1_k, \tag{6}$$

- if $s_k^{v3} = 0.5$ (gray),
  $w2_k$ is output of error diffusion at k-th pixel in G2,

where s_k^{v3} denotes the value obtained by tri-leveling s_k (see in Fig.5). The operation of tri-leveling an image is as follows:

$$x \to x^{v3} = \begin{cases} 0 & (0 \leq x < q_1) \\ 0.5 & (q_1 \leq x < q_2) \\ 1 & (q_2 \leq x \leq 1) \end{cases}, \tag{7}$$

where x denotes the intensity of a pixel in an image, q_1 and q_2 denote thresholds of tri-leveling x and have the inequality $q_1 < q_2$. By this rule, the domain in C corresponding to a dark domain in S becomes black, and the domain in C corresponding to a light domain in S becomes gray, and the domain in C corresponding to a domain like gray in S becomes darker than gray. Therefore, C tends to become a darkish image over all.

D. *Example*

We show an example of results obtained by using Fu's and Au's method in Fig.6. In this example, S is not an image with binary or ternary tones but a gray-scale image. W1 and W2 are apparently high quality and also C is reconstructed by superposing W1 and W2 with apparently high quality (see in Fig.6). In this case, appropriate transformations are given to input images. The intensities of each pixel in G1

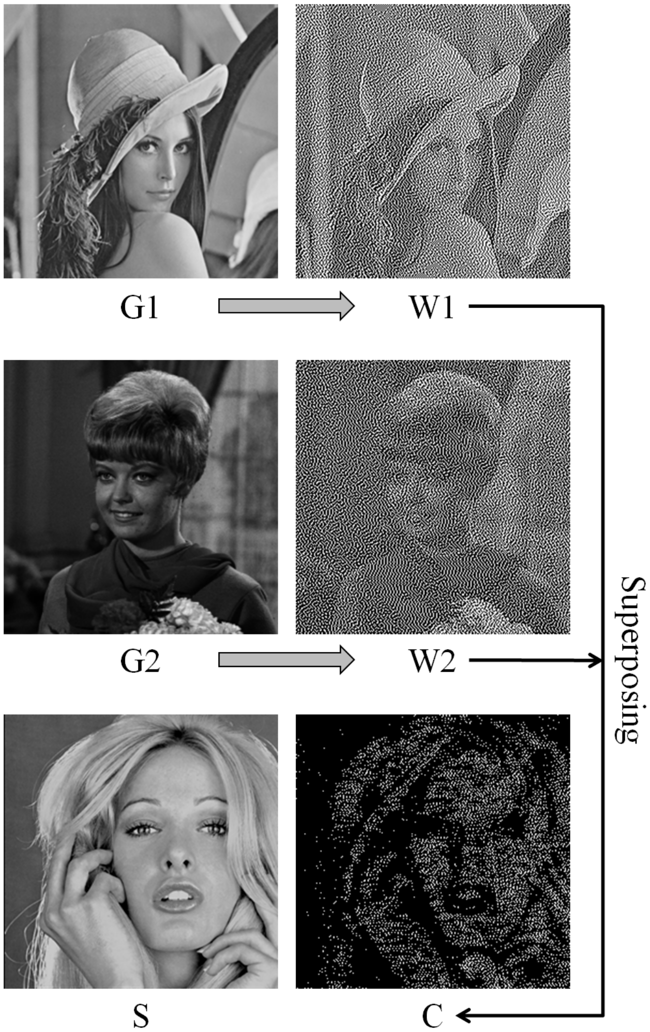

Fig. 6. An example of results by Fu's and Au's method

and G2 are converted to around 0.5 by the following affine transformation,

$$x' = 0.45x + 0.275, \tag{8}$$

where x denotes the intensity of a pixel in an image. The intensity of a pixel in S is transformed into below 0.45 by the following affine transformation,

$$x' = 0.45x. \tag{9}$$

The thresholds of quantization to ternary image for S are $q_1 = 0.25$ and $q_2 = 0.75$. The threshold of quantization to binary image for G1 and G2 is $T = 0.5$. The parameter of embedding data is $\Delta u = 0.067$.

III. Formulation of Image Data Hiding Scheme Using a Stochastic Method

A. Preliminary

We formulate Fu's and Au's method by using a stochastic method in this section. One of ways to characterize images is to get relative frequency distributions of pixel intensities for images. We consider pixel intensities for an image to be a

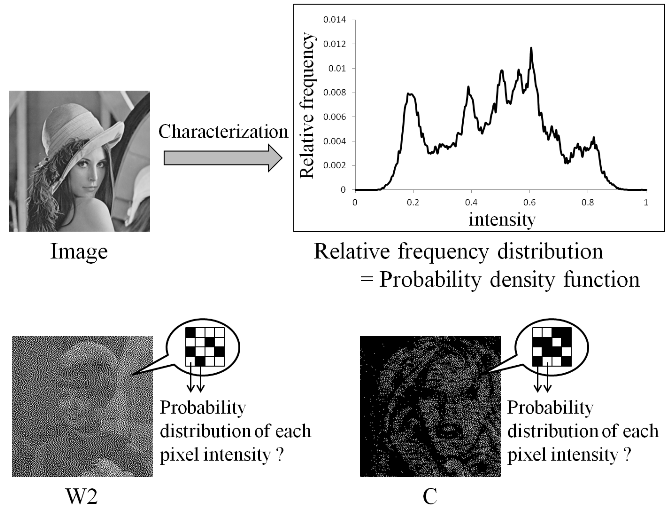

Fig. 7. The concept of formulation

random variable. Then, the variable has the probability density function obtained from the relative frequency distribution of pixel intensities for the image. Theoretically, the relative frequency distribution of pixel intensities for an image is equal to the probability density function of pixel intensities for the image since pixel intensities are continuous values on $[0, 1]$. We try to obtain probability distributions of each pixel intensity for W2 and C (see in Fig.7). Note that random variables corresponding to variables in error diffusion have probability density functions because these random variables have continuous values.

For the sake of simplicity, we omit intensity transformations for input images. They are not essence of the formulation. If it is necessary to include appropriate transformations in the formulation, replace x with x' in the formulation, where x denotes the intensities of each pixel in input images, and

$$x' = Tr(x) \tag{10}$$

and Tr denotes an appropriate transformation. In the paper, the probability density function for a continuous random variable $\mathcal{X}$ is expressed in $f_{\mathcal{X}}$.

Let $\mathcal{U}_k$ be the random variable corresponding to u_k in Equation (1), then, for example, the probability satisfied condition (4) is as follows:

$$P(T - \Delta u < \mathcal{U}_k < T + \Delta u) = \int_{T-\Delta u}^{T+\Delta u} f_{\mathcal{U}_k}(u)du. \tag{11}$$

Let $\mathcal{S}$ and $\mathcal{S}^{v3}$ be the random variables corresponding to s_k and s_k^{v3}, respectively. Then the probabilities that a pixel in S becomes black, gray and white by tri-leveling the pixel value are as follows:

$$\begin{aligned} P_b^{\mathcal{S}^{v3}} &= P(\mathcal{S}^{v3} = 0) \\ &= P(0 \le \mathcal{S} < q_1) = \int_0^{q_1} f_{\mathcal{S}}(x)dx, \end{aligned} \tag{12}$$

$$\begin{aligned} P_g^{\mathcal{S}^{v3}} &= P(\mathcal{S}^{v3} = 0.5) \\ &= P(q_1 \le \mathcal{S} < q_2) = \int_{q_1}^{q_2} f_{\mathcal{S}}(x)dx, \end{aligned} \tag{13}$$

$$\begin{aligned} P_w^{\mathcal{S}^{v3}} &= P(\mathcal{S}^{v3} = 1) \\ &= P(q_2 \leq \mathcal{S} \leq 1) = \int_{q_2}^{1} f_{\mathcal{S}}(x)dx, \end{aligned} \tag{14}$$

respectively.

B. Formulation of output images in error diffusion

We consider to formulate W2. In Fu's and Au's method, W1 is produced from G1 by using regular error diffusion, so obtaining probability distributions of each pixel intensity for W1 is rather easy. However, it is difficult to obtain them for W2 because W2 is produced from G2 by using error diffusion with applying the rule of embedding data. The distributions depends on S and W1. We get probability distributions of each pixel intensity for W2 by the following steps,

Step 1. Getting probability density functions of errors arise in error diffusion for G1 and G2, respectively.
Step 2. Getting probability density functions of accumulative errors arise in error diffusion.
Step 3. Getting probability distributions of each pixel intensity for W1.
Step 4. Getting probability distributions of each pixel intensity for W2.

Step 1 is as follows: We analyze the statistical behavior of errors arise in error diffusion by using relative frequency distributions of the errors. Let $\mathcal{E}_k$, $\mathcal{W}1_k$ and $\mathcal{W}2_k$ be the random variables corresponding to e_k in Equation (3), $w1_k$ and $w2_k$, respectively. The ways to obtain probability density functions of errors arise in error diffusion for G1 and G2 are different from each other.

- The case of G1.
  There are two cases on u_i $(i < k)$. The probability density function of $\mathcal{E}_i$ in each case is as follows:
  1) The case of $-\infty < u_i < T$.
     $w1_i$ is 0. By Equation (3), $e_i = u_i$. We obtain $-\infty < e_i < T$ by rewriting the domain of u_i to it of e_i. Let $\mathcal{E}_i^{(1)}$ be the random variable corresponding to e_i in the domain, and the probability density function of $\mathcal{E}_i^{(1)}$ is equal to it of $\mathcal{U}_i$, i.e.,
     $$f_{\mathcal{E}_i^{(1)}}(e) = f_{\mathcal{U}_i}(e), \quad -\infty < e < T. \tag{15}$$
  2) The case of $T \leq u_i < \infty$.
     $w1_i$ is 1. By Equation (3), $e_i = u_i - 1$. We obtain $T - 1 \leq e_i < \infty$ by rewriting the domain of u_i to it of e_i. Let $\mathcal{E}_i^{(2)}$ be the random variable corresponding to e_i in the domain, and the probability density function of $\mathcal{E}_i^{(2)}$ is equal to it of $\mathcal{U}_i - 1$, i.e.,
     $$\begin{aligned} f_{\mathcal{E}_i^{(2)}}(e) &= f_{\mathcal{U}_i - 1}(e) \\ &= f_{\mathcal{U}_i}(e+1), \quad T - 1 \leq e < \infty. \end{aligned} \tag{16}$$
     Note that for arbitrary random variable $\mathcal{X}$ and scalar a, we have
     $$f_{\mathcal{X}-a}(x) = f_{\mathcal{X}}(x+a). \tag{17}$$
  By Equation (15) and (16), the probability density function of $\mathcal{E}_i$ is
  $$f_{\mathcal{E}_i}(e) = f_{\mathcal{E}_i^{(1)}}(e) + f_{\mathcal{E}_i^{(2)}}(e). \tag{18}$$
- The case of G2.
  There are four cases on u_i $(i < k)$. The probability density function of $\mathcal{E}_i$ in each case is as follows:
  1) The case of $-\infty < u_i \leq T - \Delta u$.
     $w2_i$ is 0. By Equation (3), $e_i = u_i$. We obtain $-\infty < e_i \leq T - \Delta u$ by rewriting the domain of u_i to it of e_i. Let $\mathcal{E}_i^{(1)}$ be the random variable corresponding to e_i in the domain, and the probability density function of $\mathcal{E}_i^{(1)}$ is equal to it of $\mathcal{U}_i$, i.e.,
     $$f_{\mathcal{E}_i^{(1)}}(e) = f_{\mathcal{U}_i}(e), \quad -\infty < e \leq T - \Delta u. \tag{19}$$
  2) The case of $T + \Delta u \leq u_i < \infty$.
     $w2_i$ is 1. By Equation (3), $e_i = u_i - 1$. We obtain $T + \Delta u - 1 \leq e_i < \infty$ by rewriting the domain of u_i to it of e_i. Let $\mathcal{E}_i^{(2)}$ be the random variable corresponding to e_i in the domain, and the probability density function of $\mathcal{E}_i^{(2)}$ is equal to it of $\mathcal{U}_i - 1$, i.e.,
     $$\begin{aligned} f_{\mathcal{E}_i^{(2)}}(e) &= f_{\mathcal{U}_i - 1}(e) \\ &= f_{\mathcal{U}_i}(e+1), \quad T + \Delta u - 1 \leq e < \infty. \end{aligned} \tag{20}$$
  3) The case of $T - \Delta u < u_i < T$.
     $w2_i$ is 0 in regular error diffusion, but it is possible that $w2_i$ is 1 with the rule of embedding data. Therefore, we use the expected value of $w2_i$ instead of $w2_i$. Let
     $$P_b^{\mathcal{W}t_k} = P(\mathcal{W}t_k = 0), \quad t = 1, 2, \tag{21}$$
     $$P_w^{\mathcal{W}t_k} = P(\mathcal{W}t_k = 1), \quad t = 1, 2, \tag{22}$$
     and let $\langle w2_i \rangle$ be the expected value of $w2_i$, then,
     $$\langle w2_i \rangle = P_b^{\mathcal{W}2_i} \cdot 0 + P_w^{\mathcal{W}2_i} \cdot 1, \tag{23}$$
     where
     $$P_b^{\mathcal{W}2_i} = P_b^{\mathcal{S}^{v3}} P_w^{\mathcal{W}1_i} + P_g^{\mathcal{S}^{v3}} + P_w^{\mathcal{S}^{v3}} P_b^{\mathcal{W}1_i}, \tag{24}$$
     $$P_w^{\mathcal{W}2_i} = P_b^{\mathcal{S}^{v3}} P_b^{\mathcal{W}1_i} + P_w^{\mathcal{S}^{v3}} P_w^{\mathcal{W}1_i}. \tag{25}$$
     We obtain $T - \Delta u - \langle w2_i \rangle < e_i < T - \langle w2_i \rangle$ by replacing $w2_i$ in Equation (3) with $\langle w2_i \rangle$ and rewriting the domain of u_i to it of e_i. Let $\mathcal{E}_i^{(3)}$ be the random variable corresponding to e_i in the domain, and the probability density function of $\mathcal{E}_i^{(3)}$ is equal to it of $\mathcal{U}_i - \langle w2_i \rangle$, i.e.,
     $$\begin{aligned} f_{\mathcal{E}_i^{(3)}}(e) =& f_{\mathcal{U}_i - \langle w2_i \rangle}(e) \\ =& f_{\mathcal{U}_i}(e + \langle w2_i \rangle), \\ & T - \Delta u - \langle w2_i \rangle < e < T - \langle w2_i \rangle. \end{aligned} \tag{26}$$
  4) The case of $T \leq u_i < T + \Delta u$.
     $w2_i$ is 1 in regular error diffusion, but it is possible

that $w2_i$ is 0 with the rule of embedding data. Therefore, we use the expected value of $w2_i$ in the same as case 3. Then, $\langle w2_i \rangle$ is given by Equation (23), where

$$P_b^{\mathcal{W}2_i} = P_b^{\mathcal{S}^{v3}} P_w^{\mathcal{W}1_i} + P_w^{\mathcal{S}^{v3}} P_b^{\mathcal{W}1_i}, \tag{27}$$

$$P_w^{\mathcal{W}2_i} = P_b^{\mathcal{S}^{v3}} P_b^{\mathcal{W}1_i} + P_g^{\mathcal{S}^{v3}} + P_w^{\mathcal{S}^{v3}} P_w^{\mathcal{W}1_i}. \tag{28}$$

We obtain $T - \langle w2_i \rangle \le e_i < T + \Delta u - \langle w2_i \rangle$ by replacing $w2_i$ in Equation (3) with $\langle w2_i \rangle$ and rewriting the domain of u_i to it of e_i. Let $\mathcal{E}_i^{(4)}$ be the random variable corresponding to e_i in the domain, and the probability density function of $\mathcal{E}_i^{(4)}$ is equal to it of $\mathcal{U}_i - \langle w2_i \rangle$, i.e.,

$$\begin{aligned} f_{\mathcal{E}_i^{(4)}}(e) =& f_{\mathcal{U}_i - \langle w2_i \rangle}(e) \\ =& f_{\mathcal{U}_i}(e + \langle w2_i \rangle), \\ & T - \langle w2_i \rangle \le e < T + \Delta u - \langle w2_i \rangle. \end{aligned} \tag{29}$$

By Equation (19), (20), (26) and (29), the probability density function of $\mathcal{E}_i$ is

$$f_{\mathcal{E}_i}(e) = f_{\mathcal{E}_i^{(1)}}(e) + f_{\mathcal{E}_i^{(2)}}(e) + f_{\mathcal{E}_i^{(3)}}(e) + f_{\mathcal{E}_i^{(4)}}(e). \tag{30}$$

Step 2 is as follows: We analyze the statistical behavior of ER_k in Equation (1) by using relative frequency distributions of ER_k. Let $\mathcal{ER}_k$ be the random variable corresponding to accumulative error in error diffusion. Then, by Equation (1),

$$\mathcal{ER}_k = \sum_{i \in J_k} j_i \mathcal{E}_i. \tag{31}$$

Here, for arbitrary random variable $\mathcal{X}$ and scalar a, we have

$$f_{a\mathcal{X}}(x) = f_{\mathcal{X}}(\tfrac{1}{a}x), \quad a \neq 0. \tag{32}$$

Also for arbitrary random variables $\mathcal{X}$ and $\mathcal{Y}$, let Δy be a small positive constant, then,

$$\begin{aligned} & f_{\mathcal{X}+\mathcal{Y}}(x) \\ &= \sum_{t=-\infty}^{\infty} f_{\mathcal{X}+t\Delta y}(x) f_{\mathcal{Y}}(t\Delta y)\Delta y \\ &= \sum_{t=-\infty}^{\infty} f_{\mathcal{X}}(x - t\Delta y) f_{\mathcal{Y}}(t\Delta y)\Delta y \\ &= \int_{-\infty}^{\infty} f_{\mathcal{X}}(x-y) f_{\mathcal{Y}}(y) dy, \quad \Delta y \to 0. \end{aligned} \tag{33}$$

By Equation (32) and (33), for $j_m\mathcal{E}_m$ and $j_n\mathcal{E}_n$ for any $m, n \in J_k$, the probability density function of $j_m\mathcal{E}_m + j_n\mathcal{E}_n$ is

$$\begin{aligned} & f_{j_m\mathcal{E}_m + j_n\mathcal{E}_n}(e) \\ &= \int_{-\infty}^{\infty} f_{j_m\mathcal{E}_m}(e-y) f_{j_n\mathcal{E}_n}(y) dy, \\ &= \int_{-\infty}^{\infty} f_{\mathcal{E}_m}(\tfrac{1}{j_m}(e-y)) f_{\mathcal{E}_n}(\tfrac{1}{j_n}y) dy. \end{aligned} \tag{34}$$

By repeating the operation in (34), we obtain the following probability density function of $\mathcal{ER}_k$, i.e.,

$$\begin{aligned} & f_{\mathcal{ER}_k}(e) \\ &= \int_R \int_R \cdots \int_R f_{\mathcal{E}_{i'}}(\tfrac{1}{j_{i'}}(e - \sum_{\substack{i \in J_k \\ i \neq i'}} y_i)) \prod_{\substack{i \in J_k \\ i \neq i'}} f_{\mathcal{E}_i}(\tfrac{1}{j_i} y_i) dy_i, \end{aligned} \tag{35}$$

where the number of integrals is $|J_k| - 1$.

Step 3 is as follows: We obtain probability distributions of each pixel intensity for W1 from the probability density function of $\mathcal{U}_k$. Let $\mathcal{G}1$ be the random variable corresponding to intensities of G1. Then, by Equation (1),

$$\mathcal{U}_k = \mathcal{G}1 + \mathcal{ER}_k. \tag{36}$$

By Equation (33), we have

$$f_{\mathcal{U}_k}(u) = \int_{-\infty}^{\infty} f_{\mathcal{G}1}(u-e) f_{\mathcal{ER}_k}(e) de. \tag{37}$$

Therefore, the probability that k-th pixel in W1 becomes white is

$$P_w^{\mathcal{W}1_k} = P(T \le \mathcal{U}_k < \infty) = \int_T^{\infty} f_{\mathcal{U}_k}(u) du. \tag{38}$$

Note that $ER_1 = 0$ when $k = 1$. It means that ER_1 is distributed at only 0. So, the probability density function of $\mathcal{ER}_1$ is delta distribution, i.e.,

$$f_{\mathcal{ER}_1}(e) = \delta(e). \tag{39}$$

Therefore, we obtain

$$\begin{aligned} f_{\mathcal{U}_1}(u) &= \int_{-\infty}^{\infty} f_{\mathcal{G}1}(u-e) f_{\mathcal{ER}_1}(e) de \\ &= \int_{-\infty}^{\infty} f_{\mathcal{G}1}(u-e) \delta(e) de \\ &= \int_{-\infty}^{\infty} f_{\mathcal{G}1}(e) \delta(u-e) de \\ &= f_{\mathcal{G}1}(u). \end{aligned} \tag{40}$$

Step 4 is as follows: We obtain probability distributions of each pixel intensity for W2 from the probability density function of $\mathcal{U}_k$ and probability distributions of each pixel intensity for W1 and S, i.e., the probability that k-th pixel in W2 becomes white is

$$\begin{aligned} P_w^{\mathcal{W}2_k} =& P(T - \Delta u < \mathcal{U}_k < T + \Delta u) P_b^{\mathcal{S}^{v3}} P_b^{\mathcal{W}1_k} \\ & + P(T - \Delta u < \mathcal{U}_k < T + \Delta u) P_w^{\mathcal{S}^{v3}} P_w^{\mathcal{W}1_k} \\ & + P(T \le \mathcal{U}_k < T + \Delta u) P_g^{\mathcal{S}^{v3}} \\ & + P(\mathcal{U}_k \ge T + \Delta u) \\ =& \left(P_b^{\mathcal{S}^{v3}} P_b^{\mathcal{W}1_k} + P_w^{\mathcal{S}^{v3}} P_w^{\mathcal{W}1_k} \right) \int_{T-\Delta u}^{T+\Delta u} f_{\mathcal{U}_k}(u) du \\ & + P_g^{\mathcal{S}^{v3}} \int_T^{T+\Delta u} f_{\mathcal{U}_k}(u) du + \int_{T+\Delta u}^{\infty} f_{\mathcal{U}_k}(u) du, \end{aligned} \tag{41}$$

where

$$f_{\mathcal{U}_k}(u) = \int_{-\infty}^{\infty} f_{\mathcal{G}2}(u-e) f_{\mathcal{ER}_k}(e) de, \quad (42)$$

$$f_{\mathcal{U}_1}(u) = f_{\mathcal{G}2}(u) \quad (43)$$

and $\mathcal{G}2$ denotes the random variable corresponding to intensities of G2.

C. Formulation of superposing two halftone images

We consider intensity distributions of C. In Fu's and Au's method, C is produced by superposing W1 and W2. We can obtain the stochastic behavior of pixel values of C by using intensity distributions of W1 and W2. Here, k-th pixel in C becomes white when k-th pixels in W1 and W2 are both white. Let $\mathcal{C}_k$ be the random variable corresponding to intensity of k-th pixel in C. Then, by the results in the previous subsection, the probability that k-th pixel in C becomes white is

$$\begin{aligned} P_w^{\mathcal{C}_k} =& P(\mathcal{C}_k = 1) \\ =& P(T - \Delta u < \mathcal{U}_k < T + \Delta u) P_w^{\mathcal{S}^{v3}} P_w^{\mathcal{W}1_k} \\ &+ P(T \le \mathcal{U}_k < T + \Delta u) P_g^{\mathcal{S}^{v3}} P_w^{\mathcal{W}1_k} \\ &+ P(\mathcal{U}_k \ge T + \Delta u) P_w^{\mathcal{W}1_k} \\ =& P_w^{\mathcal{W}1_k} \left\{ P_w^{\mathcal{S}^{v3}} \int_{T-\Delta u}^{T+\Delta u} f_{\mathcal{U}_k}(u) du \right. \\ &+ P_g^{\mathcal{S}^{v3}} \int_{T}^{T+\Delta u} f_{\mathcal{U}_k}(u) du \\ &\left. + \int_{T+\Delta u}^{\infty} f_{\mathcal{U}_k}(u) du \right\}. \quad (44) \end{aligned}$$

IV. Conclusion

Finally, we conclude this paper. We have reviewed a formulation for Fu's and Au's data hiding scheme using a stochastic method in this paper. Fu's and Au's data hiding scheme is based on error diffusion, and demonstrates relatively good performance for secret data like natural gray-scale images by giving appropriate transformations to original input data. The formulation is based on relative frequency distributions of intensities of images. As a results, we have shown the stochastic behavior of pixel values of W2 and C.

Formulating data embedding algorithms would allow us to investigate behavior of the algorithm theoretically. For example, if the amount of embedded secret data is little, then qualities of output halftone images by error diffusion will be high and the quality of extracted data will be poor, and vice versa. By theoretical analysis by the formulation, we might be able to know this trade-off. Also, we want to know relationships between distributions of pixel intensities for input images and quality of extracted data. By simulations, we might be able to find easily pairs of input images such that secret data can be embedded into halftone images without affecting their perceptual qualities and the embedded data can be restored with apparently high quality when the halftone images are overlaid. On the other hand, we might be able to optimize parameters affect the performance of data embedding algorithm.

The concept of the formulation is basic, but theoretical studies by formulating is important trial in this field. In the future works, it will be necessary to perform a verification of the formulation through several simulations, and to propose more refined conception of formulation. Also we will apply the formulation to other image data hiding schemes.

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