Application of the Multi segment function in hydraulic systems

Claude Ziad Bayeh

Abstract--- The Multi Segment function is a mathematical function developed by the author in order to describe the continuity of non derivable linear functions. The form of this function is as its name indicate, it is composed of many linear segments connected together to form a single function. The main goal of developing this function is to be applied in the hydraulic systems and in the design of pipelines for pressurized water supply distributions underground. The Multi segment function is very important in the hydraulic systems in which the pipelines can be run parallel to the earth surface in a way to minimize the depth of the path way of the pipeline under the ground. Moreover, one can get the total length of the pipeline by using a simple formula developed in this paper. This function can be simply programmed using software as Matlab, Microsoft Excel or any other software.

Key-words---Multi Segment Function, hydraulic application, programmable function.

I. INTRODUCTION

HYDRAULIC is a topic in applied science and engineering dealing with the mechanical properties of liquids [1], [2] [3]. Fluid mechanics provides the theoretical foundation for hydraulics [4], [5], which focus on the engineering uses of fluid properties [6], [7], [8]. In fluid power, hydraulic is used for the generation, control, and transmission of power by the use of pressurized liquids [9]. Hydraulic topics range through most science and engineering disciplines, and cover concepts such as pipe flow, dam design, fluidics and fluid control circuitry, pumps, turbines, hydropower, computational fluid dynamics, flow measurement, river channel behavior and erosion [10-14].

The faced problem is how to describe the pipeline that run almost parallel to the earth surface in a simple mathematical function that can be used manually and it can be programmed using simple software as Matlab or Microsoft Excel in order to visualize the result. This problem is solved by developing a formula named "Multi segment function" and the role of this formula is to describe the path way of the pipeline under the surface of the earth. One can know the depth from the surface of the earth for any point of the pipeline at any level.

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II. DEVELOPMENT OF THE MSF

In this section, the Multi Segment function is developed using pure mathematical equations. The main target of this equation is to collect many segments related together to a non linear curve which can described a really curve presented in mathematics.

The distance between segments is equal to T_i with $T_i = x_{i+1} - x_i$ (refer to figure 1). If the distance between T_i is not very small then the MSF (Multi Segment function) behave as a discrete function that works in some point as the curve $f(x)$ (refer to figure 1.1). One can deduce if the distance of a segment T_i is very small $T_i \to 0$ then the Multi Segment Function is describing exactly the same curve $f(x)$ (refer to the figure 1.2).
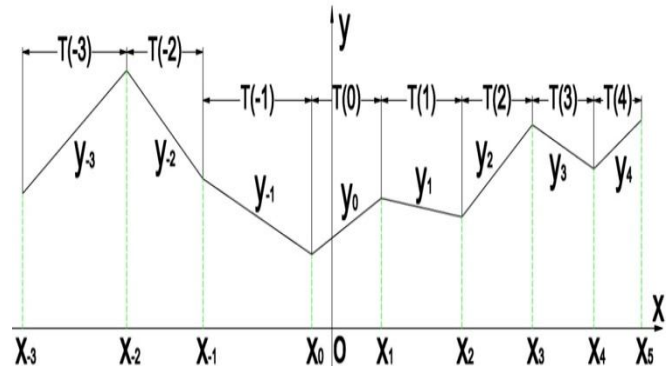

Fig. 1: represents the Multi segment function "MSF" in a limited interval
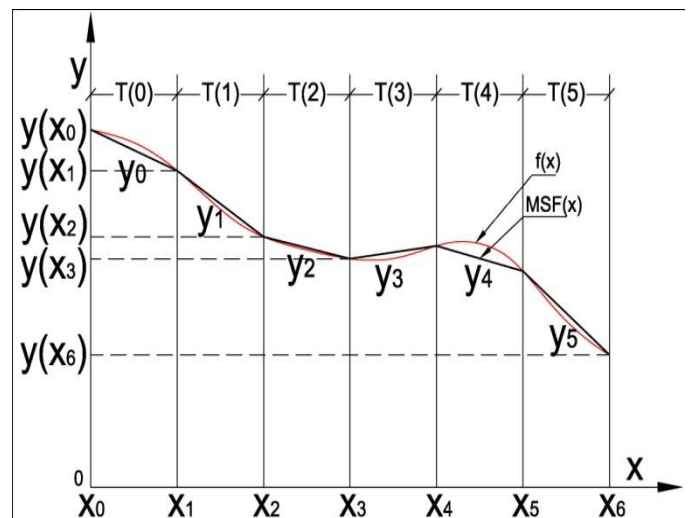

Fig. 1.1: represents the Multi segment function "MSF" in a limited interval with the function $f(x)$
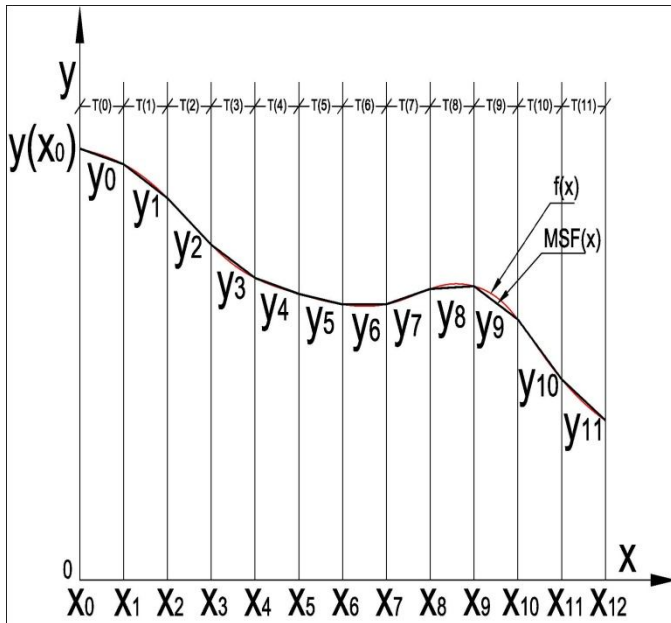
Fig. 1.2: represents the Multi segment function "MSF" in a limited interval with the function $f(x)$

Suppose that x_i is the first point of the segment y_i and x_{i+1} is the end point of the same segment (refer to figure 2).
And suppose that the difference between x_i and x_{i+1} is called T_i therefore $T_i = x_{i+1} - x_i$ with $i \in \mathbb{Z}$ (figure 1)

Let's take $x_i = x_{i-1} + T_{i-1}$ with $i \in \mathbb{Z}$
And $x_{i+1} = x_i + T_i$
$x_{i+1} = x_{i-1} + T_{i-1} + T_i$

Therefore $x_k = x_m + \sum_{i=m}^{k-1} T_i$ with $k > m$ $\qquad$ (1)
The form of a line is $y(x) = ax + b$ therefore the form of a segment of the same line is described by
$y_i(x) = a_i x + b_i$ with $x \in [x_i; x_{i+1}]$ $\qquad$ (2)
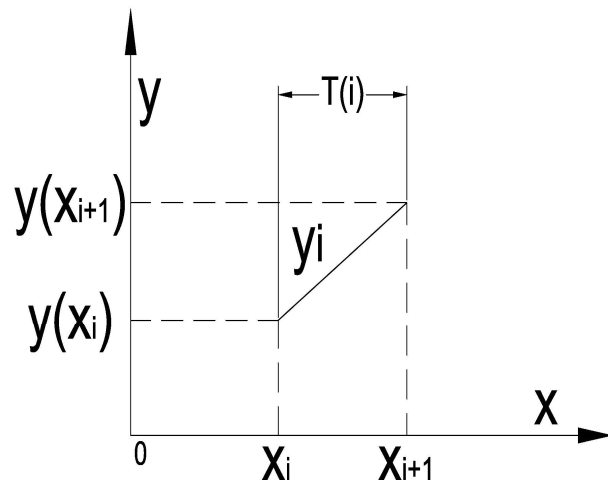As it is in the figure 2

Fig. 2: represents $y_i(x) = a_i x + b_i$ with $x \in [x_i; x_{i+1}]$

For the equation (2), one can find the two parameters a_i and b_i

We have $y_i(x_i) = y_{i-1}(x_i)$ and $y_i(x_{i+1}) = y_{i+1}(x_{i+1})$ because it is a sequence of collected segments (refer to figure 1).

$y_i(x_i) = a_i x_i + b_i$ and $y_{i-1}(x_i) = a_{i-1}x_i + b_{i-1}$
$a_i x_i + b_i = a_{i-1}x_i + b_{i-1}$

$y_i(x) = (a_i x + b_i)$ with $x \in [x_i; x_{i+1}]$
$\Rightarrow \begin{cases} y_i(x_i) = a_i x_i + b_i \\ y_i(x_{i+1}) = a_i x_{i+1} + b_i \end{cases}$
$\Rightarrow a_i = \frac{y_i(x_{i+1}) - y_i(x_i)}{x_{i+1} - x_i} = \frac{y_i(x_{i+1}) - y_i(x_i)}{T_i}$ $\qquad$ (3)
$\Rightarrow b_i = y_i(x_i) - x_i \cdot \frac{y_i(x_{i+1}) - y_i(x_i)}{x_{i+1} - x_i}$
$= y_i(x_i) - x_i \cdot \frac{y_i(x_{i+1}) - y_i(x_i)}{T_i}$ $\qquad$ (4)

In this case we use the Rectangular function as defined below:
$rect_{T_i}(x - t_i) = \begin{cases} 1 \ if \ x \in [t_i - \frac{T_i}{2}; t_i + \frac{T_i}{2}] \\ 0 \ if \ x \notin [t_i - \frac{T_i}{2}; t_i + \frac{T_i}{2}] \end{cases}$

The rectangular function is used in order to segment the linear function $y_i(x) = a_i x + b_i$. In this way every segment is separated from the other one by varying only the parameter t.

The main target of the MSF is to collect all segments formed by different curves $y_i(x)$ and have the width of the rectangular function $rect_{T_i}(x - t_i)$ as formed in the previous figure such as (figure 1 and figure 1.1)

The multi segment function is written as below:
$MSF(x) = \sum_{i=-\infty}^{\infty} \left(rect_{T_i}\left(x - \left(x_i + \frac{T_i}{2}\right)\right) \cdot \left(\frac{y_i(x_{i+1}) - y_i(x_i)}{T_i} x + \right.\right.$
$\left.\left. y_i(x_i) - x_i \cdot \frac{y_i(x_{i+1}) - y_i(x_i)}{T_i}\right)\right)$ $\qquad$ (5)

$MSF(x) =$
$\sum_{i=-\infty}^{\infty} \left(rect_{T_i}\left(x - \left(x_i + \frac{T_i}{2}\right)\right) \cdot \left(\frac{y_i(x_i + T_i) - y_i(x_i)}{T_i} x + y_i(x_i) - \right.\right.$
$\left.\left. x_i \cdot \frac{y_i(x_i + T_i) - y_i(x_i)}{T_i}\right)\right)$ $\qquad$ (6)

Therefore the final formula is written as below:

$$MSF(x) =$$

$$\sum_{i=-\infty}^{\infty} \left(rect_{T_i}\left(x - \left(x_i + \frac{T_i}{2}\right)\right) \cdot \left(\frac{y_i(x_i+T_i)-y_i(x_i)}{T_i}(x - x_i) + \right.\right.$$

$$\left.\left. y_i(x_i)\right)\right) \qquad (7)$$

Remark: For $lim_{T_i \to 0}\left(MSF(x)\right) = y(x)$

A. Example

In this example a study case is done in the hydraulic field using the multi segment function in order to create a pipeline that connect the point A with x_0 to the end of line at the point B (refer to figure 3).

The curve in green color (the upper one in figure 3) represents the surface of the ground which is complicated and not linear. This will be a challenge for engineers in order to put pipeline using very advanced software and put a huge amount of money in order to buy this software. But the case in this paper is different, because the author developed a simple formula that can describe any complicated function by putting just the coordinate system of the surface of the ground brought by the topology. This will be simply programmed using simple software such as Matlab and Microsoft Excel. In the section VI the author developed a program using MSF in order to facilitate the calculation and to be used by engineers.

The curve in red color (the lowest curve in the figure 3) is the depth of the pipeline determined and required by engineers for example they need a depth of 2 meter $\pm 0.5m$ as an error. so the curve can be described by using the MSF and calculate the depth of each point of the segment in order to be in the safe side.

This simple function that determine if the curve is in the safe side or not, it is just been as following $G(x) - MSF(x) \le \varepsilon$ with $G(x)$ is the surface formed parallel to theground (in the red color) and $MSF(x)$ is the pipeline under the ground and ε is the error put by the engineers for example $\pm 0.5m$.

The main target is to form a pipeline with a depth equal to 2 meter for example, and with a variable depth that not exceed $\pm 0.5m$. So the figure 3.1, figure 4 and figure 5 show different depth and segments formed by the MSF function and show how one can change the width of the segment T_i in order to not exceed the needed depth in the ground.
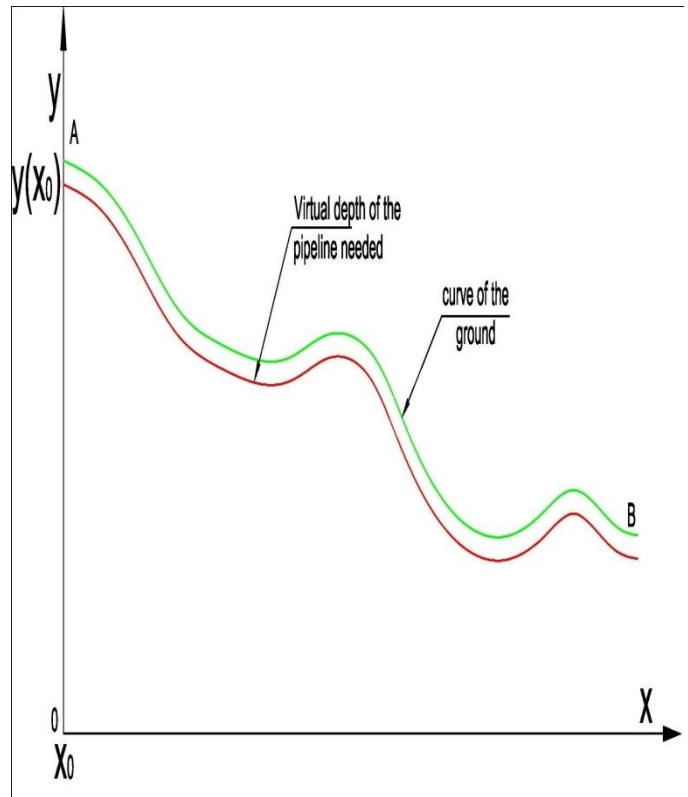

Fig. 3: represents an example of the surface of the earth (in green color) and the optimal depth needed (in the red color).
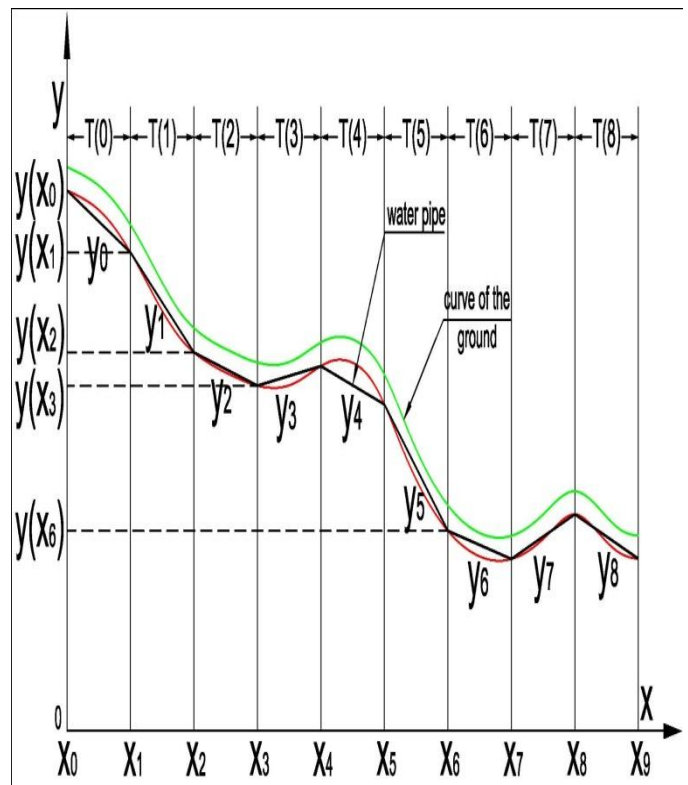

Fig. 3.1: represents an example of the MSF function (in black color) and the surface of the earth (in green color) and the optimal depth needed (in the red color).

• In the case of figure 3.1, T_i has a constant value and the depth of the pipeline in a certain point is not the same for whole points (refer to figure 3.1 between the point x_4 and x_5). In this case, the error exceed the value put by the engineers ($\pm 0.5m$) between points x_4 and x_5. In this example, the slope of the earth is very large so if the engineers accept to create a path way with a big depth then it is ok. If not, they must vary the value of T_i as in the figure 4.

Fig. 4: represents an example of the MSF function (in black color) and the surface of the earth (in green color) and the optimal depth needed (in the red color).

• In the case of figure 4 the slope of the earth is the same as the figure 3.1, but in this case T_i is divided by two and it has a constant value and the depth of the pipeline in a certain point is almost equal to whole points as in the figure 4. In this example the slope of the earth is very large so the solution is provided by reducing the value of T_i to the half, this method is very accurate but at the same time it is very costly because we have much more segments. So the optimal solution is to provide the optimal number of segments and the depth is almost equal for whole points in a certain acceptable degree and at the same time with an acceptable cost. This solution will be shown as the following in the figure 5.
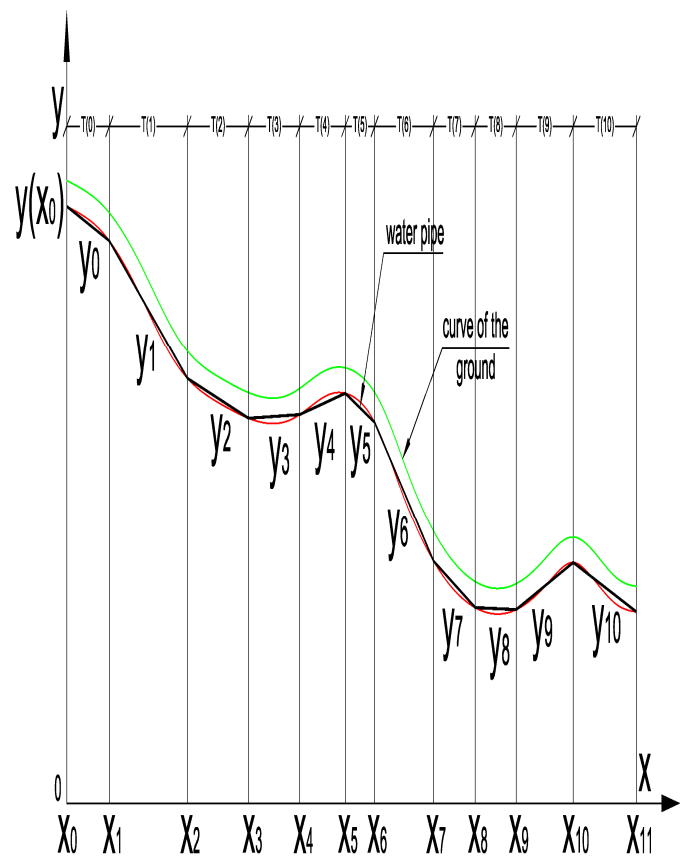

Fig. 5: represents an example of the MSF function (in black color) and the surface of the earth (in green color) and the optimal depth needed (in the red color).

• In the case of figure 5 the slope of the earth is the same as the figure 3.1, but in this case T_i has variable values in order to obtain the minimum number of segments with an optimization of the depth of the pipeline, the depth of the pipeline in a certain point is almost equal to whole points as in the figure 6 (the depth in a certain point should not overpass a limited value indicated by engineers). In this example the slope of the earth is very large so the solution is provided by optimizing the value of T_i in order to obtain good accuracy and at the same time to obtain the optimal cost and the minimum number of segments.

One can easily remark that when the width of T_i becomes very small, the MSF curve will be parallel to the curve of the earth. This is very important when creating the path way of the pipeline in the earth in which the pipeline will be parallel to the surface of the earth.
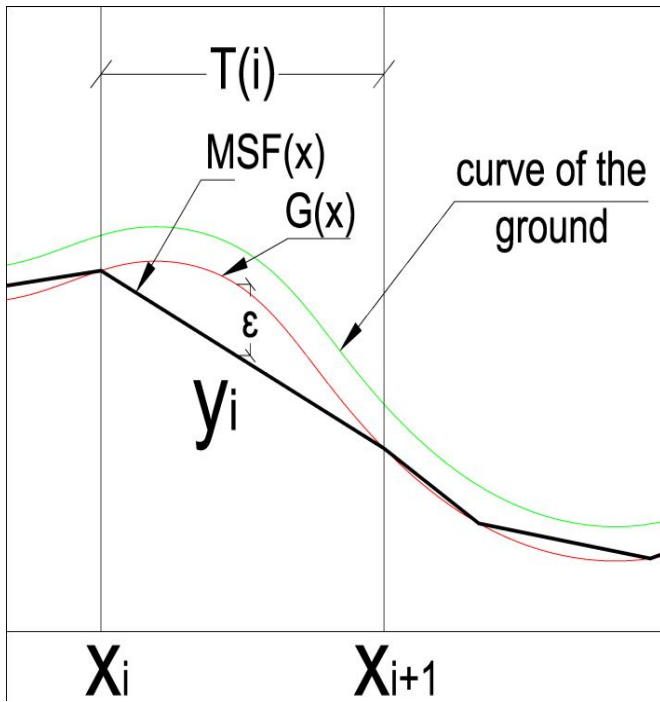
Fig. 6: represents an example of the MSF function (in black color) and the surface of the earth (in green color) and the optimal depth needed (in the red color).

In order to obtain the optimal solution we have to insure that
$$|MSF(x) - G(x)| \leq \varepsilon \quad (8)$$

With ε is a value specified by the engineers as an optimal depth not to overpass it in the path way of the pipeline. If the equation (8) is verified for each segment therefore we are in the safe side. Else, we have to reduce the value of T_i.

III. PARTICULAR CASES

A. Particular Case 1

In this case we have a limited curve that begin with point x_0 and end at a point x_n
$$MSF(x) =$$
$$\sum_{i=-\infty}^{\infty} \left(rect_{T_i}\left(x - \left(x_i + \frac{T_i}{2}\right)\right) \cdot \left(\frac{y_i(x_i+T_i)-y_i(x_i)}{T_i}(x - x_i) + y_i(x_i)\right)\right) \quad (7)$$

If one consider that the first point of the function begin with $x = 0$, then a reference point x_0 will be considered as the first point of the (ox) axis.

x_k is a point in the rang, thus $x_k = x_0 + \sum_{i=0}^{k-1} T_i$ with $k > 0$, and x_n is the end point.

Therefore the form of the expression will be as the following:

$$MSF(x) =$$
$$\sum_{i=0}^{n-1} \left(rect_{T_i}\left(x - \left(x_i + \frac{T_i}{2}\right)\right) \cdot \left(\frac{y_i(x_i+T_i)-y_i(x_i)}{T_i}(x - x_i) + y_i(x_i)\right)\right) \quad (9)$$
with $x_i = x_0 + \sum_{j=0}^{i-1} T_j$ and $i > 0$, with $n \in \mathbb{N}^*$

B. Particular Case 2

If one consider that the first point of the function begin with $x = 0$, then a reference point x_0 will be considered as the first point of the (ox) axis. Then the equation (9) is used with
$$MSF(x) =$$
$$\sum_{i=0}^{n-1} \left(rect_{T_i}\left(x - \left(x_i + \frac{T_i}{2}\right)\right) \cdot \left(\frac{y_i(x_i+T_i)-y_i(x_i)}{T_i}(x - x_i) + y_i(x_i)\right)\right) \quad (9)$$

If $T_i = T$, therefore $x_n = x_0 + \sum_{i=0}^{n-1} T_i$
$\Rightarrow T = \frac{x_n-x_0}{n}$, in this case it is simple to choose x_0, x_n and n
T is chosen in a way to obtain n greater than 1 and $n \in \mathbb{N}$.

Therefore the form of the expression will be as the following:
$$MSF(x) =$$
$$\sum_{i=0}^{n-1} \left(rect_{T_i}\left(x - \left(x_i + \frac{x_n-x_0}{2n}\right)\right) \cdot \left(\frac{y_i\left(x_i+\frac{x_n-x_0}{n}\right)-y_i(x_i)}{\frac{x_n-x_0}{n}}(x - x_i) + y_i(x_i)\right)\right) \quad (10)$$
with $x_i = x_0 + \sum_{j=0}^{i-1} T_j$ and $i > 0$, with $n \in \mathbb{N}^*$
in the equation (10), we have:
x_0 is known
x_n is known
$y_i(x_i)$ is known
$y_i\left(x_i + \frac{x_n-x_0}{n}\right)$ is known
n is determined by the engineer in order to obtain the suitable form of the curve MSF with an acceptable error.
x can be determined for each point of the curve MSF and $G(x)$.

So this function is very simple to program and it facilitate the selection of the length of each segment of the pipeline.

IV. PRACTICAL EXAMPLE

• Let's consider that in a certain region, the surface of the earth is equal to the following equation
$y(x) = -x^2 + 2x + 10$. We have to dig a pathway for the main pipeline from point (x=0) to Point (x=4) with a minimum depth equal to 1 unit (the previous values are taken in order to visualize them on a graph). The diffraction of the pipeline is been every 1 unit of length (horizontally).

Therefore by using the equation of MSF we obtain the following result

$MSF(x) =$

$\sum_{i=0}^{3}\left(rect_{T_i}\left(x - \left(x_i + \frac{T_i}{2}\right)\right) \cdot \left(\frac{y_i(x_i+T_i)-y_i(x_i)}{T_i}(x - x_i) + \right.\right.$

$\left.\left. y_i(x_i)\right)\right) = y(x_i) - 1$

With $T_i = 1$ unit

$x_{i+1} = x_i + T_i = x_i + 1$

$x_0 = 0; x_1 = 1; x_2 = 2; x_3 = 3;$ and $x_4 = 4$

$y_0(x_0) = y(x) - 1 = 9$

$y_1(x_1) = y(x) - 1 = 10$

$y_2(x_2) = y(x) - 1 = 9$

$y_3(x_3) = y(x) - 1 = 6$

$y_3(x_4) = y_4(x_4) = y(x_4) - 1 = 1$

Therefore the MSF function can be written as following:

$MSF(x) = rect_1\left(x - \left(\frac{1}{2}\right)\right) \cdot (x + 9) + rect_1\left(x - \right.$

$\left(\frac{3}{2}\right)\right) \cdot (-x + 11) + rect_1\left(x - \left(\frac{5}{2}\right)\right) \cdot (-3x + 15) +$

$rect_1\left(x - \left(\frac{7}{2}\right)\right) \cdot (-5x + 21)$

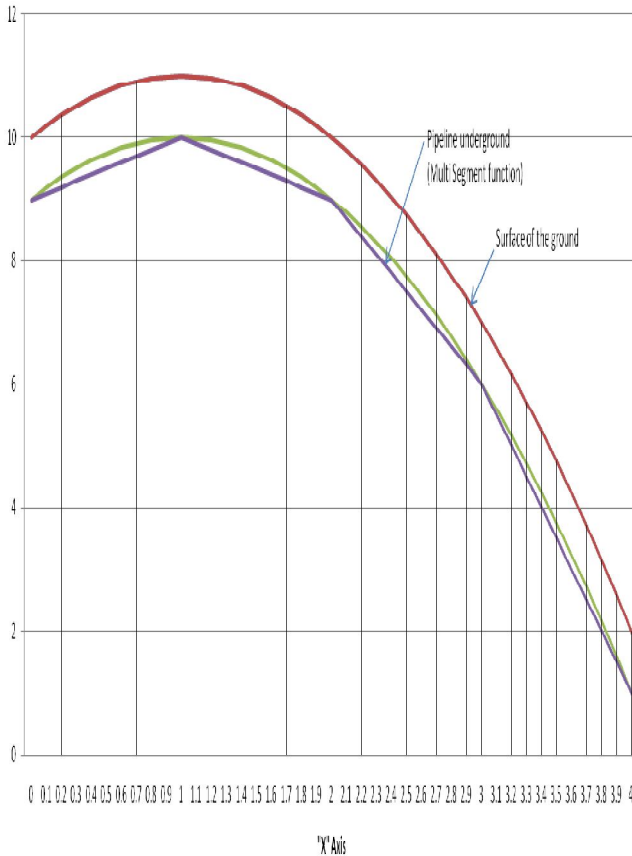The following figure is the result of the MSF (pipeline) and the earth surface equation

Fig. 7: represents the MSF function (Pipeline) and the Earth surface equations

As we have seen that it is very easy to calculate the form of the MSF function.

V. LENGTH OF THE MSF FUNCTION

In this section, the author calculates the length of the formed segments of the function MSF.

The total length or the partial length of the segments will facilitate the selection of the pipes and their accessories; this will give them an accurate of the selection of the pipes and their accessories.

The main goal of introducing this study is to find the total length of the needed pipeline used to connect two points.
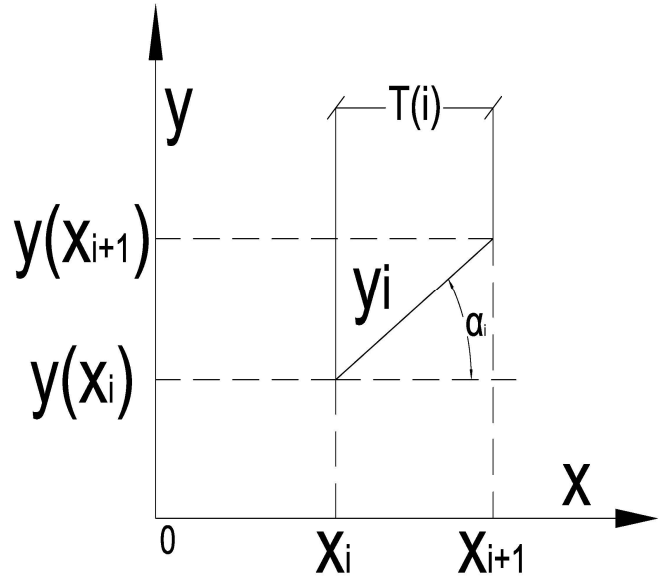

Fig. 8: represents $y_i(x) = a_i x + b_i$ with $x \in [x_i; x_{i+1}]$

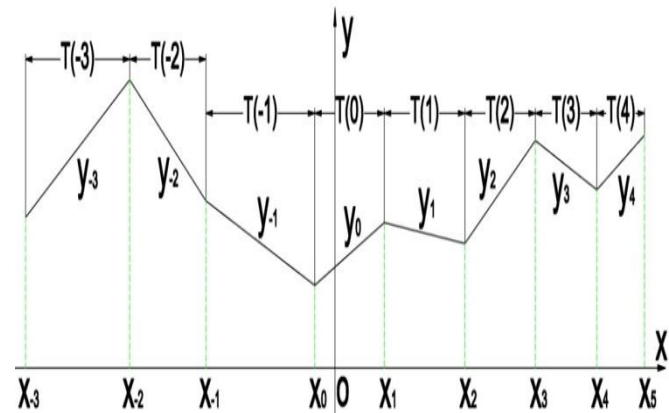Fig. 8.1: represents the length of the Multi segment function "MSF" in a limited interval

Considering that $\cos(\alpha_i) = \frac{x_{i+1}-x_i}{l_i}$ with l_i is the length of the segment formed between $[x_i; x_{i+1}]$ (refer to figure 8).

Therefore, $l_i = \frac{x_{i+1}-x_i}{\cos(\alpha_i)}$ with $\alpha_i = \arctan\left(\frac{y_{i+1}-y_i}{x_{i+1}-x_i}\right)$

$$\Rightarrow l_i = \frac{x_{i+1} - x_i}{\cos(\alpha_i)} = \frac{x_{i+1} - x_i}{\cos\left(\arctan\left(\frac{y_{i+1}-y_i}{x_{i+1}-x_i}\right)\right)} \qquad (11)$$

The total length of the total segments is presented as following:

$$L = \sum_{i=a}^{b} l_i = \sum_{i=a}^{b} \frac{x_{i+1} - x_i}{\cos\left(\arctan\left(\frac{y_{i+1}-y_i}{x_{i+1}-x_i}\right)\right)} \qquad (12)$$

Remark:

Consider that $\Delta x = x_{i+1} - x_i$

$$lim_{\Delta x \to 0} \sum_{i=a}^{b} \frac{x_{i+1} - x_i}{\cos\left(\arctan\left(\frac{y_{i+1}-y_i}{x_{i+1}-x_i}\right)\right)} = \int_{a}^{b} \sqrt{1 + (f'(x))^2} \qquad (13)$$

Which is the total length of a function $f(x)$ between two points a and b.

VI. MSF FUNCTION PROGRAMMED IN MATLAB SOFTWARE

In this section, the author developed a simple software programmed in Matlab in order to use it to describe a curve and try to draw the MSF function that gives a collected series of linear segmented functions.

```
%-----------------------------------------------------------
%MULTI SEGMENT FUNCTION MSF(x)
%Introduced by Claude Ziad Bayeh in 2009-09-15
% Segment function
i=1;
s=0.001; %step
%i=1:k1+k2+1; %to keep 'k' and 'i' at the same dimension, the
index of the matrix begin with '1'

fprintf('---SEGMENT FUNCTION Introduced by Claude Ziad
Bayeh in 2009-09-15---\n');
fprintf('----------------------------------------------------------------
-------\n');
repeat='y';
while repeat=='y'

   fprintf('Enter the first point of MSF(x), x(0):\n');
   X0=input('x(0)=');
   fprintf('Enter the end point of MSF(x), x(n):\n');
   Xn=input('x(n)=');
   if Xn <= X0,
      Xn
      error('ATTENTION: ERROR X(n) must be greater than
X(0)');
   end;
   x=X0:s:Xn;
```

```
fprintf('Write the expression of f(x)
(e.g.:x.^2+cos(x)+exp(x.^2+1):\n');
   expression_x=input('f(x)=','s');
   Function_x=eval(expression_x); % 'eval': Execute string
with MATLAB expression.

   fprintf('enter the number of sampling (n):\n');
   n=input('n=');
   if n <1,
      n
      error('ATTENTION: ERROR (n) must be greater than
Zero');
   end;
   fprintf('the corresponding period is T=(Xn-X0)/n:\n');
   T=(Xn-X0)./n;
   %Body of the Program
   Xi=X0;
   for i=0;1;n-1,
      for x=Xi:s:Xi+T,
         Y=(Function_x((Xi+T+k1)./s+1)-
Function_x((Xi+k1)./s+1)).*(x-
Xi)./(T)+Function_x((Xi+k1)./s+1);
      end;
      plot(x,Y);
      hold on;
      Xi=Xi+T;
   end;

   axis([-15 15 -15 15]);
   grid on;%grid Minor (for more details)/ grid on (for less
details)
   fprintf('Do you want to repeat ?\nPress y for "Yes" or any
key for "No"\n');
   repeat=input('Y/N=','s'); %string input
end;
%-----------------------------------------------------------
```

VII. CONCLUSION

The Multi Segment function is a mathematical function developed by the author in order to describe the continuity of non derivable linear functions. The main goal of introducing this function is to help engineers to describe the pipelines in a simple expression that can be used in order to know the depth of the pipeline under ground in every point. Moreover this function is easily programmed using software as Matlab, Microsoft Excel, Labview or any other software. The use of software will simplify the calculus and visualize the result in which the engineers will vary the parameters in order to obtain the optimal solution of the function. The optimal solution is to obtain the minimum cost and minimum worked days to have the optimal path way underground for the pipeline. The Multi Segment function, is one of the ideal solutions to solve these

problems in the hydraulic field, for this reason, the author has developed this formula. The Multi Segment Function can be used in other fields as Signal processing, mathematics, physics or any other field. As it is a purely mathematical function and can be applied in numerous fields.

REFERENCES

[1] Bruce E. Larock, Roland W. Jeppson, Gary Z. Watters, "Hydraulics of Pipeline Systems", *CRC Press,* Boca Raton London New York Washington, D.C., ISBN: 0-8493-1806-8, (2000).

[2] Schlumberger , "Completions Hydraulics Handbook", *Schlumberger American Edition*, (2000).

[3] Acheson D. J., "Elementary Fluid Dynamics", *Oxford: Clarendon Press*.(1990)

[4] Batchelor G. K., "An Introduction to Fluid Dynamics", *Cambridge University Press*, (1970).

[5] Lighthill J., "An Informal Introduction to Theoretical Fluid Mechanics", *Oxford University Press*, (1986).

[6] Reif F., "Fundamentals of Statistical and Thermal Physics", *New York: McGraw-Hill*, (1959).

[7] J.H. Ferziger, M. Peric, "Computational Methods for Fluid Dynamics", *Springer*, (1996).

[8] P. Wesseling, "Principles of Computational Fluid Dynamics", *Springer*, (2001).

[9] R. L¨ohner, "Applied CFD Techniques: An Introduction Based on Finite Element Methods", *John Wiley & Sons*, (2001).

[10] Dhruvesh P. Patel, Mrugen B. Dholakia, "Feasible Structural and Non-Structural Measures to Minimize Effect of Flood in Lower Tapi Basin", *WSEAS TRANSACTIONS on FLUID MECHANICS*, Issue 3, Volume 5, (July 2010), pp.104-121.

[11] Gabriella Bognar, "Analytic Solutions to a Boundary Layer Problem for Non-Newtonian Fluid Flow Driven by Power Law Velocity Profile*",* *WSEAS TRANSACTIONS on FLUID MECHANICS*, Issue 1, Volume 6, (January 2011), pp.22-31.

[12] A. J. Templeman, M. G. Rasul, "Large Eddy Simulation of Pressure Distribution of Fluid Flow Over Ridges of Circular, Parabolic and Rectangular Shapes", *WSEAS TRANSACTIONS on FLUID MECHANICS*, Issue 2, Volume 6, (April 2011), pp.113-122.

[13] Claudiu Stefan Nitescu, Anca Constantin," Systematic Hydraulic Study for the Preliminary Sizing of the Surge Tanks Mounted Close to the Pumping Station", *WSEAS TRANSACTIONS on SYSTEMS,* Issue 10, Volume 9, (October 2010), pp.1029-1038.

[14] Olimpiu Stoicuta, Marin Silviu Nan, Gabriel Dimirache, Nicolae Buda, Dan Liviu Dandea, "Research regarding the use of new systems to control fluid flow in Pipelines", *WSEAS TRANSACTIONS on FLUID MECHANICS*, Issue 3, Volume 5, (July 2010), pp. 144-154.

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