Minimax Robust RHC Method for Two Mobile Robots Cooperative Carrying Task Problem

Tohru Kawabe

Abstract— In this paper, a robust receding horizon control (RHC) method and its application to a cooperative carrying task problem by two mobile robots is discussed. In the problem, a following robot must be controlled autonomously and it should hold constraint conditions of relative position against structured uncertainties and bounded disturbances anytime. Then the proposed robust RHC method is based on the minimax optimization with bounded constraint conditions. The proposed method generates the velocity and direction angle adequately to hold the conditions. A numerical example is shown to demonstrated the effectiveness of the method.

Keywords– Robust Control, Minimax optimization, Receding horizon control, Cooperative caring task, Mobile robot

I. Introduction

In last few decades, receding horizon control (RHC) has been widely accepted in the industry. In the standard RHC formulation, the current control action is obtained by solving a finite or infinite horizon quadratic cost problem at every sample time using the current state of the plant as the initial state [1]. One of the significant merits of RHC is easy handling of constraints during the design and implementation of the controller.

On the other hand, a drawback of RHC is its explicit lack of robust property with respect to model uncertainties or disturbances since the on-line minimized cost function is defined in terms of the nominal systems.

A possible strategy for robust RHC is solving the so-called minimax problem, namely minimization problem over the control input of the robust performance measure maximized by plant uncertainties or disturbances.

One of the early works on robust RHC was proposed by Campo and Morari [2], and further developed by Zheng and Morari [3] for SISO FIR plants. Kothare *et al.* solve minimax RHC problems with state-space uncertainties through LMIs [4]. Cuzzola *et al.* improve the Kothare's method [4] to reduce conservativeness in [5]. Furthermore, other methods of minimax RHC for systems with model uncertainties or disturbances can be found in [6], [7]. Also, some works of minimax RHC for systems with external disturbances in [8], [9], [10].

These methods are, however, based on infinite horizon quadratic cost functions, since it is rather hard to solve the minimax finite quadratic cost problems. The issue of minimax robust RHC therefore still deserves further attention[11].

T. Kawabe is with Department of Computer Science, Graduate School of Systems and Information Engineering, University of Tsukuba, Tsukuba 305-8573, Japan (e-mail: *kawabe@cs.tsukuba.ac.jp*).

By the way, recently, according to the progress of control theories and robot technologies, robots are expected to work for human support, or instead of human, under the dangerous condition. For example, rescue robots[12], which save the people life at the disaster e.g., big earthquake and so on, has been researched and developed in the world-wide. Also, many working robot in factories[13] or in construction work[14] are developed.

Under such situations, if two or more mobile robots can work cooperatively and autonomously, we can expect that the work efficiency is improved and the work plan is able to be flexible. From such a viewpoint, there have been many researches about the control problems with multiple mobile robots for various tasks. For example, the formation control using information of the relative distance and angle between leading robot and each following robot has been developed by [15]. The tracking control considering collision avoidance among followers by [16] is also targeted multiple robots. Anyway, powerful and effective control method is need for these problems [17], [18].

In this paper, therefore, new robust control method based on minimax RHC for the cooperative carrying task problem of two two-wheeled mobile robots with model uncertainties. In this problem, the most important constraint is that the following robot must be controlled anytime to hold the condition of relative position with given margin. The following robot is controlled by using only relative position information (without using absolute position information). The relative position must be within the restricted range in any situation to prevent a carried thing from dropping. The minimax RHC seems to be a best way to improve the control performance meeting with such severe requirements. Numerical examples are given to demonstrate the effectiveness of the proposed method.

This paper is organized as follows. In section 2, the robot model and the cooperative carrying task is introduced. Then, in section 3, the robust control problem of cooperative carrying task is formulated. In section 4, the proposed method is shown as main contribution and in section 5, numerical simulation results are given. Finally, in section 6, concluding remarks and future works are stated.

II. Robot model and cooperative carrying task

The two-wheeled robot has two motors which rotate independently. Although there are many control methods using velocities and angular velocities as manipulated variables[19],

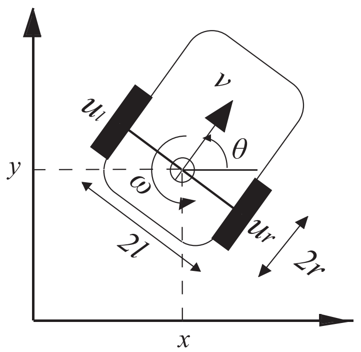

Fig. 1. Two-wheeled robot

TABLE I
DEFINITION OF ROBOT PARAMETERS

I_w	Inertia moment $[Nms^2/rad]$
M	Weight $[kg]$
I_v	Inertia moment about rotation center $[Nms^2/rad]$
l	Distance between wheel and rotation center $[m]$
c	Viscosity coefficient of friction $[Nms/rad]$
r	Wheel radius $[m]$
ϕ_r/ϕ_l	Rotation angle of left/right wheel $[rad]$

the dynamic model of the robot is used in this paper. Therefore, motor torques are set as manipulated variables[20], then the robot is torque-controlled and has two independent inputs. We assume the center of gravity (C.G.) of the robot corresponds to center of the two wheels, and let the position of C.G.set $(x,\ y)$, and θ denotes robot's direction (see fig. 1). The dynamic model of robot can be described following state space model eq. (1)[21]. Controlled variable v and ω are the velocity of C.G. and angular velocity respectively, u_r and u_l is right and left motors torques. The definition of parameters is shown in table. I.

$$\begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} + \begin{bmatrix} b_1 & b_1 \\ b_2 & -b_2 \end{bmatrix} \begin{bmatrix} u_r \\ u_l \end{bmatrix} \quad (1)$$

where

$$a_1 = \frac{-2c}{Mr^2 + 2I_\omega},\ a_2 = \frac{-2cl^2}{I_v r^2 + 2I_\omega l^2},$$
$$b_1 = \frac{r}{Mr^2 + 2I_\omega},\ b_2 = \frac{rl}{I_v r^2 + 2I_\omega l^2}$$

Generally, parameters, a_1, a_2, b_1, b_2, have uncertainties originated in the measuring errors of physical parameters or modeling errors. Therefore we need to take into account the uncertainties.

Controlled variable v and ω are the velocity of C.G. and angular velocity respectively, u_r and u_l is right and left motors torques. The definition of parameters is shown in tab. I. The relation between $(v,\ \omega)$ and $(x,\ y,\ \theta)$ is described in eq. (2).

$$\dot{x} = v\cos\theta,\ \dot{y} = v\sin\theta,\ \dot{\theta} = \omega \quad (2)$$

Input torques u_r and u_l change v, and ω according to eq. (1), v and ω change x, y, and θ according to eq. (2), too. Thus, we need to get proper torques which lead the robot to a desired position.

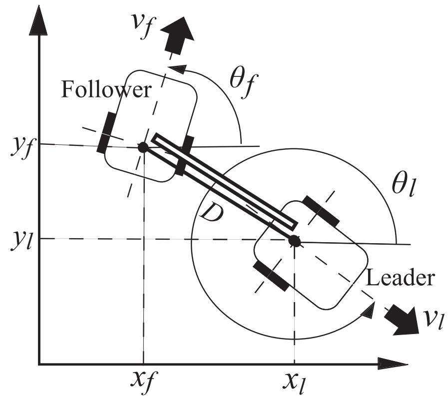

Fig. 2. Cooperative carrying task by Leader and Follower

Fig. 2 shows the cooperative carrying task. We assume that working environment is horizontal flat surface and there is no obstacle. A mobile robot's moving path is determined by the leading robot , which is assumed to be controlled by human, called "*Leader*", and the following robot is called "*Follower*".

Follower calculates its relative position $(x,\ y)$, direction θ and velocities $(v,\ \omega)$ from the communication with *leader* (by using *leader*'s position and direction). The control purpose is to generate *follower*'s input torques u_{fr} and u_{fl} which converge relative distance D to be desired distance D^d without violation of the position constraint described eq. (3) wherever *leader* is.

$$D_{min} \leq D \leq D_{max} \quad (3)$$

Leader's and *follower*'s behaviors are summarized as follows.

Leader: runs arbitrarily (driven by a human), and position, direction and velocities at each sampling time is expressed by $(x_l,\ y_l,\ \theta_l)$, $(v_l,\ \omega_l)$ respectively.

Follower: calculate own input torques u_{fr} and u_{fl} using relative position, direction and velocities expressed by $(x_f,\ y_f,\ \theta_f)$, $(v_f,\ \omega_f)$ respectively from the communication with the *leader*.

III. PROBLEM FORMULATION

A. Control law

Generally, the nonholonomic two-wheeled robot is not able to be controlled by the continuous feed-back law. About this point, Astolfi has been proposed the discontinuous feed-back law [22]. Astolfi's control law leads the robot to the origin wherever the robot is initially.

Astolfi's control law

The robot's position and direction $(x,\ y,\ \theta)$ are converted three variables $(\rho,\ \alpha,\ \phi)$ according to eq. (4) (see fig. 3).

$$\rho = \sqrt{x^2+y^2}$$
$$\alpha = -\theta + \arctan\frac{-y}{-x} \tag{4}$$
$$\phi = \frac{\pi}{2} - \theta$$

Then we can get v and ω which converge $(x_f,\ y_f,\ \theta_f)$ to (0, 0, $\pi/2$) according to eqs. (5) and (6). K_ρ, K_α and K_ϕ are the control parameters.

$$v = K_\rho \rho \tag{5}$$
$$\omega = K_\alpha \alpha + K_\phi \phi \tag{6}$$

Although this method is a superior method for the normal tracking control problem of one mobile robot, it's not applicable to the cooperative carrying task problem with two robots as it is. If we use the Astolfi's law as it is in the problem, *follower* may crash to *leader*, also maybe dropping the carrying object. Since Astolfi 's law aims to make a robot reach the fixed origin without taking account of carrying object.

Therefore, we should modify and extend it with taking account of the *leader*'s movement and carrying object as follows.

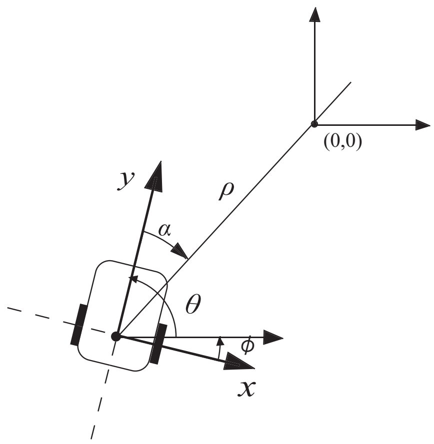

Fig. 3. Polar coordinate model

Firstly, coordinate conversion which changes $(x_l,\ y_l,\ \theta_l)$ to origin (0, 0, $\pi/2$) is done. After this conversion to $(x_f,\ y_f,\ \theta_f)$, it changes to $(x'_f,\ y'_f,\ \theta'_f)$ as shown in fig. 4. Then, $(x'_f,\ y'_f,\ \theta'_f)$ is converted to $(\rho',\ \alpha',\ \phi')$ according to eq. (7).

$$\rho' = \sqrt{(x'_f)^2+(y'_f)^2}$$
$$\alpha' = -\theta'_f + \arctan\frac{-y'_f}{-x'_f} \tag{7}$$
$$\phi' = \frac{\pi}{2} - \theta'_f$$

ρ' is equal to D. Our purpose is to converge ρ' to D^d. If we apply Astolfi's control law without any modification, it converges ρ' to 0 (*follower* collides *leader*). To avoid the collision we should converge $\rho' - D^d$ (see fig. 4) to 0. Moreover, we should take account in the Leader's velocity. From these points, we modify eq. (5) to eq. (8). The first term of right side in eq. (8) works to converge ρ' to D^d, and the second term compensate the effect of v_l.

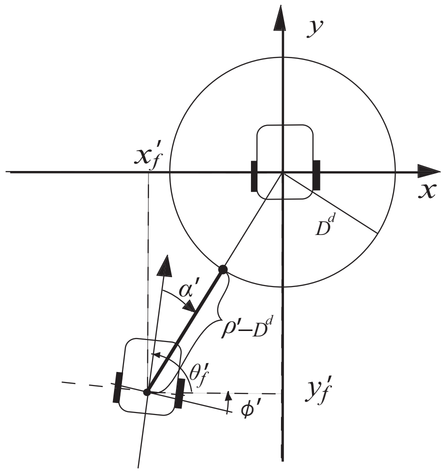

Fig. 4. Coordinate after conversion

$$v_f^d = K_\rho\left(\rho' - D^d\right) + v_l \tag{8}$$

We do not modify eq. (6) because changing ω affect to follow Leader too much. Therefore, we use eq. (9), which is same eq. (6).

$$\omega_f^d = K_\alpha \alpha' + K_\phi \phi' \tag{9}$$

B. Generation of constraints

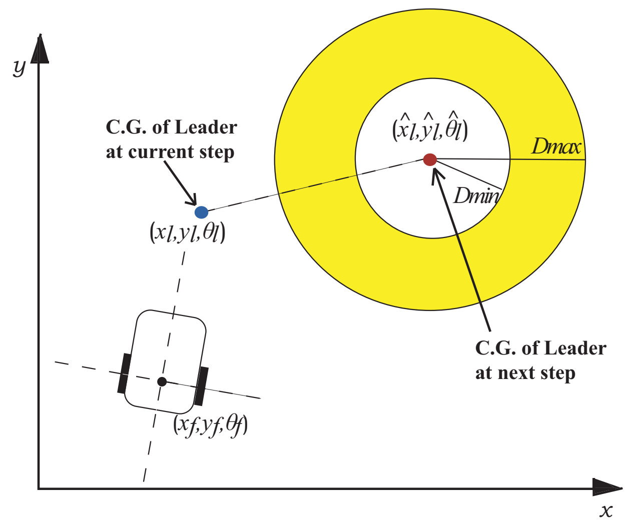

Fig. 5. The feasible region of *follower*

Follower must follow *leader* satisfying relative position condition at all time. In other words, *follower* must be inside of the yellow region, namely inside of circle $C2$ and outside of the circle $C1$ as shown in fig. 5, if any physical constraints due to the motor performance do not exist. The center of the $C1$ and $C2$ is predictive position and direction of *leader* $(\hat{x}_l, \hat{y}_l, \hat{\theta}_l)$ at next time-step, according to eq. (14)(T_s denotes sampling interval). The radius of the circle $C1$ is D_{min} and $C2$ is D_{max}.

This is feasible position area of *follower* at next step include physically impossible area. However, *follower* has constraint conditions due to motor performance actually. They are expressed with max values of velocity and angular velocity respectively. Therefore, the actual position is restricted smaller than this yellow region as follows.

Firstly, the *leader*'s position and direction at next step are predicted as

$$\begin{aligned} \hat{x}_l &= x_l + v_l \ \cos\theta_l \ T_s \\ \hat{y}_l &= y_l + v_l \ \sin\theta_l \ T_s \\ \hat{\theta}_l &= \theta_l + \omega_l \ T_s. \end{aligned} \tag{10}$$

Then, we derive the velocity constraints due to motor performance described as follows.

$$(v_{fmin} \ \leq) \ v_f \ \leq \ v_{fmax} \tag{11}$$

In fig. 6, we first consider the line segment from (x_f, y_f) to

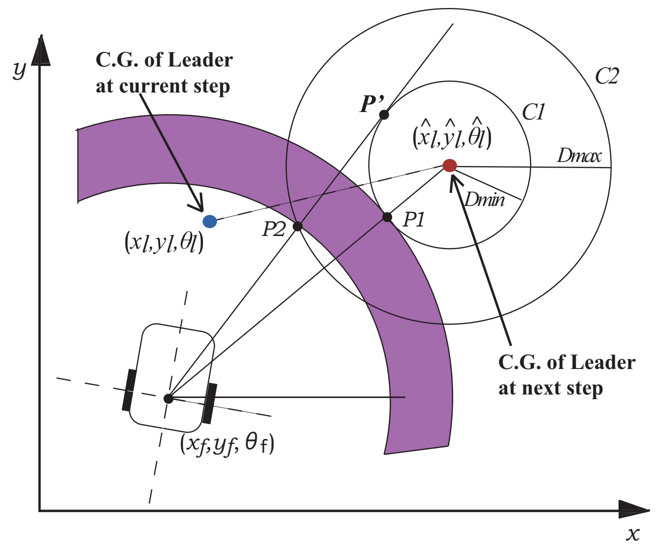

Fig. 6. The region restricted by velocity constraint

$(\hat{x}_l, \hat{y}_l)$, and we set $P1$ the intersection point of the segment with $C1$. $P1$ uses to obtain v_{fmax}, and is obtained by the smaller solution of following simultaneous equations eq. (12).

$$\begin{aligned} \sqrt{(\hat{x}_l - x_f)^2 + (\hat{y}_l - y_f)^2} &= D \\ (x_{P1} - \hat{x}_l)^2 + (y_{P1} - \hat{y}_l)^2 &= D_{min}^2 \\ (x_{P1} - x_f)^2 + (y_{P1} - y_f)^2 &= (D - D_{min})^2 \end{aligned} \tag{12}$$

Then we consider tangent line from (x_f, y_f) to point of tangency P', and we set $P2$ the intersection point of the line with $C2$. $P2$ uses to obtain v_{min}. Now, P' is needed to obtain $P2$, and is obtained by one of the solution of following simultaneous equations eq. (13).

$$\begin{aligned} (x_{P'} - \hat{x}_l)^2 + (y_{P'} - \hat{y}_l)^2 &= D_{min}^2 \\ (x_{P'} - \hat{x}_l)(x_f - \hat{x}_l) + (y_{P'} - \hat{y}_l)(y_f - \hat{y}_l) &= D_{min}^2 \end{aligned} \tag{13}$$

Then, $P2$ is obtained by one of the solution of following simultaneous equations eq. (eq:p2).

$$\begin{aligned} (x_{P2} - \hat{x}_l)^2 + (y_{P2} - \hat{y}_l)^2 &= D_{max}^2 \\ y_{P2} - y_f &= \tan\theta_1 (x_{P2} - x_f) \\ \tan\theta_1 &= \frac{y_{P'} - y_f}{x_{P'} - x_f} \end{aligned} \tag{14}$$

Finally, we calculate the velocity constraint as shown in eq. (15). Since v_{fmax} is obtained by dividing the distance between $(x_f, \ y_f)$ and $(\hat{x}_l, \ \hat{y}_l)$ in the time T_s. If v_f is not larger than v_{fmax}, *follower* does not approach than D_{min} at next step (after T_s seconds). On the other hand, if v_f is not smaller than v_{fmin}, *follower* does not part from D_{max} at next step.

$$\begin{aligned} v_{fmax} &= \frac{\sqrt{(x_{P1} - x_f)^2 + (y_{P1} - y_f)^2}}{T_s} \\ v_{fmin} &= \frac{\sqrt{(x_{P2} - x_f)^2 + (y_{P2} - y_f)^2}}{T_s} \end{aligned} \tag{15}$$

Next, the angular velocity constraint is also described as eq. (16).

$$(\omega_{fmin} \ \leq) \ \omega_f \ \leq \ \omega_{fmax} \tag{16}$$

In fig. 7, let's consider $P3$ which is obtained as well as $P2$

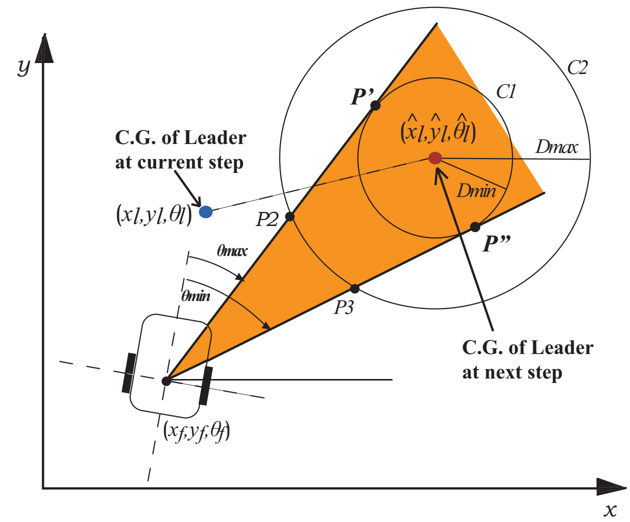

Fig. 7. The region restricted by angular velocity constraint

using the other solution P'' of eq. (13). The relative angles θ_{max} and θ_{min} are set as shown in fig. 7, respectively. Then, ω_{fmax} and ω_{fmin} are calculated by following eq. (17). Since ω_{fmax} is obtained by θ_{max}/T_s, *follower*'s direction turns between P2 and P3 at next step if v_f satisfy the constraint.

$$\omega_{fmax} = \frac{\theta_{max}}{T_s} = \frac{\arctan\frac{(y_{P2}-y_f)}{(x_{P2}-x_f)}-\theta_f}{T_s}$$
$$\omega_{fmin} = \frac{\theta_{min}}{T_s} = \frac{\arctan\frac{(y_{P3}-y_f)}{(x_{P3}-x_f)}-\theta_f}{T_s} \quad (17)$$

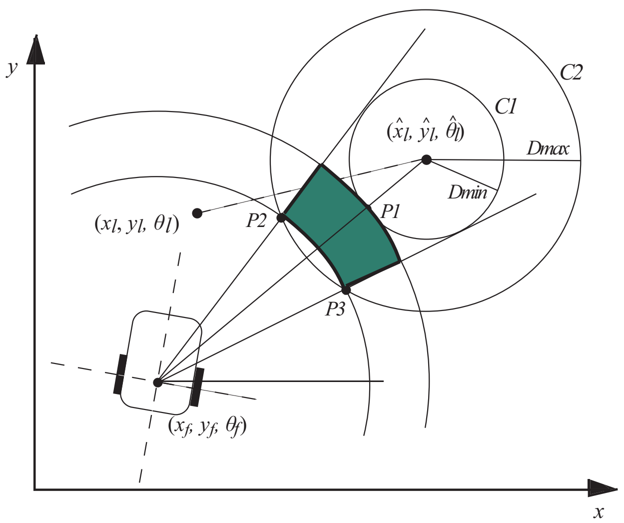

Fig. 8. The actual region restricted by all constraints

Finally, *follower* 's actual reachable position at next step without dropping the carrying object is restricted by velocity and angular velocity constraints due to physical motor performance into the green region in fig. 8. Namely, *follower* must be controlled to be in this green region at every next step.

C. Minimax RHC problem

Eq. (18) is expressed by the discretized dynamic model with uncertainties by sampling time T_s as follows.

$$z_f(k+1) = (A_d + L\triangle R_A)z_f(k) + (B_d + L\triangle R_B)u_f(k) \quad (18)$$

where

$$z_f(k) = \begin{bmatrix} v_f(k) \\ \omega_f(k) \end{bmatrix},\ u_f(k) = \begin{bmatrix} u_{fr}(k) \\ u_{fl}(k), \end{bmatrix}$$

and where $\triangle$ is a diagonal structured uncertainties parameters matrix satisfied $\triangle^T\triangle \le I$. L, R_A and R_B are constant matrices. All these vectors and matrices have appropriate dimensions. Then, we can transform this system as

$$z_f(k+1) = A_d z_f(k) + B_d u_f(k) + Lw(k) \quad (19)$$
$$\eta(k) = R_A z_f(k) + R_B u_f(k) \quad (20)$$

where $w(k)(=\triangle\eta(k))$. Moreover, assume that the uncertainties has a generalized range constrained as follows;

$$w^T(k+j)P_w w(k+j) \le 1 \quad (j=0,\cdots,N-1) \quad (21)$$

where $P_w,(P_w \succ 0)$ are positive symmetric matrix for a weight of constraint. For this systems, the quadratic performance measure with finite horizon with positive weighting constant matrices Q and R $(Q,R \succ 0)$ as :

$$J(k) = \sum_{i=0}^{Hp-1} \| z_f(k+i+1) - z^d(k) \|_Q^2 + \| u_f(k+i) \|_R^2 \quad (22)$$

is used. Where $z^d(k) = \begin{bmatrix} v_f^d(k) \\ \omega_f^d(k) \end{bmatrix}$, $z_{fmin}(k) = \begin{bmatrix} v_{fmin}(k) \\ \omega_{fmin}(k) \end{bmatrix}$ and $z_{fmax}(k) = \begin{bmatrix} v_{fmax}(k) \\ \omega_{fmax}(k) \end{bmatrix}$.

Finally RHC controller solves the following minimax optimization problems to get the input torques of *follower* against the model uncertainties.

$$\min_{\hat{u}_f} \max_{w(k+j|k)} J(k) \quad (23)$$

subject to

$$z_f(k+i+1) = A_d z_f(k+i) + B_d u_f(k+i)$$
$$z_{fmin}(k) \le z_f(k+i) \le z_{fmax}(k)$$
$$w^T(k+j)P_w w(k+j) \le 1 \quad (24)$$
$$i = 0,1,\cdots,Hp-1$$

IV. How to solve the minimax RHC problem

At each step k the following state feedback is employed;

$$u_f(k+j|k) = -F_{k+j} z_f(k+j|k) \quad (j=0,1,\cdots N-1) \quad (25)$$

where F_{k+j} is a feedback gain matrix. Then, introducing the following vectors.

$$\begin{aligned} Z &:= \begin{bmatrix} z_f(k+1|k) & z_f(k+2|k) & \cdots & z_f(k+N|k) \end{bmatrix}^T \\ U &:= \begin{bmatrix} u_f(k|k) & u(k+1|k) & \cdots & u(k+N-1|k) \end{bmatrix}^T \\ W &:= \begin{bmatrix} w(k|k) & w(k+1|k) & \cdots & w(k+N-1|k) \end{bmatrix}^T \\ \Lambda &:= \begin{bmatrix} \eta(k|k) & \eta(k+1|k) & \cdots & \eta(k+N|k) \end{bmatrix}^T \end{aligned}$$

and using state space equation recursively, we can derive

$$Z = \tilde{A}x(k) + \tilde{L}W \quad (26)$$
$$\Lambda = \tilde{R}_F\tilde{A}x(k) + \tilde{R}_F\tilde{L}W \quad (27)$$

where

$$\tilde{R}_F := R_A - R_B F$$

$$F := \begin{bmatrix} -F_k & 0 & \dots & 0 \\ 0 & -F_{k+1} & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & -F_{k+N-1} \end{bmatrix}$$

$$\tilde{A} := \begin{bmatrix} A_d \\ (A_d - B_d F_0)A \\ \vdots \\ (A_d - B_d F_0)^{N-2} A_d \end{bmatrix}$$

$$\tilde{L} := \begin{bmatrix} L & 0 & \dots & 0 \\ (A_d - B_d F_0)L & L & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ (A_d - B_d F_0)^{N-2} L & (A_d - B_d F_0)^{N-3} L & \dots & L \end{bmatrix}$$

Hence, we can transform the minimax problem (23) to

$$\min_F \gamma \tag{28}$$

$$\textbf{subject to} \quad \begin{aligned} & \max_W \quad \Pi \leq \gamma \\ & z_f(k+i+1) = A_d z_f(k+i) + B_d u_f(k+i) \\ & z_{fmin}(k) \leq z_f(k+i) \leq z_{fmax}(k) \\ & w^T(k+j) P_w w(k+j) \leq 1 \\ & \qquad (j = 0, \cdots, N-1) \end{aligned}$$

where $\gamma > 0$ (scalar parameter) and where;

$$\Pi := \left\{ \| \tilde{A} z_f(k) + \tilde{L} W \|_{\hat{Q}}^2 + \| FZ \|_{\hat{R}}^2 \right\},$$

$$\hat{Q} := \begin{bmatrix} Q & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & Q \end{bmatrix}, \qquad \hat{R} := \begin{bmatrix} R & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & R \end{bmatrix}$$

To eliminate the maximization procedure, we have to remove W term in the first constraint. For this, in the first place, following basis for all variables and transformation matrices are defined.

$$\zeta := \begin{bmatrix} z_f(k) & W^T & 1 \end{bmatrix}^T \tag{29}$$

$$Z = H_z \zeta \quad (H_x := \begin{bmatrix} \tilde{A} & \tilde{L} & 0 \end{bmatrix}) \tag{30}$$

$$FZ = H_u \zeta \quad (H_u := \begin{bmatrix} F\tilde{A} & F\tilde{L} & 0 \end{bmatrix}) \tag{31}$$

$$\Lambda = H_\eta \zeta \quad (H_\eta := \begin{bmatrix} \tilde{R}_F \tilde{A} & \tilde{R}_F \tilde{L} & 0 \end{bmatrix}) \tag{32}$$

$$1 = (H_1 \zeta)^T (H_1 \zeta) \quad (H_1 := \begin{bmatrix} \mathbf{0} & \mathbf{0} & 1 \end{bmatrix}) \tag{33}$$

By using these, we can express the first constraint condition

$$\max_W \left\{ \| H_z \zeta \|_{\hat{Q}}^2 + \| H_u \zeta \|_{\hat{R}}^2 \right\} \leq (H_1 \zeta)^T \lambda (H_1 \zeta) \tag{34}$$

Please take notice that both the left side and the right side of this inequality are expressed by the quadratic forms and they have positive scalar values. Hence, if the inequality is hold by maximum values of W and Λ in left side, this inequality must be hold by any other values of them. This fact means that we can eliminate the maximization procedure in the first constraint. We can only check the following condition instead of the first constraint.

$$\left\{ \| H_z \zeta \|_{\hat{Q}}^2 + \| H_u \zeta \|_{\hat{R}}^2 \right\} \leq (H_1 \zeta)^T \gamma (H_1 \zeta) \tag{35}$$

In the second place, $H_w(j)$ is defined. This matrix pick out the suitable block from W and satisfy the relation of $w(k+j) = H_w^{(j)} \zeta$. Then, we can derive

$$(H_w^{(j)} \zeta)^T P_w (H_w^{(j)} \zeta) \leq (H_1 \zeta)^T (H_1 \zeta) \quad (j = 0, \cdots, N-1). \tag{36}$$

Then, constraints with w and max can be transformed into

$$\forall \zeta \neq 0 \; ; \; \zeta^T \left(H_1^T \gamma H_1 - H_z^T \hat{Q} H_z - H_u^T \hat{R} H_u \right) \zeta \geq 0 \tag{37}$$

$$\textbf{subject to} \quad \zeta^T (H_1^T H_1 - (H_w^{(j)})^T P_w H_w^{(j)}) \zeta \geq 0 \quad (j = 0, \cdots, N-1). \tag{38}$$

Then, we can transform the original minimax problem to the following one by using S-procedure [23].

$$\min_{F_0} \gamma \tag{39}$$

$$\textbf{subject to} \quad \begin{aligned} & H_1^T \gamma H_1 - H_x^T \hat{Q} H_x - H_u^T \hat{R} H_u \\ & \quad - \sum_{j=0}^{N-1} [\tau_j^w S_j^w + \tau_j^u S_j^u + \tau_j^\eta S_j^\eta] \succeq 0 \\ & (j = 0, \cdots, N-1) \end{aligned}$$

where

$$S_j^w = (H_1^T H_1 - (H_w^{(j)})^T P_w H_w^{(j)}),$$
$$S_j^u = (H_1^T H_1 - (H_u^{(j)})^T P_u H_u^{(j)}),$$
$$S_j^\eta = (H_1^T H_1 - (H_\eta^{(j)})^T P_\eta H_\eta^{(j)}),$$

and where τ_j^w, τ_j^u, τ_j^η and τ_j^z are positive semi-definite scalars. It must be noted that this transformation satisfies only a sufficient condition of S-procedure, since S-procedure is not the so-called "*lossless*" in this case. We can not therefore avoid that the design results are slightly conservative. Nevertheless, we can expect the reduction of conservativeness in design result by this technique in contrast with the results by preexisting methods. Because the conservativeness caused by S-procedure is too small to put a matter for practical purposes.

Finally, using "*Schur-complement*" [24], we can transform the minimization problem (23) into the following problem which can be solved easily by using some optimization tools.

$$\min_{F_0,\tau} \gamma \quad (40)$$

$$\textbf{subject to} \quad \begin{bmatrix} H_1^T \gamma H_1 - \Sigma & H_x^T & H_u^T \\ H_x & \hat{Q}^{-1} & 0 \\ H_u & 0 & \hat{R}^{-1} \end{bmatrix} \succeq 0$$

$$\tau_j \geq 0 \quad (j = 0, \cdots, N-1)$$

where

$$\Sigma := \sum_{j=0}^{N-1} [\tau_j^w S_j^w + \tau_j^u S_j^u + \tau_j^\eta S_j^\eta]$$

and where τ_j^w is positive semi-definite scalars.

It must be noted that this transformation satisfies only a sufficient condition of S-procedure, since S-procedure is not the so-called "*lossless*" in this case. We can not therefore avoid that the design results are slightly conservative. Nevertheless, we can expect the reduction of conservativeness in design result by this technique in contrast with the results by preexisting methods. Because the conservativeness caused by S-procedure is too small to put a matter for practical purposes.

Then the proposed method based on minimax RHC algorithm which calculates *follower* motor torques is summarized as follows.

Step 1 Set initial values to all parameters.

Step 2 The controller get *leader*'s information $(x_l,\ y_l,\ \theta_l)$, $(v_l,\ \omega_l)$, and *follower*'s information $(x_f,\ y_f,\ \theta_f)$, $(v_f,\ \omega_f)$ at current time-step k.

Step 3 The controller calculates *follower*'s reference velocities according to eqs. (8) and (9), and conditions according to eqs. (15) $\sim$ (17).

Step 4 The controller calculates input torques u_{fr} and u_{fl} by minimax RHC.

Step 5 *Follower* runs by the input torque in Step 3-2. Return to Step 2 after T_s seconds.

V. Numerical Example

In this section, a example that illustrate the effectiveness of the proposed method is given. The parameters of robot shown in table I are $I_w = 0.005$, $I_v = 0.05$, and $r = 0.05$. The control parameters are initially set $K_\phi = -0.3$, $K_\rho = 1.5$, and $K_\alpha = 1.5$. Now we assume the following perturbations of l and c

$$\begin{aligned} l &\in \{l | 0.08 \leq l \leq 0.12\}, \\ c &\in \{c | 0.03 \leq c \leq 0.07\} \end{aligned} \quad (41)$$

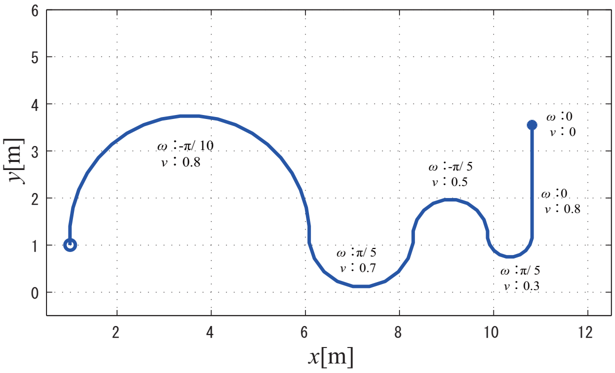

Fig. 9. Circular path

The weights of the cost function in eq. (22) are

$$Q = \begin{bmatrix} 150 & 0 \\ 0 & 20 \end{bmatrix}, \quad R = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

The initial positions and directions of *leader* and *follower* are $(x_l,\ y_l,\ \theta_l) = \left(1,\ 1,\ \frac{\pi}{2}\right)$ and $(x_f,\ y_f,\ \theta_f) = \left(1,\ 0.5,\ \frac{\pi}{2}\right)$ respectively.

Let's desired relative distance $D^d = 0.5$, and one example result of path called "Circular path" is shown from several simulations. This path is shown in fig. 9. *Leader* changes own velocities on the path.

The relative distance constraint condition is set $D_{max} = 0.6$[m] and $D_{min} = 0.4$[m].

Firstly, we assume no perturbations of parameters l and c, namely values of l and c are fixed 0.1 and 0.05 respectively.

The result of *follower*'s path and the time fluctuation of D are shown in figs. 10 and 11.

From these figures, we can see that *follower* can follow *leader* and there is no violation of relative distance constraint. This means *follower* can follow *leader* from start to goal without dropping the carrying object. The carrying task is well done in this nominal case.

Next, the worst case results are shown against the perturbation of l and c in eq. (41) in figs. 12 and 13.

In fig. 13, the red line reaches to upper bound dash line at 26 sec. , the value is 0.600[m] which is maximum value of constraint. This means *follower* is too far to *leader* for almost dropping the object, but it was just safe. Hence, the two robots can carry the object against perturbations. Then we can see that the good robust performance of the proposed method.

VI. Conclusion

In this paper, the minimax robust RHC based control method for cooperative carrying task problem of two mobile robots has been proposed. The method get the optimal torques of *follower* robot under constraint conditions. Simulation results have been illustrated to indicate the good robust performance.

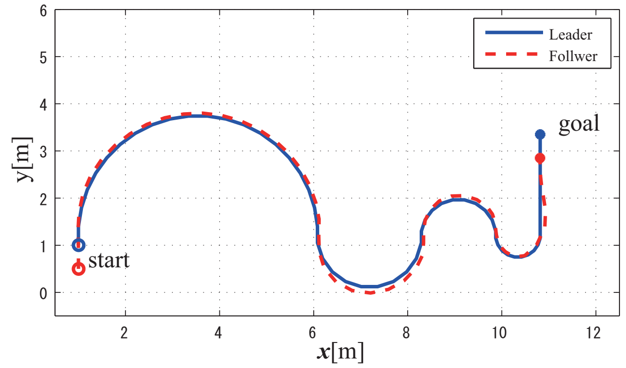

Fig. 10. Paths of *leader* and it follower without perturbation of l and c
[nominal result]

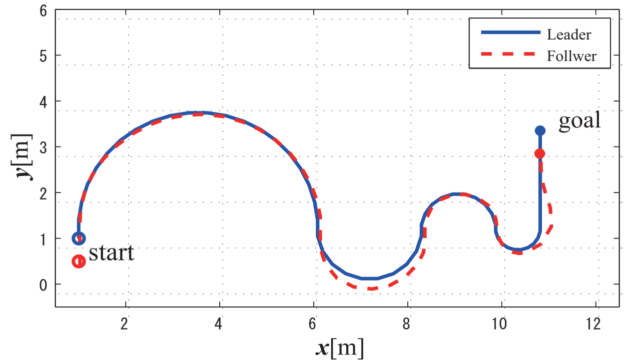

Fig. 12. Paths of *leader* and *follower* with perturbation of l and c
[robust result (worst case)]

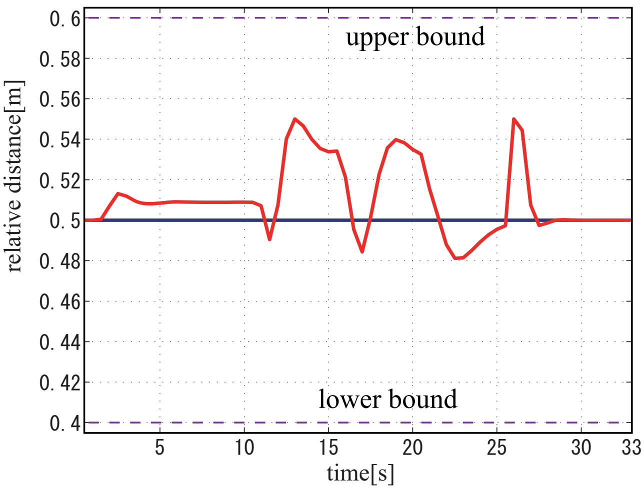

Fig. 11. Time fluctuation of relative distance without perturbation of l and c
[nominal result]

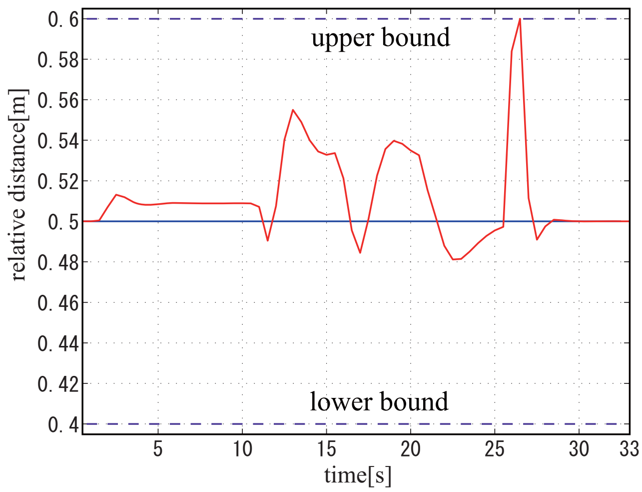

Fig. 13. Time fluctuation of relative distance with perturbation of l and c
[robust result (worst case)]

To realize the more practical systems, we need to consider the effective method of the Digital-to-Analog (DA) conversion of control inputs. In the control problem of this paper, the continuous-time objects (robots) are controlled by a discrete-time controller (computer). In such system, the Analog-to-Digital(AD) and the DA conversions of signals are indispensable operations. In this paper, the conventional zero-order hold is assumed to be used for the DA conversion on the assumption that the analog signals in each sampling interval are considered as constant values. However, to improve the control performance, it's very important to take account of the behavior of systems in the sampling intervals. For this point, we have proposed the adaptive DA converter which switch the sampling functions optimally according to the system status [25]. Hence, we will apply it to the proposed method and verify the improvement of the performance as future.

APPENDIX

The proposed approach is easily extended the systems with other constraints which are specified by ellipsoidal bounds, for example, state estimation errors and so on as follows.

In the case that $z(k)$ is not full measured and we need to estimate $z(k)$, where the bound of estimation error $e(k) = z(k) - \hat{z}(k)$ is guaranteed an ellipsoidal set as:

$$e^T(k)P_e e(k) \leq 1 \tag{42}$$

where P_e is a positive symmetric matrix for weight. This specification of estimation error is standard one. Now we introduce H_e as:

$$H_e := [1 \ 0 \cdots 0 \ \ -\hat{z}(k)] \ , \tag{43}$$

then the relation of $e(k) = H_e\zeta$ is hold. And the condition below is also hold.

$$\zeta^T(H_1^T H_1 - H_e^T P_e H_e)\zeta \geq 0 \ . \tag{44}$$

Since this condition has same form as other constraints, we can include this condition into the condition by using a new variable τ_e. Furthermore, in this case, a new output equation with measurement noise $\psi(k)$ is needed as follows.

$$y(k) = Cx(k) + \psi(k) \quad (\psi^T(k)P_\psi \psi(k) \leq 1) . \qquad (45)$$

We can also include this constraint into the condition of problem by using a new variable τ_ψ.

Moreover, Although every constraint used in this paper has been specified by the ellipsoidal bound which has one single center, it can be extended to the intersection of ellipsoidal bounds, for example:

$$w(k) \in \cup_{l=1\cdots N_1} \{ w : \begin{bmatrix} w \\ 1 \end{bmatrix} P_{w,l} \begin{bmatrix} w \\ 1 \end{bmatrix} \leq 1 \} .$$

However, it should be noted that this extension cause the rise of computational complexity due to the increase of the number of variables (τ_*) of S-procedure.

REFERENCES

[1] C.E. Garcia, D.M. Prett and M. Morari, "Model Predictive Contrl: Theory and Practice - a survey", *Automatica*, 25(3), 1989, pp. 335-348.

[2] P.J. Campo and M. Morari, "Robust Model Predictive Control", in *Proc. 1987 American Contr. Conf.*, 1987, pp. 1021-1026.

[3] Z.Q. Zheng and M. Morari, "Robust Stability of Constrained Model Predictive Control", *Proc. 1993 American Contr. Conf.* , 1993, pp. 379-383.

[4] M.V. Kothare, V. Balakrishnan and M. Morari, "Robust Constrained Model Predictive Control using Linear Matrix Inequalities", *Automatica*, 1996, 32(10), pp. 1361-1379.

[5] F.A. Cuzzola, J.C. Geromel and M. Morari, "An Improved Discrete-time Robust Approach for Constrained Model Predictive Control", in *Proc. 2001 European Contr. Conf.*, 2001 (Pre-print).

[6] J.C. Allwright and G.C. Papavasiliou, "On linear programming and robust model-predictive control using impulse-responses", *Syst. Control Lett.*, 18, 1992, pp.159-164.

[7] J.H. Lee and Z. Yu, "Worst-case Formulation of Model Predictive Control for Systems with Bounded Parameters", *Automatica*, 33(5), pp. 768-781, 1997.

[8] A. Bemporad, "Reducing Conservativeness in Predictive Control of Constrained Systems with Disturbances", *Proc. 37th IEEE Conf. Decision and Contr.*, 1998, pp. 1384-1389.

[9] A. Bemporad and A. Garulli, "Output-feedback Predictive Control of Constrained Linear Systems via Set-membership State Estimation", *Int. Journal of Control*, 73(8), 2000, pp. 655-665.

[10] P.O.M. Scokaert and D.Q. Mayne, "Min-Max Feedback Model Predictive Control for Constrained Linear Systems", *IEEE Trans. Automat. Contr.*, 43(8), 1998, pp. 1136-1142.

[11] D.Q. Mayne, J.B. Rawlings, C.V. Rao and M. Scokaert, "Constrained model predictive control: Stability and optimality", *Automatica*, 36(6), 2000, pp. 789-814.

[12] S. Tadokoro, "Robot Technologies for Assisting Search and Rescue in Disasters", *Journal of Society of Automotive Engineers of Japan*, Vol. 64(5), pp.48-51, 2010 (in Japanese).

[13] S. Sakakibara, "Robots Working at Factories : Industrial Robots, Now and the Future", *Journal of the Robotics Society of Japan*, Vol. 27(3), pp.263-264, 2009 (in Japanese).

[14] Y. Yanagihara, et al., "Development of the construction waste management RT system with the use of Next-generation Manipulator", *Journal of the Robotics Society of Japan*, Vol. 27(8), pp.842-850, 2009 (in Japanese).

[15] J.P. Desai, J. Ostrowski and V. Kumar, "Controlling formations of multiple mobile robots", *Proc. IEEE Intl. Conf. Robotics and Automation*, pp.2864-2869, 1998.

[16] K. Kon, H. Fukushima and F. Matsuno, "Model Predictive Formation Control with Multiple Mobile Robot Considering Collision Avoidance", *Trans. Society of Instrument and Control Engineers (Japan)* , Vol. 42(8), pp.877-883, 2006 (in Japanese).

[17] R.M. Murray, "Recent research in cooperative control of multi vehicle systems", *Journal of Dynamic Systems Measurement and Control-Transactions of The Asme*, Vol.129, pp.571-583, 2007

[18] T. Keviczkya, F. Borrelli, G. J. Balas "Decentralized receding horizon control for large scale dynamically decoupled systems", *Automatica*, Vol. 42(12), pp.2105-2115, 2006

[19] M. Sampei and M. Ishikawa, "State Estimation and Output Feedback Control of Nonholonomic Mobile Systems using Time-Scale Transformation ", *Trans. of the Institute of Systems, Control and Information Engineers (Japan)*, Vol. 13(3), pp.124-133, 2000 (in Japanese).

[20] R. Fierro, F.L. Lewis, "Control of a Nonholonomic Mobile Robot : Backstepping Kinematics into Dynamics", *Journal of Robotic Systems*, Vol. 14(3), pp.149-163, 1997.

[21] J. Tang, K. Watanabe, M. Nakamura and N. Koga, "Fuzzy Gaussian Neural Network Controller and Its Application to the Control of a Mobile Robot", *Trans. Japan Society of Mechanical Engineers, Series C*, Vol. 59(564), pp.2290-2297, 1993 (in Japanese).

[22] A. Astolfi, "Exponential stabilization of a wheeled mobile robot via discontinuous control", *ASME Journal of Dynamic Systems Measurement and Control*, **121**-1, pp.121-125, 1999.

[23] S. Boyd and C.H. Barratt, "*Linear Controller Design: Limits of Performance*", Prentice-Hall, 1991.

[24] K. Zhou, J.C. Doyle and K. Glover, "*Robust and optimal control* ", Prentice-Hall, 1996.

[25] T. Motoyama, T. Kawabe, et al., "New Integrated Design Approach of RHC with Adaptive DA Converter", *WSEAS Transaction on Systems*, Issue 5, Vol. 5, pp.981-988, 2006.