The HOSVD Based Domain and the Related Image Processing Techniques

András Rövid, László Szeidl and Péter Várlaki

Abstract— In the framework of the paper an improved method for image resolution enhancement is introduced. The enhancement is performed through another representation domain, where the image function is expressed with the help of polylinear functions on higher order singular value decomposition (HOSVD) basis. The paper gives a detailed description on how to determine the polylinear functions corresponding to an image and how to process them in order to obtain a higher resolution image. Furthermore, the proposed approach and the Fourier-based approach will be compared from the point of view of their effectiveness.

Keywords— Image resolution, HOSVD, approximation, numerical reconstruction.

I. INTRODUCTION

Nowadays the importance of image processing and machine vision-based applications is increasing significantly. In order to perform the desired task more accurately and reliably, the related algorithms should be developed[1]. An important factor regarding the processing is the applied data representation domain, in which the processing is performed. Numerous tasks can be performed more efficiently in one domain then in another one. Therefore, the investigation of new data representation approaches or domains can be considered as an important research topic. Representing an image in other domain may give new possibilities regarding its processing. In many cases these representations are related to expressing the image intensity function as a combination of simpler functions (components) having useful predefined properties. Let us mention some applications and some concepts of the related methods and algorithms from the field of image processing, where the applied domain plays a significant role.

The image resolution enhancement, filtering [2]-[5], image compression [6]–[9] etc. can be performed much more effectively when working in another image representation domain [11]. In the frequency domain for instance, the image compression is much more efficient than in the spatial one. There are image processing approaches indirectly related to the image representation domain, which aim, in case of compressed images for example, to directly process the compressed image, avoiding the compression and decompression prior and after the processing [10]. On the other hand, to represent the image in frequency domain without meaningful quality decline, relatively large number of trigonometric components is needed.

Another well known application is the resolution enhancement of an image [12][13][14][15], investigated in the framework of this paper in more details, focusing on its realization and effectiveness in the proposed domain. Before going into details, let us summarize the most frequently used methods and concepts applicable for this purpose.

Numerical reconstruction or recovering of a continuous intensity surface from discrete image data samples is considered, for example, when the image is resized or remapped from one pixel grid to another one. In case of image enlargement the color components or in case of grayscale images the intensity of missing pixels should be estimated. One common way for estimating the color values of such pixels is interpolating the discrete source image. There are several issues which affect the perceived quality of the interpolated images: sharpness of edges, freedom from artifacts and reconstruction of high frequency details [24].

There are numerous methods approximating the image intensity function based on the color and location of known image points, such as the bilinear, bicubic or spline interpolation all working in the spatial domain of the input image. Depending on their complexity, these methods use anywhere from 0 to 256 (or more) adjacent pixels when interpolating. The more adjacent pixels they include, the more accurate they can become, but this comes at the expense of much longer processing time [26].

Bilinear Interpolation determines the value of a new pixel based on a weighted average of the 4 pixels in the nearest 2 x 2 neighborhood of the pixel in the original image. The averaging has an anti-aliasing effect and therefore produces relatively smooth edges with hardly any jaggies [26].

The bicubic interpolation considers the closest 4x4 neighborhood of known pixels. Since these are at various distances from the unknown pixel, closer pixels are given a higher weighting in the calculation. It produces noticeably sharper images than the bilinear one, and is perhaps the ideal combination of processing time and output quality. This is the method most commonly used by image editing software [26]. Because of the spatial domain, the image in these cases is represented as a set of discrete color values without any useful predefined properties.

There are also image enlargement methods based on partial differential equations [22], multiscale geometric representations [23], some are merging a set of low-resolution images into a high-resolution image [21], and some methods are extending the above basic interpolation methods by considering also the image local features [25].

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A new data representation domain is introduced in the paper, connected to the higher order singular value decomposition (HOSVD), which is a generalized form of the well known singular value decomposition (SVD). As shown in upcoming sections, any n-variable smooth function can be expressed with the help of a system of orthonormal one-variable smooth functions on HOSVD basis. The main aim of the paper is to numerically reconstruct these one-variable functions (so called eigenfunctions) using the HOSVD and to show that how this approach can give support for certain image processing tasks and problems. First of all, it shows how the image can be expressed in the HOSVD based domain and how the most characteristic tasks like the above mentioned resolution enhancement, edge detection, data compression and filtering can be performed using the proposed approach. Furthermore, the comparison of the Fourier based representation domain to the proposed HOSVD based one will be performed.

The paper is organized as follows: Section II gives a closer view on how to express a multidimensional function using polylinear functions on HOSVD basis, and how to reconstruct these polylinear functions, Section III shows how this representation can be applied in image processing for resolution enhancement, while in section IV the proposed method is compared to the well known Fourier transformation from the approximation error point of view. Section V shows the experimental results and finally, conclusions are reported.

II. Theoretical background

The approximation methods of mathematics are widely used in theory and practice for several problems. If we consider an n-variable smooth function

$$f(x),\ x=(x_1,...,x_N)^T,\ x_n\in[a_n,b_n]\,,\ 1\le n\le N,$$

then we can approximate the function $f(x)$ with a series

$$f(x)=\sum_{k_1=1}^{I_1}...\sum_{k_N=1}^{I_N}\alpha_{k_1,...,k_n}p_{1,k_1}(x_1)\cdot...\cdot p_{N,k_N}(x_N). \quad (1)$$

where the system of orthonormal functions $p_{n,k_n}(x_n)$ can be chosen in classical way by orthonormal polynomials or trigonometric functions in separate variables and the numbers of functions I_n playing role in (1) are large enough. With the help of Higher Order Singular Value Decomposition (HOSVD) a new approximation method was developed in [17] and [18], [20], [19] in which a specially determined system of orthonormal functions can be used depending on function $f(x)$, instead of some other systems of orthonormal polynomials or trigonometric functions.

Assume that the function $f(x)$ can be given with some functions $\widetilde{w}_{n,i}(x_n), x_n\in[a_n,b_n]$ in the form

$$f(x)=\sum_{k_1=1}^{I_1}...\sum_{k_N=1}^{I_N}\alpha_{k_1,...,k_n}\widetilde{w}_{1,k_1}(x_1)\cdot...\cdot\widetilde{w}_{N,k_N}(x_N). \quad (2)$$

Denote by $\mathcal{A}\in\mathbb{R}^{I_1\times...\times I_N}$ the N-dimensional tensor determined by the elements $\alpha_{i_1,...,i_N},\ 1\le i_n\le I_n,\ 1\le n\le N$ and let us use the following notations (see : [16]).

- $\mathcal{A}\boxtimes_n\mathbf{U}$: the n-mode tensor-matrix product,
- $\mathcal{A}\boxtimes_{n=1}^N\mathbf{U}_n$: the multiple product as $\mathcal{A}\boxtimes_1\mathbf{U}_1\boxtimes_2\mathbf{U}_2...\boxtimes_N\mathbf{U}_N$.

The n-mode tensor-matrix product is defined by the following way. Let $\mathbf{U}$ be a $K_n\times M_n$-matrix, then $\mathcal{A}\boxtimes_n\mathbf{U}$ is an $M_1\times...\times M_{n-1}\times K_n\times M_{n+1}\times...\times M_N$-tensor for which the relation

$$(\mathcal{A}\boxtimes_n\mathbf{U})_{m_1,...,m_{n-1},k_n,m_{n+1},...,m_N}\overset{def}{=}\sum_{1\le m_n\le M_n}a_{m_1,...,m_n,...,m_N}U_{k_n,m_n}$$

holds. Detailed discussion of tensor notations and operations is given in [16]. We also note that we use the sign $\boxtimes_n$ instead the sign $\times_n$ given in [16]. Using this definition the function (2) can be rewritten as a tensor product form

$$f(x)=\mathcal{A}\boxtimes_{n=1}^N\widetilde{w}_n(x_n), \quad (3)$$

where $\widetilde{w}_n(x_n)=(\widetilde{w}_{n,1}(x_n),...,\widetilde{w}_{n,I_n}(x_n))^T,\ 1\le n\le N$. Based on HOSVD it was proved in [17] and [18] that under mild conditions the (3) can be represented in the form

$$f(x)=\mathcal{D}\boxtimes_{n=1}^N w_n(x_n), \quad (4)$$

where

- $\mathcal{D}\in\mathbb{R}^{r_1\times...\times r_N}$ is a special (so called core) tensor with the properties:
  1) $r_n=rank_n(\mathcal{A})$ is the n-mode rank of the tensor $\mathcal{A}$, i.e. rank of the linear space spanned by the n-mode vectors of $\mathcal{A}$:
  $$\{(a_{i_1,...,i_{n-1},1,i_{n+1},...,i_N},...,a_{i_1,...,i_{n-1},I_n,i_{n+1},...,i_N})^T:\ 1\le i_j\le I_n,\ 1\le j\le N\},$$
  2) all-orthogonality of tensor $\mathcal{D}$: two subtensors $\mathcal{D}_{i_n=\alpha}$ and $\mathcal{D}_{i_n=\beta}$ (the n-th indices $i_n=\alpha$ and $i_n=\beta$ of the elements of the tensor $\mathcal{D}$ keeping fix) orthogonal for all possible values of n,α and β : $\langle\mathcal{D}_{i_n=\alpha},\mathcal{D}_{i_n=\beta}\rangle=0$ when $\alpha\ne\beta$. Here the scalar product $\langle\mathcal{D}_{i_n=\alpha},\mathcal{D}_{i_n=\beta}\rangle$ denotes the sum of products of the appropriate elements of subtensors $\mathcal{D}_{i_n=\alpha}$ and $\mathcal{D}_{i_n=\beta}$,
  3) ordering: $\|\mathcal{D}_{i_n=1}\|\ge\|\mathcal{D}_{i_n=2}\|\ge\cdots\ge\|\mathcal{D}_{i_n=r_n}\|>0$ for all possible values of n ($\|\mathcal{D}_{i_n=\alpha}\|=\langle\mathcal{D}_{i_n=\alpha},\mathcal{D}_{i_n=\alpha}\rangle$ denotes the Kronecker-norm of the tensor $\mathcal{D}_{i_n=\alpha}$).
- Components $w_{n,i}(x_n)$ of the vector valued functions
  $$w_n(x_n)=(w_{n,1}(x_n),...,w_{n,r_n}(x_n))^T,\ 1\le n\le N,$$
  are orthonormal in L_2-sense on the interval $[a_n,b_n]$, i.e.
  $$\forall n:\int_{a_n}^{b_n}w_{n,i_n}(x_n)w_{n,j_n}(x_n)dx=\delta_{i_n,j_n},\quad 1\le i_n,j_n\le r_n,$$

where $\delta_{i,j}$ is a Kronecker-function ($\delta_{i,j}=1$, if $i=j$ and $\delta_{i,j}=0$, if $i\ne j$)

The form (4) was called in [17] and [18] HOSVD canonical form of the function (2).

Let us decompose the intervals $[a_n, b_n]$, $n = 1..N$ into M_n number of disjunct subintervals $\triangle_{n,m_n}$, $1 \leq m_n \leq M_n$ as follows:

$$\xi_{n,0} = a_n < \xi_{n,1} < \ldots < \xi_{n,M_n} = b_n,$$
$$\triangle_{n,m_n} = [\xi_{n,m_n}, \xi_{n,m_n-1}).$$

Assume that the functions $w_{n,k_n}(x_n)$, $x_n \in [a_n, b_n]$, $1 \leq n \leq N$ in the equation (2) are piece-wise continuously differentiable and assume also that we can observe the values of the function $f(x)$ in the points

$$y_{i_1,\ldots,i_N} = (x_{1,i_1}, \ldots, x_{N,i_N}),\ 1 \leq i_n \leq M_n. \tag{5}$$

where

$$x_{n,m_n} \in \triangle_{n,m_n}, \quad 1 \leq m_n \leq M_n,\ 1 \leq n \leq N$$

Based on the HOSVD a new method was developed in [17] and [18] for numerical reconstruction of the canonical form of the function $f(x)$ using the values $f(y_{i_1,\ldots,i_N})$, $1 \leq i_n \leq M_n$, $1 \leq i_n \leq N$. We discretize function $f(x)$ for all grid points as:

$$b_{m_1,..,m_N} = f(\mathbf{y}_{m_1,..,m_N}).$$

Then we construct N dimensional tensor $\mathcal{B} = (b_{m_1,\ldots,m_N})$ from the values $b_{m_1,..,m_N}$. Obviously, the size of this tensor is $M_1 \times \ldots \times M_N$. Further, we discretize vector valued functions $\mathbf{w}_n(x_n)$ over the discretization points x_{n,m_n} and construct matrices $\mathbf{W}_n$ from the discretized values as:

$$\mathbf{w}_n = \begin{pmatrix} w_{n,1}(x_{n,1}) & w_{n,2}(x_{n,1}) & \cdots & w_{n,r_n}(x_{n,1}) \\ w_{n,1}(x_{n,2}) & w_{n,2}(x_{n,2}) & \cdots & w_{n,r_n}(x_{n,2}) \\ \vdots & & \ddots & \vdots \\ w_{n,1}(x_{n,M_n}) & w_{n,2}(x_{n,M_n}) & \cdots & w_{n,r_n}(x_{n,M_n}) \end{pmatrix} \tag{6}$$

Then tensor $\mathcal{B}$ can simply be given by (4) and (6) as

$$\mathcal{B} = \mathcal{D} \boxtimes_{n=1}^{N} \mathbf{W}_n. \tag{7}$$

Consider the HOSVD decomposition of the discretization tensor

$$\mathcal{B} = \mathcal{D}^d \boxtimes_{n=1}^{N} \mathbf{U}^{(n)} \tag{8}$$

where $\mathcal{D}^d$ is the so-called core tensor, and $\mathbf{U}^{(n)} = \begin{pmatrix} U_1^{(n)} & U_2^{(n)} & \ldots & U_{M_n}^{(n)} \end{pmatrix}$ is an $M_n \times M_n$-size orthogonal matrix ($1 \leq n \leq N$).

Let us introduce the notation: $\widetilde{r_n}^d = rank_n \mathcal{B}$, $1 \leq n \leq N$ and consider the $\widetilde{r_1}^d \times \cdots \times \widetilde{r_N}^d$-size reduced version $\widetilde{\mathcal{D}}^d = (\mathcal{D}^d_{m_1,\ldots,m_N}, 1 \leq m_n \leq r_n, 1 \leq n \leq N)$ of the $M_1 \times \cdots \times M_N$-size tensor $\mathcal{D}^d$. The following theorems were proved in [17] and [18]. Denote

$$\Delta = \max_{1 \leq n \leq N} \max_{1 \leq i_n \leq M_n} (\xi_{n,m_n} - \xi_{n,m_n-1}) \quad \text{and}$$

$$\rho = \prod_{n=1}^{N} \rho_n,\ \rho_n = (b_n - a_n)/M_n.$$

Theorem 1: If Δ is sufficiently small, then $\widetilde{r_n}^d = r_n$, $1 \leq n \leq N$ and the convergence $\sqrt{\rho}\widetilde{\mathcal{D}}^d \to D$, $\Delta \longrightarrow 0$ is true.

Let us denote the elements of matrix $\mathbf{U}^{(n)}$ by $U_{i,k}^{(n)}$ and introduce the step functions $u_{n,i}(x), 1 \leq i \leq r_n$ in the following way

$$u_{n,i}(x) = \frac{1}{\sqrt{\rho_n}} U_{i,k}^{(n)} I(x \in \Delta_{n,k}), 1 \leq k \leq M_n$$

Theorem 2: If $\Delta \longrightarrow 0$ then

$$\int_{a_n}^{b_n} (w_{n,i}(x) - u_{n,i}(x))^2 \mathrm{d}x \to 0, \quad 1 \leq i \leq r_n, 1 \leq n \leq N$$

III. Resolution Enhancement on HOSVD basis

Based on the previous section the image resolution can be efficiently increased as follows:

Let $f(x), x = (x_1, x_2, x_3)^T$ represent the image function, where x_1 and x_2 correspond to the vertical and horizontal coordinates of the pixel, respectively. x_3 is related to the color components of the pixel, i.e. the red, green and blue color components in case of *RGB* image. Function $f(x)$ can be approximated (based on notes discussed in the previous section) in the following way:

$$f(x) = \sum_{k_1=1}^{I_1} \sum_{k_2=1}^{I_2} \sum_{k_3=1}^{I_3} \alpha_{k_1,k_2,k_3} \widetilde{w}_{1,k_1}(x_1) \cdot \widetilde{w}_{2,k_2}(x_2) \cdot \widetilde{w}_{3,k_3}(x_3). \tag{9}$$

The red, green and blue color components of pixels can be stored in a $m \times n \times 3$ tensor, where n and m correspond to the width and height of the image, respectively. Let $\mathcal{B}$ denote this tensor. The first step is to reconstruct the functions $\widetilde{w}_{n,k_n}, 1 \leq n \leq 3, 1 \leq k_n \leq I_n$ based on the HOSVD of tensor $\mathcal{B}$ as follows:

$$\mathcal{B} = \mathcal{D} \boxtimes_{n=1}^{3} \mathbf{U}^{(n)} \tag{10}$$

where $\mathcal{D}$ is the so called core tensor. Vectors corresponding to the columns of matrices $\mathbf{U}^{(n)}, 1 \leq n \leq 3$ as described in the previous section are representing the discretized form of functions $\widetilde{w}_{n,k_n}(x_n)$ corresponding to the appropriate dimension n, $1 \leq n \leq 3$.

Our goal is to demonstrate the effectiveness of image scaling in the HOSVD based domain.

Let $s \in \{1, 2, \ldots\}$ denote the number of pixels having to be injected between each neighbouring pixel pair in horizontal and vertical directions. First, let us consider the first column $U_1^{(1)}$ of matrix $\mathbf{U}^{(1)}$. Based on the previous sections, it can be seen, that the value $\widetilde{w}_{1,1}(1)$ corresponds to the 1st element

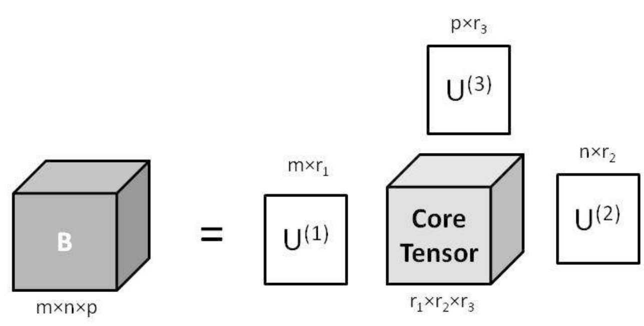

Fig. 1. Illustration of the core tensor and the corresponding orthonormal matrices.

of $U_1^{(1)}$, $\widetilde{w}_{1,1}(2)$ to the 2nd element,..., $\widetilde{w}_{1,1}(M_n)$ to the M_nth element of $U_1^{(1)}$. To scale the image in the HOSVD-based domain, the $\mathbf{U}^{(i)}, i = 1..2$ matrices should be updated, depending on s, as follows: The number of columns remains the same, the number of lines will be extended according to the factor s. Let us denote the such obtained matrix as $\mathbf{V}^{(1)}$. For example let consider the column $U_1^{(1)}$ of $\mathbf{U}^{(1)}$. The elements of $V_1^{(1)}$ are determined as follows: $V_1^{(1)}(1) := U_1^{(1)}(1)$, $V_1^{(1)}(s+2) := U_1^{(1)}(2)$, $V_1^{(1)}(2s+3) := U_1^{(1)}(3)$,..., $V_1^{(1)}((M_n-1)s+M_n) := U_1^{(1)}(M_n)$. The missing elements of $V_1^{(1)}$ can be determined by interpolation. In the paper the cubic spline interpolation was applied. The remaining columns should be processed similarly. After every matrix element has been determined the enlarged image can be obtained using the equation (10).

IV. Fourier vs. HOSVD

The proposed approach introduced in the previous sections uses orthonormal functions $\widetilde{w}_{n,i}(x_n), x_n \in [a_n, b_n]$ (see Section II) for approximating an n-variable smooth function. We saw how these functions $\widetilde{w}_{n,i}(x_n)$ can numerically be reconstructed and what properties they have. Comparing the proposed approach to the Fourier transformation similarities can be observed in their behaviour. As it is well known, the Fourier Transformation is connected to trigonometric functions, while in case of HOSVD approach the functions $\widetilde{w}_{n,i}(x_n)$ are considered, which are specific to the approximated n-variable function. In both cases the functions are forming an orthonormal basis. Since in case of HOSVD the functions are specific ones, much fewer number of components is needed then in case of Fourier based approach to achieve the same approximation accuracy.

Let us mention some common, widely used applications of both approaches.

In case of Fourier based smoothing, some of higher frequencies from the frequency domain are dismissed, which results eliminated singularities, i.e. smoothed image.

In case of HOSVD, considering only polylinear functions corresponding to the larger singular values for certain dimensions, will have similar effect then the above mentioned low pass frequency filtering. The same concept can be used also for data compression.

In the opposite case, i.e. when maintaining only the functions corresponding to the smaller singular values, an edge detector is obtained. In case of Fourier approach detecting edges in an image is equivalent to dismissing the smaller frequency components (high pass filtering).

The examples show that in case of HOSVD much smaller number of basis functions are enough to represent the image without significant information loss. In case of Fourier based approach a much larger number of trigonometric functions are needed in order to maintain the same quality. When dismissing high frequency components, there is a frequency threshold depending on the concrete image, below which by dismissing further frequencies some kind of waves can be observed in the image as noise. In case of the proposed approach the ratio of maintained and dismissed components may be significantly smaller in order to achieve the same quality than in case of applying trigonometric functions. Vectors corresponding to the columns of matrices $\mathbf{U}^{(n)}, 1 \leq n \leq 3$ as described in the previous section contain the control points of functions $\widetilde{w}_{n,k_n}(x_n)$ corresponding to the appropriate dimension n, $1 \leq n \leq 3$. It means, that there will be as many one-variable functions for a dimension as many columns there are in the orthonormal matrix corresponding to that dimension (see Fig.). The number of these functions can be further decreased by dismissing some columns from the orthonormal matrices obtained by HOSVD (see. Fig.). Let C_n, $0 \leq C_n \leq I_n$, $n = 1..N$ stand for the number of dismissed columns in nth dimension. Dismissing some of the columns is equivalent to dismissing some of the frequencies in case of Fourier approach.

$$f(x) = \sum_{k_1=1}^{I_1-C_1} \sum_{k_2=1}^{I_2-C_2} \sum_{k_3=1}^{I_3-C_3} \alpha_{k_1,k_2,k_3} \widetilde{w}_{1,k_1}(x_1) \cdot \widetilde{w}_{2,k_2}(x_2) \cdot \widetilde{w}_{3,k_3}(x_3). \quad (11)$$

The below examples clearly show that the proposed approach has good compression capabilities, which further extends its applicability also in the field of image processing. The only disadvantage of the HOSVD based approches is their relatively high computing complexity, comparing to other methods aimed for similar purposes.

V. Examples

Part-1 (HOSVD vs. Fourier)

In this section some approximations can be observed performed by the proposed and by the Fourier-based approach. As the number of the used components decreases, the observable differences in quality become more significant. In the examples below in both the HOSVD-based and Fourier-based cases the same number of components have been used in order to show how the form of determined functions influences the quality.

Fig. 2. Original image (24bit RGB)

Fig. 3. HOSVD-based approximation using 7500 components composed from polylinear functions on HOSVD basis

Fig. 4. Fourier-based approximation using 7500 components composed from trigonometric functions

Fig. 5. Enlarged segment of Fig.3

Fig. 6. Enlarged segment of Fig.4. Occurence of waves can be observed.

Fig. 7. HOSVD-based approximation using 2700 components composed from polylinear functions on HOSVD basis

Fig. 8. Fourier-based approximation using 2700 components composed from trigonometric functions

Fig. 9. Enlarged segment of Fig.7

Fig. 10. Enlarged segment of Fig.8. Occurence of waves can be observed.

Part-2 (Resolution Enhancement)

The pictures are illustrating the effectiveness of the image zooming with the proposed approach. The resulting images are compared to the results obtained by the bilinear and bicubic image interpolation methods. Fig. 11 and 15 represent the original low resolution images. In figs. 12 – 14 the enlarged versions of the indicated rectangular area (see Fig. 11) can be followed, obtained by the above mentioned interpolation techniques and by the proposed one. Figs. 16 – 19 stand for the same sequence, but regarding the indicated rectangular area depicted in Fig. 15.

VI. Conclusion

In the present paper a new image representation domain and reconstruction technique has been introduced. The results show that how the efficiency of the certain tasks depends on the applied domain. Image zooming has been performed using the proposed technique and has been compared to other well known image interpolation methods. Using this technique the resulted image maintains the edges more accurately than the

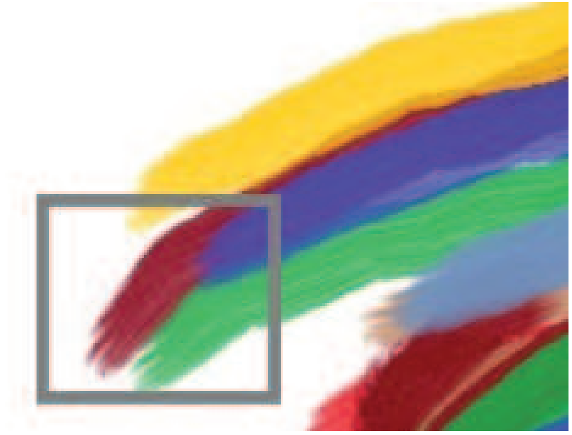

Fig. 11. The original image.

Fig. 12. The 10x enlarged rectangular area using bilinear interpolation.

Fig. 13. The 10x enlarged rectangular area using bicubic interpolation.

Fig. 14. The 10x enlarged rectangular area using the proposed HOSVD-based method. Smoother edges can be observed.

Fig. 15. The original image.

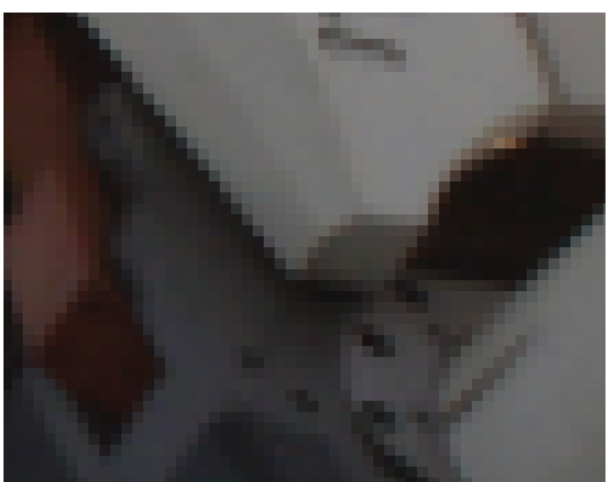

Fig. 16. The 10x enlarged rectangular area using Nearest-neighbor interpolation method.

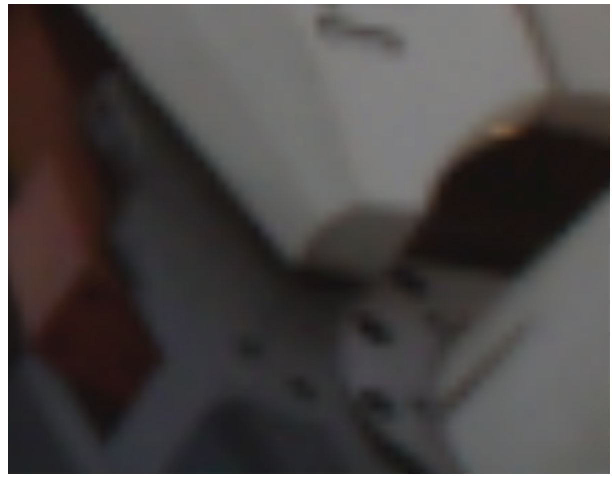

Fig. 17. The 10x enlarged rectangular area using bilinear interpolation.

Fig. 18. The 10x enlarged rectangular area using bicubic interpolation.

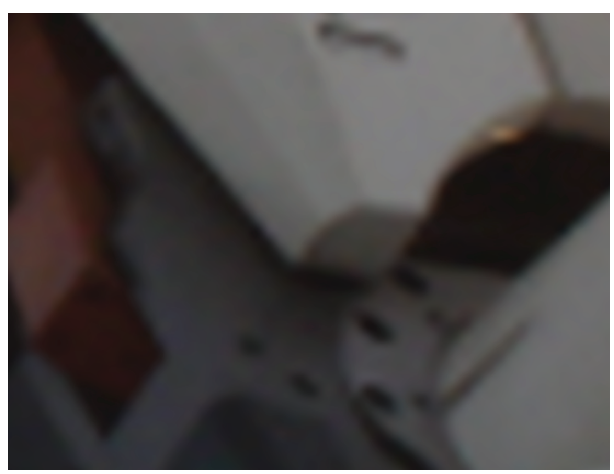

Fig. 19. The 10x enlarged rectangular area using the proposed HOSVD-based method. Smoother edges can be observed.

other well-known image interpolation methods. Furthermore, some properties of the proposed representation domain have been compared to the corresponding properties of the Fourier-based approximation. The results show that in the proposed domain some tasks can be performed more efficiently then in other domains. The proposed form of image representation can efficiently be applied also for image compression and noise filtering.

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