Radial Approach to the Emergency Public Service System Design with Generalized System Utility

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***Abstract*—** The paper is focused on methods of the public service system design, where the generalized utility is considered instead the usual disutility represented by distance. On the contrary to the former formulations, the generalized utility defined for a public service system assumes that user's utility comes generally from more than one located service center and the individual contributions from relevant centers are weighted by reduction coefficients depending on a center order. Real instances of the public service system design problem are characterized by a big number of possible facility locations. The classical approaches to the related problems make use of location-allocation model. Complexity of location-allocation problems considerably grows with the number of possible locations and so commercial IP-solvers often fail due to enormous computational time or extreme memory demands. This drawback can be overcome by the approximate covering approach based on so called radial model of the problem. Within this paper, we suggest radial formulation of the public service system design with the generalized utility and compare the approaches used location-allocation model with those, which are based on the radial formulation.

***Keywords*—** Approximate approach, generalized utility, public service system design, radial model.

I. Introduction

THE design of almost any public service system [3], [6], [11], [12], [16] includes determination of center locations, from which the associated service is distributed to all users of the system. Source of the service must be usually concentrated to a limited number of centers due to economic reasons, regardless of the case whether the service is delivered to users or the users usually travel for the service to some center. Thus the public service system structure is formed by the deployment of limited number of service centers and the associated objective in the standard formulation is to minimize some sort of disutility as the social costs, which are proportional to the distances between served objects and the nearest service centers.

A substantial drawback of the original disutility minimization is the linear proportionality of the disutility on the distance. This type of dependence may be approved only if cost of transport is considered, but it can be hardly accepted in the case of the utility, which is perceived by a user in emergency as fire, heart attack etc. In such cases, perceived utility of the service for an afflicted user sharply drops, when the service is delivered after some time-threshold.

Another simplification often used by service system designers constitutes in the assumption that a user is serviced from the nearest located service center or from the center, which offers the biggest utility to the user. The simplification can be used, if the structure of a public administration system is designed, but the assumption does not hold, when an emergency service system is designed due to random occurrence of the demand for service and limited capacity of the service centers. At the time of the current demand for service, the nearest service center may be occupied by some other user. When this situation occurs, the last demand is usually serviced from the second nearest center or from the third nearest center, if the second one is also occupied. This way, the emergency public service system can be considered a queuing system, which servicing facilities are spread over the serviced area. As the underlying p-median problem is NP-hard even without the mentioned stochastic formulation, there is almost no chance to develop an algorithm of the emergency public service system design, which takes into consideration both location and queuing properties of the problem.

There were several attempts to comply the randomness of the demands by avoiding the queuing part of the problem [4] suggested so-called double coverage approach, which ensures that each user is located in the given radius from the nearest service center and that the number of users located in the radius from other service center is maximal. Based on the generalized concept of public service system utility [9], we suggested another approach to the emergency public service system design with the limited capacity of the service centers, when the location-allocation model was used to describe the problem for a subsequent optimization process. The associated solving technique for the modeled service design problem can be performed by various disposable tools. Depending on scientific background and information support a designer can

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choose either the way, when the resulting decision support tool is "tailored" directly to his/her concrete problem, or the way, when a commercial IP-solver is used. When the first way is followed, the tool provider can base the developed tool either on special exact location method [1], [7], [14] or he/she can make use of an algorithm from the broad family of metaheuristics [15], [17], [18], [19]. In the both cases, the design, development, programming and testing of the tool take term of several month. The second way makes use of ready commercial IP-solvers [9]. This way avoids the long time of the tool development and thus the time of an application can be considerably reduced. That is why; we concentrate on the second way in this paper, which includes usage of a commercial IP-solver for the emergency public service system design.

Concentrating on the commercial IP-solver usage, a potential designer must face the complexity of the problem, when an optimal solution is sought. It was found that the number of possible service center locations seriously impacts the computational time in location-allocation models [13]. The necessity of solving larger instances of the design problem leads to the approximate approach, which can enable to solve real-sized problems in admissible time, what was proved for the classical p-median problems by [1], [5], [7], [10]. The suggested approximate approach is adjusted to the generalized utility model. The approach is based on the upper bound minimization and performs as a heuristic, where the lower bound of the optimal value of the objective function is easy to obtain.

The remainder of the paper is organized as follows. Section 2 introduces a utility contribution function which models the contribution of a given service center to utility of a given user. Section 3 describes the generalized model of individual user's utility considering more than one contributing centers and Section 4 describes the location-allocation mathematical programming models for the generalized utility. Section 5 contains the radial formulation of the problem and the associated approximate approach. Sections 6 and 7 contain numerical experiments, comparison of the both approaches and the final conclusions.

II. Service Center Contribution to User's Utility

The original approaches to the public service system are based on the total social cost minimization. The total cost expensed by one user to reach a source of service is derived from user's distance to the nearest service center. It is considered that the social cost depends on the distance linearly. The social cost is often replaced by the notion of user's disutility, which is also proportional to the distance between the user location and the nearest service center location. This way, the original approaches lead to formulating and solving the classical p-median problem. Contrary to the original approaches, we do not consider user's disutility based on social cost, but we suppose that a service center located somewhere in the neighborhood of a user contributes to the total general utility perceived by the user. The size of contribution depends on the time-distance between the user and the nearest service center, but this dependence is not linear. If the time-distance is small, the utility contribution is near to some maximal value u^{max}, and then it slowly decreases. When the time distance comes near to some threshold t_{krit}, in the neighborhood of the threshold, the value of utility contribution sharply drops to a neighborhood of zero, and then it asymptotically converges to zero. To model the above described dependence, we introduce the utility contribution function *u(t)*. The function *u(t)* depends on the time distance *t* of the user from the service center accordingly to the expression described by (1).

$$u(t) = \frac{C_0}{1+e^{\frac{t-t_{krit}}{T}}} \tag{1}$$

The symbol t_{krit} represents some time-threshold, where the utility contribution from the service center decreases most steeply. The positive shaping parameter *T* makes the decrease of the function steeper if it takes a value near to zero, what is demonstrated in Fig. 1 for three different values of the parameter. Using different values of *T* the shape of the utility contribution function may change from almost linear function to a step function.

The constant C_0 determines the maximal value u^{max} of the contribution accordingly to (2).

$$u^{\max} = \frac{C_0}{1+e^{\frac{-t_{krit}}{T}}} \tag{2}$$

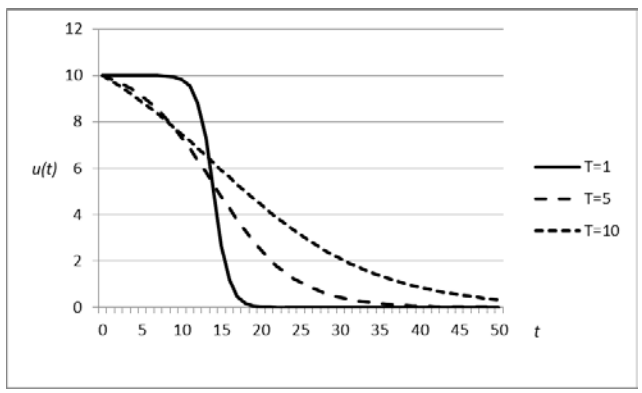

Fig. 1 various shapes of the utility contribution function

III. Generalized Model of User's Utility

The generalized model of the public service system utility for an individual user is based on the summation of utility contributions from a given number of located service centers.

If I_1 denotes the set of all located service centers in the public service system and t_{ij} denotes the travelling time from any user located at position *j* to the service center location *i*, then the utility of the system for each user located at *j* can be expressed by the following terms:

Let us introduce the mapping $\Theta : R^n \rightarrow R^n$ such that a vector $[u_1, u_2, \ldots, u_n]$ is mapped to the vector $[u_{\tau(1)}, u_{\tau(2)}, \ldots, u$

$_{\tau(n)}$], where the values of the components decrease $u_{\tau(1)} \geq u_{\tau(2)} \ldots \geq u_{\tau(n)}$. Then, the symbol $\Theta_k(u_i : i=1, \ldots, n)$ denotes the k-*th* component of the resulting n-tuple. If this denotation is applied to a set $\{u_k : k=1..n\}$, then is the biggest element and $\Theta_n(u_i : i=1, \ldots, n)$ is the smallest one of the set.

Let us denote $U_j(I_1)$ the utility, which a user located at j obtains from the public service system given by the set I_1 of located service centers. The generalized utility $U_j(I_1)$ is defined by (3), where r denotes the given number of service centers, which take part on the utility for the user at j.

$$U_j(I_1) = \sum_{k=1}^{r} q_k \Theta_k(u(t_{ij}) : i \in I_1) \tag{3}$$

The reduction coefficients q_k for $k=1..r$ are positive real values, which meet the following inequalities $q_1 \geq q_2 \geq \ldots \geq q_r$. These coefficients reduce the individual contributions from the relevant r biggest contributors accordingly to size of the contributions. I.e. the biggest contribution is reduced less than the second biggest contribution; the second contribution is reduced less than the third biggest contribution etc. If we realize that the biggest contribution comes from the nearest service center due to property of $u(t)$, then the coefficients q_k can model the situation when more distant service center is less important for the user not only for the distance but even for the fact that some other service center is placed closer to the user.

The influence of the reduction coefficients is demonstrated in Fig. 2, where full big circles denote the located service centers and small full circles denote user locations. The demonstration is performed for parameter values $r = 2$, $q_1 = 1$, and $q_2=0.5$.

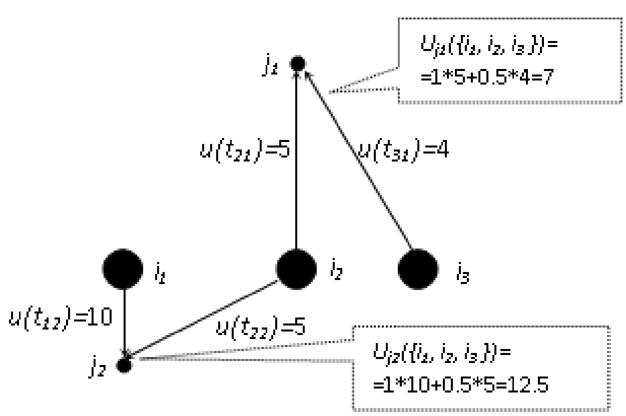

Fig. 2 enumeration of the generalized utility with two reduction coefficients

The public service system design problem with the system optimal utility for users is formulated as the task of service centers determination so that the sum of user utilities is maximal and the total number of located centers does not exceed a given number p. To describe the problem, we denote by J the set of user locations and by I the set of possible center locations. Let b_j denote the number of the users located at j and let $|I_1|$ denote cardinality of the set I_1 of locations occupied by service centers. Then, the problem can be formulated in the following combinatorial form.

$$\max\{\sum_{j \in J} b_j U_j(I_1) : I_1 \subset I, |I_1| \leq p\} \tag{4}$$

In the combinatorial formulation, such a set I_1 of at most p possible locations is searched for, to maximize sum of utilities over all users.

IV. System Utility Optimal Design Problem

To formulate the public service system design problem (4) with the system optimal generalized utility taking into consideration r contributing service centers, we denote the set of users' locations by symbol J as above, and the set of possible service center locations is denoted by symbol I.

At most p locations from I must be chosen so that the sum of users' utilities is maximum. The network time distance of a possible location i from user location j is denoted as t_{ij}. The decisions, which determine the designed system, can be modeled by further introduced decision variables. The variable $y_i \in \{0,1\}$ models the decision on service center location at place I, where $i \in I$. The variable takes the value of 1 if a facility is located at i and it takes the value of 0 otherwise.

In addition, the allocation variables $z_{ijk} \in \{0,1\}$ are introduced for each $i \in I$ and $j \in J$ and $k=1, \ldots, r$ to describe by the value of one that the service center located at i contributes to the utility of each user located at j as the k-*th* biggest contributor. Then the location-allocation model of the problem (4) can be written as follows.

$$\textit{Maximize} \quad \sum_{j \in J} b_j \sum_{k=1}^{r} q_k \sum_{i \in I} u(t_{ij}) z_{ijk} \tag{5}$$

$$\textit{Subject to} \quad \sum_{i \in I} y_i \leq p \tag{6}$$

$$z_{ijk} \leq y_i \quad \textit{for } i \in I,\ j \in J,\ k = 1,\ldots,r \tag{7}$$

$$\sum_{i \in I} z_{ijk} = 1 \quad \textit{for } j \in J,\ k = 1,\ldots,r \tag{8}$$

$$\sum_{k=1}^{r} z_{ijk} \leq 1 \quad \textit{for } j \in J,\ i \in I \tag{9}$$

$$y_i \in \{0, 1\} \quad \textit{for } i \in I \tag{10}$$

$$z_{ijk} \in \{0, 1\} \quad \textit{for } i \in I,\ j \in J,\ k = 1,\ldots,r \tag{11}$$

In the model, the objective function (5) gives the system utility value. The constraint (6) limits the number of located facilities by p. The link-up constraints (7) assure that the users' locations are assigned by variable z_{ijk} only to the located service centers. The allocation constraints (8) ensure that each user location j for each subscript k is assigned to exactly one of the possible service center locations. The constraints (9) prevent a solver against multiple assigning service center location i to user location j for different subscript values of k.

A feasible solution depicted in Fig. 2 is demonstrated in Fig. 3 using the above-mentioned variables, where only non-zero variables are shown.

Each feasible solution $(\boldsymbol{y}, \boldsymbol{z})$ may locate at most p service centers due to (6) and it is not allowed to assign any user location j for any subscript (order) k to the possible service center location I unless a service center is located at location i

due to (7). It follows from (8) that exactly one service center location is assigned to a given user *j* for the given order *k*. In addition, the service center location assigned to the user *j* must differ for different *k*. The proper assignment of service center location to the given order *k* for given *j* is ensured by the maximization process.

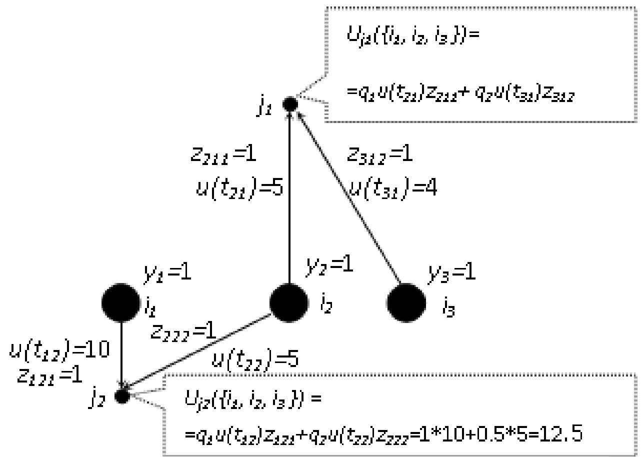

Fig. 3 demonstration of a feasible solution of the problem (5)−(11)

In the case, when the reduction coefficients are the same, the model (5) – (11) can be reduced by the following way, where the allocation variables $z_{ij}\in\{0,1\}$ for each $i\in I$ and $j\in J$ are introduced to describe by the value of one that the service center located at *i* contributes to the utility of each user located at *j*. The location variables y_i for $i\in I$ are used in the same meaning as above. Then the location-allocation model can be written as follows.

$$\text{Maximize} \quad \sum_{j\in J} b_j \sum_{i\in I} u(t_{ij}) z_{ij} \tag{12}$$

$$\text{Subject to} \quad \sum_{i\in I} y_i \le p \tag{13}$$

$$z_{ij} \le y_i \quad for\ i\in I,\ j\in J \tag{14}$$

$$\sum_{i\in I} z_{ij} = r \quad for \quad j\in J \tag{15}$$

$$y_i \in \{0, 1\} \quad for\ i\in I \tag{16}$$

$$z_{ij} \in \{0, 1\} \quad for\ i\in I,\ j\in J \tag{17}$$

In this simpler model, the objective function (12) gives the system utility value. The constraint (13) limits the number of located facilities by *p*. The link-up constraints (14) assure that the users' locations are assigned by variable z_{ij} only to the located service centers. The constraints (15) ensure that each user's location is assigned to exactly *r* of the possible service centers.

Note that both models can The models can be rewritten to the minimization problems by introducing so called disutility contribution $d_{ij}=u^{max}-u(t_{ij}\ )$. We can derive the following equality for objective function (5) under constraints (8).

$$\begin{aligned}\sum_{j\in J} b_j \sum_{k=1}^{r} q_k \sum_{i\in I} d_{ij} z_{ijk} &= \\ = u^{\max} \sum_{k=1}^{r} q_k \sum_{j\in J} b_j - \sum_{j\in J} b_j \sum_{k=1}^{r} q_k \sum_{i\in I} u(t_{ij}) z_{ijk}\end{aligned} \tag{18}$$

Similar derivation with (12) under constraints (15) gives equality (19).

$$\begin{aligned}\sum_{j\in J} b_j \sum_{k=1}^{r} q_k \sum_{i\in I} d_{ij} z_{ijk} &= \\ = u^{\max} \sum_{k=1}^{r} q_k \sum_{j\in J} b_j - \sum_{j\in J} b_j \sum_{k=1}^{r} q_k \sum_{i\in I} u(t_{ij}) z_{ijk}\end{aligned} \tag{18}$$

Now we can deal with of the problem (20), (6)-(11) instead of (5) – (11), or we can use model (21), (13)-(17) instead of (12) – (17).

$$\text{Minimize} \quad \sum_{j\in J} b_j \sum_{k=1}^{r} q_k \sum_{i\in I} d_{ij} z_{ijk} \tag{20}$$

Subject to (6)-(11).

$$\text{Minimize} \quad \sum_{j\in J} b_j \sum_{i\in I} d_{ij} z_{ij} \tag{21}$$

Subject to (12)-(17).

To obtain optimal decisions on service center locations in a serviced area, some of the mathematical programming methods can be used. The both location-allocation models constitute such mathematical programming problems, which resist to any attempt at fast solution for bigger instances. Nevertheless, it is known that large instances of the covering problem are easy to solve by common optimization software.

V. Radial Formulation for Generalized Utility

The necessity of solving large instances of the p-median problem has led to the radius formulation [1], [2], [5], [7]. This approach avoids assigning the individual user location to some of located service centers and deals only with information, whether some service center is or is not located in a given radius from the user. This approach leads to the model similar to the set covering problem, which is easily solvable even for large instances by a common optimization software tools. We made use of this concept and adapted it to the problems (20), (6)-(11) and (21), (13)-(17).

In these problem formulations, there is minimized sum of disutility values d_{ij} computed for each pair (*i*, *j*) of a possible center location $i\in I$ and a user location $j\in J$. The disutility values form a matrix $\{d_{ij}\}$. Both the discussed location-allocation models use, the allocation variables $z_{ijk}\in\{0,1\}$ or $z_{ij}\in\{0,1\}$ to determine the disutility values, which enter the objective function value (20) or (21) respectively. The keystone of the approximate approach consists in the assignment relaxation of a possible service center to a user because these assignments require introducing the big series of the allocation variables. In the following approximate approach we try to approximate the disutility value for a user and the associate service centers unless the possible center locations must be determined. To this purpose, we partition the range [0, $max\{ d_{ij}: i\in I, j\in J \}$] of all possible disutility values of the former location-allocation problems into v+1 zones. The zones are separated by finite ascending sequence of values D^1, D^2, …, D^v referred as the dividing points, where $0 = D_0 < D_1$ and $D_v < D_m = max\{ d_{ij}: i\in I, j\in J \}$. We introduce a numbering of these zones so that the zone s corresponds with the interval (D_s, D_{s+1}], the zone with subscript 0 corresponds with the interval (D_0, D_1] and so on, till the v-th zone, which corresponds with interval (D_v, D_m]. A width of the s-th interval is denoted by e_s for s= 0, …, v. To describe the system of radii formed by the system of dividing points, a system of zero - one constants is defined so that the constant a_{ij}^s is equal to 1 if and only if the disutility contribution d_{ij} for a user from location j from the possible center location i is less or equal to D_s, otherwise a_{ij}^s is equal to 0. The relation between the system of dividing points and the system of constants is shown in Fig. 4, where u^{max} is set at the value of 11 to be consistent with utility values used in Fig. 2 and Fig. 3.

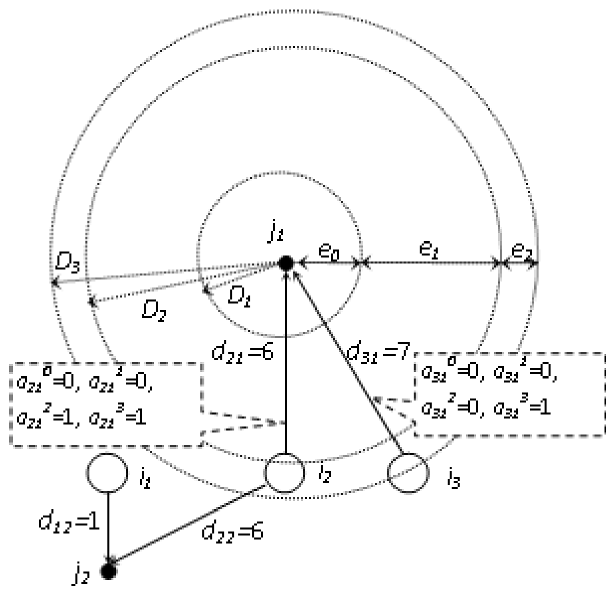

Fig. 4 system of radii and elements of incidental matrix $\{a_{ij}^s\}$

Using the above introduced location variables $y_i\in\{0,1\}$ for $i\in I$, where variable y_i takes the value of 1 if a facility is located at i and it takes the value of 0 otherwise, the expression (22) models the number of the service centers located in the radius D_s from the user location j.

$$\sum_{i\in I} a_{ij}^s y_i \tag{22}$$

If an auxiliary integer variable w_{js} is introduced, used in constraint (23) and pushed down by an optimization process, then the variable obtains the value equal to the number of service centers, which belong to the r closest centers to j and lie outside the radius D^s.

$$w_{js} + \sum_{i\in I} a_{ij}^s y_i \geq r \tag{23}$$

This way, the resulting value of w_{js} gives information that w_{js} relevant disutility values are bigger than D_s. Then, lower and upper bounds of the sum $d_{i1,j}+ d_{i2,j} +…+ d_{ir,j}$ of relevant disutility values from j to the r nearest service centers i_1, i_2, …, i_r can be expressed as $e_0w_{j1} + e_1w_{j2} + e_2w_{j3} + e_3w_{j4} +…+ e_{v-1}w_{jv}$ and $e_0w_{j0} + e_1w_{j1} + e_2w_{j2} + e_3w_{j3} +…+ e_vw_{jv}$ respectively.

The approximation is demonstrated in Fig. 5, which is derived from system of radii depicted in Fig. 4 and for service center location from Fig. 3. It can be found that it is possible to express the lower and upper bounds of sum $d_{21}+ d_{31}$ of relevant utilities as $e_0w_{j1} + e_1w_{j2} + e_2w_{j3}= 2e_0+e_1$ and $e_0w_{j0} + e_1w_{j1} + e_2w_{j2}=2e_0+2e_1+e_2$ respectively.

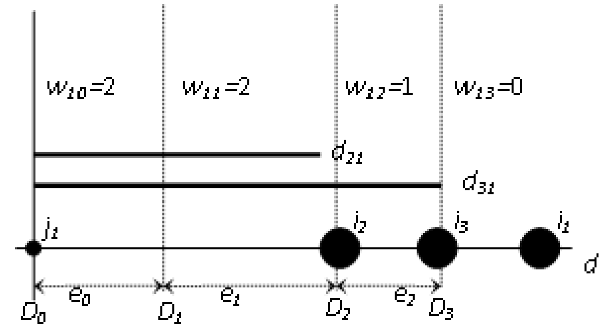

Fig. 5 sum of relevant disutility values for user 1

After these preliminaries the model (21), (13) – (17) can be rewritten into the following radial form, in which the upper bound of (21) is minimized. A radial-type weighted covering model can be formulated similarly to [7], [10] as follows:

$$Minimize \sum_{j\in J} b_j \sum_{s=0}^{v} e_s w_{js} \tag{24}$$

$$Subject\ \ to \sum_{i\in I} y_i \leq p \tag{25}$$

$$w_{js} + \sum_{i\in I} a_{ij}^s y_i \geq r \quad for\ j\in J,\ s=0,...,v \tag{26}$$

$$y_i \in \{0, 1\} \quad for\ i\in I \tag{27}$$

$$w_{js} \in Z^+ \quad for\ \ j\in J,\ s=0,...,v \tag{28}$$

In the model (24) – (28), the objective function (24) gives the upper bound of the sum of disutility values. The constraint (25) puts a limit p on the number of located facilities. The constraints (26) ensure that variable w_{js} expresses the number of the service centers outside the radius D_s from the user location j, which remains to the number r.

We note that the location – allocation model (21), (13) – (17) is only simplification of the model (20), (6)-(11), in which the more general disutility is taken into consideration. In the model (20), (6)-(11), the reduction coefficients q_k are used to weight the smallest relevant disutility by the biggest coefficients, the second smallest disutility by the second biggest coefficient etc. To be able to distinguish and weight the individual relevant disutility values, we introduce auxiliary zero-one variables x_{jsk} for $j\in J$, s=0, …, v, k=1, …, r and express each variable w_{js} by sum of variables x_{jsk} over k=1, …, r. Then, we approximate the problem (20), (6)-(11) by model (29)-(33).

$$Minimize \sum_{j\in J} b_j \sum_{s=0}^{v} e_s \sum_{k=1}^{r} q_k x_{jsk} \qquad (29)$$

$$Subject \ \ to \ \sum_{i\in I} y_i \le p \qquad (30)$$

$$\sum_{k=1}^{r} x_{jsk} + \sum_{i\in I} a_{ij}^{s} y_i \ge r \quad for \ j\in J,\ s=0,\dots,v \qquad (31)$$

$$y_i \in \{0, 1\} \quad for \ i \in I \qquad (32)$$

$$x_{jsk} \in \{0,1\} \quad for \ \ j\in J,\ s=0,\dots,v,\ k=1,\dots,r \qquad (33)$$

The constraints (31) ensure that the sum of variables x^{k}_{js} over k expresses the number of the service centers outside the radius D_s from the user location j, which remains to the number r. The constraint (30) has the same meaning as constraint (25).

Validity of the assertion that the expression (34) is an upper bound of the sum $q_1d_{i1,j}+ q_2d_{i2,j} +\dots+ q_rd_{ir,j}$ of weighted relevant disutility values from j to the r nearest service centers $i_1, i_2, \dots, i_r$, follows from the next reasoning.

$$\sum_{s=0}^{v} e_s \sum_{k=1}^{r} q_r x_{jsk} \qquad (34)$$

As can be seen in the definition of w_{js} the portion of sum of disutility values for the interval $(D_s, D_{s+1}]$ is represented by product e_sw_{js}. The number w_{js} is expressed as a sum of r zero-one variables x_{jsk} over k=1, …, r, where exactly w_{js} of them is equal to one. As individual variables x_{jsk} are not addressed to service centers or any other objects, then those of the variables x_{jsk} get the values of one, which correspond with the smallest coefficients q_k.

As concern the model sizes, we can easily find that the location – allocation model (21), (12)-(17) uses $|I|*(|J|+1)$ decision variables and $(|I|+1)*|J|+1$ constraints, where $|I|$ denotes number of the possible service center locations and $|J|$ denotes the number of user locations. The corresponding approximate model (24)-(28) contains $|I|+(v+1)*|J|$ variables and $(v+1)*|J|+1$ constraints. It follows that if we want to keep the approximate model at moderate size, then the number v of the dividing points must be in order less than the number $|I|$ of possible locations.

The location–allocation model (20), (6)-(11) uses $|I|*(r*|J|+1)$ decision variables and $(r+1)*|I|*|J|+r*|J|+1$ constraints. The corresponding approximate model (29)-(33) contains $|I|+r*(v+1)*|J|$ variables and $(v+1)*|J|+1$ constraints. Even here the number v of the dividing points must be in order less than the number of possible locations.

VI. Numerical Experiments

To compare the both approaches to the public service system design with the generalized utility with and without reduction coefficients, we suggested two groups of numerical experiments. In the first group of experiments, the location – allocation and approximate approaches without reduction coefficients q_k are compared, i.e. models (21), (12)-(17) and (24)-(28) are used and the associated computational processes are run. In the second group, we deal with generalized utility, which comprises the general reduction coefficients q_k, i.e. models (20), (6)-(11) and (29)-(33) are used. Each group consists of five series of experiments, which differ in used form of the utility contribution (1), where t_{krit} is set to one of the values 10, 12, 14, 16 and 18 minutes for an individual series. Each series contains five instances differentiating in the number r of service centers contributing to the utility of a user. The number varies from one to five in each series. The utility expression (2) in the first group of experiments used the same reduction coefficients, where q_k=1. In the second group of experiments, the values of q_k decrease with the subscript k.

In the experiments, the shaping parameter T was set to the value of 1 and the coefficient C_0 was set to the value of 100. All instances were derived from real emergency health care system, which was originally designed for region of Zilina. The system covers demands of 315 communities - towns and villages spread over the region by p=31 ambulance vehicles, where each of them being located represents one service center. The communities were considered as elements of the set J of users' locations and also as elements of the set I of possible service center locations. The time distances t_{ij} were computed using the road network distances for the average speed of 60 kilometer per hour. The dividing points for the approximate approach were deployed equidistantly in this preliminary experiments and the number v of dividing points was set at the value of 20.

To solve the problems described by models, the optimization software FICO Xpress 7.3 (64-bit, release 2012) was used and the experiments were run on a PC equipped with the Intel® Core™ i7 2600 CPU processor with the parameters: 3.4 GHz and 16 GB RAM.

The obtained results and associated computational times of the first group of experiments are plotted into tables 1-5 accordingly to increasing value of t_{krit}.

Table 1 *Results of experiments for* $t_{krit} = 10$

r	TL	ObjL	TR	ObjR	Hm	Gap
1	6.1	646260	0.2	646260	14	0.4
2	4.9	1089140	0.2	1089140	8	0.1
3	5.3	1369610	0.2	1366819	6	0.2
4	5.8	1576450	0.2	1576450	4	0.0
5	6.3	1752843	0.2	1751980	4	0.0

After the optimal service center locations were determined by the both approaches for a given instance, the associated objective function values were computed accordingly to (5), to be able to compare them.

Table 2 *Results of experiments for* $t_{krit} = 12$

r	TL	ObjL	TR	ObjR	Hm	Gap
1	7.9	674443	0.5	672919	22	0.2
2	6.1	1186469	0.6	1181554	10	0.4
3	5.6	1527969	0.2	1526927	10	0.1
4	6.5	1767670	0.2	1767003	6	0.0
5	6.5	1967083	0.2	1966327	4	0.0

Table 3 *Results of experiments for* $t_{krit} = 14$

r	TL	ObjL	TR	ObjR	Hm	Gap
1	9.2	686360	0.2	685350	20	0.1
2	5.3	1261935	0.2	1260535	8	0.1
3	5.6	1661851	0.2	1657934	8	0.2
4	7.3	1958645	0.2	1956450	14	0.1
5	6.2	2199235	0.3	2199235	0	0.0

Table 4 *Results of experiments for* $t_{krit} = 16$

r	TL	ObjL	TR	ObjR	Hm	Gap
1	6.8	690018	0.3	689142	30	0.1
2	5.3	1315748	0.4	1314503	16	0.1
3	5.9	1778564	0.3	1777925	6	0.0
4	5.7	2145949	0.3	2145949	0	0.0
5	6.5	2439842	0.3	2439756	2	0.0

Table 5 *Results of experiments for* $t_{krit} = 18$

r	TL	ObjL	TR	ObjR	Hm	Gap
1	6.1	690905	0.3	690174	39	0.1
2	5.5	1352035	0.3	1352030	14	0.1
3	5.9	1878611	0.6	1875151	12	0.2
4	5.7	2313446	0.3	2313360	2	0.0
5	6.5	2665325	0.3	2665325	0	0.0

The rows of tables correspond to individual values of the parameter *r*, what is the number of located service centers, which are taken into account as contributors to the user's utility. The columns of the tables correspond with the resulting characteristics. The labels "TL" and "TR" denote the obtained computational times in seconds for the location-allocation and the approximate radial approach respectively. The associated optimal objective functions (system utility) for the approaches are denoted as "ObjL" and "ObjR". To describe the further differences between obtained designs, we evaluated also so called Hamming distances between the resulting vector ***y*** obtained for the location-allocation approach and the vector of location variables obtained for the approximate covering approach. This parameter is referred as "Hm". The label "Gap" denotes the difference between objective function value obtained by the approximate approach and the exact objective function value by the location-allocation approach. This difference is given in percentage, where the exact objective function value represents hundred percent.

The results of the second group of experiments, where optimization of problems (20), (6)-(11) and (29)-(33) were run, are plotted into tables 6-10, which are organized in the same way as the first group tables. The second group of experiments were performed for the same parameters as the first group but the reduction coefficients, which was set accordingly to $q_k=1/k$ for $k=1,\dots, r$ in the individual instances. After the optimal service center locations were determined by the both approaches for a given instance, the associated objective function values were computed accordingly to (12), to be able to compare them.

Table 6 *Results of experiments for* $t_{krit} = 10$

r	TL	ObjL	TR	ObjR	Hm	Gap
1	7.3	646260	0.2	643793	14	0.4
2	17.7	844986	0.3	844882	2	0.0
3	29.7	921069	0.5	919430	10	0.2
4	44.6	956182	0.5	955376	4	0.1
5	61.2	971100	0.6	970354	6	0.1

Table 7 *Results of experiments for* $t_{krit} = 12$

r	TL	ObjL	TR	ObjR	Hm	Gap
1	9.1	674443	0.4	672919	22	0.2
2	21.4	912955	0.9	912284	10	0.1
3	30.4	1013899	0.4	1013845	2	0.0
4	45.8	1057018	0.5	1056215	8	0.1
5	61.2	1076367	0.6	1076323	2	0.0

Table 8 *Results of experiments for* $t_{krit} = 14$

r	TL	ObjL	TR	ObjR	Hm	Gap
1	10.7	686360	0.2	685350	20	0.1
2	18.9	964036	0.4	962622	10	0.1
3	30.3	1084417	0.5	1082795	8	0.1
4	46.8	1143450	0.6	1142194	10	0.1
5	63.5	1173001	0.6	1172127	8	0.1

Table 9 *Results of experiments for* $t_{krit} = 16$

r	TL	ObjL	TR	ObjR	Hm	Gap
1	8.1	690018	0.3	689142	30	0.1
2	18.9	998316	0.4	997380	18	0.1
3	32.8	1140027	1.2	1138179	12	0.2
4	48.3	1217572	0.6	1216298	10	0.1
5	60.7	1260818	0.6	1260604	2	0.0

Table 10 *Results of experiments for* $t_{krit} = 18$

r	TL	ObjL	TR	ObjR	Hm	Gap
1	7.4	690905	0.3	690174	39	0.1
2	18.3	1020139	0.4	1019544	20	0.1
3	33.2	1185276	0.5	1183473	18	0.2
4	59.9	1282263	0.6	1281181	8	0.1
5	61.8	1337663	0.7	1337159	2	0.0

The odd integers of the Hamming distance in tables 5 and 10 for instances with $r=1$, were caused by the solutions of the approximate problems, where the constraints (25) and (30) were fulfilled as inequalities on the contrary to the exact problems, where the associated constraints were fulfilled as equalities..

VII. Conclusion

We have explored solvability of the generalized utility formulation for the public service system design and tried to find the loss of accuracy, under which the exact approach can be replaced by the approximate covering approach. Comparing the reported results in Tables 1-5 and 6-10, we can conclude that the suggested approximate approach is able to solve the instances almost at the same accuracy as the original approach. The corresponding gap was less than half percent in each solved instance and in the most of instances was less than two tenths of percent. The average computational time of the approximate approach was at least ten times smaller than the time of the exact approach. Further research in this field can be focused on a non-equidistant deployment of the dividing points.

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