Stabilized column generation for the construction of Rotations with resource constraints

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Abstract— **Column generation has proven to be efficient in solving the linear programming relaxation of large scale instances of the construction of Rotations with resource constraints. However difficulties arise when the instances are highly degenerate. Recent research has been devoted to accelerate column generation while remaining within the linear programming framework. This paper presents an efficient approach to solve the linear relaxation of the construction of Rotations with resource constraints. It combines column generation, preprocessing variable fixing, and stabilization. The outcome shows the great potential of such an approach for degenerate instances.**

Keywords- column generation, crew pairing problem, linear programming

I. Introduction

In the airline industry, optimization and automation of the construction crew rotations is a major financial and organizational challenge. The problem is to cover cost of all flights of the company, programmed over a given time, with crews trained staff cockpit (pilot, co-pilots) and flight attendants (stewardesses, stewards) . At intervals of several days (of the order of the week), each crew from the base to which he is assigned, connects a number of flights and returned to base. This sequence of flights back to the base is called rotation. The construction of rotations of an airline is extremely restricted by international regulations, national and domestic labor, and the limited availability of resources.

These constraints make the problem particularly difficult to solve. The use of models and optimization software for this problem enables large companies to make substantial financial gains. It is not uncommon that a reduction of one percent on the total cost of rotations translates into tens of millions of dollars of savings for large companies [1], where research, basic and applied abundant on the subject. The general problem of construction of Rotations with resource constraints (RC-PCR) can be formulated as a feasible flow problem minimum cost in a multiple network, with additional variables and constraints of resources.

Finally, note that resource constraints make the problem (PCR-CR) NP-hard. The terms of network construction rotations eligible for calculating the cost of rotations and the associated mathematical program are presented in Section 2, Section 3 provides an overview of classical techniques of resolution, including the method of column generation, whose associated sub problem is treated in Section 4, in addition to our contribution. Section 5 presents some numerical results. To Section 6 concludes the paper.

II. Presentation of the problem

All flights covered by the crews is denoted V = (1, ..., N). The flight schedule and schedules are established partners on an almost-certain about the period of the order of the month or week depending on company size. The term flight Associate each element i ∈ Vest in some cases unfair, since i can actually represent a sequence of flights aggregate and indivisible, i.e., can not be covered by a single crew in its entirety. Often too, the task to cover a crew is not the only flight but a flight service may start before and end after the flight itself, in order to include the time of preparing the aircraft and the accompanying time passengers, for example. However, we maintain this terminology theft, for readability. We know for each flight i ∈ V:

(i) the departure time $t^{\nearrow}(i)$,
(ii) the time of arrival $t^{\searrow}(i)$,
(iii) the airport of departure $a^{\nearrow}(i)$,
(iv) the airport of arrival $a^{\searrow}(i)$.

A rotation should begin and end at one of the bases of the company. The set B of bases is usually composed of large platforms called interconnection hubs. The problem building rotations in the air has often resource constraints on rotations. To address these constraints, valid for each rotation individually, a classical model associates with each crew subnet constructed as follows.

The Problem of Construction of Rotation Constraint Resources (PCR-CR) can be modeled, if the cost function is linear, the Linear Programming in mixed variables. We have a feasible flow problem minimum cost on all subnets with varying binary variables and continuous flow of resources:

$$
\equiv \begin{cases}
\min \sum_{k=1}^{K} \sum_{(i,j)\in A^k} c_{ij}^k x_{ij}^k & (1)\\
\sum_{k=1}^{K} \sum_{j:(i,j)\in A^k} x_{ij}^k \geq 1 \ for\ i \in V = \{1,\ldots\ldots,n\} & (2)\\
\sum_{i:(o^k,i)\in A^k} x_{o^k i}^k = 1 \ for\ k \in \mathcal{K} & (3)\\
\sum_{i:(i,d^k)\in A^k} x_{id^k}^k = 1 \ for\ k \in \mathcal{K} & (4)\\
\sum_{i:(i,j)\in A^k} x_{ij}^k = \sum_{l:(j,l)\in A^k} x_{jl}^k \ for\ j \in V^k & (5)\\
T_i^{k,q} + t_i^{k,q} - T_j^{k,q} \leq M\left(1 - x_{ij}^k\right) \ for\ (i,j) \in A^k, k \in \mathcal{K}, q \in \mathcal{Q} & (6)\\
a_i^{k,q} \leq T_i^{k,q} \leq b_i^{k,q} \ for\ i \in V^k, k \in \mathcal{K}, q \in \mathcal{Q} & (7)\\
x_{ij}^k \in \{0,1\}, T_i^{k,q} \geq 0 \ for\ (i,j) \in A^k, k \in \mathcal{K}, q \in \mathcal{Q} & (8)
\end{cases}
$$

The objective (1) minimizes the total cost of rotations. Constraints (2) express the coverage of each flight by at least one crew if only one crew is allowed per flight is forced to equal. Constraints (3-5) define a path structure in the subnet G^k : transition from a flow unit (3 or 4) and flux conservation at the vertices (5). Constraints (6-7) are the constraints associated with each rotation. Constraint (1.6), where M> 0 is a parameter very large, can also be found under the following nonlinear form:

$$x_{ij}^k\left(T_i^{k,q} + t_i^{k,q} - T_j^{k,q}\right) \leq 0 \quad (i,j) \in A^k, k \in \mathcal{K}, q \in \mathcal{Q} \qquad (9)$$

The inequality in (6) or (9) states that the waiting is allowed for the crew, otherwise the constraint is written to equality. This constraint yields accumulated consumption of resource q at node j, since we have:

$$T_j^{k,q} = \max(a_i^{k,q}, T_i^{k,q} + t_i^{k,q})$$

Constraints (7) are constraint bounds at the nodes of the network (time windows for example). Note that the constraints (3-7) are local constraints valid for one subnet G^k.

Only coverage constraints (2) are global constraints linking the K sub-networks. The relaxation of these binding constraints and the decomposition of the original problem by sub-network will be an interesting option for resolution.

Finally, note that resource constraints (6-7) make the problem (PCR-CR) NP-hard. Even the feasibility problem associated is NP-complete.

III. SOLVING APPROACHES

A. Principles of decomposition

There are two types of constraints in the system (2) - (7):

(i) the coverage constraints (2), said bonding or global, binding all crews $k = 1, \ldots., K,$

(ii) the constraints (3) - (7) of each crew $k \in \{1, \ldots., K\}$ and defining a legal road.

The matrix associated with constraints (3) - (7) is block diagonal, and the objective (1) is separable (for linear), solving the continuous relaxation of this model may be based on the decomposition of Dantzig-Wolfe . In this type of decomposition, the constraints (3) - (7) define K independent sub-problems and global constraints (2) are stored in the master problem. In a schema type column generation, it is alternately solving the master problem and the K sub-problems. For a complete solution, this scheme can be applied at each node of the search tree. The major difficulty lies in solving sub-problems whose state space can grow exponentially with the number of resources Q, making essential use of heuristics. On the other hand, the convergence of the scheme of column generation is sensitive to the quality of solutions provided by the resolution of these sub-problems; the effective resolution of instances from real industry needs to find a good compromise between Quality solutions and time resolution of sub-problems. In what follows, we detail the general principle of column generation for the problem (PCRCR).

B. Column Generation, master problem and sub problem

The methods of column generation [3] have been successfully applied to problems of construction of rotations [2], [5]. In this approach, the master problem is reformulated by a coverage problem (PC) (Set Covering and Set Partitioning under the constraint of flight coverage is unequal or equal):

$$
(PC) \equiv \begin{cases}
\min \sum_{r\in\mathcal{R}} c_r x_r & (10)\\
\sum_{r\in\mathcal{R}} a_{ir} x_r \geq 1 \quad i \in V = \{1,\ldots\ldots,n\} & (11)\\
x_r \in \{0,1\} \qquad r \in \mathcal{R} & (12)
\end{cases}
$$

Or $\mathcal{R}$ designate all eligible rotations satisfying the resource constraints and sequence between flights, c_r represents the cost of the rotation $r \in \mathcal{R}$, $a_{ir} = 1$ if and only if the rotation r covers flight i, and the variable x_r indicates binary choice whether or not the rotation r in the solution.

We note $(\overline{PC})$ continuous relaxation of problem (PC) where the integrity constraints (12) are replaced by $x_r \geq 0$ for $r \in \mathcal{R}$. The total number of allowable rotations $|\mathcal{R}|$ is generally an exponential function of the number $n = |V|$ flights to cover the complete list of $\mathcal{R}$ is to be avoided. However, it is possible to quickly find a reasonable solution to optimal $(\overline{PC})$ without generating a small subset of rotations (i.e., columns of the matrix of constraints). The principle is as follows. $\mathcal{R}^0$ is a feasible solution for (PC), comprising a small number of rotations of $\mathcal{R}$, and generated by any heuristic. We can solve the Linear Programming (e.g., the Simplex algorithm) it $(\overline{PC}^0)$, which is the restriction of $(\overline{PC})$ the subset of rotations $\mathcal{R}^0$. This resolution also provides a vector of multipliers or dual variables $(\delta_1^0, \dots\dots, \delta_n^0)$ associated with no flights to be covered. The optimality criterion that all rotations are positive reduced cost at the optimum led to investigate the rotation of lower cost reduces negative, let

$$r^0 = arg \min_{r \in \mathcal{R}} \left(c_r - \sum_{i=1}^{n} \delta_1^0 a_{ir} \right) \qquad (13)$$

If we can find in reasonable time the rotation r^0 , then we can revive the resolution of the hedging program $(\overline{PC})$ on all $\mathcal{R}^1 = \mathcal{R}^0 \cup \{r^0\}$, adding column a_{r^0} to the matrix of constraints. Overall, we solve at each iteration t the restricted master problem $(\overline{PC}^t)$:

$$(\overline{PC}^t) \equiv \begin{cases} \min \sum_{r \in \mathcal{R}^t} c_r x_r & (14) \\ \sum_{r \in \mathcal{R}^t} a_{ir} x_r \geq 1 \quad i \in V = \{1, \dots\dots, n\} & (15) \\ x_r \geq 0 \qquad r \in \mathcal{R}^t & (16) \end{cases}$$

such that $\mathcal{R}^t = \mathcal{R}^{t-1} \cup \{r^{t-1}\}$
or, if δ^{t-1} denotes the vector of multipliers associated with no flights in the resolution$(\overline{PC}^{t-1})$, l the rotation r^{t-1} lower cost of reduced negative is defined by

$$r^{t-1} = arg \min_{r \in \mathcal{R}} \left(c_r - \sum_{i=1}^{n} \delta_i^{t-1} a_{ir} \right) \qquad (17)$$

The term generation of columns from the addition of column a_{r^t} to the matrix of constraints of the master problem at each iteration t. This iterative process of solving the master problem (14-16) and the sub-problem (17) is stopped when all rotations are positive reduced cost in solving the problem by - a sign that the continuous optimum is reached -- either iteration s as:

$$\min_{r \in \mathcal{R}} \left(c_r - \sum_{i=1}^{n} \delta_i^s a_{ir} \right) \geq 0$$

A variant of this method to accelerate the process in practice [6], is to add at each iteration a subset of rotations additional cost reduces negative instead of the single best rotation of sub-problem (17). The desired maximum size of this subset of incoming columns can be set so as to evolve during the algorithm. The overall complexity of the method is highly dependent on the complexity of the sub-problem that resource constraints make it NP-hard. It is however often possible to resolve in a reasonable time by an implicit enumeration $\mathcal{R}$ by exploiting the graph structure of the sub-problem and using variants of algorithms shortest path.

C. Resolution of sub-problem for column generation

Noting that in the case of several sub-networks $= 1, \dots\dots, K$, under the resolution of problem (1.21) is decomposed by sub-networks, we omit the index k and the graph of the problem will be denoted as $G = (\{o\} \cup V \cup \{d\}, A)$.

The problem of shortest path with resource constraints (RC-CCP), is Formula as follows:

$$(PCCCR) \begin{cases} \min \sum_{(i,j) \in A} c_{ij} x_{ij} & (18) \\ \sum_{i:(o,i) \in A} x_{oi} = 1 & (19) \\ \sum_{i:(i,d) \in A} x_{id} = 1 & (20) \\ \sum_{i:(i,j) \in A} x_{ij} = \sum_{i:(i,j) \in A} x_{ji} \qquad j \in V = \{1, \dots\dots, n\} & (21) \\ x_{ij}\left(T_i^q + t_i^q - T_j^q\right) \leq 0 \quad (i,j) \in A, q \in Q & (22) \\ a_i^q \leq T_i^q \leq b_i^q \quad i \in V, q \in Q & (23) \\ x_{ij} \in \{0,1\}, T_i^q \geq 0 \ (i,j) \in A, q \in Q & (24) \end{cases}$$

To resolve this problem, Desrochers and Soumis [9] offer a dynamic programming algorithm type pulling.

Definition 1 A path from each origin o to node j, we associate a label $(C_j, T_j) = (C_j, T_j^1, \dots, T_j^Q)$ representing the state of its resources and cost.

Definition 2 Let (C_j, T_j) and (C'_j, T'_j) two labels associated with two feasible paths P and $P'$from o to j. We say that(C_j, T_j) dominates (C'_j, T'_j) (or alternatively that P dominates P' and there $(C_j, T_j) \leq (C'_j, T'_j)$ (or as $P \leq P'$) if and only if $C_j \leq C'_j$ et $T_j^q \leq T'^q_j$, $\forall q \in Q$.

Definition 3 A label associated with a feasible path from o to d, is called effective if it is minimal in the sense of the order relation ≤. A path is said to be efficient if it is associated to a label effective.

The dynamic programming algorithm (DPA) proceeds in three stages. In each node j ∈ V, it does the following:
1. Extension of roads (generation of labels)
2. Filtering (test for feasibility)
3. Dominance (removing labels inefficient).

For a given node j, labels are created by extending those present at the nodes i such that $(i,j) \in A$. Thus, a new label (C_j, T_j) is given by

$$C_j = C_i + c_{ij}$$

$$T_j^q = \max\{a_j^q, T_i^q + t_{ij}^q\}, q \in Q$$

Considering that all the predecessors of node j ∈ V have already milked the dominant node j can be interpreted as determining the Pareto optimal problem of a multicriteria $|Q| + 1$ functions:

$$\begin{cases} \min_{i;(i,j)\in A} \left(C_i + c_{ij}\ ; \max\{a_j^q, (T_i^q + t_{ij}^q)\}, q \in Q\right) \\ T_i^q + t_{ij}^q \le b_j^q \quad , \qquad q \in Q \end{cases}$$

Let v^* its optimal value.

In a recent work Nagih and Soumis [4] propose a method of aggregation of resources for PCC-CR by projection, in each node simultaneously using an algorithm of dynamic programming and Lagrangean relaxation.

D. Algorithm

Step0 initialization PMR^0
Step1 (Solve PMR^k) by the simplex method
$\rightarrow (z_{PM}^k, x^k, \delta^k)$
Step2 (Solve SP^k)
- meter update costs: $c_{ij} = c_{ij} - \delta_j$.
- calculate the Lagrange multipliers u_{ij}^k .
- calculate the solution $maxL(u_{ij}^k)$, uses APD-L
- calculate feasible solutions $\Phi(u_{ij}^k)$, uses APD-LND
- test, if min $\Phi(u_{ij}^k) \ge 0$ then stop, go to Step 5.
otherwise go to Step 3.

Step3 generated the best solution and a negative cost subset of complementary solutions that can be calculated by two techniques (selection, or resolution) $\rightarrow X^k$.
Step4 put $PMR^k = PMR^k \cup \{X^k\}$.
Returned to Step 1.
Step5 $\rightarrow (z_{PM}^k, x^k)$ Optimal solution.

IV. NUMERICAL RESULTS

This section presents the preliminary evaluation of our approach to the problem of construction of vehicle routing with a single resource.

Solomon's 100-customer Euclidean (VRPTW) instances are used to test our algorithm. In these instances, the travel time and the Euclidean distance between two customer locations are the same and this value is truncated to two decimal places. There are six different classes of instances depending on the geographic location of the customers (R : random; C : clustered ;RC : mixed) and width of the scheduling horizon (1 : short horizon ; 2 : long horizon). In this work, instances of type 1 are discarded due to the short horizon that does not allow a significant number of routes to be sequenced to form a workday. Results are thus reported for R2 , C2 and RC2 . Due to the limitations of our exact approach, the computational study focuses on instances obtained by taking only the first 25 customers from each original instance.

Solomon's (VRPTW) test instances are modified to fit our problem. In particular, a value tmax to limit route duration is needed. This value was first set to 100 in the case of R2 and RC2, and 200 in the case of C2. The value is larger for C2 because the service or dwell time at each customer is 90, as opposed to 10 for R2 and RC2. Finally, a gain of 1 is associated with each customer and weighted by an arbitrarily large constant to maximize first the number of served customers, and then minimize the total distance.

The results for the instances with reduced time windows are shown in Table1. In the table1, a particular instance is identified by its class and its index followed by a dot and the number of customers considered. For example RC202.25 is the second instance of class RC2, where only the first 25 customers are considered. In these table, column Problem is the identifier of the problem instance, ItrGC is the total number of iteration of (PM) solved by Simplex,Col is the total number of columns generated during the branch-and-price algorithm, T(ssp) is the computation time in seconds and Obj. is the total distance.

TABLE I.

Problems	ItrGC	Col	T(ssp)	Obj
RC201.25	123	609	0.9	967.9
RC202.25	110	1132	221.0	961.6
RC203.25	713	2589	2566.2	751.3
RC205.25	218	944	5.4	974.9
RC206.25	444	1703	4.6	977.1
RC207.25	3119	13989	418.4	819.6
R201.25	218	577	1.0	772.8
R202.25	108	1030	127.0	694.0
R205.25	1326	4930	60.1	761.2
R210.25	71	918	121.4	704.6
R211.25	57	1150	42.9	623.7
C201.25	329	3448	5.1	679.5
C202.25	4023	13860	782.8	677.3

V. METHOD OF SEPARATION

The method of column generation is used to solve the relaxed problem at node U. The hybrid of simplex algorithm (method exists in the library ILOG) with a method called Pricing. If the solution is fractional then a separation method is applied to the problem P^u. It is to subdivide the whole set of solutions Su into two disjoint sets, this has the effect of eliminating the achievability of the fractional solution for both new issues that are the son of P^u.

VI. CONCLUSION

In this section devoted to solving the problem of construction of Rotations with resource constraints (PCR-CR), we have mainly developed approaches to column generation and decomposition master problem and sub-problem. The difficulty of solving sub-problem is directly related to the number of resources, we particularly studied the techniques of reduction of space resources, and the concept of reduction is a key element of the effectiveness of the overall resolution of issue. Indeed, if in a strategic planning perspective the computation time may be less critical than the overall cost of rotations, however in an operational setting the gain on the time resolution of sub-problem becomes a major issue. The prospects of research on this problem are numerous. Among these are the problems of rebuilding a robust solution after disruption by any hazard, planning initially constructed. These re-optimization problem of growing interest among engineers in charge of planning in the large transport companies and open up avenues of research particularly interesting and promising.

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