Multi-layer Thermal Analysis for VLSI Chips: A New Technique and Its Mathematical Foundations

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Abstract—We present a new technique of multi-layer thermal analysis for VLSI chips. It performs two dimensional, steady-state analysis of thermal conduction and heat generation. Its key component is a new direct method of solving huge systems of linear equations derived from thermal conduction equations. We implemented our technique in C and compared its performance to that of the most effective iterative method of ICCG of LASPACK. Our experimental results demonstrate the superiority of our program by the factors of 3.25 and 6.4 while keeping smaller residuals by 5 and 1 order(s) of magnitude, respectively.

Keywords—thermal analysis, VLSI, Laplace equation, Poisson equation, direct method, ICCG.

I. INTRODUCTION

As semiconductor technology advances in feature size reduction, speed-up, and power reduction, thermal analysis of VLSI chips comes to play more and more important roles in their performances [6][18]. Since the late 1990s, a sizable amount of research activities have been made on the effect of thermal conduction from silicon substrate and heat generated from metal layers on the characteristics of VLSI chips in the post-90 nm era. They are categorized in two major approaches [13].

The first approach starts with a physical equation such as a thermal conduction equation. It then discretizes the equation and solves the generated system of linear equations.

Such a system is then solved by iterative methods such as Incomplete Cholesky Conjugate Gradient (ICCG) method [5] or direct methods such as LU decomposition (LUD) method as well as a special purpose SPICE-like algorithm [1] or a simple tridiagonal band matrix solver, called the Thomas algorithm [14].

In one way, the equation is modeled by finite element method and analyzed by a general purpose solver, called ANSYS through an iterative method or a direct method [4]. Another way uses finite difference method to discretize the equation and analyzes the resultant system of linear equations by a special purpose SPICE-like algorithm [1] or a tridiagonal matrix solver, called the Thomas algorithm in combination with a speed-up technique, called the ADI (Alternative Direction Implicit) method [14].

These methods provide accurate solutions by way of solving thermal equations and are easy to handle different heat sources and a variety of boundary conditions. However, when dealing with large scale thermal analysis for, say an entire chip, the size of the coefficient matrix generated becomes huge and its analysis gets extremely difficult [13].

A new trend of research has recently emerged on thermal analysis for entire VLSI chips with complex shapes and multi-layer structures. This second approach utilizes analogy between heat and electricity to model heat transfer mechanism as RC networks analyzes them using SPICE-like [2][6][17] or a full-chip-scale circuit simulator [1][9].

These methods can be combined with circuit and parasitic analyses and hence it is feasible to realize chip design that takes thermal analysis into consideration. However, the RC networks used become huge and their reduction and approximation are required. As a result, the accuracy of simulation may degrade [13].

It should be noted that a third approach was very recently proposed. It used Green function to describe thermal conduction. However, it cannot be used for transient thermal analysis [15].

We propose a new multi-layer thermal analysis technique along the line of the first approach. It utilizes a new direct method that requires less time and memory than even most iterative methods.

The application of finite difference methods to two or three dimensional Laplace and/or Poisson equations generates large scale systems of linear equations. The coefficient matrices of such systems have a special structure, called block tridiagonal band matrices. Most of the methods and tools thus far proposed use iterative methods for solving the tridiagonal systems of linear equations since such methods require less time and memory than direct methods [4][8][16]. However, the former cannot obtain as accurate solutions as the latter due to an error caused by forced termination of computation [5]. Thus, the direct methods are still preferred in certain situations. Furthermore, in the case of transient thermal analysis, LUD is most often used for easy substitution iteration.

Our technique uses a direct method for block tridiagonal band coefficient matricies. It was derived from a general case linear system solver, called Partial Solution Method (PSM) [12] and is known as Symbolic PSM (S-PSM) in the area of computational fluid dynamics [3]. It was applied to thermal conduction analysis for two and four adjacent materials, where thermal conduction was described by Laplace equations [10].

We demonstrated that S-PSM can be applied to multiple layer materials whose thermal conduction is described by a

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