

delay could cause a stable equilibrium to become unstable and cause the populations to fluctuate. One requires realistic mathematical models that should be quantitatively and qualitatively consistent with the biological phenomena and experimental data. We have seen that delay models of real-phenomena have more interesting dynamics than equations that lack memory-effects.

Sensitivity functions clearly demonstrate the measure of the importance of the input parameters. We have remarked how these functions enable one to assess the relevant time intervals for the identification of specific parameters and enhance the understanding of the role played by specific model parameters in describing experimental data.

The literature on this subject is very broad and we cannot quote many interesting papers, as an exhaustive list of references is not possible in this short entry.

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