# Analysis of Non-linear attributes of Cooperative MIMO Channel using State Space Modeling

Ankumoni Bora, Kandarpa Kumar Sarma and Nikos Mastorakis

*Abstract*— Stochastic wireless channels have significant nonlinear contents which in a restricted form are handled by traditional channel estimation and modeling approaches. The working of most of the traditional channel modeling approaches is based on the adoption of a linearized representation of the path gains which truncates a sizable portion of the contents. The outcome is a distinct fall in the quality of service (QoS) and degradation of the system performance. Non-linear components in the channel are better approximated by considering analytical methods which retain more of these path gain segments. The state space modeling is a viable approach as it encompasses higher order terms in the representation and thereby prevents discarding of important channel components. Here, we discuss the application of state space modeling to cooperative wireless channels and provide an analytical treatment of such a propagation medium configured for high data rate cooperative communication. The analytical treatment is compared with simulation results using two estimation techniques namely zero forcing (ZF) and least mean square (LMS) equalizer.

*Keywords*— Cooperative communication, MIMO, State space equation, ZF, LMS.

## I. INTRODUCTION

Nonlinear behaviour is an important attribute that characterize a wireless communication channel. Often it is found that these play a significant role to lower the performance of data recovery systems due to the abundant random behaviour and unaccounted degradations contributed by the non-linear terms present in the channel modeled using certain mathematical, analytical or computational toll. Traditionally, data recovery systems in a wireless system attempt to discard the non-linear effects in the signal which are acquired while propagating through the channel. The channel is approximated using mathematical and computational tools which nearly adopt a linearized or piecewise linear approach to remove the non-linear attributes of the channel. It results in oversimplification of the channel, contributes towards lowering the efficiency of the receiver and wasting a sizable portion of the bandwidth by demanding more pilot carriers during data recovery. The signal reception quality degrades due to the nonlinearity present in the channel. In general, there are a number of approaches to remove the nonlinear aspects of a channel. Since most of these result in lowering of the receiver and spectral efficiencies, there have been certain recent attempts which have considered non-linear aspects of the channel to be essential [1] while recovering the transmitted data. To consider non-linear components in the channel, better analytical methods are required. Though Volterra model is a popular choice, the state space modeling is a viable approach as it encompasses higher order terms in the representation and thereby prevents discarding of important channel components. Further, the state space model enables the incorporation of initial, final and intermediate states in the calculation which provides a detailed picture. Some related works are discussed here.

Authors discuss a wireless channel model using state space analysis in [2]. Two wireless channels namely, Rayleigh and Rician channels are modeled. For the state space matrices, known parameters are considered in this work. In [3], authors report the design of a MIMO channel using the state space technique. Authors compare state space model with FIR model and obtain improved results in comparison to FIR model. In [4], a channel equalizer with state space is discussed. This equalizer has smaller symbol error rate than FIR based technique. Authors considered linear channel in all the above cases. But in real time applications, all channel coefficients vary nonlinearly. The normal propagation behaviour of a wireless channel changes as the fading increases. Due to this, during recovery of signals the general tools related to the channel, requires lot of reference symbols. The dependency of reference symbols is increased as nonlinear behaviour increases. This is more prominent as fading increases. It reduces the bandwidth availability. Therefore, to design data recovery systems with higher reliability and better efficiency, channels non-linear behaviour is required to be considered.

Ankumoni Bora and Kandarpa Kumar Sarma are with the Department of Electronics and Communication Technology, Gauhati University, Guwahati-781014, Assam, India. (e-mail: ankuele@gmail.com kandarpaks@gmail.com).

Nikos Mastorakis is with the Technical University Sofia, Bulgaria (e-mail: mastor@tu-sofia.bg).

There are several methods to analyze channel nonlinearities. These are Volterra model [5], state space model [6] etc. State space modeling is a realistic approach among all nonlinear methods. In the representation of the state space method, the higher order terms are incorporated into the analysis. It can prevent discarding of important channel components. State space modeling is better since it allows contributions from higher order terms in the calculation and it considers initial and zero state conditions. It can be applied to time invariant non linear systems and multiple input multiple output (MIMO) systems. Additionally, with state space analysis, the internal state of the system becomes observable and controllable.

Though state space analysis includes detailed mathematical treatment, its use in nonlinear channels continues to an evolving area of research. The properties of a channel depend on the propagation parameters of wireless communication medium. Path gain, delay spread, path loss, coherence bandwidth, input signal quality etc are some primary parameters. In all practical works, analytical modeling is a basic approach to design a system. To solve a given problem, it enables a detailed analysis of the steps required. For any system, we can design state of the particular system. State of a system represents a set of the system variables. These variables are named as state variable. According to [6], a system can be defined as a set of state variables, $x_i$ (t), i=1…n. Here, the variables consist of the system variables with initial time $t_0$ and the system input. The system output also can be modeled with the input and system variables depending on the state condition. The state equations are the mathematical expressions to represent a state of a system. The state equations are actually differential equations. The derivatives of these equations are the differentiation of state variables and the input of the system with respect to time.

In our work, we consider the nonlinear feature of the channel. This analysis is done in case of the propagation region existing between source and relay and relay and destination. In this wireless medium, non-linear aspects are important. Nonlinear channel modeling becomes less tedious with state space analysis. Here, we have derived a state space cooperative wireless channel model. We provide an analytical treatment of a stochastic wireless set-up configured for high data rate cooperative communication. Taylor series expansion is used to linearize the nonlinear state space based channel. The channel is compared with a Rayleigh frequency selective channel after linearization. First, the Rayleigh channel is estimated by using a zero forcing (ZF) equalizer. After the ZF equalization, to compare, the same operation is done using least mean square (LMS) equalizer. Actually these two equalizers are used to get the channel coefficients so that the coefficients can be used in state matrix. ZF and LMS equalizers are common techniques used in wireless communication systems and enjoy widespread acceptance. These are easy to implement and serve as benchmark techniques. The state space based channel is identified by Numerical algorithm for Subspace State space System Identification (N4SID) and Multivariable Output Error State Space (MOESP). At the end of this work, phase variations for two operations are compared. Experimental results show that the proposed analytical treatment and computational modeling is a viable option for visualization of appropriate designs to improve quality of service (QoS) of wireless communication systems despite the use of nonlinear aspects of the wireless channels.

The rest of the paper is organized as follows. In section 2, we discuss about the related theoretical concepts. The proposed state space modeling of a cooperative wireless channel is discussed in section 3. Later, in section 4, experimental details and results are discussed. Section 5 concludes the paper.

## II. THEORITICAL BACKGROUND

In this section, we briefly discuss about some of the related theoretical concepts used in the work.

### *A.* Path Gain

In wireless channel, path gain is an important parameter. In the free space propagation [6], the path gain in dB is the ratio of receiver ($P_r$) and transmitter power ($P_t$). The equation for Path gain is

Path gain= $10\log\frac{P_r}{P_t}$ … (1)

In the Line Of Sight (LOS) propagation, the transmission and reception of information is done in a very short distance. In LOS propagation of a signal, the obstructions due to buildings, mountains etc. are almost tends to zero. Therefore, the propagation effects like diffraction, scattering, reflection etc. can be ignore. According to [6], the LOS path gain for two isotropic transmit- receive (Tx-Rx) antennas is expressed as

$$PG_{LOS} = \left[\frac{\lambda}{4\pi R}\right]^2 \left|2\sin\left(2\pi\frac{h_t h_r}{\lambda R}\right)\right|^2 \quad \dots (2)$$

Here, $\lambda$ *is the transmission wavelength,*

$h_t, h_r$ *are Tx, Rx antenna height respectively*

and R is the distance between antennas.

In case of NLOS environment, the propagating signals are severely affected by large numbers of obstructions like buildings, mountains etc. Therefore, in NLOS propagation, main propagation effects are diffraction, reflection, scattering etc. Because of these effects, the path gain model also varied. The path gain equation for NLOS environment is given as [7]

$$PG_{LOS} = \left[\frac{\lambda}{4\pi(R_0+R_1)}\right]^2 \left|2\sin\left(2\pi\frac{h_t h_r}{\lambda(R_0+R_1)}\right)\right|^2 \frac{D^2}{R_0 R_1}(R_0 + R_1) \dots (3)$$

Here, D is the diffraction coefficient,

$R_0$ is the distance from Tx antenna to the diffraction point and $R_1$ is the distance from diffraction point to the Rx antenna.

In case of CMIMO, there are two main channel part. So the path gain also changes in each path. Due to this fact, the path gain equation is modified for each path.

$$PG_{LOS1} = \left[\frac{\lambda 1}{4\pi(R_0+R_1)}\right]^2 \left|2\sin\left(2\pi\frac{h_t h_{r1}}{\lambda 1(R_0+R_1)}\right)\right|^2 \frac{{D_0}^2}{R_0 R_1}(R_0 + R_1) \dots (4)$$

$$PG_{LOS2} = \left[\frac{\lambda 2}{4\pi(R_2+R_3)}\right]^2 \left|2\sin\left(2\pi\frac{h_r h_{r1}}{\lambda 1(R_2+R_3)}\right)\right|^2 \frac{{D_1}^2}{R_2 R_3}(R_2 + R_3) \dots (5)$$

where,

$D_0$ is the diffraction coefficient for channel 1, $D_1$ is the diffraction coefficient for channel 2, $R_0$, $R_1$ are distances from Tx antenna to diffraction point and diffraction point to the relay respectively and $R_2$, $R_3$ are distances from Relay to diffraction point and diffraction point to the Rx antenna respectively.

*B. Delay spread*

The channel parameters have important role to characterize a wireless channel. The variation of channel parameters can vary the wireless channel properties. Delay spread is a one kind of channel parameter of a wireless channel. This parameter depends on the difference of time of arrival of the channel LOS component and multipath channel component. According to [8], the delay spread can be measured by the root mean square (RMS) delay spread. The mean excess delay equation of the channel is given as [8]

$$\overline{\tau} = \frac{\sum {a_k}^2 \tau_k}{\sum {a_k}^2} \dots (6)$$

where $a_k$ is the amplitude of the nth path

$and\ \tau_k\ is\ the\ \ delay\ time.$

The RMS delay spread of the channel is given as

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\overline{\tau})^2} \dots (7)$$

where, $\overline{\tau^2} = \frac{\sum {a_k}^2 {\tau_k}^2}{\sum {a_k}^2}$

In cooperative communication, for the two channels the amplitude of the $n^{th}$ path and time delay are varied. Therefore, for the two channel paths, delay spread also will vary.

*C.* State Space Modeling

The simple state space model of a wireless channel is shown in figure 1.
This is the block diagram for a state space model to realize a proper channel. The system internal state is expressed by an equation known as the state equation. The output of a state space system is consist of a combination of the current input and the current state of the system. These two equations are called state equations. Usually, in control system state space modeling is mostly used. Now a days, in communication it is used in case of linearization process of nonlinear system model like NARMA model [9]. The state equations for a channel

$$\dot{X} = Ax + Bu$$
$$Y = Cx + Du \dots (8)$$

where, x is the state of the system, u is the input signal of the system, Y is the output signal of the system, A is the state matrix, B is the input matrix, C is the output matrix and D is the direct feed through matrix.
The State Space modeling set-up for a wireless channel is shown in Figure 1.

As describe earlier, the state space modeling is a standard mathematical modeling to explain a system. In control system state space model gives the mathematical form of a physical system with the set of input, output and state variables. The state equations with state variables $x_1(t) \dots x_n(t)$ and inputs $u_1(t) \dots u_n(t)$ are represented as follows:

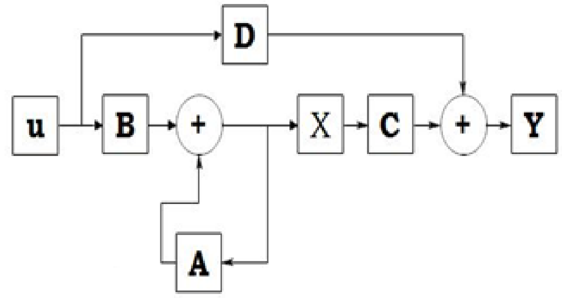


Figure 1: State Space model of a system

$$\dot{x}_1 = f(x_1, u_1, t)$$
$$\dot{x}_2 = f(x_2, u_2, t)$$
$$\vdots$$
$$\dot{x}_n = f(x_n, u_n, t) \dots (9)$$

where, $\dot{x} = \frac{dx}{dt}$.

The linear state space model can be represented in a matrix form. Let us consider three state variables $x_1$, $x_2$ and $x_3$ and inputs $u_1$, $u_2$ and $u_3$ of a system. The state equations are,

$$\dot{x}_{1=} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + b_{11}u_1 + b_{12}u_2 + b_{13}u_3$$

$$\dot{x}_{2=} a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + b_{21}u_1 + b_{22}u_2 + b_{23}u_3$$

$$\dot{x}_{2=} a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + b_{31}u_1 + b_{32}u_2 + b_{33}u_3 \dots (10)$$

Here, $a_{ij}$ and $b_{ij}$ are coefficients of state variable and input respectively.
The matrix-vector form of these equations as follows

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

The vector form of this is as shown in eq (11)

$$\dot{X} = AX + BU \dots (11)$$

Similarly the output equation also can be written in state space form. The output state equation depends on the state variables and input of the system. In similar way, the vector form of output equation is as follows

$$\dot{Y} = CX + DU \dots (12)$$

*D.* Cooperative wireless channel

In modern communication system, MIMO is a most popular technique due to the efficiency and ability to fight fading. In MIMO communication, due to the large number of antennas, the transmission power is equally distributed among several spatially spaced channel paths as a result of which capacity is increased. But beyond a certain limit, the capacity saturates which demands complementary solutions to enhance the limits further to compensate for the resource requirement and make the arrangement cost effective. As fading increases, computational complexities of the system also rises further complicating the issues involved. It degrades the quality of service (QoS). To improve channel quality and capacity enhancement, another advanced MIMO version is deployed. This is called Cooperative MIMO (CMIMO). This CMIMO is also known as virtual MIMO because this system is formed by dynamic selection of the source node and cooperative nodes and the creation and deletion of relays in a sharing arrangement to enhance the spectral efficiency. A large number of relays maybe there in between transmission and reception nodes. These relays work as a virtual processing and antenna slots. Relay can receive information from transmitter and send it to destination receiver. These relay nodes behave as transreceiver and antenna in CMIMO communication system. A schematic of such an arrangement is shown in Figure 2.

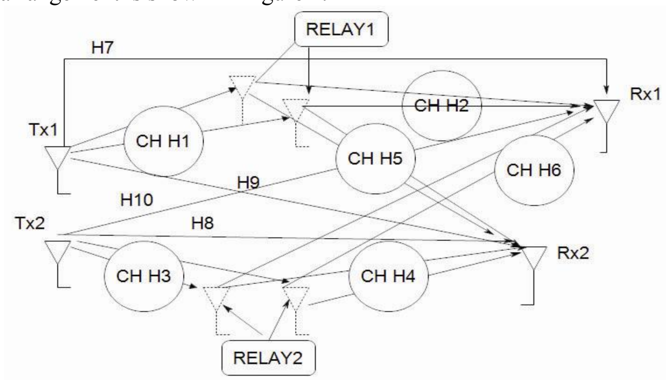


Figure 2: Cooperative MIMO channel

The state space based channel representation between base station and relay and relay and receiver set-up is shown in figure 3.
The severely faded cooperative channel is assumed as a nonlinear. The channel environment in consideration is in real time. Therefore, the channel response of the proposed model also should be nonlinear. Two channel paths are considered in cooperative setup i,e. source to relay and relay to destination. Each channel is assumed as Rayleigh frequency selective fading. The NLOS Rayleigh channel has two components, namely Inphase and Quadrature phase component.
The received signal is defined as

$$Y(t)=\sum_{1}^{N}\left[\left\{I_n\left(t,\tau_n(t)\right)\right\}\cos(\omega_{ct})-\left\{Q_n\left(t,\tau_n(t)\right)\right\}\sin(\omega_{ct})\right]s\left(t-\tau_n(t)\right)+V_{I(t)}\cos(\omega_{ct})-V_{Q(t)}\sin(\omega_{ct})\ \ldots(13)$$

***State space model for the channel (BS-RS):*** For the propagating channel between base station (BS) and Relay station (RS), channel parameters like the path gain and delay spread are considered as channel variables.
The state equations for the two state nonlinear channels,

$$\dot{x}_1=f_1\left(x_1,\ x_{2,}\ D_,\ P,w,t\right)\ldots(14)$$
$$\dot{x}_2=f_2\left(x_1,\ x_{2,}\ D_,\ P,w,t\right)\ldots(15)$$

The Taylor series expansion of nonlinear function of equation (31)

$$\begin{aligned}f_1(x_{1,}x_{2,}P,D,w,t)&=f_1\left(x_{1s,}x_{2s,}P,D,w,t\right)+\frac{\partial f_1}{\partial x_1}(x_1-x_{1s})\\&+\frac{\partial f_1}{\partial x_2}(x_2-x_{2s})+\frac{\partial f_1}{\partial P}(P-P_s)\\&+\frac{\partial f_1}{\partial D}(D-D_s)+\frac{\partial f_1}{\partial w_0}(w_0-w_s)\\&+\frac{1}{2}[\frac{\delta^2 f_1}{\delta {x_1}^2}\ (x_1-x_{1s})^2+\frac{\delta^2 f_1}{\delta {x_2}^2}\ (x_2-x_{2s})^2\\&+\frac{\delta^2 f_1}{\delta P^2}\ (P-P_s)^2+\frac{\delta^2 f_1}{\delta D^2}\ (D-D_s)^2\\&+\frac{\delta^2 f_1}{\delta {w_0}^2}\ (w_0-w_{0s})^2]\\&+\frac{\delta^2 f_1}{\delta x_1\delta x_2}(x_1-x_{1s})(x_2-x_{2s})\\&+\frac{\delta^2 f_1}{\delta x_1\partial P}(x_1-x_{1s})(P-P_s)\\&+\frac{\delta^2 f_1}{\delta x_1\partial D}(x_1-x_{1s})(D-D_s)\\&+\frac{\delta^2 f_1}{\delta x_1\delta w_0}(x_1-x_{1s})(w_0-w_{0s})\\&+higher\ terms\ \ldots(16)\end{aligned}$$

To make it linearization neglect the higher terms

$$\begin{aligned}f_1\left(x_{1,}x_{2,}P,D,w,t\right)&=f_1\left(x_{1s,}x_{2s,}P,D,w,t\right)+\frac{\partial f_1}{\partial x_1}(x_1-x_{1s})\\&+\frac{\partial f_1}{\partial x_2}(x_2-x_{2s})+\frac{\partial f_1}{\partial P}(P-P_s)\\&+\frac{\partial f_1}{\partial D}(D-D_s)+\frac{\partial f_1}{\partial w_0}(w_0-w_s)\ldots(17)\end{aligned}$$

Similarly for equation (15)

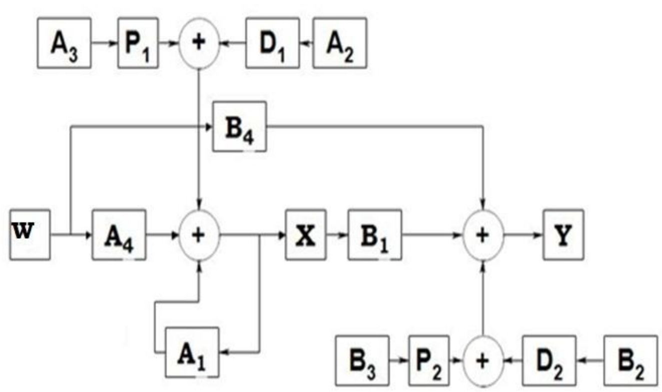


Figure 3: State space representation of the channel between base station and relay and relay and receiver

## III. PROPOSED STATE SPACE MODELING OF A COOPERATIVE WIRELESS CHANNEL

Here we discuss about the proposed state space model of the CMIMO channel. Certain mathematical analysis and state space model estimation using MOESP and N4SID approaches are the primary components of this section.

### *A.* Proposed State Space Model

$$f_2(x_1, x_2, P, D, w, t) = f_2(x_{1s}, x_{2s}, P, D, w, t) + \frac{\partial f_2}{\partial x_1}(x_1 - x_{1s}) + \frac{\partial f_2}{\partial x_2}(x_2 - x_{2s}) + \frac{\partial f_2}{\partial P}(P - P_s) + \frac{\partial f_2}{\partial D}(D - D_s) + \frac{\partial f_2}{\partial w_0}(w_0 - w_s) \dots (18)$$

The state space model of these equations (17) and (18)

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \begin{bmatrix} x_1 - x_{1s} \\ x_2 - x_{2s} \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial P} & 0 \\ 0 & \frac{\partial f_2}{\partial P} \end{bmatrix} [P - P_s \quad 0] + \begin{bmatrix} \frac{\partial f_1}{\partial D} & 0 \\ 0 & \frac{\partial f_2}{\partial D} \end{bmatrix} [D - D_S \quad 0] + \begin{bmatrix} \frac{\partial f_1}{\partial w_0} \\ \frac{\partial f_2}{\partial w_1} \end{bmatrix} [\omega_0 - \omega_s] \dots (19)$$

Therefore,

$$\dot{X}_I = A_1 X + A_2 P + A_3 D + A_4 W \dots (20)$$

For the output inphase component

$$Y_{I1} = g_1(x_1, x_2, D, P, w, t) \dots (21)$$
$$Y_{I2} = g_2(x_1, x_2, D, P, w, t) \dots (22)$$

After Taylor series expansion and linearization the output state space model is as follows

$$Y = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} \end{bmatrix} \begin{bmatrix} x_1 - x_{1s} \\ x_2 - x_{2s} \end{bmatrix} + \begin{bmatrix} \frac{\partial g_1}{\partial P} & 0 \\ 0 & \frac{\partial g_2}{\partial P} \end{bmatrix} [P - P_s \quad 0] + \begin{bmatrix} \frac{\partial g_1}{\partial D} & 0 \\ 0 & \frac{\partial g_2}{\partial D} \end{bmatrix} [D - D_S \quad 0] + \begin{bmatrix} \frac{\partial g_1}{\partial w_0} \\ \frac{\partial g_2}{\partial w_1} \end{bmatrix} [w - w_s]$$

...(23)

The output Inphase state equation

$$Y_I = B_1 X + B_2 P + B_3 D + B_4 W \dots (24)$$

The quadrature components for both state input and output equations are

$$\dot{X}_Q = A_{Q1} X + A_{Q2} P + A_{Q3} D + A_{Q4} W \dots (25)$$
$$Y_Q = B_{Q1} X + B_{Q2} P + B_{Q3} D + B_{Q4} W \dots (26)$$

Now applying Laplace transforms,

$$Y_I(s) = B_1 [sI - A]^{-1} [A_2 P(s) + A_3 D(s) + A_4 W(s)] + B_2 P(s) + B_3 D(s) + B_4 W(s)] \dots (27);$$
$$Y_I(S) = H_I(S) + C_1 P(S) + C_2 D(S) \dots (28)$$

And

$$Y_Q(S) = H_Q(S) + C_3 P(S) + C_4 D(S) \dots (29)$$

Here, $H_I(S) = B_1 [sI - A]^{-1} A_4 + B_4 \dots (30)$

$$H_Q(S) = B_{Q1} [sI - A]^{-1} A_{Q4} + B_{Q4} \dots (31)$$

A similar type of state space channel model can be deduced for the next propagating channel path of cooperative set-up i,e from relay to destination. The previous channel output is considered as input of the next channel in this case.

B. State Space Model Estimation using MOESP and N4SID

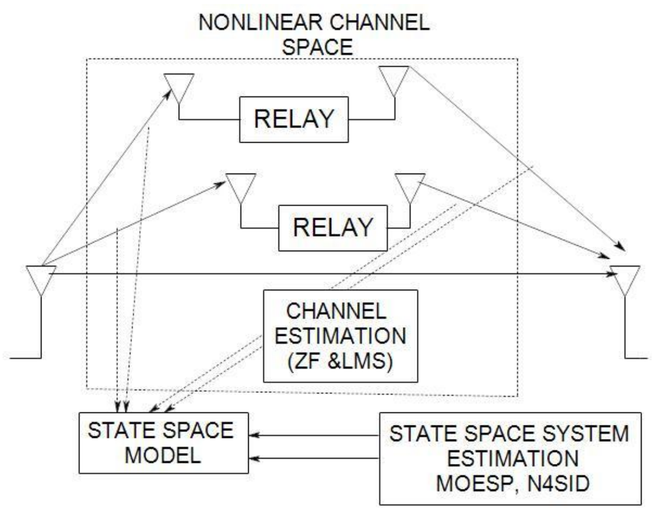


Figure 4: Block diagram for the state space estimation system in CMIMO

The Numerical algorithm for Subspace State Space System Identification (N4SID) and Multivariable Output Error State Space (MOESP) are two well-known techniques used to estimate a state space based system. These subspace algorithms are popular since these are used to estimate, predict and filter a state space systems. The block diagram for the state space estimation system in CMIMO is shown in Figure 4.

Table 1: Parameters for system estimation using N4SID and MOESP

| Parameters | Values |
|---|---|
| Input state matrix | 2x2 matrix and elements are Rayleigh coefficients |
| Output matrix | 2x2 matrix |
| Feed through | None |
| Number of free coefficients | 40 |
| Sample time | 1sec |

In our case, Rayleigh channel coefficients are determined from general estimation process like ZF and LMS. The coefficients thus obtained are applied in the state space matrix and then estimated using N4SID and MOESP algorithms. This state space model can be treated as channel between transmitter to relay and relay to the receiver.

IV. EXPERIMENTAL DETAILS AND RESULTS

In this section, we include the results obtained from the experiments performed and include related discussions.

First, as discussed earlier section, the state space based nonlinear channel is linearized by using Taylor series expansion. The linearized channel is compared with a Rayleigh frequency selective channel.

We have estimated the path gains of a Rayleigh frequency selective channel using the parameters shown in Table 1. To estimate the order of the system, Singular Value Decomposition (SVD) method is used. The stability of the system can be determined by the position of the poles and zeros inside the unit circle. For a range of experiments performed, figure 3 shows the output of the proposed state space model for a wireless channel. The values nearly match. At first, the estimation of this Rayleigh channel is done by using ZF equalizer and later the same operation is done by using LMS equalizer. In case of transmitter to the relay station and relay to the destination, the same channel model can be applied.

As already described, the N4SID and MOESP [12] algorithms are necessary for state space based system identification and estimation. After that the simulated output is compared with measured output. The measured and simulated output for 1000 samples is shown in Figure 5. The best fitness value for this system is 71.68%. Figure 6 gives the N4SID based SVD results. In this case, the best order is 2. Figure 7 shows the modeled and simulated output for 500 samples. In this case, the best fit value is 62.53%. Figure 8 shows the SVD model of this system for 500 samples. In this case also the best order is 2.

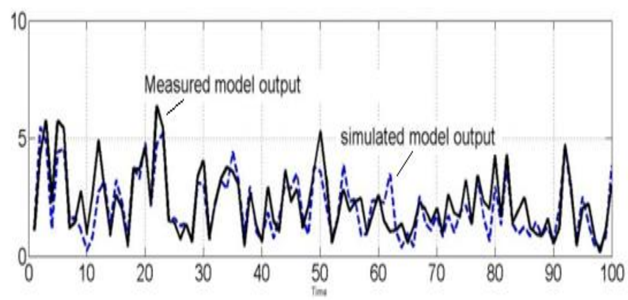


Figure 5: Measured and simulated output of state space model of 1000 samples for the first analysis (with ZF).

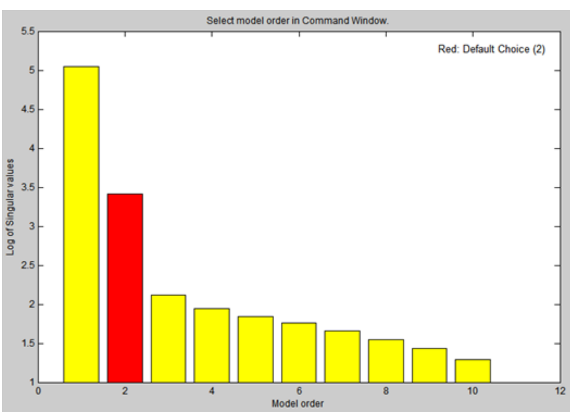


Figure 6: SVD with best fit value 62.53% of 1000 samples for the first analysis (with ZF).

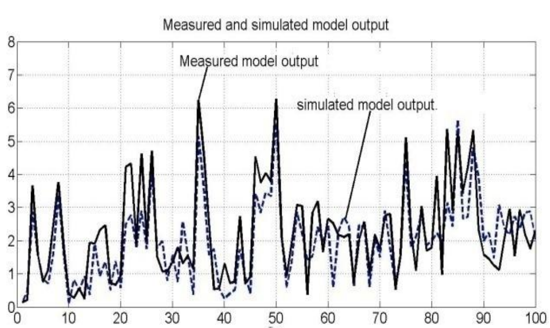


Figure 7: Modeled and simulated output with best fit value 62.53% of 500 samples for the first analysis (with ZF).

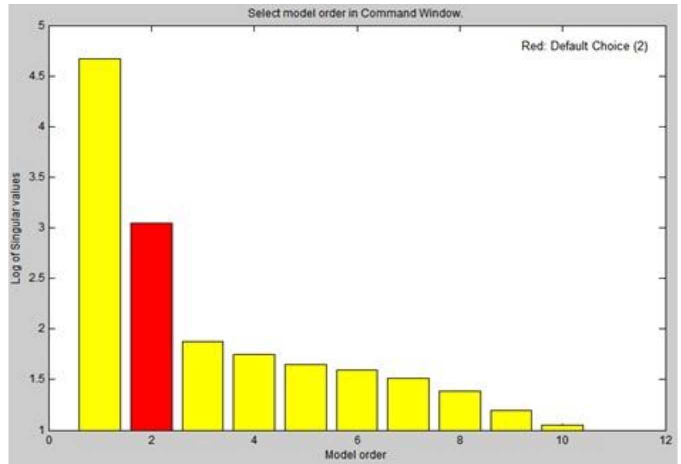


Figure 8: N4SID based SVD for 500 samples in the first analysis (with ZF).

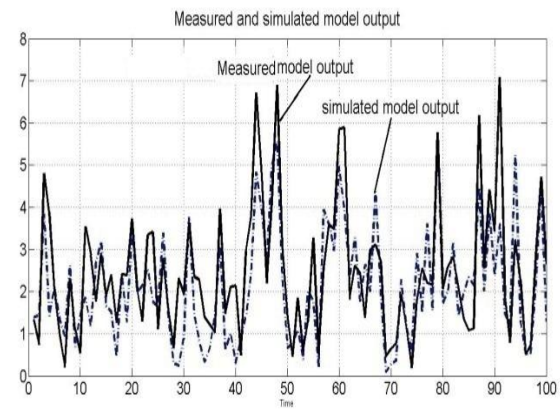


Figure 9: Measured and simulated output of state space model for 100 samples in the first analysis (with ZF).

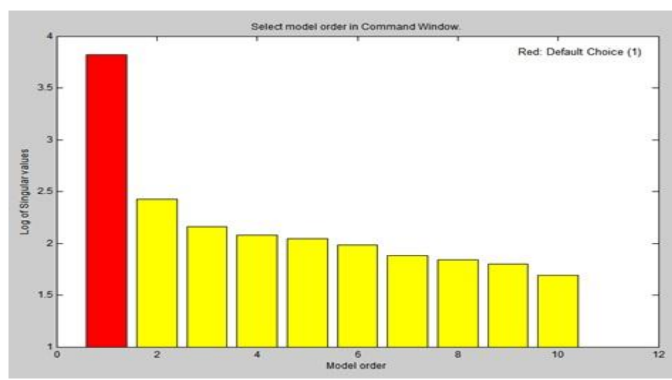


Figure 10: N4SID based SVD for 100 samples in the first analysis (with ZF).

Similarly, for 100 samples, we get the best fit value is 61.25%. Figure 9 gives the modeled and simulated output. The SVD model of this system for 500 samples is shown in Figure 8. Figure 13 shows the unit circle of this system for 1000 samples. The position of poles and zeroes in a unit circle reflects the stability of a system. As shown by Figure 13, since all poles and zeroes are inside the unit circle, the stability of the system is quiet distinct. Therefore, this system with 1000 samples can be treated as stable system in comparison to other two cases. But in case of 100 and 500 samples, systems demonstrate instability. The results have already been discussed in [13].

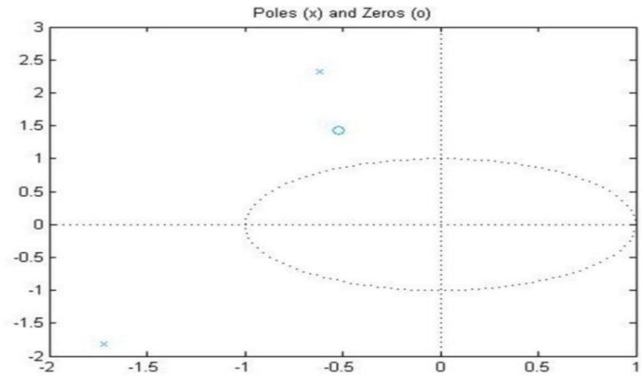


Figure 11: Unit circle for 100 sample based on N4SID algorithm in the first analysis (with ZF).

The state matrix values of the system state equation are obtained from the Rayleigh channel coefficient. The parameters for the Rayleigh channel estimation as shown in table 1. These are identical for both ZF and LMS equalizers.

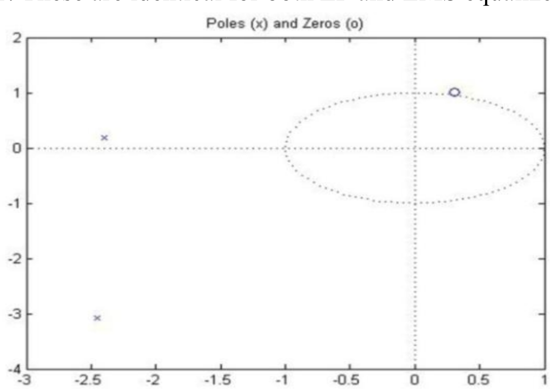


Figure 12: Unit circle for the N4SID based system for 500 samples in the first analysis (with ZF).

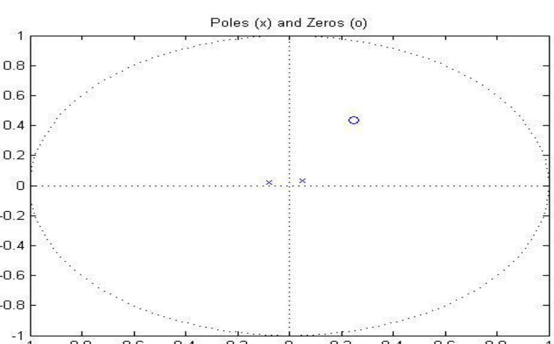


Figure 13: Unit circle for the N4SID based system for 1000 samples in the first analysis (with ZF).

Table 2: Parameters for Rayleigh channel estimation in the first analysis

| Parameters | values |
|---|---|
| No of frames | 100 |
| Length of frame | 1000 |
| Length of pilot | 100 |
| Modulation | BPSK |
| Channel | Rayleigh channel |
| SNR | -20 to 20dB |
| Noise | AWGN |
| Equalizer | Zero forcing |

Table 3: Parameters for Rayleigh channel estimation for second analysis

| Parameters | Values |
|---|---|
| No of frame | 100 |
| Length of frame | 1000 |
| Length of pilot | 100 |
| Modulation | BPSK |
| Channel | Rayleigh channel |
| SNR | -20 to 20 dB |
| Noise | AWGN |
| Equalizer | LMS |

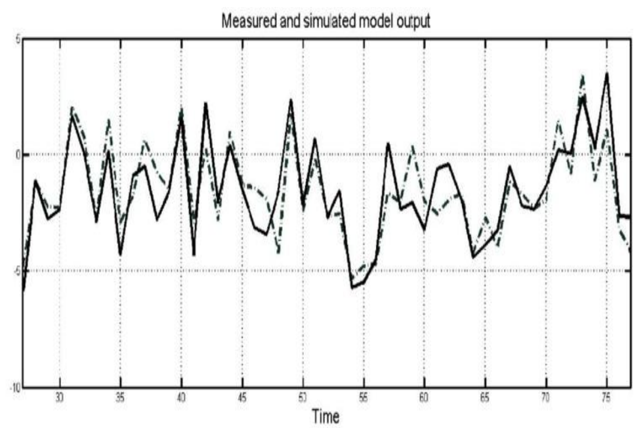


Figure 14: Measured and simulated output of state space model for 100 samples for second analysis (with LMS).

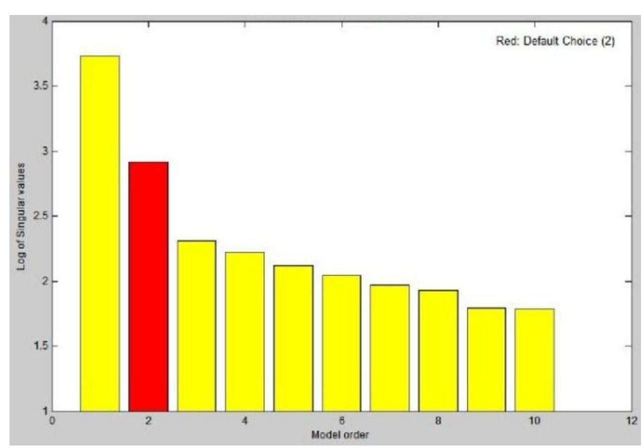


Figure 15: N4SID based SVD for 100 samples in the second analysis (with LMS).

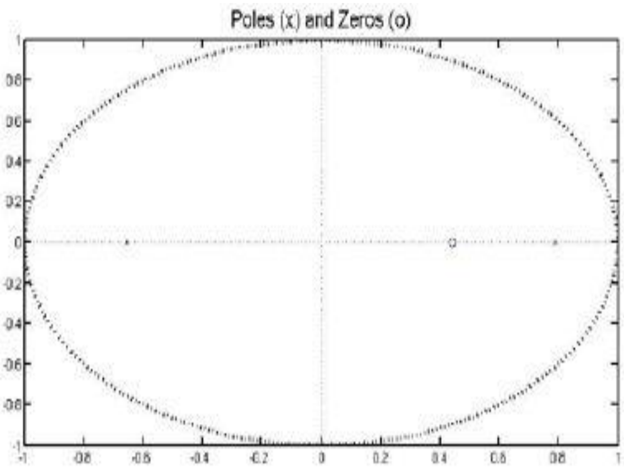


Figure 16: Unit circle for 100 samples in second analysis (with LMS).

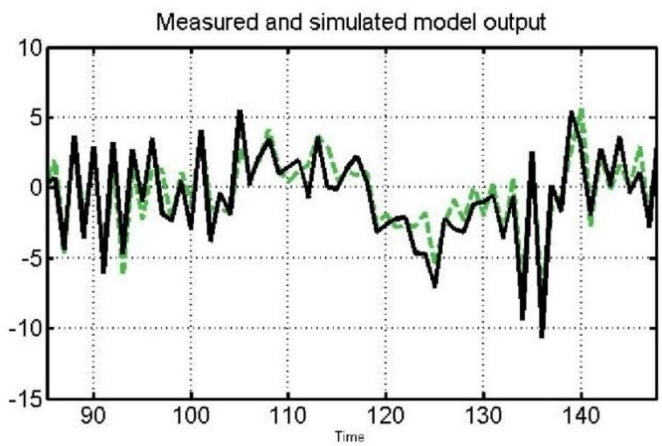


Figure 17: Measured and simulated output of state space model for 500 samples in second analysis (with LMS).

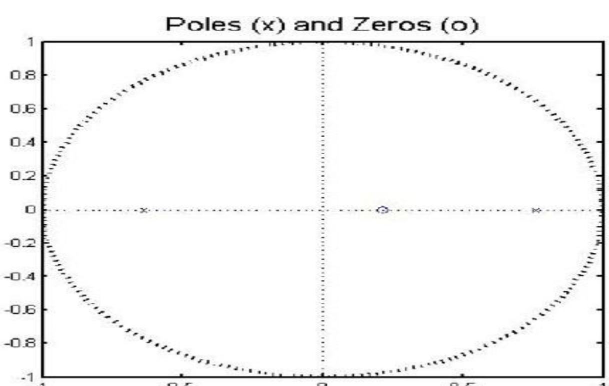


Figure 18: Unit circle for 500 samples in second analysis (with LMS).

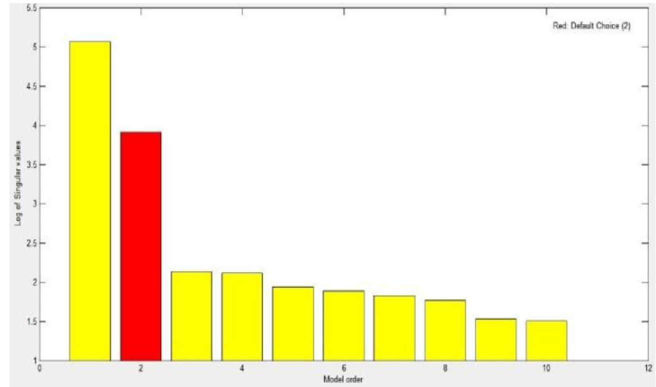

Figure 19: N4SID based SVD for 500 samples in the second analysis (with LMS).

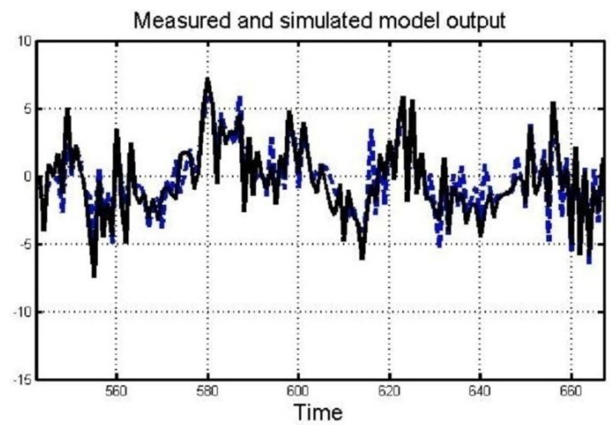


Figure 20: Measured and simulated output of state space model for 1000 samples in second analysis (with LMS).

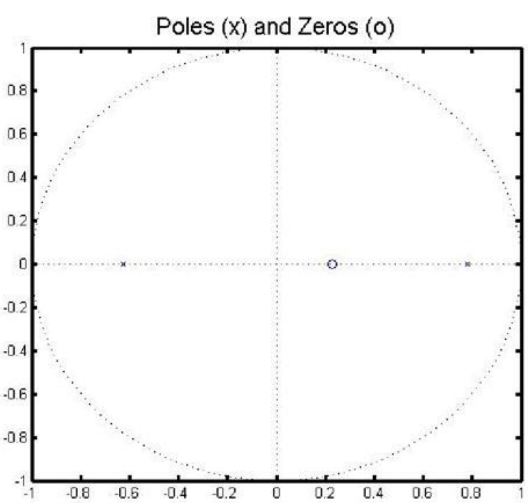


Figure 21: Unit circle for 1000 samples in second analysis (with LMS).

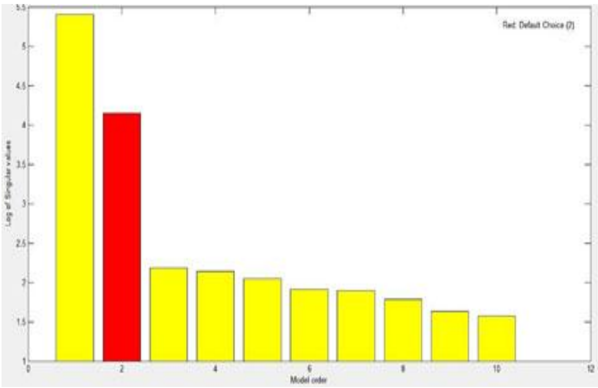

Figure 22: N4SID based SVD for 1000 samples in the second analysis (with LMS).

In the second analysis, as describe before, three ranges of samples are considered. The best fit value for 100 samples is 57.17%. Similarly, for 500 and 1000 samples the best fit values are 53.94% and 50.17% respectively. The measured and simulated model output for 100 samples in the second analysis shown in Figure 14. Similarly, for 100 samples the unit circle is shown in Figure 16. It is seen that in the second analysis, the unit circle shows stable result for 100 samples. The measured and simulated model output for 500 samples and 1000 samples in the second analysis are shown in Figures 17, 20 respectively. Similarly, for 500 and 1000 samples the unit circles are shown in Figures 18 and 21 respectively. For the first and second analysis, the variation of phase with frequency is shown in Figures 23 and 24 respectively.

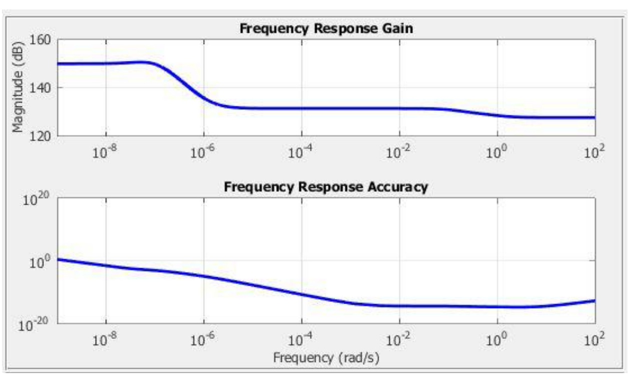


Figure 23: Phase variation with respect to frequency for first analysis (with ZF).

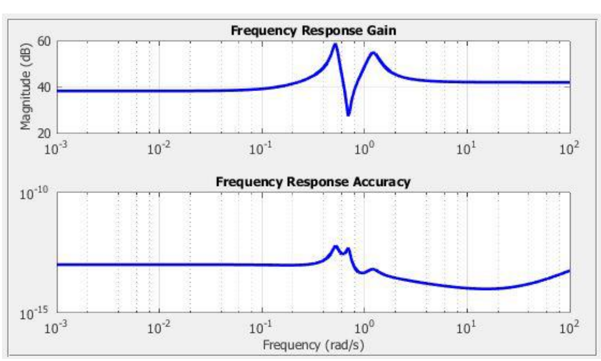


Figure 24: Phase variation with respect to frequency for second analysis (with LMS).

From Figures 23 and 24, it is clear that that the phase variation becomes almost constant after midpoint. It means that the system delay of the corresponding state space becomes constant after midpoint.

As already described, the Voltera model is a popular approach of analysis to model a nonlinear channel. We compare the performance of the proposed approach to that of the Voltera model in terms of computational loads associated. The Voltera based design is discussed in [10]. The expression of Voltera model (2.1) in [11] is modified for 5$^{th}$ order and written as follows:

$$y(n) = h_0 + \sum_{m_1=0}^{M_1} h_1(m_1) * x(n-m_1) + \sum_{m_1=0}^{M_2}\sum_{m_2=0}^{M_2} h_1(m_1,m_2) * x(n-m_1) * x(n-m_2) + \sum_{m_1=0}^{M_3} h(m_1,m_2,m_3) * x(n-m_1) * x(n-m_2) * x(n-m_3) + \sum_{m_1=0}^{M_4}\sum_{m_2}^{M_4} h_1(m_1,m_2,m_3,m_4) x(n-m_1) \ldots x(n-m_4) + \sum_{m_1=0}^{M_5}\sum_{m_2=0}^{M_5} h_1(m_1,m_2,m_3,m_4,m_5)\, h_1(m_1,m_2,m_3,m_4,m_5) * x(n-m_1) * \ldots x(n-m_5) \ldots (32)$$

The eq. (32) is a 5$^{th}$ order linearized Voltera model. This expression is compared with our state space based output expression (28). The comparison of the throughout analysis of state space output equation and Voltera model is listed in Table 3.

Table 3: Comparison of normal Voltera model with the analytical state space model.

| Nonlinear model | No of Product operations | No of summation operations | Computational complexity |
|---|---|---|---|
| Voltera model | 63 | 32 | More complex |
| StateSpace model | 20 | 10 | Less complex |

From Table 3, it is clear that the computational complexity of general Voltera model is more than the proposed analytical state space model. Thus from the estimated value analysis (Figures 5, 7, 9 for ZF and 14, 17 and 20 for LMS), stability plots (Figures 11, 12, 13 for ZF and 16, 18, 21 for LMS), best order measurement (Figures 6, 8, 10 for ZF and 18, 19, 22 for LMS), phase response (Figures 23 (ZF) and 24 (LMS)) and the comparative depiction of computational load as shown in Table 3, the advantages offered by the state space model is obvious.

## V. CONCLUSION

This paper presents a state space based stochastic non-linear channel modeling. Rayleigh frequency selective fading is considered. This work modeled a channel in between BS-RS as part of a high data rate cooperative communication setup. The system identification is done by two algorithms i,e. N4SID and MOESP. The proposed model has been simulated for a frequency selective channel. After simulation that channel is compared with the measured signal. This operation is done for two channel equalizers i,e ZF and LMS to compare the results. This work helps to confirm the validity of the proposed state-space model for a non-linear channel.

## References


[1] T. Bouilloc and G. Favier , *Nonlinear channel modeling and identification using baseband Voltera- Parafac models,* pp 1492-1498, December, 2011.

[2] X. Li, State *Space Estimation for Wireless Fading Channels,* MS Thesis, Dept. of System Science, University of Ottawa, Canada, 2002.

[3] C Zhang and R. R Bitmead., *State Space Modeling for MIMO Wireless Channels*, IEEE International Conference on Communications, vol. 4, pp. 2297 – 2301, 2005.

[ 4] C. Zhang and R.R. Bitmead, *MIMO Equalization with State-Space Channel Models*, IEEE Trans. on signal processing, vol. 56, No. 10, pp 5222-5231, October, 2008.

[5] G. Mileounis, P. Koukoulas and N. Kalouptsidis, *Input–output identification of nonlinear channels using PSK, QAM and OFDM inputs*, Published in IEEE International Conference on Acoustics, Speech and Signal Processing ( ICASSP 2008), pp 3589-3592, Las Vegas, Nevada, March, 2008.

[6] D. G. Schultz and J. L. Melsa, *State Functions and Linear Control Systems*, McGraw-Hill, New York 1967.

[7] J.S. Lu Bertoni, L Henry, C. Chrysanthou and J. Boksiner , *Simplified Path Gain Model For Mobile to Mobile Communications in an Urban High -Rise Environment*, Published in Sarnoff Symposium, 2010 IEEE, pp 1-6, April 2010.

[8] T. S. Rappaport., *Wireless Communications: Principles and Practice*, 2nd Ed., Pear son Education, New Delhi 2004.

[9] M. Boruah, K. K. Sarma and N. Mastorakis, *Adaptive NARMA Equalization of Nonlinear ITU Channels,* pp(149-153), 19th International conference on circuits, systems, Communications and computers (CScc-2015), Zakynthos Island, Greece, 16-20 July, 2015.

[10] N. Saidi, O. Taouali and H. Messaoud, *Modelling of a SISO and MIMO non linear communication channel using two modelling techniques*, WSEAS transactions on circuits and systems,2009.

[11] P. S. Kumar, *Various Nonlinear Models and their Identification, Equalization and Linearization,* Mtech thesis, Dept of *Electronics* and Communication Engineering, NIT, Rourkela, 2006-2008.

[12] I. W. Jamaludin, N. A. Waheb, N. S. Khalid, S. Sahlan, Z Ibrahim and M. F. Rahmat, *N4SID and MOESP Subspace Identification Methods*, 2013 IEEE 9th International Colloquium on Signal Processing and its Applications, pp 140-145, Kuala Lumpur, Malaysia, March 2013.

[13] A. Bora, K. K. Sarma, and N. Mastorakis, *Cooperative Nonlinear Stochastic Wireless Channel Modeling using State-Space Analysis,* 19th International conference on circuits, systems, Communications and computers (CScc-2015), pp (143-148), Zakynthos Island, Greece, 16-20 July, 2015.


**Ankumoni Bora** completed her MTech degree in Signal Processing and Communication from Gauhati University, Guwahati, Assam, India in 2015. Earlier she completed Msc from same university. Her areas of interest includes non-linear channel modeling, mobile communication.

**Kandarpa Kumar Sarma** is currently an Associate Professor at Department of Electronics and Communication Technology, Gauhati University, Guwahati, Assam, India. He obtained MTech in Signal Processing in 2005 and later PhD from Department of Electronics and Electrical Engineering, Indian Institute of Technology Guwahati, Assam, India. His areas of interest include MIMO system, mobile and wireless communication, soft computing, speech processing, antenna design and document image analysis. He is a senior member of IEEE (USA) and fellow of IETE (India).

**Prof. Dr. Nikos E. Mastorakis** received his B.Sc. and M.Sc. (Diploma) in Electrical Engineering from the National Technical University of Athens (Greece) and the Ph.D. in Electrical Engineering and Computer Science from the same university. He also received the B.Sc. (Ptychion) in Pure Mathematics from the National University of Athens, Greece. He also studied Medicine in Medical School of Athens of the same university. He have served as special scientist on Computers and Electronics in the Hellenic (Greek) Army General Staff (1993-1994) and taught several courses in the Electrical and Computer Engineering Department of the National Technical University of Athens (1998-1994). He has also served as Visiting Professor at the University of Exeter, School of Engineering (UK, 1998), Visiting Professor in the Technical University of Sofia (Bulgaria, 2003-2004) while he is now Professor in the Technical University of Sofia (Bulgaria, http://elfe.tu-sofia.bg/elfe/staff.htm, http://elfe.tu-sofia.bg/elfe/curriculum4.htm and http://elfe.tu-sofia.bg/elfe/curriculum3.htm and also Professor in the department of Computer Science at the Military Institutions of University Education (MIUE) -Hellenic Naval Academy, Greece.
Prof. Dr. Nikos Mastorakis was the first that solved with several different approaches the former unsolved problem of Multivariable Factorization and published it. He was also the first scholar that completely solved the problem of stability for Multidimensional Systems using Genetic Algorithms. Also, was the first that constructed Electronic Musical Instrument with the spaces of the Byzantine music. He is an active researcher in Applied Mathematics and Computer Science (Systems Theory, Control, Optimization Theory, Algorithms Theory, Signal Processing, Robotics, Computational Intelligence). The editor of over than 200 Books and the author of 5 books, Dr. Mastorakis has published more than 600 papers (see below) in international books, journals and conferences. An active reviewer of 26 International Journals and member of the Editorial Board of 13 International Journals and Editor of International Book Series: (Editor of the series “Electrical and Computer Engineering” (WSEAS Press) and Editor of the series “Mathematics and Computers in Science and Engineering” (WSEAS-Press), Member of the Editorial Board of “Advances in Computation: Theory and Practice” by NOVA), Dr. Mastorakis has received several awards (Royal Society of England, Hellenic National Research Foundation, etc) for his academic studies and his scientific research.
Prof. Dr. Nikos Mastorakis is the Editor-in-Chief in many International Journals. He was the General Chairman in more than 30 International Conferences. He has organized more than 40 Special Sessions, 3 Workshops and has given many plenary lectures. He is also member of IEEE (Senior Member), New York Academy of Sciences, of A.F. Communications and Electronics Association, American Association for the Advancement of Science and other smaller scientific societies.
Dr. Mastorakis is a registered professional electrical and mechanical engineer. He is also Honorary Professor, University of Cluj, ROMANIA http://outstanding.wseas.us He has received the Prize of Excellence from Romanian Academy of Science, Bucharest, ROMANIA http://outstanding.wseas.us and he is also Professor at the ASEI (Military Institutes of University Education), Hellenic Naval Academy, GREECE since 1994 http://www.hna.gr.