Contours of Combinatorial Super-pixel Grouping for Object Extraction

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***Abstract*—We propose a unified contours grouping approach for object extraction via super-pixel, which has strong contour support in the image. For this purpose, we first develop a fast sub-segments algorithm. We then propose a new cost function that makes effective promote spatially coherent sets of super-pixels with object boundary. Finally, we use a grouping strategy that combines our sub-segments into highly-accurate super-pixel by exploring efficiently their gap space. We evaluate the proposed method against two leading contour closure approaches in the literature on the BSDS500. The results demonstrate that the proposed object extraction method performs both good accuracy and time efficiency against other state-of-the-art methods.**

***Keywords*-super-pixel grouping; object extraction; sub-segments; contour;**

I. Introduction

Detecting a number of unknown objects in cluttered scenes is an important and difficult problem in computer vision research and finding a specified object or area is a pre-process to the following steps in image processing. If multiple objects are obtained from the image, we can significantly improve segmentation, surveillance, and semantic analysis etc. In particular, it also can help doctors make correct diagnosis from medical images.

The main idea of extracting objects is to use the perceptual grouping method, such as virtual link to complement a set of fragmented contours into a cycle and then separating an salient object from its background. What makes the problem particularly hard is the intractable number of cycles, which may exist in the contour extracted from the image of a real scene.

In this paper, we introduce a framework that builds a regularization function for efficiently search for an optimal closure contour from another perspective. An overview of the proposed approach is illustrated in Figure 1. We restrict object boundaries close to the boundaries of super-pixel, and the ideal case is that missing boundaries can follow the boundary of super-pixel.

In framework, our first is to reduce the problem of finding cycles to the problem of finding a closure boundary that align well with some subset of super-pixels boundary which has strong contour support in the *P-map* image. Furthermore, the accuracy of *P-map* image is a pivotal role in searching, because many details will interfere with the real contours and cause inaccurate result. However, there does not exist a method where all details will vanish and only true object edges remain. This is why we use point information of all scaled images in a probability map, where all relevant information exists for further (sub-segment) analysis. An algorithm proposed in [1], called SC, which is very time-consuming to compute probabilities map and super-pixel, especially in large data set. Therefore, we use a new method called *HACDL* which based on local cues from pixels and global cues from saliency [2] and is quite fast. On average, *HACDL* method takes 20 seconds but *gPb* method [3] [4] of SC algorithm takes 180 seconds in each image. In addition, *HACDL* algorithm achieves the highest average precision. After the formation of sub-segments, the proposed object extraction method can rely on the scale information in searching. In order to improve the accuracy of the searching for object boundary over sub-segments, some pre-processing steps are introduced in [5]–[7]. So we use the least relevant Sub-Segments filter in sub-segments preprocessing step.

The reformulation needs a mechanism to obtain super-pixel subsets which are spatially coherent [1]. It is a property of cost function that computes the ratio of perimeter to area. We build a ratio cost function based on Stahl and Wang [6] and Levinshtein [1] to operate on super-pixels rather than contours. In addition, we consider similar pixels which is not considered in [1]. The function represents that boundaries of super-pixel and contour will have more prominently spatial coherence. Next, we use five features to illustrate the "gap function" which is the distance between image contour and super-pixel boundary, the strength of nearby image contour, the orientation and curvature of those pixels on the two boundaries and the similar pixel factor. Those features play an important role in our reformulation.

Finally, we use parametric maxflow [8] to get the global optimum based on our cost function, which contains area and gap factor. Those solutions are the complete object boundary which have largest set of super-pixel area and least gap. Moreover, the parametric maxflow not only generates the minimum cost solution, but also generates serial cost solutions [8].

Therefore, *SC* algorithm in [1] may be more reasonable if they had considered least relevant filter in preprocessing and Similar Pixel Feature in gap function.

II. Related Work

Several approaches have been proposed by researchers to extract object from images. Wang had summarized some methods detailed in [6] and then we will introduce them briefly in the next. The earliest attempt at finding salient bound-

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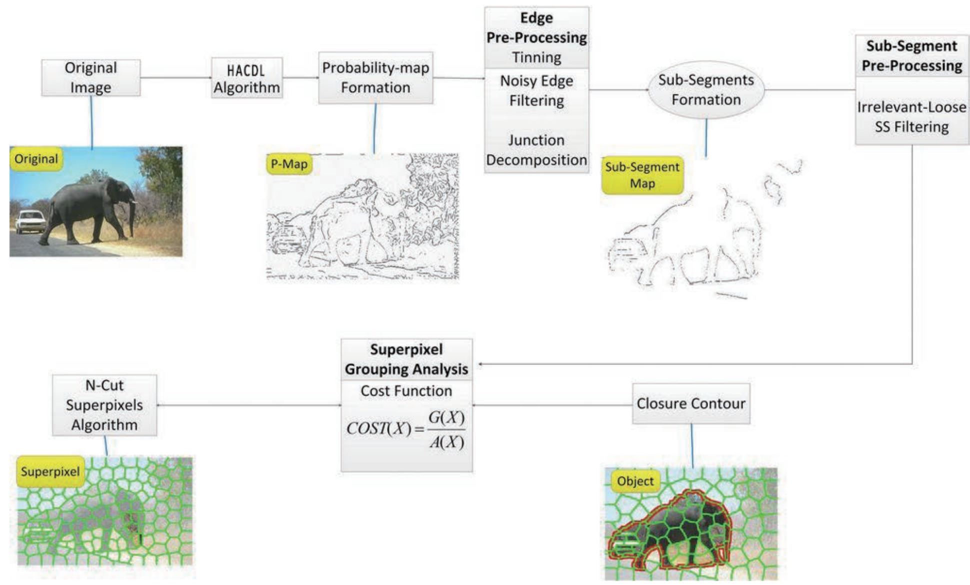


Fig. 1: Overview of the proposed approach: (1) contour image: it is the Probability-map obtained by *M-mPb* and Compass Edge Detector as the first step; (2) Sub-Segment: many details edge in P-map that un-useful to object extraction, therefore use preprocessing step to remove some details, noisy edge, and irrelevant *SS* that remain accuracy contour fragments. (3) Super-pixel Segmentation: super-pixel ensure that target boundaries are reasonably well approximated by super-pixel boundaries. (4) Super-pixel Grouping Analysis: a cost function construct reflects the extent to which the super-pixel boundary is supported by evidence of a real image contour. (5) Object: find a global optimal cycle base on cost function that make the largest set of super-pixels bounded by contours that have the least gaps and complement.

aries was based on edge detection [9]–[11] and edge-linking methods [12] which use edge or local search techniques to link clustered edges into closed boundaries. However, it is not certain whether those boundaries are accurate. In recent years, Kiranyaz [5] use uniform cost search algorithm to make automatic object extraction based on linking clustered boundary fragments.

A closed contour is represented by a parameterized curve, which is a classical model, called 'snake' [13]. However, this kind of method has a larger challenge from the change of topology and the presence of corners. To resolve these problems, level set approach has been proposed by Osher. This approach can handle topological changes by the curve in an implicit form. Another salient measure was proposed based on Bayesian variational problem, including the Theater-Wing model [14] and the Region Competition model [15], but it is usually difficult to find optimal solutions.

Due to the drawbacks of aforementioned methods, graph theoretic methods were introduced to solve these problems. Graphic method is to find a boundary for partitioning the graph, which makes the cost function optimal. These methods include Minimum Cut [16], Normalized Cut [17], Average Cut [18], Ratio Cut [19] and Jermyn and Ishikawa algorithm [20]. But graph constructed by these methods always use pixels or small regions as vertices, which makes it difficult to consider many Gestalt cues.

Our aim is to obtain closure boundary. Therefore, in the context of graph-based optimization algorithms, the constrain corresponds to finding cycles in a graph. Elder and Zucker [21] define boundary saliency that connect between two adjacent contour fragments, and find optimal closure boundary by using the shortest path algorithm. Williams and Thornber method [22] [23] has the similar model and use spectral analysis techniques and a strongly-connected-component algorithm to obtain closure boundary. Wang et al. [6] use the Minimum Weight Perfect Matching to identify the alternate cycle with minimum cycle ratio that operate on contour fragments, however, Jermyn [20] works directly with pixels in a 4-connected image grid. Nevertheless, if a measure only depends on the total boundary gap, it is insufficient for perceptual closure, and the distribution of gaps along the contour is also important to analysis perceptual closure, which was argued by Elder and Zucker. However, those methods suffer from the high complexity of choosing the right closure from a sea of contour fragments. In addition, some elegant combinatorial optimization measures are available, a *MCG* method was proposed by Pont-Tuset for image segmentation and object proposal generation for recognition in [24].

In recent years, super-pixel is becoming increasingly popular for use in computer vision applications. The idea of super-pixel was originally developed by Ren and Malik [25]. Some research makes contour grouping based on super-pixel, such as Levinshtein, [26] constrain the symmetric parts to be collections of super-pixels, and [1] obtain optimal contour closure base on super-pixel grouping; Zhang [27] propose a super-pixel cluster saliency object detection method based on LSSC and ULRR. We will draw on this idea of super-pixel grouping, and we will improve its performance by changing gap function.

In this paper, our goal is to find closed object boundary in an efficient manner. Drawn on [6] [1] [26], we use sub-segments and super-pixel to constrain the search space of the resulting closure. Moreover, the gap computation is also easy by super-pixel boundary. On the optimization side, we will use parametric maxflow problem method as used in [1] to obtain a global optimum of closure cycle.

III. Problem Formulation

We refer to the process of identifying a subset of fragments that is produced by preprocessing and build a cost function to form a closed boundary, which can get a global optimal cycles solution. The framework is mentioned in Sec. I

A. Contour Image and Sub-Segments Preprocessing

In the framework, we should obtain P-map first. Complications and degradations in the segmentation accuracy begin to occur when the image gets more and more ”detailed”. Therefore, we use the probability of a pixel belonging to a contour map as the scale information. An algorithm for estimating the probability of a pixel on contour boundary called *gPb* was proposed, which combines multiple local features in a probabilistic framework that contains two main components: the *mPb* detector based on local image analysis at multiple scales and the *sPb* detector based on *mPb* and the normalized cut segmentation results, which is the spectral of affinity matrix. The *gPb* used in [1] is time-consuming, although *gPb* has high accuracy with a higher F-measure. So instead we use an new efficient and effective algorithm for contour detection. After we obtain P-map, the next step is sub-segments formation and postprocessing. In this problem, we filter the least relevant sub-segments by an experimental formula to reduce noisy disturbance and then use maxflow operation to obtain the finally results.

1) Contour Detection via a holistic approach: In order to obtain the object efficiently and effectively, a contour extraction algorithm based on both local cues from pixels and global cues from saliency was proposed. We first set the local and global cues as input features, and then consider the self-similarity as new feature (we call it the *HACDL* method). A more detailed description of the algorithm can be found in Ref [2].

In addition, the computational cost is as small as about 20 seconds for each image, while the *gpb* is about 180 seconds. We note that both the *gPb* and the *HACDL* methods are able to extract object contour with fewer spurious edges. Therefore, in order to grouping efficiently and effectively, we choose the proposed *HACDL* method instead of *gPb* to achieve contour image.

2) Least Relevant Sub-Segments Filter: Because the P-map has some incorrect results because noisy disturbance, and the sub-segments based on inaccurate P-map maybe result in a wrong closure boundary. Therefore we hope to find a method that can alleviate its destruction.

Once all the sub-segments are formed from the edge pixels of the P-map, the most relevant sub-segments that bear the major object boundaries are usually longer with higher probability. Therefore, the relevance, *R*, of a sub-segment *SS*, can then be expressed as Eq.(1) [5]:

$$R\left(SS\right)=\sum_{e\in SS}p\left(e\right) \tag{1}$$

where $p(e)$ is the probability factor of an edge pixel e, in SS. Sorting all the sub-segments formed over the P-map and removing the least relevant ones, the threshold is chosen by an empirical Equation (2).

$$N_T=\{x_1|\left|x_1-100\right|<\left|x_2-100\right|,x_1,x_2\in(N\_mean,N\_std)\} \tag{2}$$

where, N_mean is the mean of all sub-segments, N_std is the standard deviation. So, a more important sub-segment map is obtained by this processing.

B. Super-pixels Grouping Analysis

Our framework reduces grouping complexity by restricting closure to lie along super-pixel boundaries. According to Fig 1, we will make super-pixel grouping analysis to construct cost function after obtain sub-segments and superpixel.

We define closure cost function as Equation (3), which draw on Stahl and Wang [6], Levinshtein [1].

$$COST(\mathbf{X})=\frac{G(\mathbf{X})}{A(\mathbf{X})} \tag{3}$$

where $\mathbf{X}$ is a vector indicator that labels all superpixels of image *I* as figure (1) or ground (0). $G(\mathbf{X})$ is the boundary gap along the perimeter of $\mathbf{X}$, and $A(\mathbf{X})$ is its area. the boundary gap is defined to be $G(\mathbf{X})=P(\mathbf{X})-E(\mathbf{X})$, which is a measure of the difference between boundary of $\mathbf{X}$ and contour fragments. where $P(\mathbf{X})$ is the length of $\mathbf{X}$, and $E(\mathbf{X})$ is the number of boundary $\mathbf{X}$, which satisfies the rules in Section IV. In order to solve cost function conveniently, we use a method

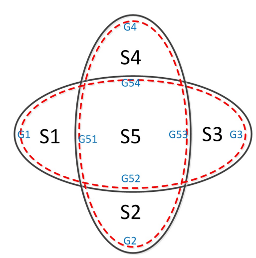


Fig. 2: Boundary gap computation over super-pixel graph. S_1, S_2, S_3, S_4, and S_5 correspond to super-pixels that were selected. G_i and G_{ij} are the boundary gap of super-pixel i and the gap on the edge between super-pixels i and j respectively. The gap along the outermost ring is then $G_{1234}=\sum_{i=1}^{4}G_i-2(G_{51}+G_{54}+G_{52}+G_{53})$

in [8]. Let X_i be a binary variable indicator for the *i-th* super-pixel, P_i be the perimeter length of super-pixel i and P_{ij} be the length of the shared edge between super-pixel i and j. Similarly, let E_i be the conditional edges of super-pixel *i's* boundary, and E_{ij} be the conditional edge for the shared boundary between super-pixel *i* and *j*. Then let $G_i = P_i - E_i$ and $G_{ij} = P_{ij} - E_{ij}$ as the boundary gaps between super-pixel and contour fragment. Above all, the final formula is Eq. (4):

$$Cost(\mathbf{X}) = \frac{\sum_i G_i X_i - 2\sum_{i<j} G_{ij} X_i X_j}{\sum_i A_i X_i} \quad (4)$$

Eq. (4) illustrate that a optimal closure boundary should have a small gap between boundary of super-pixel and sub-segments. We wish the true closure boundary is along super-pixel boundary. Most of other grouping approaches are to complement the missing contour fragments and find the missing fragments between two solid fragment endpoints. However, those ideas are complex because they try to find missing fragments in a sea of possible super-pixel boundary. On the contrary, we inverse those ideas and try to use super-pixel as basis to find a closure contour along the boundary. The gaps measure will be discussed detailed in the next section. It is the reason that the smaller the gaps, the better it is. Because it may exist some shared boundary of super-pixels that can be computed in gaps if we only use G_i for every internal boundary.Finally, the gaps will be wiped out. We subtract the gaps for all internal boundaries.In general, we are always sensitive for an object with larger area, and then we add the individual areas of all the super-pixels as the denominator. By this formation, the area not only promote spatial coherence but also promote compactness.

IV. Gap Measure and Optimization Framework

The critical part is the gap measure and the method to find optimal closure contour. Levinshtein et al proposed a gap measure in [1] that can obtain a good results at most time. However, it maybe fail when a pixel information is lost in contour image and preprocessing step. So we introduce a feature that is in terms of the similar point feature. The example as Fig.3.

A. Gap Measure

The gap as defined aforementioned, here, incorporate multiple contour features for gap computation. For a pair of super-pixels i and j, the gap on the edge between them is $G_{ij} = P_{ij} - E_{ij}$, Where $P_{ij} = |LB_{ij}|$, the $|LB_{ij}|$ is the length of super-pixel boundary (i, j), and $E_{ij} = \sum_{p \in LB_{ij}} E_{ij}^p$, where $E_{ij}^p = [Logistic\,(f^p) > T_e]$ is an edge indicator for pixel p, in which f^p is a feature vector for the pixel p, *Logistic* is a logistic regressor, and T_e is a threshold that can determine whether current pixel belongs to closure contour. This feature vector consists of five features (see Fig.3.) In all five features mentioned above, four features are defined as [1] and the fifth feature is Eq. (5):

$$f_d(p,q) = \frac{2}{3\pi}\left\{\arccos\left[d_{e1}\,(p,q)\right] + \arccos\left[d_{e2}\,(p,q)\right]\right\} \quad (5)$$

where:

$$d_{e1}(p,q) = D^{'}(p) \cdot L(p,q) \qquad d_{e2}(p,q) = D^{'}(q) \cdot L(p,q) \quad (6)$$

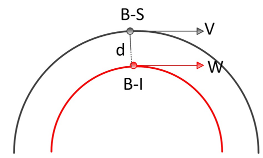


Fig. 3: Five features illustrate. B-S is boundary of super-pixel, B-I is boundary of image object. Black curves correspond to super-pixel boundaries, while the red curve correspond to detected image edges. All features are computation at super-pixel boundary pixel B-S: Distance between *B-S* and *B-I*, this is mean if the pixel at super-pixel boundary is closed to the detected image edges, it is more likely the edges along super-pixel boundary; Strength use image edge strength at *B-I*; Alignment, computed as the absolute value of the cosine of the angle between *v* and *w*; the fourth feature is curvature that computed as the squared curvature at *B-S*; Final feature is similar pixel feature, we use *p* and *q* instead of *B-S* and *B-I* in Eq. (5).

$$L(p,q) = \begin{cases} \frac{1}{\|p-q\|}(q-p), & if \;\; D^{'}(p)\cdot(q-p) \geq 0 \\ \frac{1}{\|p-q\|}(p-q), & if \;\; D^{'}(p)\cdot(q-p) < 0 \end{cases} \quad (7)$$

$$D^{'}(p) = (-I_y(p), I_x(p)) \qquad D^{'}(p) = (-I_y(p), I_x(p)) \quad (8)$$

Then we use $f_d(p,q)$ as the similar feature, which indicate two pixels are similar when $f_d(p,q)$ is smaller. All the pixels are from original image, and the fifth feature reflect contour edge or missing edge information. Next, we use logistic classifier over a feature vector to obtain E_{ij}.

B. A Parametric Maxflow Problem

In [6], it construct a graph to find a perfect matching that make the solution be the optimal cycle constraints on ration minimum. However, instead of minimizing the ratio in Eq. (3), we reduce it to a parametric energy function $E(\mathbf{X}, \lambda) = G(\mathbf{X}) - \lambda A(\mathbf{X})$. Therefore, it can obtain optimal solution according to optimal λ. the constraints on the ratio guarantee that the resulting difference is the global minimization.

Ratio minimization can be reduced to solving a parametric maxflow problem and making the method in [8] applicable for minimizing the ration $Cost(\mathbf{X})$. The method can not only find one solution, but also can find others. The details can refer to [1]. In our experiments we choose 150 superpixels in each image.

V. Evaluation

We evaluate the proposed method(*MSC*) against two other contour grouping methods: one version of ratio contours (*RRC*) [6] and the other method of super-pixel grouping [1], called *SC*. We provide a qualitative evaluation on various images (see Fig 5), as well as a quantitative evaluation on the Berkeley Segmentation Data Set (BSDS500) which includes 500 images with human labeled segmentation results, but the human segmentation is not binary image. We use 300 images for training and 200 for test [3].

A. Quantitative Evaluation

For a quantitative evaluation of the results, we use Variation of information (*VI*) metric, which measures the distance between two segmentations in terms of their average conditional entropy, Segmentation Covering, *RI* [3] and F-measure as the benchmark. However, the ground truth in BSDS500 is several segmentations but not binary segmentation. So the quantitative evaluation is not like binary segmentation benchmark. We average all of the image F-measures in BSDS500 and show some results in the following.

The whole benchmark is based on [3]. We chose the best parameters for all three algorithms and fixed them for the entire experiments. For *RRC*, we used $\lambda = 0$ and $\alpha = 1$, the all sets as [6]. For *SC*, we used 150 superpixels and other sets as [1]. For our method, we fixed the number of super-pixel to 150 and set $T_e = 0.05$ as *SC*.

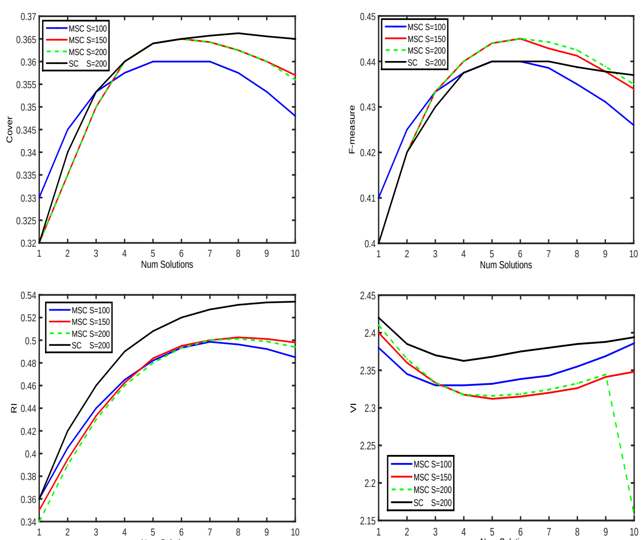


Fig. 4: Evaluate *MSC* results use Cover , *RI*, *VI* and F-measure Measures

Fig 4 shows the super-pixel number influence the *MSC* results. As above mentioned, Fig 4.1 shows that the cover is larger than that of other number of super-pixels along with the increasing of solutions under 150 super-pixels. Fig 4.2 shows that different number of super-pixel could not largely influence on the results of *RI*. Through [3], when the measures of the cover and *RI* are much larger and the *VI* is much smaller, the final result much well. In Fig 4.4, we find the change is not critical for *F-measure* under 150 or 200 super-pixels.

All these evaluation results are based on the number of solutions, we set the number of the solutions to 10 in maxflow computation, but the optimal result is not automatic selection. So, we manually choose best solutions from whole results, and evaluate those results to compare with *SC* results under 100,150,200 super-pixels. The evaluation results are shown in table I.

TABLE I: The Comparison between the best *MSC* results on different super-pixels and best *SC* [1] results used in region benchmarks on the BSDS500

BSDS500				
	Cover	RI	VI	F-measure
MSC S=100	0.42	0.60	2.24	0.51
MSC S=150	0.43	0.60	2.23	0.53
MSC S=200	0.41	0.60	2.22	0.50
SC [1]	0.40	0.59	2.28	0.48

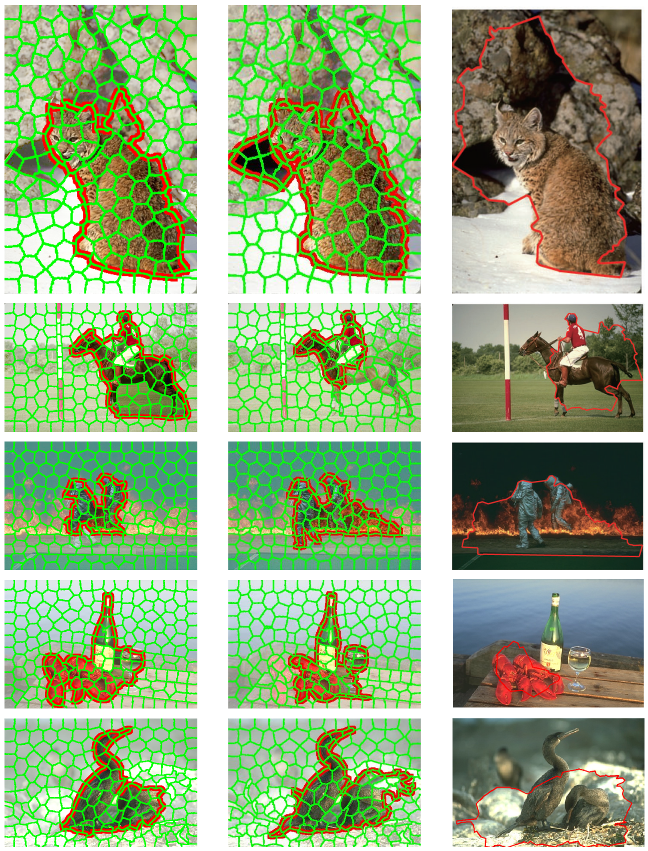
As shown in table I, we can find the *MSC* has a well performance than *SC*, because the *MSC* F-measure is 0.53 under 150 super-pixels, yet *SC* F-measure is 0.48. The other index of measures are secondary. Therefore MSC has a slighter performance than other two algorithms, such as *RRC* and *SC*. In addition, we use *HACDL* instead of *gPb* to improve time cost and hardly change any accuracy.

B. Qualitative Evaluation

In order to compare those three algorithms, we also provide a qualitative evaluation. Fig. 5 illustrates the performance of our method comparing with the other two competing approaches. We manually select best solution for each method in BSDS500. We can see our method is well than *RRC*. Because we use *HACDL* algorithm as contour detector, which is very fast and make the detected contours be closer to the true object contour that make gap computation more accuracy. We observe that our framework is more effective, and it can accurately extract object from background quicker than *SC*. This is clearly visible in the image of goose and horse, where unbroken goose (Right) was obtained by *MSC*, but it couldn't find an unbroken object in *SC* and *RRC*. However, this is not the usual case. When there is more compact contour which is not lost on gap, it will be preferred. This is the reason why the filled gap is between the horse's legs in second image. In contrast, *SC* obtains a better solution than *RRC*. However, the SC couldn't find the whole horse in second image and fifth image, because it finds many uncorrelated areas that make incorrect object boundary. *RRC* is much worse. In the third image, two person couldn't perfectly be extracted by *SC*, But *MSC* can obtain a compact result.

VI. Conclusion

In this paper, we propose a method of contour fragment grouping via super-pixel, in which boundary has strong edge support in the image. While we use super-pixel properties with an ideal scope and a convenient mechanism for incorporating appearance information, this method yields an optimal framework for closure detection that compares favorably with two leading prior approaches. Firstly, although *gPb* have high F-measure, our method, *HACDL*, can also obtain much well results with quick time, and so we use a competitively effective and efficient *HACDL* contour detector instead of *gPb* to obtain *P-map*; Secondly, we use a new irrelevant loose SS filtering formula to reduce some noisy edges jamming in Sub-Segment map postprocess; Finally, we introduce a similar pixel formula



MSC proposed *SC* [1] *RRC* [6]

Fig. 5: Sample experimental results. We compare our results (*MSC*) to two other algorithms: *SC* [1] and *RRC* [6]

to improve gap measure. We find a correct object extraction can be obtained from *MSC* algorithm through quantitative and qualitative evaluation demonstrations. In the future, we will plan to pursue a more elegant coarse-to-fine framework for object extraction by using multiple super-pixel scales to improve the precision of super-pixel, and incorporating homogeneous property in object appearance.

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